A Stochastic SOCP Optimal Power Flow With Wind Power Uncertainty

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***Abstract*—Recent research shows that non-convex AC OPF problem can be recast as a convex Semidefinite (SD) problem or Second Order Cone Programming (SOCP) problem. This paper presents a stochastic SOCP OPF (SSOCP-OPF) model for power systems connected to the wind farms. This is performed by reformulating the original non-convex OPF problem using more practical parameters of the power system. Finally, we obtain a convex optimization problem through some well-known approximations and an exact relaxation incorporating the stochastic nature of wind power. One of the advantage of convex SOCP problems, which are a general form of linear problems, is that they can be efficiently solved through Interior Point Methods (IPMs). The proposed OPF model takes advantage of both DC-OPF models (solution efficiency) and full AC-OPF models (solution accuracy). As an application of SSOCP-OPF model, we study the impact of wind power uncertainty on the transmission loss in the power systems. To evaluate the proposed stochastic model modified IEEE 30-bus test system is used. The optimization problem is coded in GAMS platform and solved using its embedded interior point optimizer MOSEK.**

***Index Terms*—Convex Optimization, Stochastic SOCP Programming, Wind Power Uncertainty.**

I. Introduction

Nowadays, due to lack of a strong interconnection between electric power systems within EU, there is a concern about the restricted power exchange. One of the reasons of improving the level of power exchange is development of renewable energy sources such as wind farms. The total capacity of wind energy installed in Europe by the end of 2009 was about $76GW$ [1]. This amount is predicted to be about $180GW$ including $35GW$ offshore, and $300GW$ including 120 GW offshore for 2020 and 2030, respectively [2]. Therefore, such wind farm connections have become the main attention point of most countries which potentially have the capability of wind power generation. However, the intermittent nature and corresponding uncertainty modeling of wind generation is one of the issues which has to be taken into account in the operation of practical power systems. This uncertainty can cause additional operating costs for energy procurement from hour/day-ahead markets. Several techniques have been employed to incorporate such uncertainty of wind power generation [3]–[6]. In this regard, a wide range of OPF formulations have been adopted in the stochastic analysis of power systems in presence of wind generation. A group of proposed stochastic OPF problems has used a convex model using a well-known DC model, where the reactive powers have not been incorporated in the model, [7], or a linearized model where nonlinear equations are approximated using Taylors expansion, [4]. While, another group has adopted the non-convex and nonlinear injection AC mismatch equations, [8], [9]. References [10], [11] have also proposed stochastic OPF problems using the conventional nonlinear programming. Reference [12] has adopted a rectangular form of nonlinear formulation. However if it is to solve an OPF problem especially for a huge number of scenarios, a relatively efficient model compared to the simple DC-OPF models which is accurate enough compared to original full AC-OPF models is needed.

This paper presents a stochastic optimization problem where the OPF model is based on a convex second order cone programming (SOCP) which at the same time takes advantage of both DC-OPF (solution efficiency) and full AC-OPF models (solution accuracy). The proposed stochastic SOCP-OPF (SSOCP-OPF) model can be adopted as a tool for different stochastic analyses in the power systems. As an application of proposed SSOCP-OPF model, we study the impact of wind power uncertainty on the transmission loss in the power systems. It is done by minimizing the expected total transmission loss in the presence of intermittent wind power generation. This process is done on an hourly basis during the real-time operation of power system. It is also assumed that forecasted probability function of the wind generation is available for the interval under consideration. According to the literature, several approaches have been proposed for wind uncertainty modeling. One approach is accurate prediction of wind power through advanced forecasting techniques with small error including physical and statistical methods [13]. Another approach is based on designing stochastic processes [14], where variable wind power is represented by a set of finite scenarios with corresponding probability of occurrence. In the present paper, we consider the latter one to model the wind power uncertainty in the proposed optimization process. The SSOCP-OPF model includes full AC system constraints with their associated limits on the generation units and transmission lines where different scenarios are adopted for modelling the intermittency of wind generation. The formulated optimisation problem can be solved using the commercially available optimisation softwares such as GAMS platform and solved using its built interior point optimizer MOSEK.

II. SSOCP-OPF Formulation

In this section the proposed stochastic optimization formulation is presented. The OPF problem used in the present

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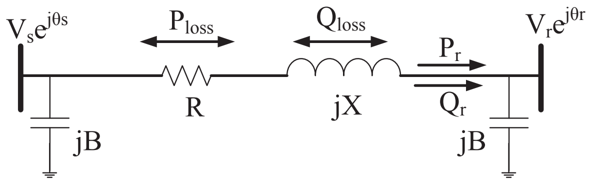

Fig. 1. The equivalent circuit of an AC line .

paper is a convex SOCP OPF formulation. The power flow equations are derived as functions of line and bus variables including line power flows and losses and voltage magnitudes (See Fig. 1). Using these variables a set of equations is derived. This set of equations was first developed in [15] for solving an economic dispatch problem of hybrid AC-DC grids. The derived equations fully representing the AC network constraints are listed as follow (in all equations s indicates the scenario index). Nodal active power mismatch constraints are derived as follow $(i = 1, ..., n_b)$:

$$P_G(i,s) - P_D(i,s) = \sum_{l=1}^{n_l} \boldsymbol{M_{PQ}}(i,l) P_r(l,s) + \sum_{l=1}^{n_l} \boldsymbol{M_l}(i,l) P_{loss}(l,s) \tag{1}$$

$$Q_G(i,s) - Q_D(i,s) = \sum_{l=1}^{n_l} \boldsymbol{M_{PQ}}(i,l) Q_r(l,s) + \sum_{l=1}^{n_l} \boldsymbol{M_l}(i,l) Q_{loss}(l,s) - B_i W(i,s) \tag{2}$$

where $W(i) = V^2(i)$ is square of voltage magnitude at i^{th} bus, W_{s_l} and W_{r_l} square of voltage magnitude at sending and receiving ends of l^{th} AC line, respectively. Elements of $\boldsymbol{M_{PQ}}$ and $\boldsymbol{M_l}$ are obtained as follow:

$$\boldsymbol{M_{PQ}}(i,l) = \begin{cases} -1 & \text{if bus i is the receiving end of line l} \\ 1 & \text{if bus i is the sending end of line l} \\ 0 & \text{if bus i is not connected to line l} \end{cases} \tag{3}$$

$$\boldsymbol{M_l}(i,l) = \begin{cases} 1 & \text{if bus i is the sending end of line l} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

n_b is the number of AC buses. n_G indicates the number of generators. n_l is the number of AC lines. P_{G_i} and Q_{G_i} are the active and reactive power generation at each AC bus. P_{D_i} and Q_{D_i} are active and reactive power loads. P_{r_l} and Q_{r_l} are active and reactive line flows at receiving end of lines. P_{loss_l} and Q_{loss_l} are active and reactive power line losses. The equations associated with drop voltage constraints are listed as follow $(l = 1, ..., n_l)$:

$$\begin{aligned} W_{se}(l,s) - W_r(l,s) = 2R_l P_r(l,s) + 2X_l Q_r(l,s) \\ + R_l P_{loss}(l,s) + X_l Q_{loss}(l,s) \end{aligned} \tag{5}$$

The equation associated with line losses $(l = 1, ..., n_l)$:

$$P_{loss}(l,s) = \frac{P_r^2(l,s) + Q_r^2(l,s)}{W_r(l,s)} R_l \tag{6}$$

The relation between active and reactive power losses is obtained as follow $(l = 1, ..., n_l)$:

$$-X_l P_{loss}(l,s) - R_l Q_{loss}(l,s) = 0 \tag{7}$$

where (6) is relaxed by replacing equality with inequality and using $P_{loss}(l,s) = 2R_l \tilde{P}_{loss}(l,s)$:

$$2\tilde{P}_{loss,AC}(l,s) W_{r_{AC_l}}(l,s) \geq P_{r,AC}^2(l,s) + Q_r^2(l,s) \tag{8}$$

By adding a linear term to the objective function, [16], [17], this relaxation is always exact meaning that this inequality is binding at the optimal solution. Expression (8) is exactly in the form of a rotated quadratic cone which is a convex set given in (18).

$$2x_1x_2 \geq x_3^2 + x_4^2, \ x_1, x_2 \geq 0 \tag{9}$$

In other conic formulations proposed in the literature either phase angle constraints are eliminated from the whole power flow equations and all voltage magnitudes are assumed to be one in all equations, [18], or they are approximated by the first-order Taylor's series to linearise these phase angle constraints, [19]. Because first-order Taylor's series is a linearisation of the original nonlinear equation, an extra iterative loop is proposed in [19]. In this paper, phase angle constraints are incorporated in the OPF problem using graph theory, stating that the phase angle difference around each independent loop in a meshed graph is zero, and two well-known approximations $sin\theta_{sr} \approx \theta_{sr}$ and $V_s V_r \approx 1$. It should be highlighted that these approximation are only applied to this equation not to all equations. The approximated phase angle constraint is then obtained as follow $(m = 1, ..., n_c)$:

$$C(m,l) X_l P_r(l,s) - C(m,l) R_l Q_r(l,s) = 0 \tag{10}$$

where $\mathbf{C}$ is a matrix with the size of $n_c \times n_l$ in which according to graph theory the number of independent loops in a graph is $n_c = n_l - n_b + 1$. The elements of $\mathbf{C}$ is obtained as follow:

$$\boldsymbol{C}(m,l) = \begin{cases} 1 & \text{line } l \text{ is in loop } m \text{ with the same direction} \\ -1 & \text{line } l \text{ is in loop } m \text{ with the opposite direction} \\ 0 & \text{line } l \text{ is not in loop } m \end{cases} \tag{11}$$

The accuracy of the results affected by this approximation have been found very good as tested on different case studies shown in [15].

III. Wind Uncertainty Modelling

Several approaches have been proposed for modeling of wind uncertainty. One approach is accurate prediction of wind power through advanced forecasting techniques with small error including physical and statistical methods [13]. Another approach is based on designing stochastic processes [14]. In this approach, variable wind power is represented by a set of finite scenarios with corresponding probability of occurrence. To generate scenarios different methods such as path based methods, movement matching and internal sampling have been proposed [20], [21]. Rayleigh PDF is one of those expressions which is widely used to model the behaviour of wind [22].

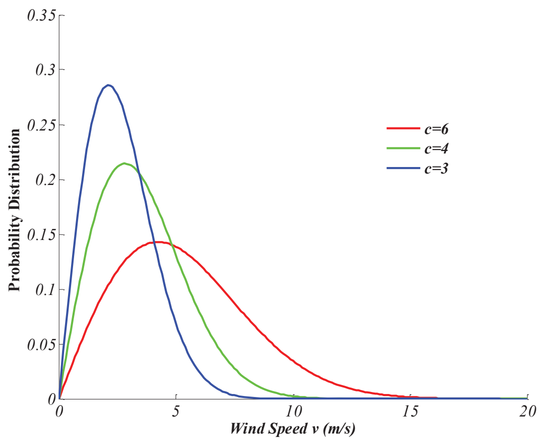

Fig. 2. Rayleigh PDF with three different scale factors.

Rayleigh PDF is a special form of Weibull PDF in (12) with shape factor $k = 2$.

$$f(v \mid k, c) = (\frac{k}{c})(\frac{v}{c})^{(k-1)} exp[-(\frac{v}{c})^k] \tag{12}$$

In this expression c is the scale factor at a given location for each wind unit. Fig. 2 shows Rayleigh PDFs with three different scale factors. In this paper we assume that the PDF for wind speed at each location is available. Then output power scenarios are to be calculated through speed-power relation curve. This curve is obtained through a set of experimentally values from manufacturer. In the literature different mathematical expressions have been proposed for this curve. A linear approximation of this curve which is used in the present paper is as follows [23]:

$$P_W = \begin{cases} 0 & \text{if } v_s < v_{in} \text{ or } v_s > v_{out} \\ P_{wr} & \text{if } v_r < v_s < v_{out} \\ (\frac{v_s - v_{in}}{v_r - v_{in}})P_{wr} & \text{if } v_{in} < v_s < v_r \end{cases} \tag{13}$$

where v_{in}, v_{out} and v_r are cut-in, cut-out and rated wind speed, respectively. P_W and P_{wr} are power output and rated power of wind turbine, respectively.

IV. SSOCP-OPF Problem

As perviously explained, the objective function in the SSOCP-OPF is to minimize the expected total transmission loss as follow:

$$\begin{aligned} \textit{Minimize} \; \sum_{s=1}^{n_s} \rho_s (\sum_{l=1}^{n_l} P_{loss}(l,s)) \\ \textit{S. t.} \end{aligned} \tag{14}$$

(1), (11), (5), (7), (8) and (10),

for $i = 1, ..., n_b$

$$\begin{aligned} \underline{V}_i^2 &\leq W(i,s) \leq \overline{V}_i^2 \\ \underline{P}_{Gi} &\leq P_G(i,s) \leq \overline{P}_{Gi} \\ \underline{Q}_{Gi} &\leq Q_G(i,s) \leq \overline{Q}_{Gi} \end{aligned} \tag{15}$$

In order to check both sending and receiving powers limits, let us define two more variables for each line as follow

$$\begin{aligned} P_{se}(l,s) &= P_r(l,s) + P_{loss}(l,s) \\ Q_{se}(l,s) &= Q_r(l,s) + Q_{loss}(l,s) \end{aligned} \tag{16}$$

These two variables together with receiving end powers are checked as the line flow constraints as follow

$$\begin{aligned} S_{L,MAX} &\geq \sqrt{P_{se}^2(l,s) + Q_{se}^2(l,s)} \\ S_{L,MAX} &\geq \sqrt{P_r^2(l,s) + Q_r^2(l,s)} \end{aligned} \tag{17}$$

These two inequalities are also in the form of standard convex cones as follow:

$$x_1 \geq \sqrt{x_2^2 + x_3^2}, \; x_1 \geq 0 \tag{18}$$

This whole problem is now in the form of SOCP problem which is a general form of linear programming accompanying some convex cones. SOCPs can be efficiently solved through Interior Point Methods (IPMs). It should be noted that the choice of objective function leads to an exact relaxation used in (8). The objective function calls for decreasing left side of inequality (8) until this inequality become binding at the optimal solution. This means the conic relaxation used in this OPF problem is always exact. The accuracy of the results affected by approximations used in the proposed SOCP OPF has been found very good as tested on different case studies in references [15], [24].

V. Case Study and Simulation Results

In order to evaluate the proposed SSOCP-OPF a modified version of IEEE 30-bus test system with a wind farm connected to bus 15 is used (See Fig. 3). We assume that the dispatch level of each generating unit is determined in a day-ahead market considering a fixed wind power output corresponding to the mean value of wind speed. Several objectives may be optimized by ISO during the real-time operation of power system. In this paper, we assume ISO is to minimize the expected total transmission loss in an hourly basis in the presence of a variable wind power generation assuming that forecasted probability function for wind speed is available for the next hour interval. The wind speed scenarios and corresponding probabilities are obtained through a sample-based approach from the available Rayleigh PDF with the scale factor $c = 8$. The wind scenarios and their corresponding probabilities are given in Table I. The output power scenarios corresponding to the wind speed scenarios, given in Table I, using power-speed relation curve in (13). These output power scenarios are then used in the SOCP OPF problem. The rated power of each turbine is considered $2MW$ and the number of wind turbines in the farm is assumed to be 50. It is assumed that the power output curve of a wind farm is sum of output powers of all the individual units within the area. It is also assumed that power factor of wind turbine is 1 (i.e. no reactive power injection/absorption by wind turbines). The wind turbine parameters used in this paper are given in Table II. The wind farm is connected to bus 15 in the grid. The dispatch results from the day ahead market which are obtained based on a constant wind power injected to bus 15

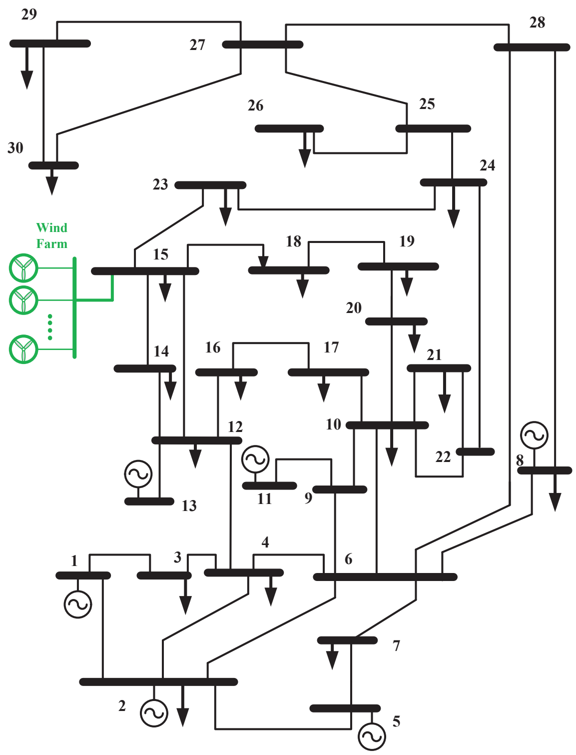

Fig. 3. Modified IEEE 30-Bus test system with a wind farm connected to bus 15.

TABLE I
WIND SPEED SCENARIOS AND CORRESPONDING PROBABILITIES

Scenario	v_s (m/s)	ρ_s
s1	0.9167	0.0512
s2	2.7500	0.1383
s3	4.5833	0.1872
s4	6.4167	0.1918
s5	8.2500	0.1626
s6	10.0833	0.1181
s7	11.9167	0.0747
s8	13.7500	0.0416
s9	15.5833	0.0205
s10	17.4167	0.0090
s11	19.2500	0.0035
s12	21.0833	0.0012
s13	22.9167	0.0004
s14	24.7500	0.0001

TABLE II
WIND TURBINE UNIT PARAMETERS

v_{in} (m/s)	v_r (m/s)	v_{out} (m/s)	P_{wr} (MW)
4	14	20	2

are given in Table III. To investigate the impact of wind power uncertainty on the transmission loss, firstly, we obtain the minimum transmission loss through a single-run of SSOCP-

TABLE III
DISPATCH RESULTS WITHOUT TRANSMISSION LOSS AND WITH A CONSTANT WIND POWER

$PG(1)$ (MW)	$PG(2)$ $(MVAr)$	P_W (MW)	ΣP_{loss} (MW)
240.58	36.98	5.83	0

TABLE IV
MINIMUM TRANSMISSION LOSS OBTAINING FROM A SINGLE-RUN SOCP OPF CONSIDERING CONSTANT WIND POWER

Bus	PG (MW)	QG $(MVAr)$	P_W (MW)
1	258.140	0	-
2	36.980	20.470	-
5	0	34.443	-
8	0	39.999	-
11	0	16.161	-
13	0	23.571	-
15	-	-	5.83
ΣP_{loss}	(MW)	17.553	
ΣQ_{loss}	$(MVAr)$	66.293	

TABLE V
EXPECTED GENERATION AND MINIMUM TRANSMISSION LOSS OBTAINING FROM SSOCP-OPF CONSIDERING VARIABLE WIND POWER

Bus	EPG (MW)	EQG $(MVAr)$	P_W (MW)
1	228.57	0	-
, 2 2	36.980	14.300	-
5	0	33.302	-
8	0	39.662	-
11	0	14.117	-
13	0	21.480	-
15	-	-	Probable Scenarios
$E\Sigma P_{loss}$	(MW)	15.198	
$E\Sigma Q_{loss}$	$(MVAr)$	54.961	
Execution time	(sec)	0.2	

OPF where the generation units but one (Generator 1) are set to their dispatch levels and the wind power output is set to its minimum probable value of $5.83MW$. The results are given in Table IV. In another stage, given the available information from day ahead market and wind speed PDF for the interval under consideration, system operator runs SSOCP-OPF to minimize the expected transmission loss. In this SSOCP-OPF, the wind power output is modeled as different scenarios obtained from wind speed scenarios and power-speed relation curve. We consider the output of generation unit 1 as control variable and can be controlled by system operator. For all scenarios power output of generator 2 is fixed at its committed dispatch level. The results are given in Table V. All values in this Table are expected values of the parameters. As it can be seen from the results obtained through the SSOCP-OPF the expected minimum loss for the specified horizon time is obtained as $15.198MW$ which is different from $17.553MW$ obtained from a single-run OPF. This difference comes from the wind power uncertainty in the interval under consideration. However, system operator tries to obtain an expected minimum transmission loss corresponding to all possible wind power outputs.

VI. Conclusion

Following the recent research on recasting the non-convex AC OPF problem as convex Second Order Cone Programming (SOCP) formats, this paper presents a stochastic SOCP OPF (SSOCP-OPF) for AC grids connected to wind farms. The proposed convex OPF problem used for the stochastic analysis of power system takes advantage of some well-known approximations and an exact relaxation. Unlike the simplified DC-OPF models, the SSOCP-OPF takes into account both transmission loss and reactive powers and voltage magnitudes. Also, unlike the conventional full AC-OPF models, the SSOCP-OPF is a convex problem and can be efficiently solved through Interior Point Methods (IPMs). The choice of new variables used for deriving the SSOCP-OPF formulation reflects more practical knowledge about the power system in the output of proposed OPF. This makes it easier to optimize practical parameters of a power system such as transmission line flows and transmission line losses by simply adding their linear combinations in the the objective function of SSOCP-OPF. Using a linear set of the new variables in the objective function, the proposed SSOCP-OPF is used to evaluate the impact of wind power uncertainty on the total transmission loss in the grid. To this end, we assume system operator is to minimize the expected transmission loss for all possible wind power outputs. The wind power uncertainty is modeled as different scenarios corresponding to wind speed scenarios obtained from PDF function.

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