Range tree

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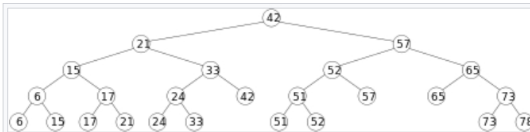
Range tree		
Type	tree	
Invented	1979	
Invented by	Jon Louis Bentley	
Time complexity in big O notation		
Algorithm	Average	Worst case
Space	$O(n \log^{d-1} n)$	$O(n \log^{d-1} n)$
Search	$O(\log^d n + k)$	$O(\log^d n + k)$

In computer science, a **range tree** is an ordered tree data structure to hold a list of points. It allows all points within a given range to be reported efficiently, and is typically used in two or higher dimensions. Range trees were introduced by Jon Louis Bentley in 1979.[1] Similar data structures were discovered independently by Lueker,[2] Lee and Wong,[3] and Willard.[4] The range tree is an alternative to the *k*-d tree. Compared to *k*-d trees, range trees offer faster query times of (in Big O notation) $O(\log^d n + k)$ but worse storage of $O(n \log^{d-1} n)$, where *n* is the number of points stored in the tree, *d* is the dimension of each point and *k* is the number of points reported by a given query.

Bernard Chazelle improved this to query time $O(\log^{d-1} n + k)$ and space complexity $O\left(n\left(\frac{\log n}{\log\log n}\right)^{d-1}\right)$.[5][6]

Contents [hide]

Data structure [edit]



An example of a 1-dimensional range tree. Each node which is not a leaf stores the maximum value in its left subtree.

A range tree on a set of 1-dimensional points is a balanced binary search tree on those points. The points stored in the tree are stored in the leaves of the tree; each internal node stores the largest value contained in its left subtree. A range tree on a set of points in *d*-dimensions is a recursively defined multi-level binary search tree. Each level of the data structure is a binary search tree on one of the *d*-dimensions. The first level is a binary search tree on the first of the *d*-coordinates. Each vertex *v* of this tree contains an associated structure that is a (*d*−1)-dimensional range tree on the last (*d*−1)-coordinates of the points stored in the subtree of *v*.

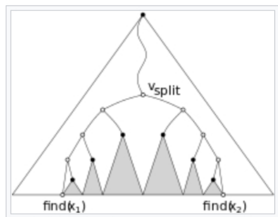
Operations [edit]

Construction [edit]

A 1-dimensional range tree on a set of *n* points is a binary search tree, which can be constructed in $O(n \log n)$ time. Range trees in higher dimensions are constructed recursively by constructing a balanced binary search tree on the first coordinate of the points, and then, for each vertex *v* in this tree, constructing a (*d*−1)-dimensional range tree on the points contained in the subtree of *v*. Constructing a range tree this way would require $O(n \log^d n)$ time.

This construction time can be improved for 2-dimensional range trees to $O(n \log n)$.[7] Let *S* be a set of *n* 2-dimensional points. If *S* contains only one point, return a leaf containing that point. Otherwise, construct the associated structure of *S*, a 1-dimensional range tree on the *y*-coordinates of the points in *S*. Let x_m be the median *x*-coordinate of the points. Let S_L be the set of points with *x*-coordinate less than or equal to x_m and let S_R be the set of points with *x*-coordinate greater than x_m. Recursively construct v_L, a 2-dimensional range tree on S_L, and v_R, a 2-dimensional range tree on S_R. Create a vertex *v* with left-child v_L and right-child v_R. If we sort the points by their *y*-coordinates at the start of the algorithm, and maintain this ordering when splitting the points by their *x*-coordinate, we can construct the associated structures of each subtree in linear time. This reduces the time to construct a 2-dimensional range tree to $O(n \log n)$, and also reduces the time to construct a *d*-dimensional range tree to $O(n \log^{d-1} n)$.

Range queries [edit]



A 1-dimensional range query [x_1, x_2]. Points stored in the subtrees shaded in gray will be reported. find(x_1) and find(x_2) will be reported if they are inside the query interval.

A range query on a range tree reports the set of points that lie inside a given interval. To report the points that lie in the interval [x_1, x_2], we start by searching for x_1 and x_2. At some vertex in the tree, the search paths to x_1 and x_2 will diverge. Let v_{split} be the last vertex that these two search paths have in common. For every vertex *v* in the search path from v_{split} to x_1, if the value stored at *v* is greater than x_1, report every point in the right-subtree of *v*. If *v* is a leaf, report the value stored at *v* if it is inside the query interval. Similarly, reporting all of the points stored in the left-subtrees of the vertices with values less than x_2 along the search path from v_{split} to x_2, and report the leaf of this path if it lies within the query interval.

Since the range tree is a balanced binary tree, the search paths to x_1 and x_2 have length $O(\log n)$. Reporting all of the points stored in the subtree of a vertex can be done in linear time using any tree traversal algorithm. It follows that the time to perform a range query is $O(\log n + k)$, where *k* is the number of points in the query interval.

Range queries in *d*-dimensions are similar. Instead of reporting all of the points stored in the subtrees of the search paths, perform a (*d*−1)-dimensional range query on the associated structure of each subtree. Eventually, a 1-dimensional range query will be performed and the correct points will be reported. Since a *d*-dimensional query consists of $O(\log n)$ (*d*−1)-dimensional range queries, it follows that the time required to perform a *d*-dimensional range query is $O(\log^d n + k)$, where *k* is the number of points in the query interval. This can be reduced to $O(\log^{d-1} n + k)$ using a variant of fractional cascading.[2][4][7]

See also [edit]

- *k*-d tree
- Segment tree
- Range searching

References [edit]

1. ^ Bentley, J. L. (1979). "Decomposable searching problems". *Information Processing Letters*. **8** (5): 244–251. doi:10.1016/0020-0190(79)90117-0.

2. ^ a b Lueker, G. S. (1978). "A data structure for orthogonal range queries". *19th Annual Symposium on Foundations of Computer Science (sfcs 1978)*. pp. 28–21. doi:10.1109/SFCS.1978.1.
3. ^ Lee, D. T.; Wong, C. K. (1980). "Quintary trees: A file structure for multidimensional database systems". *ACM Transactions on Database Systems*. **5** (3): 339. doi:10.1145/320613.320618.
4. ^ a b Willard, Dan E. *The super-b-tree algorithm* (Technical report). Cambridge, MA: Aiken Computer Lab, Harvard University. TR-03-79.
5. ^ Chazelle, Bernard (1990). "Lower Bounds for Orthogonal Range Searching: I. The Reporting Case" (PDF). *ACM*. **37**: 200–212.
6. ^ Chazelle, Bernard (1990). "Lower Bounds for Orthogonal Range Searching: II. The Arithmetic Model" (PDF). *ACM*. **37**: 439–463.
7. ^ a b de Berg, Mark; Cheong, Otfried; van Kreveld, Marc; Overmars, Mark (2008). *Computational Geometry*. doi:10.1007/978-3-540-77974-2. ISBN 978-3-540-77973-5.

External links [edit]

- Range and Segment Trees in CGAL, the Computational Geometry Algorithms Library.
- Lecture 8: Range Trees, Marc van Kreveld.
- Range Trees using PAM, the parallel augmented map library.

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