Задача 8

3.17

$$y = -\frac{e^{-2(x+2)}}{2(x+2)} \qquad x \neq -2$$

$$y' = -\frac{e^{-2(x+2)}\cdot(-2)\cdot 2(x+2) - e^{-2\ldots}}{2(x+2)^2}$$

$$= \cancel{e} \; \frac{-2e^{-2(x+2)}\cdot 2x - 2e^{-2(x+2)}}{2(x+2)^2}$$

$$-4x-4-2$$

$$= \cancel{e(e} \; \frac{e^{-2(x+2)}(-1\cdot 4x-10)}{2(x+2)^2}$$

$x = -2$

$x = -\frac{5}{2}$

$+ \quad \circ \quad + \quad \circ \quad - \quad \circ$

$\nearrow \; -2 \; \nearrow \; -\frac{5}{2} \; \searrow \; 0$

max

$$y\left(-\frac{5}{2}\right) = \frac{e^{-2\left(-\frac{5}{2}+2\right)}}{e^{2\left(-\frac{5}{2}+2\right)}} = e$$

$-2(+2$

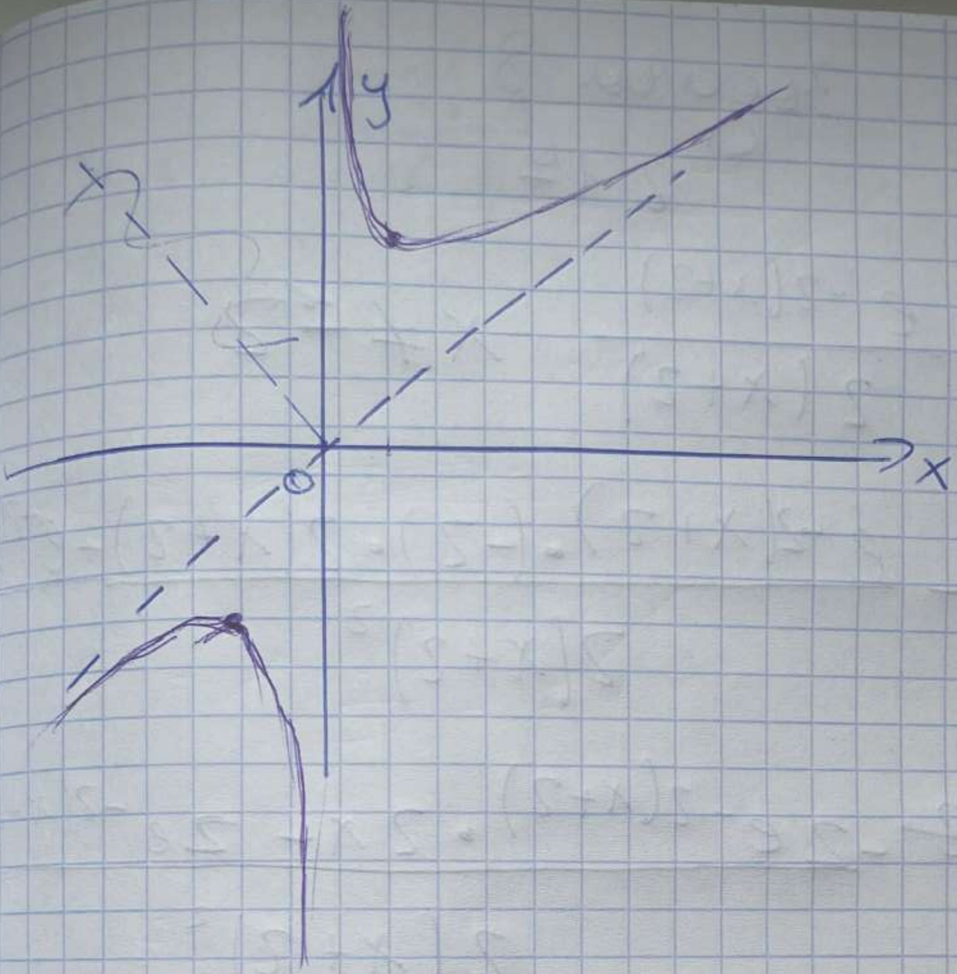
$+2$

$x+2)$

интото

$\frac{3}{1} = 3$

$=$




$\frac{1}{x} \, 3 = 0$

$x = 0 \rightarrow$ вертикальная асимптота

$$k = \lim_{x \to \pm\infty} \frac{(3x^4+1)}{x^3 \cdot x} = \frac{\infty}{\infty} = \frac{3}{1} =$$

$$b = \lim_{x \to \pm\infty} \left( \frac{(3x^4+1)}{x^3} - 3 \cdot x \right) =$$

$$= \frac{3x^4+1}{x^3} - \frac{6x^4+3}{x^3} \cdot \frac{3x^4+1}{\cancel{x^3}}$$

$$\frac{3x^4+1-3 \cdot x^4}{x^3} = \cancel{3x}\ \frac{1}{\cancel{x}^3} =$$

$y = x$

$x = 0$

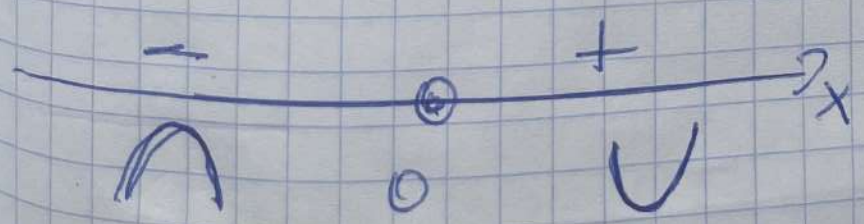

$\cancel{y(-1) = (3x\,3}$

$y(-1) = \cancel{(}\ \frac{4}{-1} = -4$

$y(1) = \frac{4}{1} = 4$

$y' = \frac{3(x^4-1)}{x^4} = 3 - \frac{3}{x^4}$

$y'' = (-3\cdot(x^{-4}))' = -3\cdot(-4)x^{-5} = \frac{12}{x^5}$



$$\cancel{y' = \frac{4 \cdot 3 \cdot 4x}{3x}}$$

$$y' = \frac{3 \cdot 4x^3 \cdot x^3 - (3x^4+1) \cdot 3x^2}{x^6} =$$

$$= \frac{3 \cdot 4x^6 - (3x^4+1) \cdot 3x^2}{x^6} = \frac{3 \cdot 4x^6 - 9x^6 - 3x^2}{x^6}$$

$$= \frac{12x^6 - 9x^6 - 3x^2}{x^6} = \frac{3x^6 - 3x^2}{x^6} =$$

$$= \frac{3x^4 - 3}{x^4}$$

$$3x^4 - 3 = 0$$

$$3(x^4 - 1) = 0$$

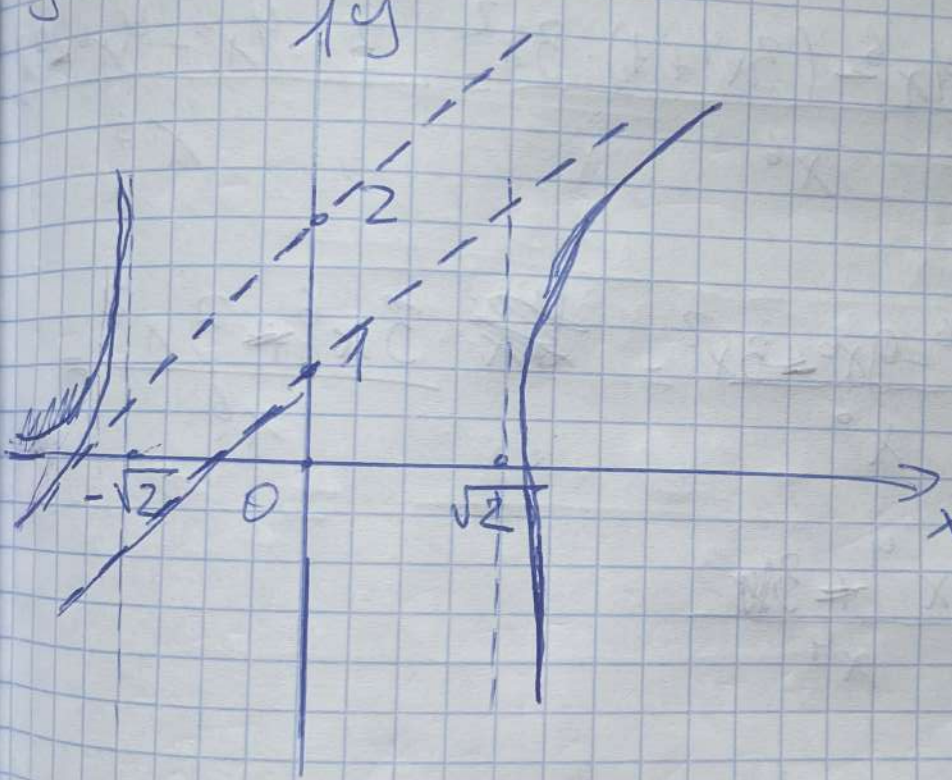
$$3(x^2 - 1)(x^2 + 1) = 0$$

$$3(x - 1)(x + 1) = 0$$

$$b=\lim_{x\to-\infty}\frac{(2x^2-1)}{\cancel{\sqrt{x^2-2}}} \mp \frac{x\cdot\sqrt{x^2-2}}{\sqrt{x^2-2}} = 3$$

$y = x+1$

$y = x+2$




Задача 7

7.17

$$y=\frac{(3x^4+1)}{x^3}$$

$D = x \neq 0$

Задача 6

6.17

$$y = \frac{(2x^2-1)}{\sqrt{x^2-2}}$$

$$\sqrt{x^2-2} = 0$$

$$x^2 - 2 = 0$$

$$x^2 = 2$$

$x = \pm\sqrt{2}$ – вертикальная асимпто

$$k = \lim_{x \to +\infty} \frac{(2x^2-1)}{\sqrt{x^2-2}\cdot x} = 1$$

$$k = \lim_{x \to -\infty} \frac{(2x^2-1)}{\sqrt{x^2-2}\cdot \cancel{x}} = -1$$

$$b = \lim_{x \to +\infty} \left( \frac{(2x^2-1)}{\sqrt{x^2-2}} - x \right) =$$

$$= \frac{\frac{(2x^2-1)}{\cancel{\sqrt{x^2-2}}} - x\sqrt{x^2-2}}{\sqrt{x^2-2}} = 1$$

$y'' = -2 + \cos(x-2) + 1\cdot\cos(x-2) +$
$+(x-2)\cdot(-\sin(x-2))$

$y''(2) = -2 + \cos 0 + 1\cdot\cos 0 + 0 =$
$= -2 + 1 + 1 = 0$

$y''' = -\sin(x-2) + \cancel{\cos}(x-2) + \cancel{\cos(x-2)\cdot}$
$\cancel{\cdot(-\sin(x-2))} + \cancel{1\cdot(-\sin(x-2)) + (x-2)\cdot}$
$\cancel{\cdot(-\cos(x-2))}$

$y'''(2) = -\sin 0 + (-\sin(0))\cdot$

$y''' = -3\sin(x-2) - (x-2)\cdot\cos(x-2)$

$y'''' = -3\cos(x-2) - 1\cdot\cos(x-2) +$
$+(x-2)\cdot(-\sin(x-2))$

$y''''(2) = -3\cdot\cos 0 + \cos 0 + 0 =$
$= -2 < 0$, следовательно $x_0 = 2$ – max

$$y(-2) = -\frac{(-2)^2}{2} + \frac{8}{-2} + 8 = -2 + (-4) + 8 =$$

$$= 2 \rightarrow \text{наибольшее}$$

$$y(-4) = -\frac{(-4)^2}{2} + \frac{8}{-4} + 8 = -8 + (-2) + 8$$

$$= -2 \rightarrow \text{наименьшее}$$

$$y(-1) = -\frac{(-1)^2}{2} + \frac{8}{-1} + 8 = -\frac{1}{2} + (-8) + 8$$

$$= -\frac{1}{2}.$$

Задача 5

5.17

$$y = 4x - x^2 + (x-2)\sin(x-2), \quad x_0 = 2$$

$\cancel{y'} =$

$$y' = 4 - 2x + 1 \cdot \sin(x-2) + (x-2) \cdot$$

$$\cdot \cos(x-2).$$

$\Rightarrow$ Ответ

3.17

$$y = -\frac{x^2}{2} + \frac{8}{x} + 8 \qquad x \neq 0$$

$$y' = -\frac{1}{2} \cdot 2x + 8 \cdot (-1)x^{-2} = \cancel{-x + 8} =$$

$$= -x + \frac{8}{x^2} = \frac{x^3}{x^2} - \frac{8}{x^2} = \frac{-x^3 - 8}{x^2} = 0$$

$$-x^3 - 8 = 0$$

$$x^3 + 8 = 0$$

$$x^3 = -8$$

$$x = -2$$

$y' = 0$

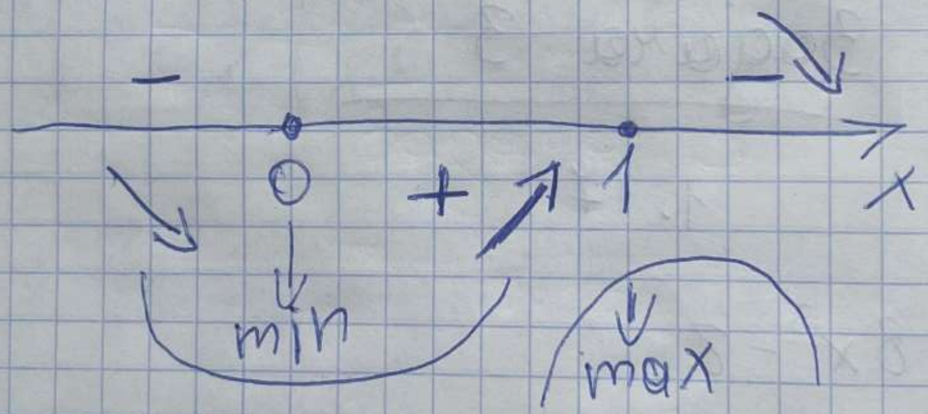
$y' = \cancel{24}x - \cancel{24}x^2 = 0$

$\cancel{y' =}$

$x(1 - x) = 0$

$\cancel{x0}$

$x = 0 ; x\ 1$




$x_0 = 0 - \min$ $\quad y_{min} = \cancel{\ } -2$

$x_0 = 1 - \max$ $\quad y_{max} = 2$

$1+\cancel{\frac{2}{}}$

$1+\frac{2}{x^2}\cancel{-1}+\cancel{x}\cdot\left(\sqrt{\frac{2}{x^2}-1}\right)\cdot\frac{2}{x^{\cancel{2}}\sqrt{2-x^2}}=$

$=\frac{2}{x^2}+\left(\sqrt{\frac{\cancel{2-x^2}}{x^{\cancel{2}}}}\right)\cdot\frac{2}{x\sqrt{\cancel{2-x^2}}}=$

$=\frac{2}{x^2}-\frac{2}{x^2}=\underline{0}$

## Задача 3

1.17

$y=12x^2-8x^3-2$

1. $D=R$
2. Общего вида

$\cancel{3.}$

$\cancel{y}$

3. $y'=12\cdot 2x-8\cdot 3x^2=24x-24x^2$

~~20.20~~

20.17

$$y=-\sqrt{\frac{2}{x^2}-1}$$

$$1+y^2+xyy'=0$$

$$\cancel{y'=-\frac{1}{2}\left(\frac{2}{x^2}\right)^{-1}}$$

$$y'=-\frac{1}{\cancel{2}}\left(\frac{2}{x^2}-1\right)^{-\frac{1}{2}}\cdot\left(-\cancel{4}^{2}\cdot x^{-3}\right)=$$

$$=\frac{2}{x^3\sqrt{\frac{2}{x^2}-1}}=\frac{2}{x^3\sqrt{\frac{2-x^2}{x^2}}}=$$

$$=\frac{2}{x^2\sqrt{2-x^2}}$$

$$= 2 \cdot \frac{t-2}{}$$

$$= 2\,\frac{\dfrac{t-2}{2\sqrt{t-3}} - \sqrt{t-3}}{(t-2)^2} = 2\,\frac{\dfrac{t-2-2(t-3)}{2\sqrt{t-3}}}{(t-2)^2} =$$

$$= 2\,\frac{4-t}{2\sqrt{t-3}\cdot(t-2)^2}$$

$$y''_{x^2} = \frac{(4-t)\cdot 2\cdot\cancel{\sqrt{t-3}}}{\cancel{\sqrt{t-3}}\cdot(t-2)^2} =$$

$$= \frac{(4-t)\cdot 2}{(t-2)^2} \Rightarrow \text{Ответ}$$

$$1$$

$$=$$

$$1$$

$$\Rightarrow$$

19.17

$$\begin{cases} x = \sqrt{t-3}, \\ y = \ln(t-2) \end{cases}$$

$$x'_t = \frac{1}{2}(t-3)^{-\frac{1}{2}} \cdot 1 = \frac{1}{2} \cdot \frac{1}{\sqrt{t-3}}$$

$$y'_t = \frac{1}{(t-2)}$$

$$y'_x = \frac{\cancel{1}}{(t-2)} \cdot \frac{\sqrt{t-3} \cdot 2}{\cancel{1}}$$

$$(y'_x)'_t = \frac{2 \cdot (\sqrt{t-3})' \cdot (t-2) - \sqrt{t-3} \cdot 1}{(t-2)^2}$$

$$= \frac{2 \cdot \left(\frac{1}{2}(t-3)^{-\frac{1}{2}} \cdot (t-2) - \sqrt{t-3} \cdot 1\right.}{(t-2)^2}$$

$x'_t = a(1\cdot\sin t + \cancel{t}\cdot\cos t + (-\sin t)) =$

$= a(\cancel{t}\cdot t\cos t)$

$y'_t = a(\cancel{\sin}\ \cancel{\cos t}\ \cancel{\mp}(1\cdot\cos t) + t\cdot(-\sin t)) =$

$= -a(t\sin t)$

$y'_x = \frac{\cancel{a}\cdot t\sin t}{-\cancel{a}\cdot t\cos t} = -\operatorname{tg} t$

$y'_{x_0} = -\operatorname{tg}\frac{\pi}{4} = -1$

касательная:

$y - \frac{\sqrt{2}}{2}\cdot a\left(1 - \frac{\pi}{4}\right) = -1\cdot\left(x - a\left(\frac{\pi}{4}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)\right)$

нормаль:

$y - \frac{\sqrt{2}}{2}\cdot a\left(1 - \frac{\pi}{4}\right) = 1\cdot\left(x - a\left(\frac{\pi}{4}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)\right)$

$$y_x = \frac{\dfrac{t}{\sqrt{t^2-1}} - \dfrac{1}{\sqrt{t^2-1}} \cdot \dfrac{1}{t}}{-\dfrac{1}{\sqrt{\left(1-\left(\frac{1}{t}\right)^2\right)}} \cdot \left(1 - (t)^{-2}\right)}$$

16.17

$$\begin{cases} x = a(t\sin t + \cos t), \\ y = a(\sin t - t\cos t), \quad t_0 = \frac{\pi}{4} \end{cases}$$

$$x_0 = a\left(\frac{\pi}{4}\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) = a\left(\cancel{\sqrt{2}}\,\frac{\pi}{4}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}\cdot a\left(\frac{\pi}{4} + 1\right)$$

$$y_0 = a\left(\cancel{\pi}\,\frac{\sqrt{2}}{2} - \frac{\pi}{4}\cdot\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}\cdot a\left(1 - \frac{\pi}{4}\right)$$

15.17

$$\begin{cases} x = \arccos\frac{1}{t} \\ y = \sqrt{t^2-1} + \arcsin\frac{1}{t} \end{cases}$$

$$x'_t = -\frac{1}{\sqrt{1-\left(\frac{1}{t}\right)^2}} \cdot \left(-1\cdot(t)^{-2}\right)$$

$$y'_t = \frac{1}{2}\cdot\left(t^2-1\right)^{-\frac{1}{2}}\cdot 2t + \frac{1}{\sqrt{1-\left(\frac{1}{t}\right)^2}}\cdot$$

$$\cdot\left(-1\cdot(t)^{-2}\right)$$

$$y'_t = \frac{1}{\cancel{2}}\cdot\left(t^2-1\right)^{-\frac{1}{2}}\cdot \cancel{2}t + \frac{1}{\sqrt{1-\left(\frac{1}{t}\right)^2}}\cdot$$

$$\cdot\left(-1\cdot(t)^{-2}\right) = \frac{t}{\sqrt{(t^2-1)}} - \sqrt{\frac{t}{t^2-1}}\cdot\frac{1}{t}$$

$$\cancel{y'_x = \frac{t}{\sqrt{(t^2-1)}} - \frac{1}{\sqrt{t^2-1}}\cdot\frac{1}{t}}$$

13.17

$$y=\frac{1}{3}(x-2)\sqrt{x+1}+\ln\left(\sqrt{x+1}+1\right)$$

$$y'=\frac{1}{3}\left(1\cdot\sqrt{x+1}+(x-2)\cdot\frac{1}{2}(x+1)\right)+\frac{1}{\left(\sqrt{x+1}+1\right)}\cdot\frac{1}{2}(x+1)^{-\frac{1}{2}}\Rightarrow \text{Ответ}$$

14.17

$$y=\operatorname{arctg}\frac{2\sin x}{\sqrt{9\cos^2 x-4}}$$

$$y'=\frac{1}{1+\left(\frac{2\sin x}{\sqrt{9\cos^2 x-4}}\right)^2}\cdot\left(\frac{2\cos x\cdot\sqrt{9\cos^2 x-4}-(2\sin x)\cdot\frac{1}{2}(9\cos^2 x-4)\cdot 9\cdot 2\cos x\cdot(-\sin x)}{9\cos^2 x-4}\right)\Rightarrow \text{Ответ}$$

$$\frac{y'}{y} = \frac{5}{2}\left(1 \cdot \ln(\sin x) + x \cdot \left(\frac{1}{\sin x} \cdot \cos x\right)\right)$$

$$y' = (\sin x)^{\frac{5x}{2}} \cdot \left(1 \cdot \ln(\sin x) + x \cdot \cancel{\tfrac{1}{\sin x}}\, ctg\, x\right)$$

Ответ

12.17

$$y = \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x-1}{\sqrt{2}} + \frac{x-1}{x^2 - 2x + 3}$$

$$\cancel{y' = \frac{1}{\sqrt{2}} \cdot \frac{1}{\left(1 + \left(\frac{x-1}{\sqrt{2}}\right)^2\right)} \cdot}$$

$$y' = \frac{1}{\sqrt{2}} \cdot \frac{1}{\left(1 + \left(\frac{x-1}{\sqrt{2}}\right)^2\right)} \cdot \frac{1}{\sqrt{2}} + \frac{1 \cdot}{}$$

$$\cdot \frac{x^2 - 2x + 3 - (x-1) \cdot (2x-2)}{(x^2 - 2x + 3)^2} \Rightarrow \text{Ответ}$$

10.17

$$y = -\frac{sh\,x}{2ch^2x} + \frac{3}{2}\arcsin(th\,x)$$

$$y' = -\frac{1}{2}\frac{(sh\,x)'\cdot ch^2x - sh\,x\cdot(ch^2x)'}{ch^4x} + \frac{3}{2}\cdot$$

$$\cdot\frac{1}{\sqrt{1-(th\,x)^2}}\left(\frac{1}{ch^2x}\right) = -\frac{1}{2}\frac{(ch\,x^3 - sh\,x\cdot 2ch\,x\cdot sh\,x)}{ch^4x} + \frac{3}{2}\cdot\frac{1}{\sqrt{1-(th\,x)^2}}\cdot\frac{1}{ch^2x}$$

Ответ:

11.17

$$y = (\sin x)^{\frac{5x}{2}}$$

$$\ln y = \ln\left((\sin x)^{\frac{5x}{2}}\right)$$

$$\ln y = \frac{5x}{2}\ln(\sin x)$$

$$\frac{y'}{y} = \frac{5}{2}\left((x)'\cdot\ln(\sin x) + x\cdot(\ln(\sin x)\right.$$

$$\frac{2\sin 17x \cdot \cos 17x \cdot 17 \cdot \cos 34x - (\sin^2 17x) \cdot}{(\cos 34x)^2}$$

$$\frac{\cdot(-\sin 34x)\cdot 34}{} \Rightarrow \text{Ответ}$$

9.17

$$y = \frac{x-3}{2}\sqrt{6x-x^2-8} + \arcsin\sqrt{\frac{x}{2}-1}$$

$$y' = \frac{1}{2}\left(\left(x-3\right)' \cdot \sqrt{6x-x^2-8} + (x-3) \cdot \left(\sqrt{6x-x^2-8}\right)'\right.$$

$$+ \frac{1}{\sqrt{1-\left(\frac{x}{2}-1\right)}} \cdot \frac{1}{2}\left(\frac{x}{2}-1\right)^{-\frac{1}{2}} \cdot \frac{1}{2} =$$

$$= \frac{1}{2}\left(\sqrt{6x-x^2-8} + (x-3) \cdot \frac{1}{2}(6x-x^2-8)^{-\frac{1}{2}} \cdot\right.$$

$$\cdot (6-2x)\Big) + \frac{1}{\sqrt{2-\frac{x}{2}}} \cdot \frac{1}{2}\left(\frac{x}{2}-1\right)^{-\frac{1}{2}} \cdot \frac{1}{2} \Rightarrow \text{Ответ}$$

7.17

$$y = \ln\cos\frac{2x+3}{x+1}$$

$$y' = \frac{1}{\cos\frac{2x+3}{x+1}} \cdot \left(-\sin\frac{2x+3}{x+1}\right) \cdot \frac{(2x+3)' \cdot (x+1) - (2x+3) \cdot (x+1)'}{(x+1)^2} =$$

$$= -\operatorname{tg}\frac{2x+3}{x+1} \cdot \frac{2 \cdot (x+1) - (2x+3) \cdot 1}{(x+1)^2}$$

Ответ

8.17

$$y = \frac{\operatorname{ctg}(\sin\frac{1}{3}) \cdot \sin^2 17x}{17\cos 34x}$$

$$y' = \frac{\operatorname{ctg}(\sin\frac{1}{3})}{17} \cdot \frac{(\sin^2 17x)' \cdot \cos 34x - (\sin^2 17x)(\cos 34x)'}{(\cos 34x)^2} = \frac{\operatorname{ctg}(\sin\frac{1}{3})}{17} \cdot$$

$$\frac{y'}{y}=\frac{1}{2}\cdot\frac{1}{2x+3}\cdot 2+\frac{1}{x-2}\cdot 1-2\cdot\frac{1}{x}$$

$$y'=\frac{\sqrt{2x+3}\,(x-2)}{x^2}\cdot\left(\frac{1}{2}\cdot\frac{1}{2x+3}\cdot 2+\frac{1}{x-2}\cdot 1-2\cdot\frac{1}{x}\right)$$

Ответ.

6.17

$$y=\ln\left(e^x+\sqrt{e^{2x}-1}\right)+\arcsin e^{-x}$$

$$y'=\frac{1}{e^x+\sqrt{e^{2x}-1}}\cdot\left(e^x+\sqrt{e^{2x}-1}\right)'+\frac{1}{\sqrt{1-e^{-2x}}}\cdot$$

$$\cdot e^{-x}\cdot(-1)=\frac{1}{e^x+\sqrt{e^{2x}-1}}\cdot\left(e^x+\frac{1}{2}\left(e^{2x}-1\right)^{-\frac{1}{2}}\cdot\right.$$

$$\left.\cdot e^{2x}\cdot 2\right)+\frac{1}{\sqrt{1-e^{-2x}}}\cdot e^{-x}\cdot(-1)\Rightarrow \text{Ответ}$$

$y(x_0) = 3$

$y' = (\cancel{\sqrt{4x-1}})^{\frac{1}{2}} = \left((4x-1)^{\frac{1}{2}}\right)' = \frac{1}{2}(4x-1)^{-\frac{1}{2}} \cdot 4$

$= \frac{2}{\sqrt{4x-1}}$

$y'(x_0) = \frac{2}{\sqrt{4x-1}}\Bigg|_{x_0 = 2,5} = \frac{2}{3}$

$y \approx 3 + \frac{2}{3} \cdot 0,06$

$y \approx 3,04 \Rightarrow$ Ответ

5.17

$y = \frac{\sqrt{2x+3}(x-2)}{x^2}$

$\ln y = \ln \frac{\sqrt{2x+3}(x-2)}{x^2} = \ln \sqrt{2x+3}(x-2) -$

$- \ln x^2 = \ln \sqrt{2x+3} + \ln(x-2) - 2\ln x =$

$= \frac{1}{2}\ln(2x+3) + \ln(x-2) - 2\ln x$

3.17

$dy = y' \cdot dx$ $\ln\frac{a}{b} = \ln a - \ln b$

$y = \ln\left|\frac{x+\sqrt{x^2+1}}{2x}\right| = \ln|x+\sqrt{x^2+1}| - \ln|2x|$

$y' = \frac{1}{|x+\sqrt{x^2+1}|} \cdot (x+\sqrt{x^2+1})' - \frac{1}{2x} \cdot 2 =$

$= \frac{1}{|x+\sqrt{x^2+1}} \cdot \left(1+\frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot 2x\right) - \frac{1}{x}$

Ответ:

$dy = \frac{1}{|x+\sqrt{x^2+1}|} \cdot \left(1+\frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot 2x\right) - \frac{1}{x} \cdot dx$

4.17

$y = \sqrt{4x-1}$, $x_1 = 2,56$

$y \approx y(x_0) + y'(x_0) \cdot \Delta x$, $\Delta x = x - x_0$

$x_0 = 2,5$

$\Delta x = x - x_0 = 0,06$

$(x^5+1)' = 5x^4$

$(x^4+1)' = 4x^3$

$$y' = \frac{5x^4\cdot(x^4+1) - (x^5+1)\cdot 4x^3}{(x^4+1)^2} =$$

$$= \frac{x^3\left(5x\cdot(x^4+1) - (x^5+1)\cdot 4\right)}{(x^4+1)^2} =$$

$$= \cancel{x}$$

$$y'(1) = \frac{5\cdot 2 - 8}{4} = \frac{2}{4} = \frac{1}{2}$$

$y - 1 = \frac{1}{2}(x-1)$ — урав. касательной

$y - 1 = -2(x-1)$ — урав. нормали

19.17

$$\lim_{x \to \frac{\pi}{4}} (\sin x + \cos x)^{\frac{1}{tg\, x}} = \lim_{x \to \frac{\pi}{4}} \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)^{1} =$$

$$= \sqrt{2} \Rightarrow \text{Ответ.}$$

20.17

$$\lim_{x \to 0} \sqrt{arctg\, x \cdot \sin^2 \frac{1}{x} + 5\cos x} =$$

$$= \lim_{x \to 0} \sqrt{0 \cdot \sin^2 \frac{1}{x} + 5} = \sqrt{0 + 5} = \sqrt{5} \Rightarrow \text{Ответ}$$

## Задача 2

2.17

$$y = \frac{x^5 + 1}{x^4 + 1}, \quad x_0 = 1$$

$$y_0 = \frac{1+1}{1+1} = \frac{2}{2} = 1 \Rightarrow y_0$$

$$K = F'(x_0)$$

$$y' = \frac{(x^5+1)' \cdot (x^4+1) - (x^5+1) \cdot (x^4+1)'}{(x^4+1)^2} =$$

$t^{\frac{t+1}{t}}$

$-1-1) =$

$\frac{t+1}{t} =$

$\cdot \frac{t+1}{t} =$

18.17

$$\lim_{x\to1}\left(2e^{x-1}-1\right)^{x/(x-1)}$$

$t=x-1 \quad x=t+1$

$$\lim_{t\to0}\left(2e^{t}-1\right)^{\frac{t+1}{t}}=\lim_{t\to0}\left(1+\left(2e^{t}-1-1\right)\right)^{\frac{t+1}{t}}=$$

$$=\lim_{t\to0}\left(1+\left(2e^{t}-2\right)\right)^{\frac{1}{2e^{t}-2}\cdot\frac{2e^{t}-2}{1}\cdot\frac{t+1}{t}}=$$

$$=\lim_{t\to0} e^{\frac{2e^{t}-2}{1}\cdot\cancel{\frac{1}{e}}\;\frac{t+1}{t}}=$$

$$=\lim_{t\to0} e^{2(e^{t}-1)\cdot\frac{t+1}{t}}=\lim_{t\to0} e^{2\cancel{t}\cdot\frac{t+1}{t}}=$$

$$=\lim_{t} e^{2}=e^{2}\Rightarrow \text{Ответ}$$

1)
$$\begin{array}{l|l} x^3 - 3x^2 + 4 & x-2 \\ \hline -(x^3 - 2x^2) & x^2 - x - 2 \\ 0 - x^2 + 4 & \\ -(-x^2 + 2x) & \\ 0 - 2x + 4 & \\ -(-2x + 4) & \\ 0 & \end{array}$$

$$\frac{-6x(x-2)}{-x-2)} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x^2+2x+4) - 6x)}{\cancel{(x-2)}(x^2-x-2)} =$$

$$= \lim_{x \to 2} \frac{0}{0} = \lim_{x \to 2} \frac{x^2 - 4x + 4}{(x-2)(x+1)} =$$

$$= \lim_{x \to 2} \frac{(x-2)}{\cancel{(x-2)}(x+1)} = \frac{0}{3} = 0 \Rightarrow \text{Ответ}$$

17.17

$$\lim_{x\to 0}\left(\frac{2^{2x}-1}{x}\right)^{x+1} = \lim_{x\to 0} e^{\ln\left(\frac{2^{2x}-1}{x}\right)^{x+1}}$$

$$= \lim_{x\to 0} e^{(x+1)\ln\left(\frac{2^{2x}-1}{x}\right)}$$

$$\ln\left(\frac{2^{2x}-1}{x}\right) = \ln\left(\frac{x\ln 4}{\cancel{x}}\right) = \ln(\ln 4) =$$

$$= \lim_{x\to 0} e^{(x+1)\ln(\ln 4)} = e^{\ln(\ln 4)} =$$

$$= e^{\ln 4} = \ln 4 \Rightarrow \text{Ответ}$$

9.17

$$\lim_{x\to 2}\frac{x^3-6x^2+12x-8}{x^3-3x^2+4} = \frac{2^3-6\cdot 2^2+12\cdot 2-8}{2^3-3\cdot 2^2+4} =$$

$$= \frac{8-24+24-8}{8-12+4} = \frac{0}{0}$$

$$\lim_{x\to 2}\frac{(x^3-8-6x(x-2)}{(x-2)(x^2-x-2)} = \lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{(x-2)(x^2-}$$

16.17

$$= \lim_{x\to 0} \left(2-e^{x^2}\right)^{1/\ln(1+tg^2(\pi x/3))} =$$

$$= \lim_{x\to 0} \left(1+(1-e^{x^2})\right)^{1/\ln(1+tg^2(\pi x/3))} =$$

$$= \lim_{x\to 0} \left(1+(1-e^{x^2})\right)^{\frac{1}{1-e^{x^2}} \cdot \frac{(1-e^{x^2})}{1} \cdot \frac{1}{\ln(1+tg^2(\pi x/3))}}$$

$$= \lim_{x\to 0} e^{\frac{(1-e^{x^2})}{\ln\left(1+tg^2\left(\frac{\pi x}{3}\right)\right)}} = \lim_{x\to 0} e^{\frac{-x^2}{tg^2\frac{\pi x}{3}}}$$

$$= \lim_{x\to 0} e^{\frac{-x^2}{\left(\frac{\pi x}{3}\right)^2}} = e^{\frac{-1}{\frac{\pi^2}{9}}} =$$

$$= e^{\frac{-9}{\pi^2}}$$ — Ответ

14.17

$$\lim_{x\to 0}\frac{7^{3x}-3^{2x}}{tg x+x^3}=\lim_{x\to 0}\frac{(7^{3x}-1)+1-3^{2x}}{x+x^3}$$

$$=\lim_{x\to 0}\frac{(7^{3x}-1)-(3^{2x}-1)}{x+x^3}=\frac{x\cdot\ln 7^3-x\cdot\ln 3^2}{x+x^3}=\lim_{x\to 0}\frac{\cancel{x}(3\ln 7-2\ln 3)}{\cancel{x}(1+x^2)}=$$

$$=3\ln 7-2\ln 3\Rightarrow \text{Ответ}$$

15.17

$$\lim_{x\to 0}\frac{e^{\sin 2x}-e^{\sin x}}{tg x}=$$

$$=\lim_{x\to 0}\frac{e^{\sin 2x}-1+1-e^{\sin x}}{x}=\lim_{x\to 0}\frac{(e^{\sin 2x}-1)-(e^{\sin x}-1)}{x}$$

$$=\lim_{x\to 0}\frac{\sin 2x-\sin x}{x}=\cancel{\frac{2x-x}{x}=2}$$

$$=\frac{2x-x}{x}=\frac{x}{x}=1\Rightarrow \text{Ответ}$$

$$= \lim_{t \to 0} \frac{\ln\left(1 + \frac{2t}{\pi}\right)}{-\frac{\sqrt{2}}{2} \cdot (-\sin t)} = \lim_{t \to 0} \frac{\frac{2t}{\pi}}{-\frac{\sqrt{2}}{2} \cdot (-t)} =$$

$$= \lim_{t \to 0} \frac{4}{\sqrt{2} \cdot \pi} = \frac{4}{\sqrt{2} \cdot \pi} \Rightarrow \text{Ответ}$$

13.17

$$\lim_{x \to 3} \frac{\ln(2x-5)}{e^{\sin \pi \cdot x} - 1} = \frac{0}{0} \Rightarrow \text{неопред.}$$

$$\lim_{x \to 3} \frac{\ln(1 + (2x-6))}{\sin \pi \cdot x} = \lim_{x \to 3} \frac{2x-6}{\sin \pi \cdot x} =$$

$$= \lim_{x \to 3} \frac{2(x-3)}{\sin \pi \cdot x} = (t = x-3;\ x = t+3) =$$

$$= \lim_{t \to 0} \frac{2t}{\sin \pi \cdot (t+3)} = \lim_{t \to 0} \frac{2t}{-\sin \pi t} = \lim_{t \to 0} \frac{2t}{-\pi t}$$

$$= \lim_{t \to 0} \cancel{2} \ -\frac{2}{\pi} \Rightarrow \text{Ответ}$$

11.17

$$\lim_{x\to 0} \frac{2\sin[\pi(x+1)]}{\ln(1+2x)} = \frac{0}{0} \Rightarrow \text{неопред.}$$

$$\lim_{x\to 0} \frac{2\sin[\pi x+\pi]}{2x} = \lim_{x\to 0} -\frac{2\sin\pi x}{2x}$$

$$= \lim_{x\to 0} -\frac{\cancel{2}\,\pi x}{\cancel{2}x} = -\pi \Rightarrow \text{Ответ}$$

12.17

$$\lim_{x\to \frac{\pi}{2}} \frac{\ln 2x - \ln\pi}{\sin\left(\frac{5x}{2}\right)\cos x} = \frac{0}{0} \Rightarrow \text{неопред.}$$

$$\lim_{x\to \frac{\pi}{2}} \frac{\ln\frac{2x}{\pi}}{\left(-\frac{\sqrt{2}}{2}\right)\cdot\cos x} = \left\{t = x - \frac{\pi}{2}\right); \left(x = t + \frac{\pi}{2}\right) = \lim_{t\to 0} \frac{\ln\frac{2\left(t+\frac{\pi}{2}\right)}{\pi}}{-\frac{\sqrt{2}}{2}\cdot\cos\left(t+\frac{\pi}{2}\right)} =$$

$$\lim_{x\to 4}\frac{(\sqrt[3]{16x}-4)\cdot((\sqrt[3]{16x})^2+\sqrt[3]{16x}\cdot 4+16)}{((\sqrt[3]{16x})^2+\sqrt[3]{16x}\cdot 4+16)(\sqrt{4+x}-\sqrt{2x})}$$

$$=\lim_{x\to 4}\frac{(16x-64)(\sqrt{4+x}+\sqrt{2x})}{((4+x)-2x)((\sqrt[3]{16x})^2+\sqrt[3]{16x}\cdot 4+16)}=$$

$$=\lim_{x\to 4}\frac{16\cancel{(x-4)}(\sqrt{4+x}+\sqrt{2x})}{(\cancel{(4-x)}(\sqrt[3]{16x})^2+(\sqrt[3]{16x}\cdot 4)+16)}=$$

$$=\lim_{x\to 4}-\frac{16(\sqrt{4+x}+\sqrt{2x})}{((\sqrt[3]{16})^2+\sqrt[3]{16x}\cdot 4+16)}=$$

$$=-\frac{16(\sqrt{4+4}+\sqrt{2\cdot 4})}{((\sqrt[3]{16\cdot 4})^2+(\sqrt[3]{16\cdot 4}\cdot 4)+16)}=-\frac{16\cdot 2\sqrt{8}}{16+16+16}=$$

$$=-\frac{\cancel{16}\cdot 2\sqrt{8}}{3\cdot\cancel{16}}=-\frac{2\sqrt{8}}{3}\Rightarrow \text{Ответ}$$

6.17

$$\lim_{n\to\infty}\left(\frac{n+3}{n+5}\right)^{n+4} = \left(\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e\right) =$$

$$= \lim_{n\to\infty}\left(\frac{n+5-2}{n+5}\right)^{n+4} = \lim_{n\to\infty}\left(1+\frac{-2}{n+5}\right)^{n+4} =$$

$$= \lim_{n\to\infty}\left(1+\frac{-2}{n+5}\right)^{\frac{n+5}{-2}\cdot\frac{-2}{n+5}\cdot(n+4)} =$$

$$= \lim_{n\to\infty} e^{\frac{-2}{n+5}\cdot(n+4)} = e^{-2} \Rightarrow \text{Ответ}$$

10.17

$$\lim_{x\to 4}\frac{\sqrt[3]{16x}-4}{\sqrt{4+x}-\sqrt{2x}} = \frac{\sqrt[3]{16\cdot 4}-4}{\sqrt{4+4}-\sqrt{2\cdot 4}}$$

$$= \frac{4-4}{\sqrt{8}-\sqrt{8}} = \frac{0}{0} \to \text{неопределённость}$$

$$=\lim_{n\to\infty}\frac{9n-5n^4+n^2+5}{n\left(\sqrt{n(n^5+9)}+\sqrt{(n^4-1)(n^2+5)}\right)}=$$

$$=\lim_{n\to\infty}\frac{-5n^4}{n\cdot(n^3+n^3)}=\frac{-5n^4}{2n^4}=-\frac{5}{2}\Rightarrow \text{Ответ}$$

5.17

$$\lim_{n\to\infty}\left[\frac{n+2}{1+2+3\ldots+n}-\frac{2}{3}\right]=\left[S_n=\frac{a_1+a_n}{2}\cdot n\right]=$$

$$=\lim_{n\to\infty}\left[\frac{n+2}{\frac{1+n}{2}\cdot n}-\frac{2}{3}\right]=$$

$$=\lim_{n\to\infty}\left[\frac{2(n+2)}{1+n\cdot n}-\frac{2}{3}\right]=$$

$$=\frac{n}{n^2}-\frac{2}{3}=-\frac{2}{3}\Rightarrow \text{Ответ}$$

$$\frac{+(n+5)^2}{} =$$

$$= \frac{(n+2)\cdot((2n+3)^2+(2n-3)\cdot(n+5)+(n+5)^2)}{(3n-1)^3+(2n+3)^3}$$

$$= \lim_{n\to\infty} \frac{7n^3}{27n^3+8n^3} = \frac{7n^3}{35n^3} = \frac{1}{5} \Rightarrow \text{Ответ}$$

3.17

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^3-7}+\sqrt[3]{n^2+4}}{\sqrt[4]{n^5+5}+\sqrt{n}} = \lim_{n\to\infty} \frac{n}{n^{\frac{5}{4}}} = \underline{0}$$

9.17

$$\lim_{n\to\infty} \frac{\sqrt{n(n^5+9)}-\sqrt{(n^4-1)(n^2+5)}}{n} = \lim_{n\to\infty} \frac{n(n^5+9)}{n(\sqrt{n}}$$

$$\frac{-(n^4-1)(n^2+5)}{n^5+9)\ \ +\sqrt{(n^4-1)(n^2+5)})} =$$

$$= \lim_{n\to\infty} \frac{n^6+9n-(n^6+5n^4-n^2-5)}{n(\sqrt{n(n^5+9)}+\sqrt{(n^4-1)(n^2+5)})} =$$

$$= \lim_{n\to\infty} \frac{\cancel{n^6}+9n-\cancel{n^6}-5n^4+n^2+5}{n(\sqrt{n(n^5+9)}+\sqrt{(n^4-1)(n^2+5)})} =$$

$$3n-1>\frac{14}{3\varepsilon}$$

$$3n>\frac{14}{3\varepsilon}+1$$

$$n>\frac{14}{9\varepsilon}+\frac{1}{3}$$

$$N(\varepsilon)=\left[\frac{14}{9\varepsilon}+\frac{1}{3}\right]\Rightarrow \text{Ответ}$$

2.17

$$\lim_{n\to\infty}\frac{(2n-3)^3-(n+5)^3}{(3n-1)^3+(2n+3)^3}$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$
$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$\lim_{n\to\infty}\frac{(2n-3-n-5)((2n-3)^2+(2n-3)\cdot(n+5)+}{(3n-1)^3+(2n+3)^3}$$

~~Задача~~

1.17

$$a_n = \frac{4+2n}{\cancel{3n-1}\ 1-3n}; \quad a = -\frac{2}{3}$$

$$\left|\frac{4+2n}{1-3n}+\frac{2}{3}\right| = \left|\frac{4+2n+2}{(1-3n)\cdot 3}\right| =$$

$$= \left|\frac{12+6n+2-6n}{(1-3n)\cdot 3}\right| = \left|\frac{14}{(1-3n)\,3}\right| =$$

$$= \left|\frac{14}{(1-3n)\,3}\right| < \varepsilon$$

$$\frac{14}{3}\cdot\frac{1}{|1-3n|} < \varepsilon$$

$$\frac{14}{3}\cdot\frac{1}{3n-1} < \varepsilon$$

$$\frac{1}{3n-1} < \frac{3}{14}\cdot\varepsilon$$