Lecture 18
Multiscale Image Representation

ECEN5283 Computer Vision

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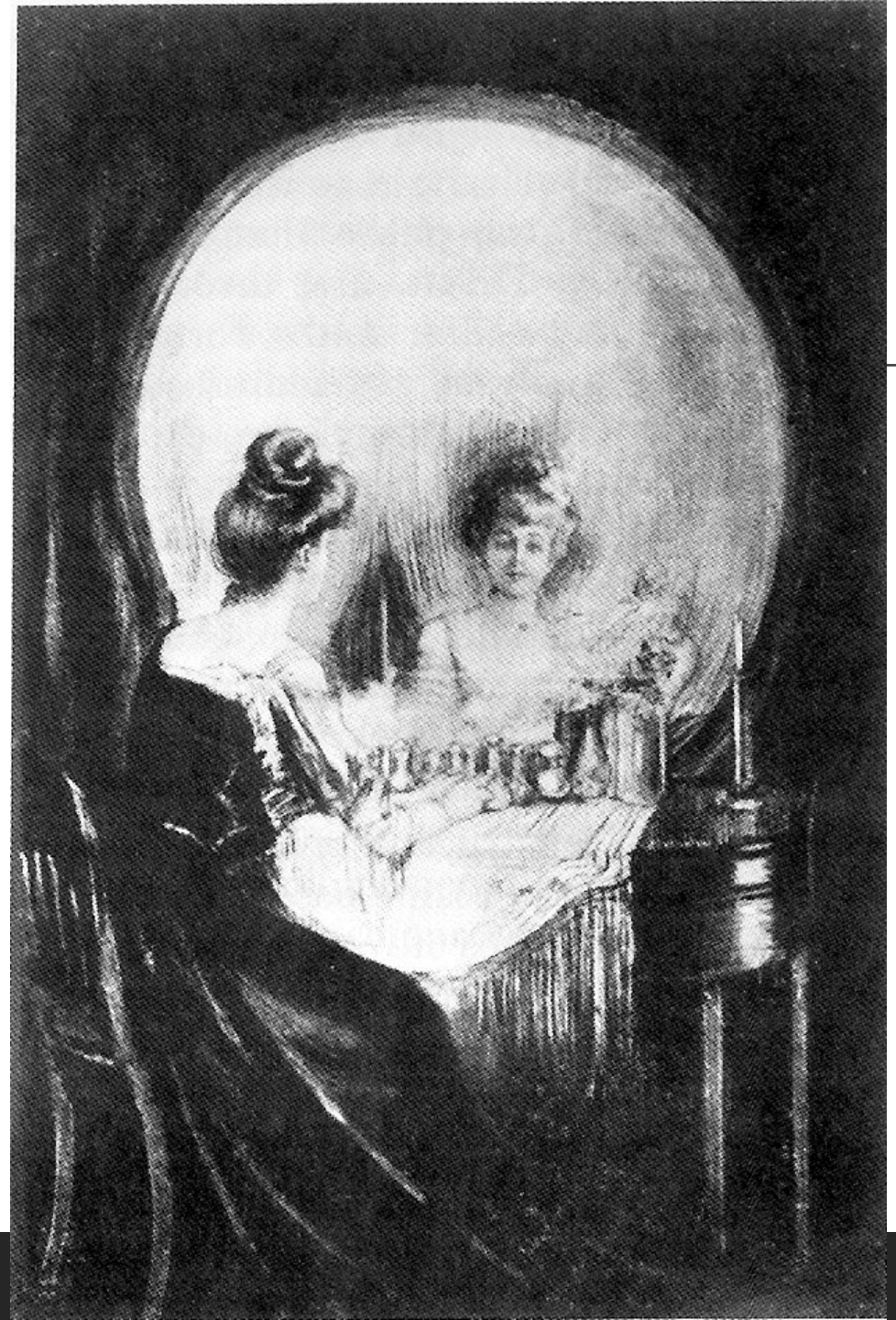
Goals

To introduce the concepts of multiscale image analysis

To create the Gaussian pyramid that supports efficient multiscale image processing

To discuss the Laplacian pyramid that plays a complementary role to the Gaussian pyramid



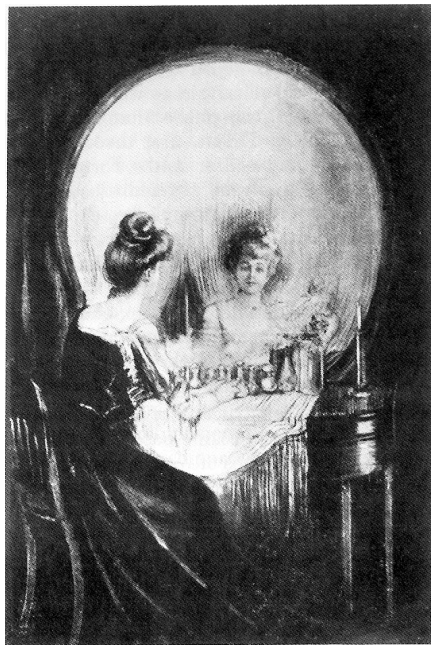
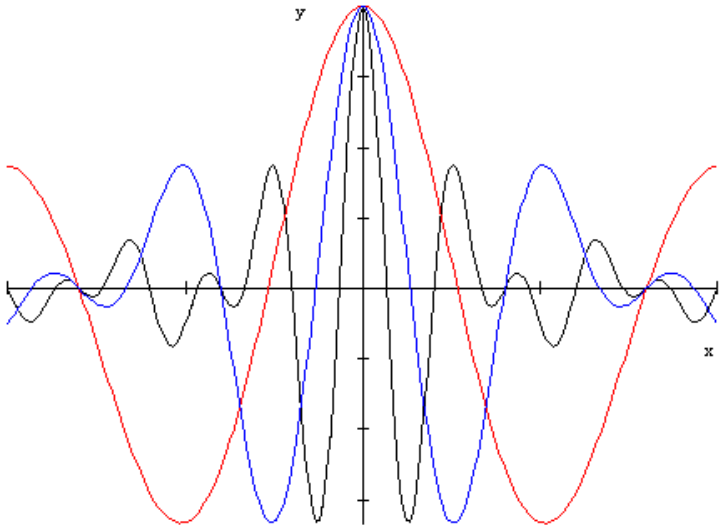


Image Scales

Images look quite different at different scales.

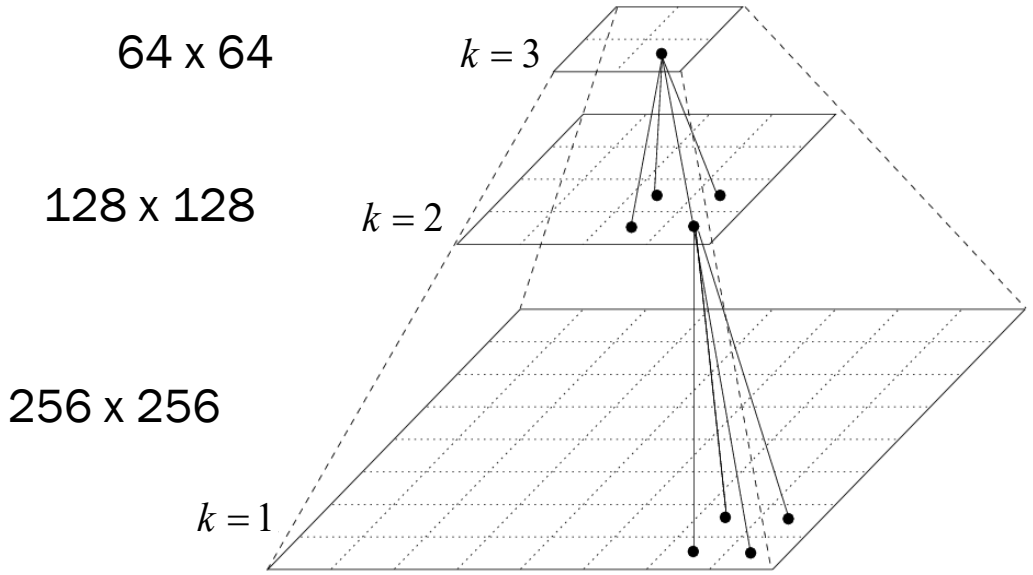
- Different scales are coded in terms of the response of a set of filters that operate at different number of pixels.
- A small scale filter is used to find details while a large scale filter can find the major structures.



Multi-scale filters

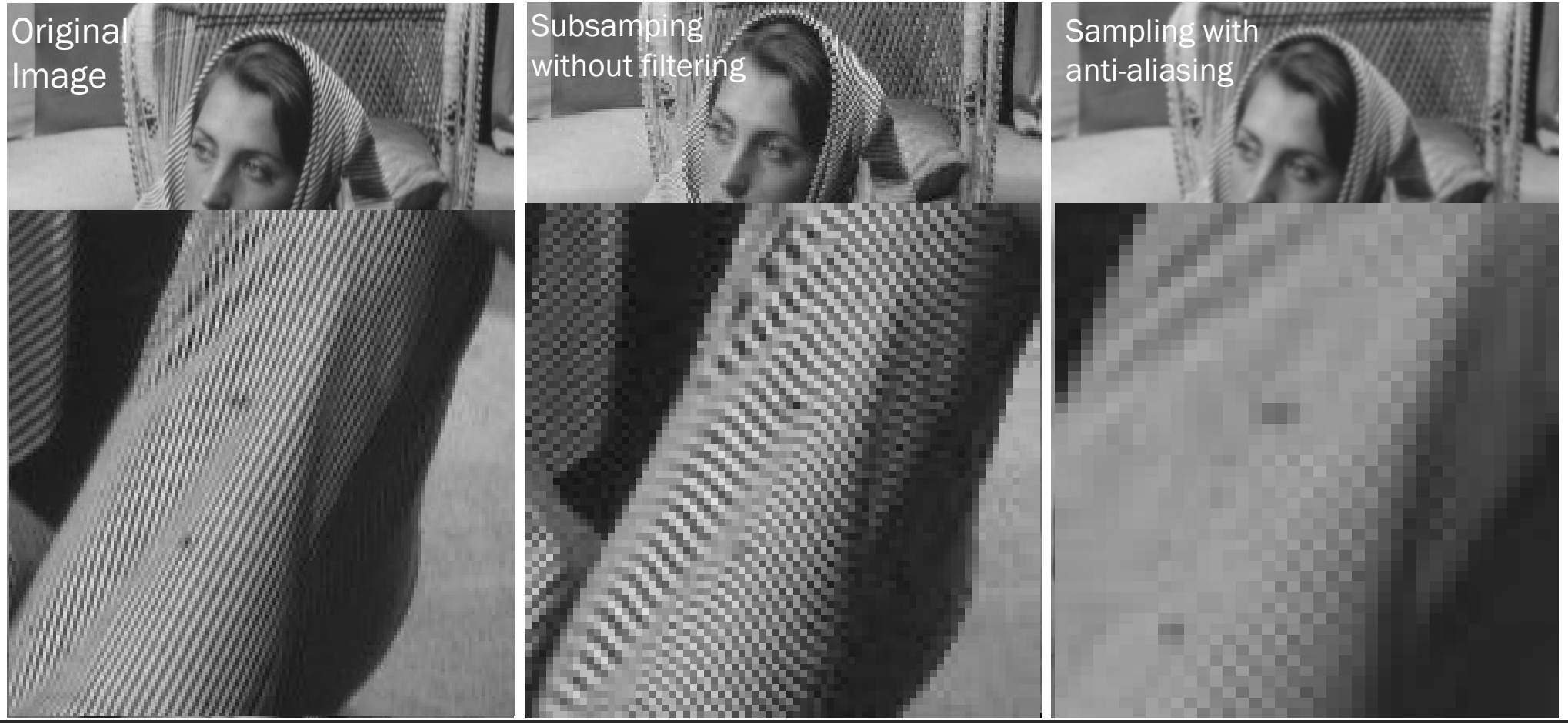
Image Pyramid

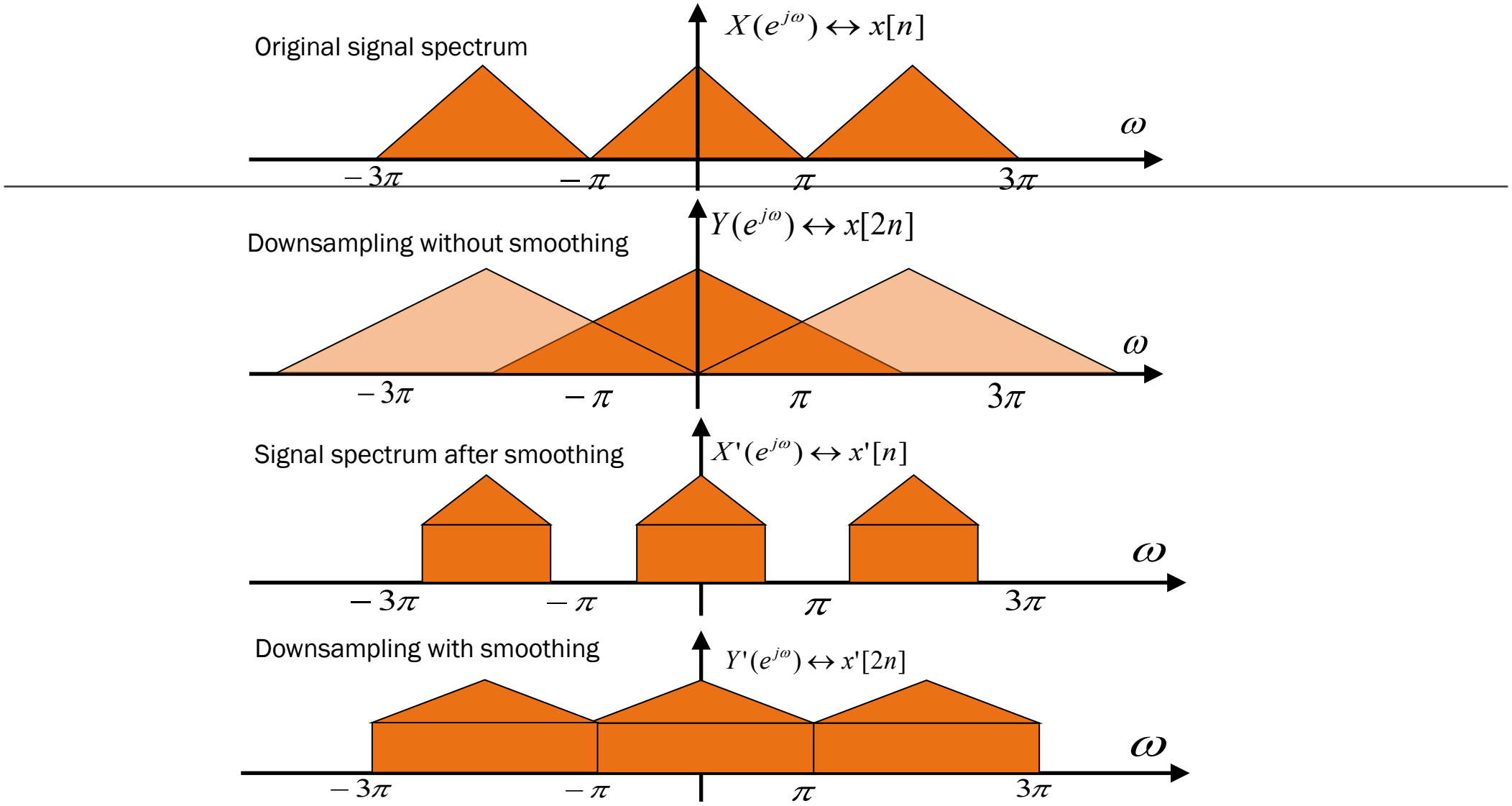
Instead of using a large scale filter, we can use a single small scale filter to a smoothed and re-sampled version of an image.



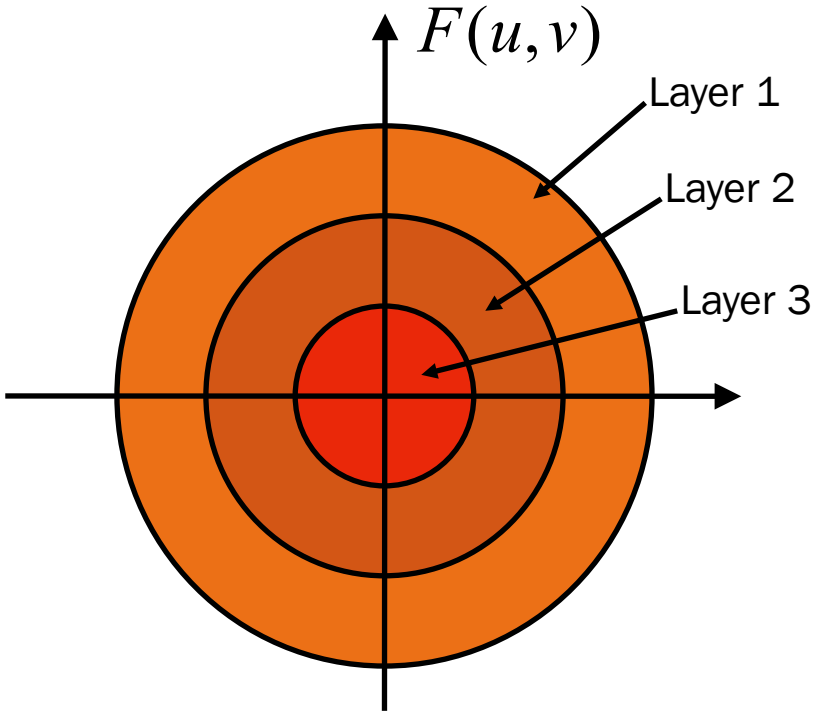
$$I_{k+1} = S\downarrow(G**I_k) = S\downarrow I_k^G \rightarrow I_{k+1}(i,j) = I_k^G(2i,2j)$$

Why smoothing before resampling?

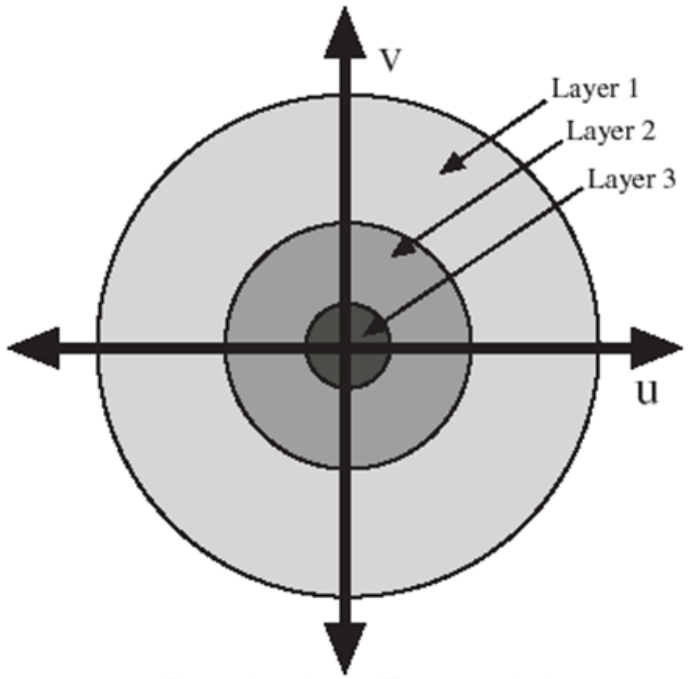



Original signal spectrum
$X(e^{j\omega}) \leftrightarrow x[n]$
ω
-3π
$-\pi$
π
3π
Downsampling without smoothing
$Y(e^{j\omega}) \leftrightarrow x[2n]$
ω
-3π
$-\pi$
π
3π
Signal spectrum after smoothing
$X'(e^{j\omega}) \leftrightarrow x'[n]$
ω
-3π
$-\pi$
π
3π
Downsampling with smoothing
$Y'(e^{j\omega}) \leftrightarrow x'[2n]$
ω
-3π
$-\pi$
π
3π

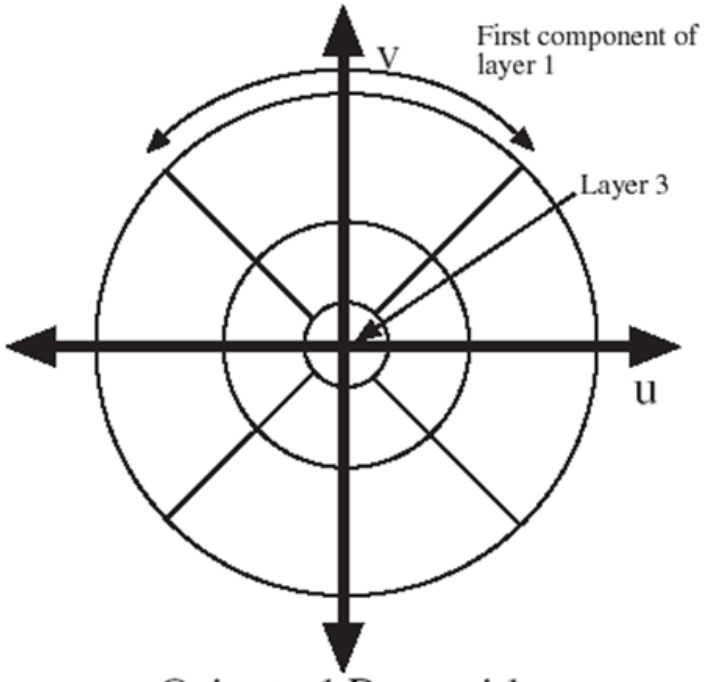
Oriented and Non-oriented Multiscale Image Representation in the Frequency Domain



Gaussian Pyramid



Laplacian Pyramid



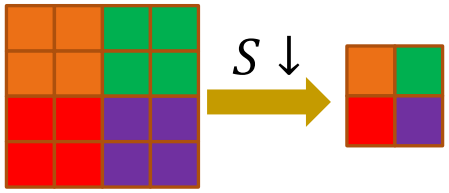
Oriented Pyramid

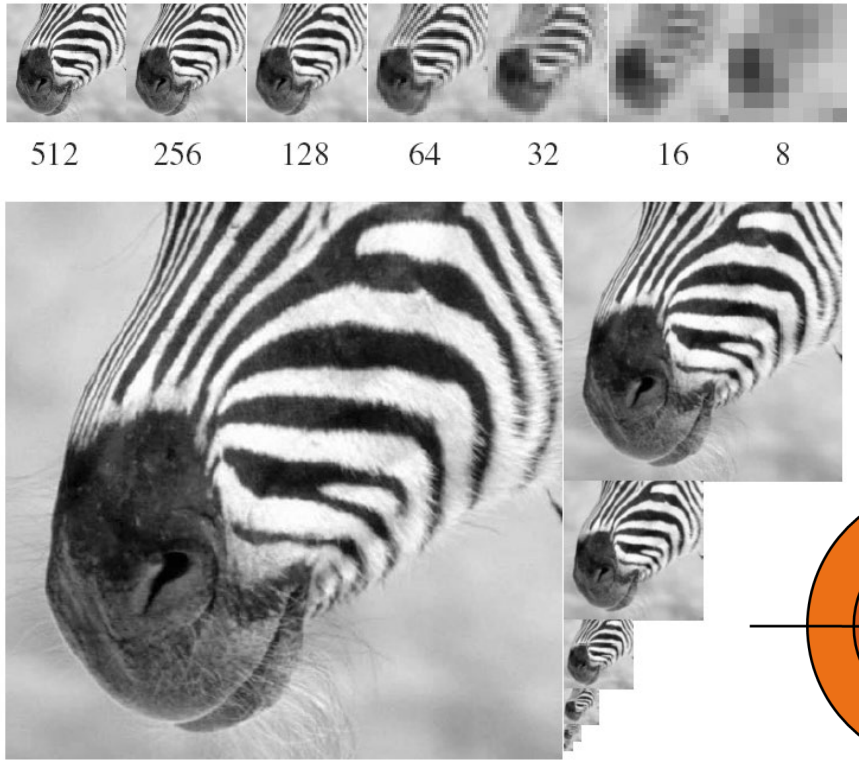
Gaussian Pyramid

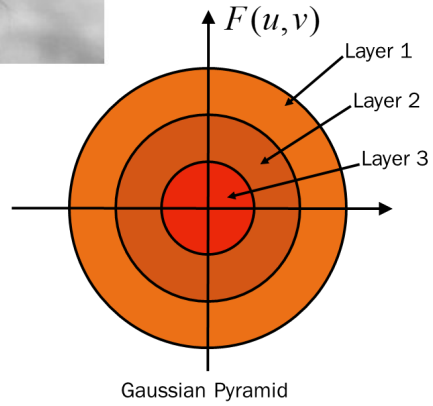
Gaussian pyramid is a collection of representation of an image at different scales or resolutions.

$$P_{\text{Gaussian}}(I)_1 = I \quad (\text{finest scale})$$

$$P_{\text{Gaussian}}(I)_{n+1} = S\downarrow (G_\delta ** P_{\text{Gaussian}}(I)_n)$$

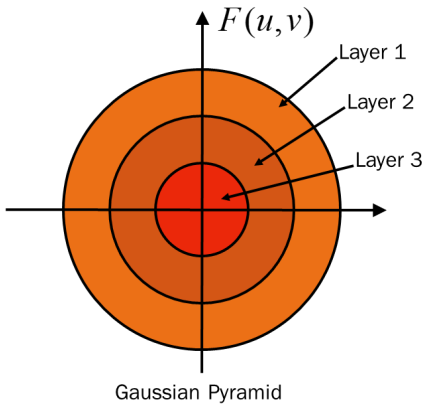


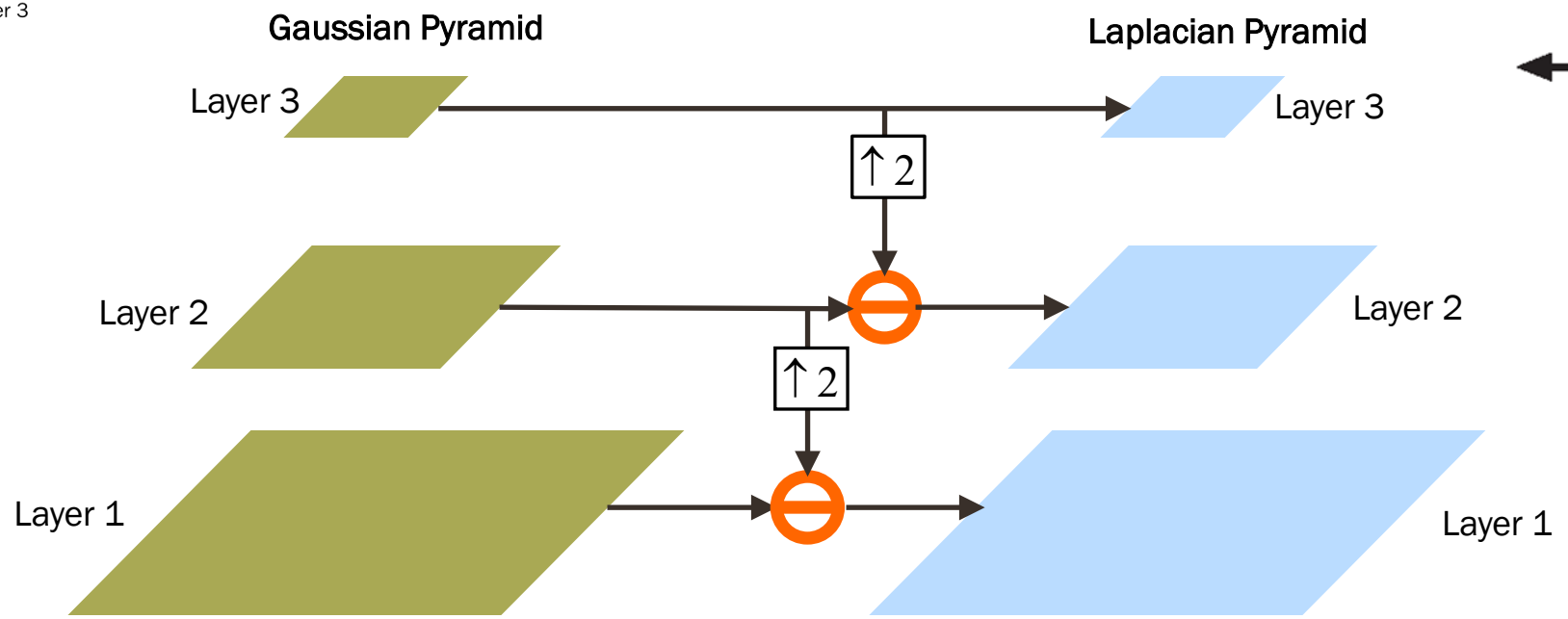


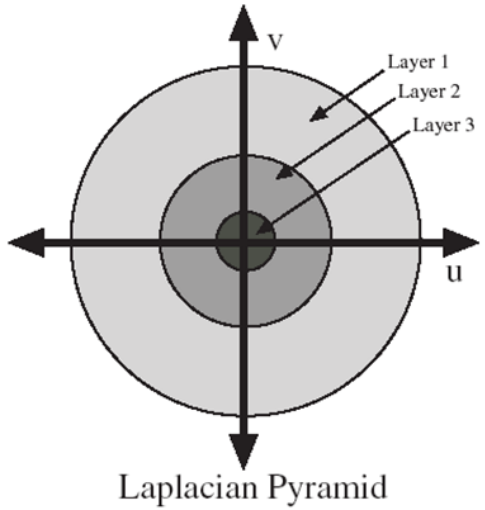


- Applications:
  - Corse-to-fine search and image segmentation

Gaussian and Laplacian Pyramids



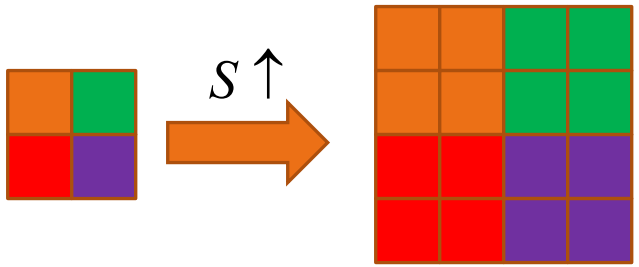




Laplacian Pyramid

Laplacian pyramid makes use of the fact that a coarser layer of Gaussian pyramid can predict the next finer scale.

- We can expand a coarser scale image by replicating pixels which involves an up-sampling operator S↑.



- Compared to Gaussian pyramid, Laplacian pyramid is less redundant and has rich high-frequency information (why?)
- We need only store the difference between this prediction and the next finer layer itself (high-frequency).

Laplacian Pyramid

$$P_{\text{Gaussian}}(I)_m \quad (k = 1,\ldots,m)$$

$$P_{\text{Laplacian}}(I)_m = P_{\text{Gaussian}}(I)_m$$

(where m is the coarest scale.)

$$P_{\text{Laplacian}}(I)_k = P_{\text{Gaussian}}(I)_k - S\uparrow\left(P_{\text{Gaussian}}(I)_{k+1}\right)$$

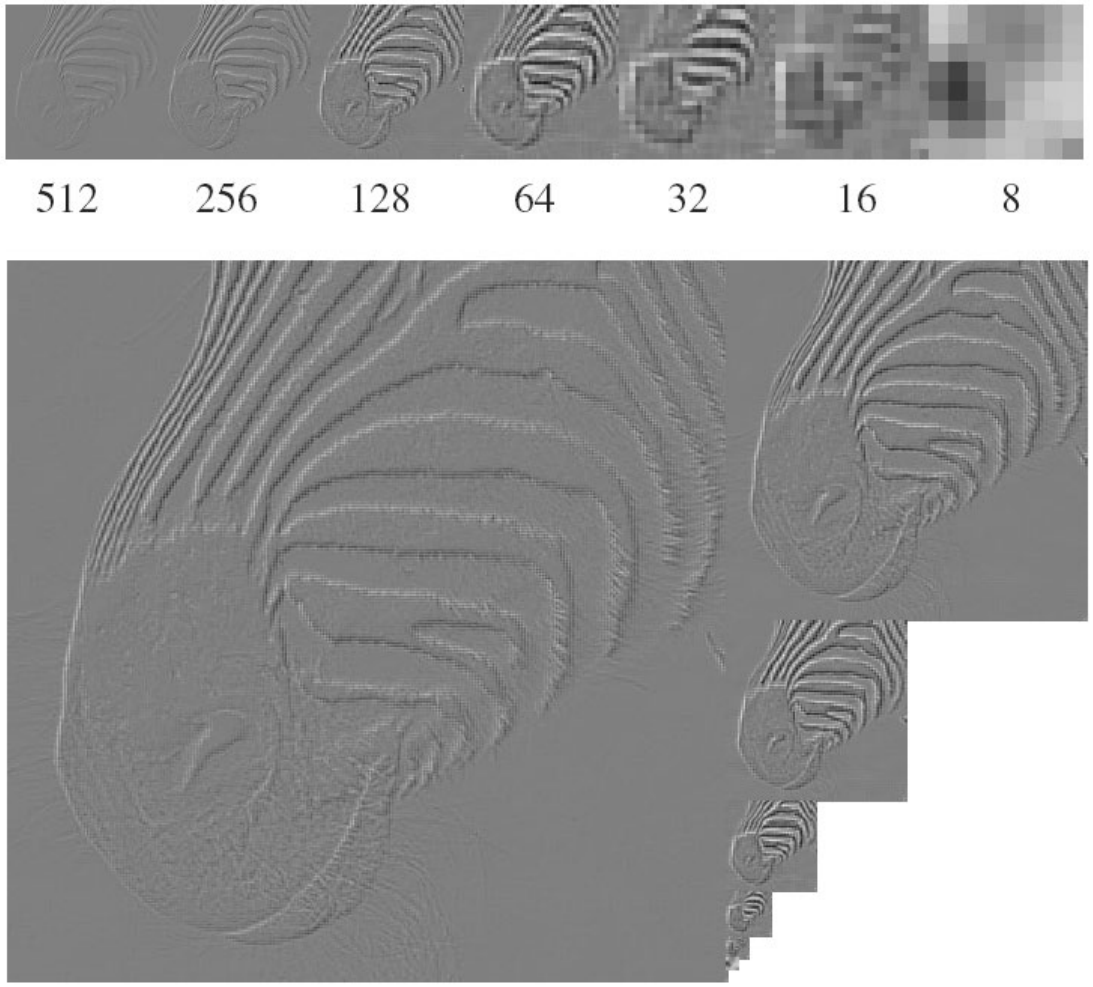
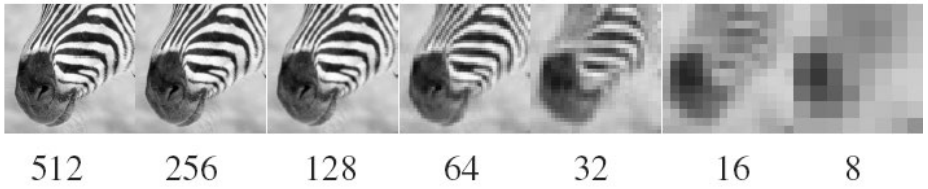
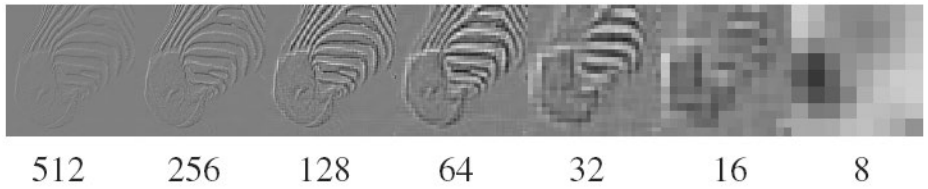
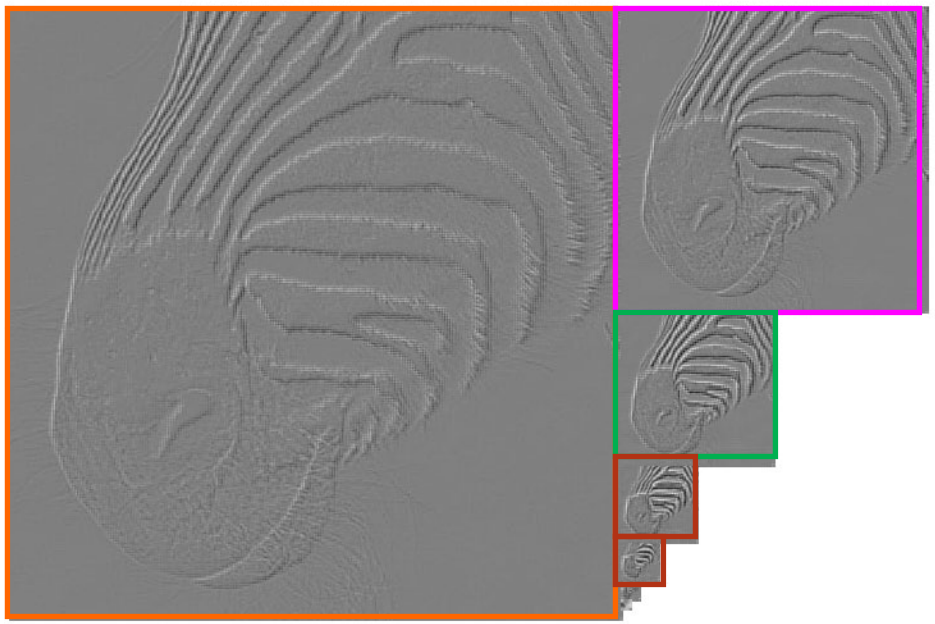


Figure 11.7. A Laplacian pyramid of images, running from 512x512 to 8x8. A zero response is coded with a mid-grey; positive values are lighter and negative values are darker. Notice that the stripes give stronger responses at particular scales, because each layer corresponds (roughly) to the output of a band-pass filter.

Laplacian-based Texture Analysis



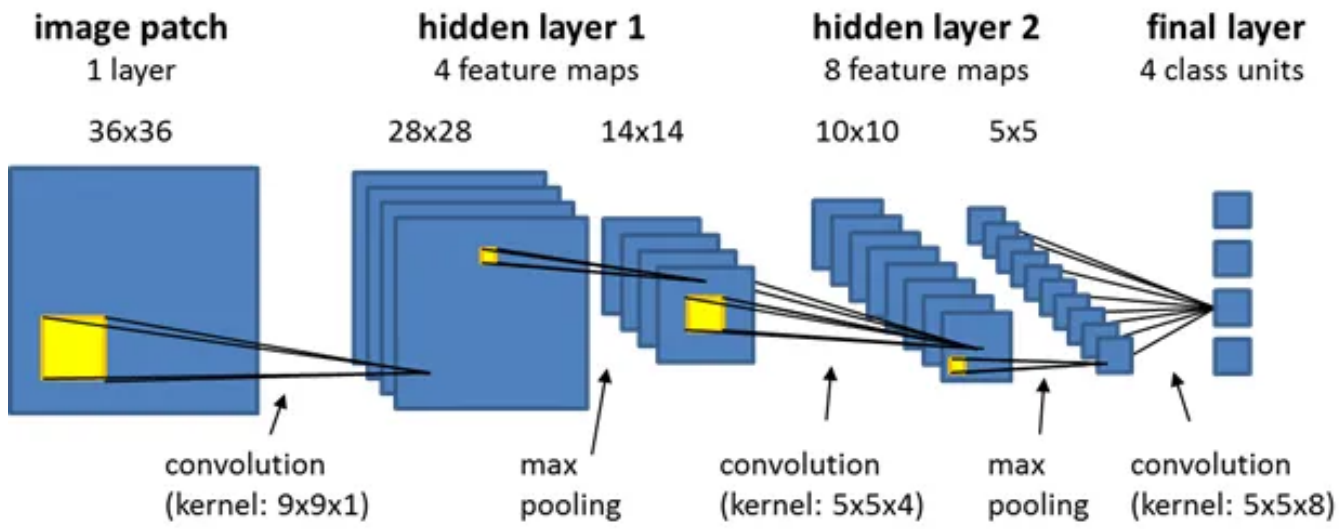




Pooling Layers in Deep Learning
Convolutional Neural Network (CNN)

Pooling layers act to subsample the input image which helps alleviate the computational load of the CNN and can help with overfitting the model.

The pooling acts only to aggregate values with varying aggregation functions, e.g., Maxpooling and AveragePooling.



https://jacobheyman702.medium.com/different-pooling-layers-for-cnn-4652a5103d62