Lecture 15
Gabor Filters

ECEN5283 Computer Vision

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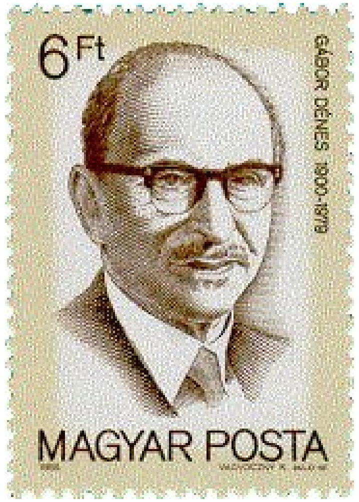
Goals

To review two issues of texture analysis using a filter bank.

To conduct texture analysis in the frequency domain.

To apply Gabor filters for texture analysis.



Dennis Gabor (original Hungarian name: Gábor Dénes), FRS, (June 5, 1900, Budapest – February 9, 1979, London) was a Hungarian physicist and inventor, most notable for inventing holography, for which he later received the Nobel Prize in Physics.

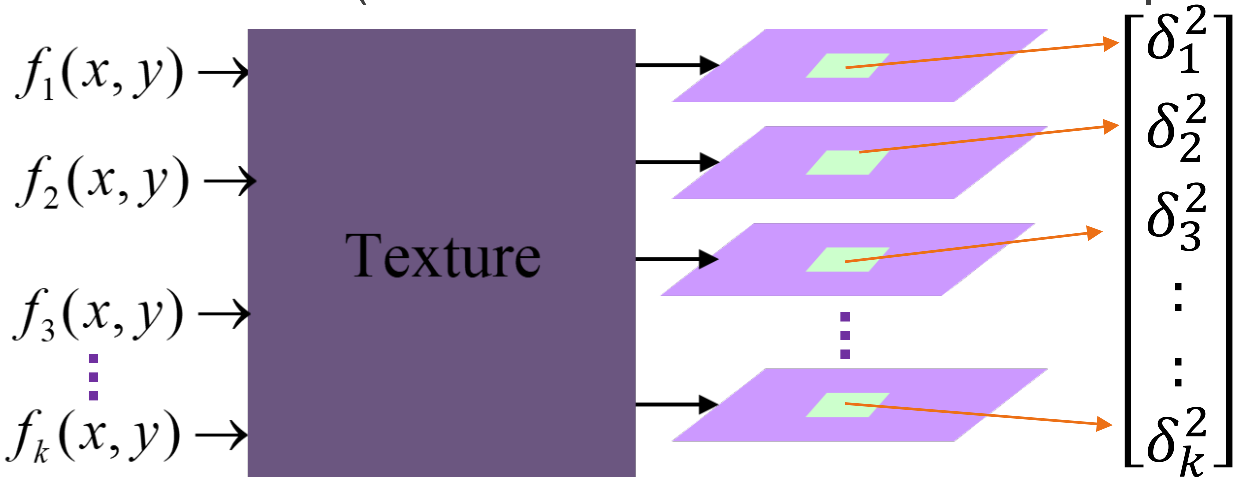
http://en.wikipedia.org/wiki/Dennis_Gabor

Filter Bank-based Texture Analysis

Texture analysis is referred to as the process of convolving an image with a range of oriented filters.

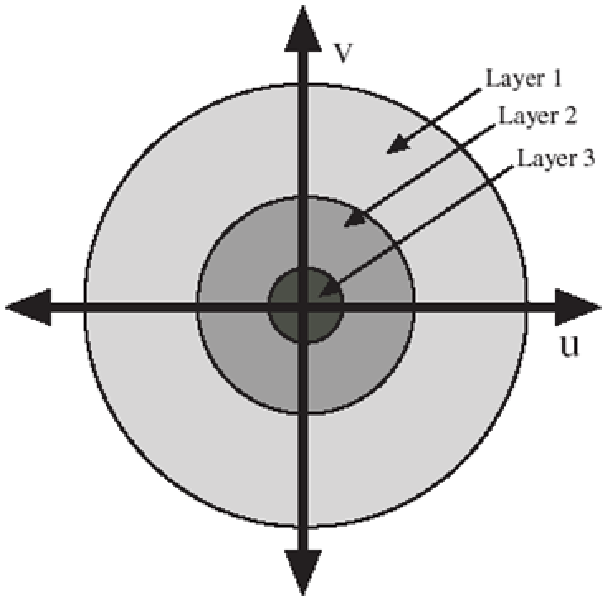
Two issues are involved for texture analysis using a filter bank.

- Statistics (mean, variance, skewness, kurtosis)
- Choice of scale (window-based statistics computation)

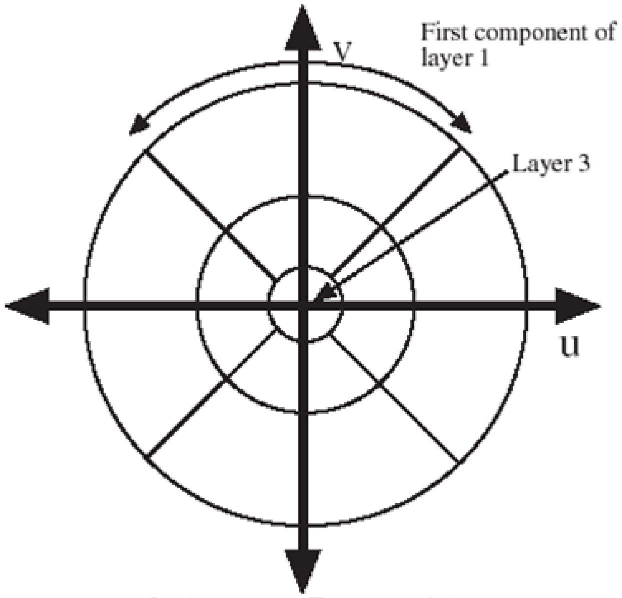


Frequency-domain Representation

The frequency-domain representation can give us some insights to a systematic design of a filter bank for texture analysis.



Laplacian Pyramid



Oriented Pyramid

Oriented filters can reveal more distinct frequency characteristics between different textures.

Frequency Transform Pairs (1)

$$g(t) = \delta(t) \leftrightarrow G(f) = 1$$

$$g(t) = 1 \leftrightarrow G(f) = \delta(f)$$

$$e^{j2\pi f_0 t} g(t) \leftrightarrow G(f - f_0)$$

Modulation

$$e^{j2\pi f_0 t} \longleftrightarrow \delta(f - f_0)$$

Fourier basis

Frequency Transform Pairs (2)

$$e^{j2\pi(u_0x+v_0y)} \xleftrightarrow{\text{2D Fourier}} \delta(u-u_0, v-v_0)$$ 2D Fourier basis

$$e^{j2\pi(u_0x+v_0y)}g(x,y) \xleftrightarrow{\text{2D Fourier}} G(u-u_0, v-v_0)$$ 2D Modulation

$$g(x,y) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \xleftrightarrow{\text{2D Fourier}} G(u,v) = \exp\left(-\frac{2\pi^2(u^2+v^2)}{1/\sigma^2}\right)$$

Gaussian function

Fourier Basis vs. Gabor Filters

The Fourier basis has not spatial selectivity but provides the best frequency selectivity.

$$f(x, y|u_0, v_0) = \exp^{j2\pi(u_0x+v_0y)} \quad \text{(Fourier basis)} \rightarrow F(u, v) = \delta(u - u_0, v - v_0)$$

Gabor filters can achieve *localized frequency characterization* by multiplying the Fourier basis elements with Gaussians.

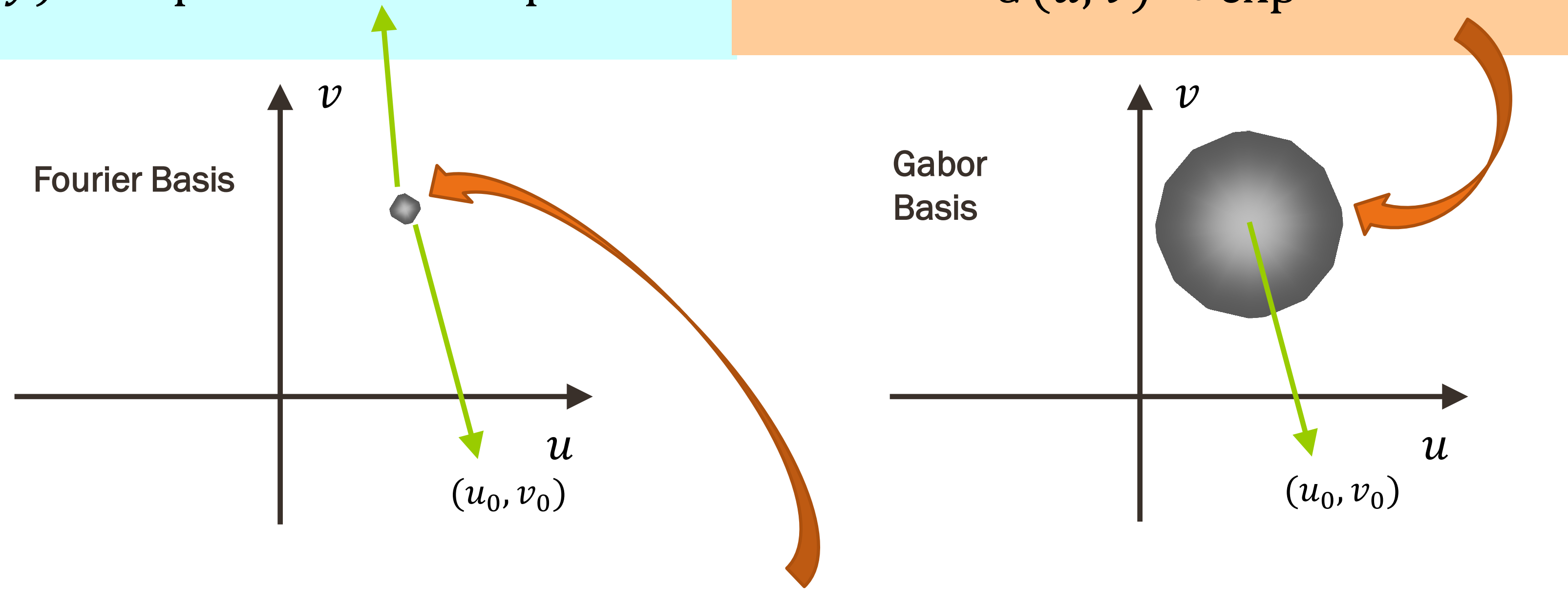
$$g(x, y|u_0, v_0) = \exp^{j2\pi(u_0x+v_0y)} \exp^{-\left\{\frac{x^2+y^2}{2\sigma^2}\right\}} \quad \text{(Gabor basis)}$$

Uncertainty principle: spatial resolution & frequency resolutions cannot be enhanced at the same time.

$$\rightarrow G(u, v|u_0, v_0) \propto \exp^{-\left\{\frac{2\pi^2[(u-u_o)^2+(v-v_0)^2]}{1/\sigma^2}\right\}}$$

Frequency-domain Comparison
Gabor Basis vs. Fourier Basis

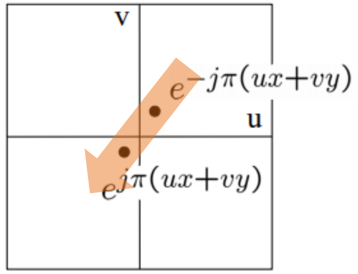
Gabor basis: $g(x,y) \propto \exp^{j2\pi(u_0x+v_0y)} \exp^{-\left\{\frac{x^2+y^2}{2\sigma^2}\right\}} \xleftrightarrow{\text{Fourier}} G(u,v) \propto \exp^{-\left\{\frac{2\pi^2[(u-u_o)^2+(v-v_0)^2]}{1/\sigma^2}\right\}}$

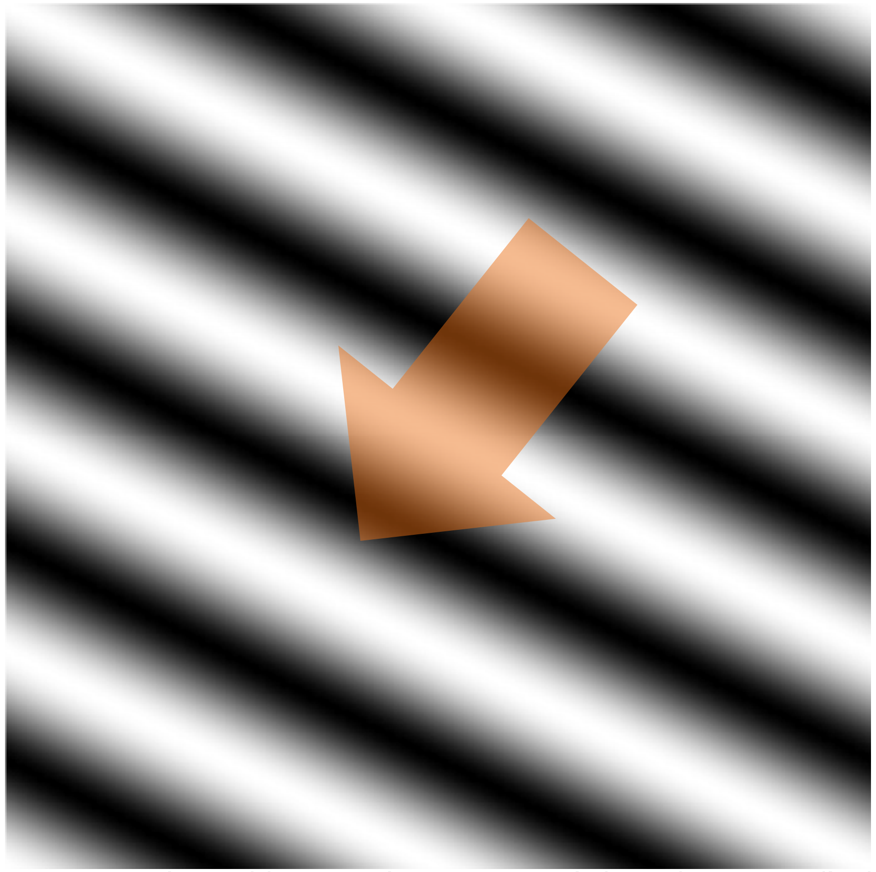


Fourier basis: $f(x,y|u_0,v_0) = \exp^{j2\pi(u_0x+v_0y)} \leftrightarrow F(u,v)=\delta(u_0,v_0)$

Fourier Basis Images (1)

To get some sense of what basis elements look like, we plot a basis element --- or rather, its real part --- as a function of x,y for some fixed u, v. We get a function that is constant when (ux+vy) is constant. The magnitude of the vector (u, v) gives a frequency, and its direction gives an orientation. The function is a sinusoid with this frequency along the direction, and constant perpendicular to the direction.

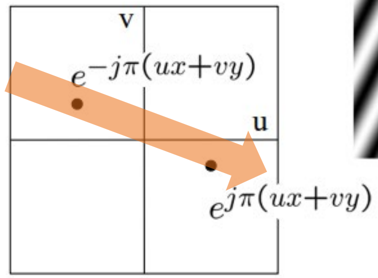




http://www.robots.ox.ac.uk/~az/lectures/ia/lect2.pdf

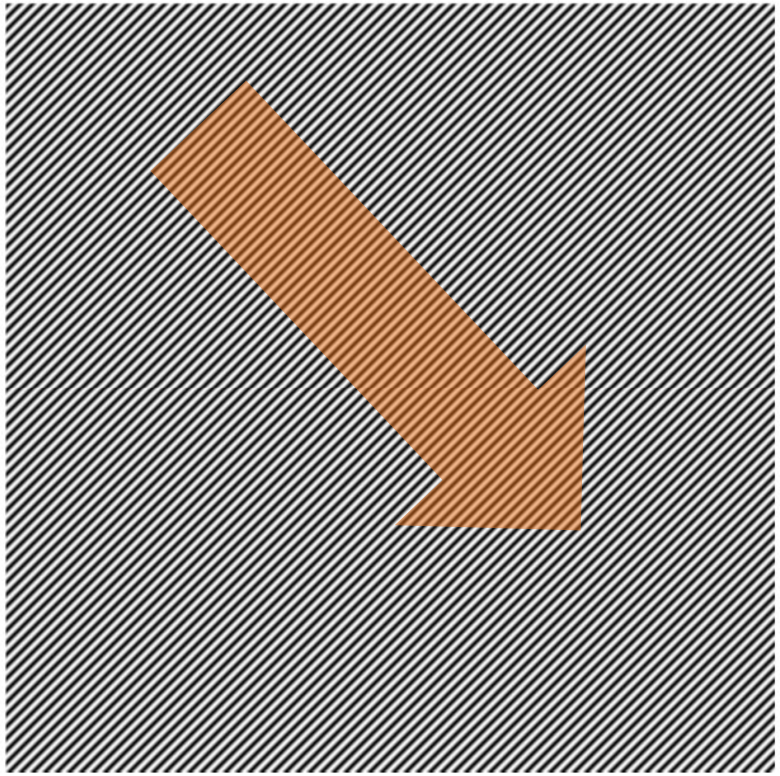
Fourier Basis Images (2)

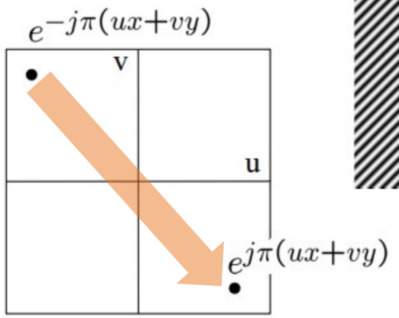
Here u and v are larger than in the previous slide.



Fourier Basis Images (3)

And larger still...





Gabor Filter Bank Design

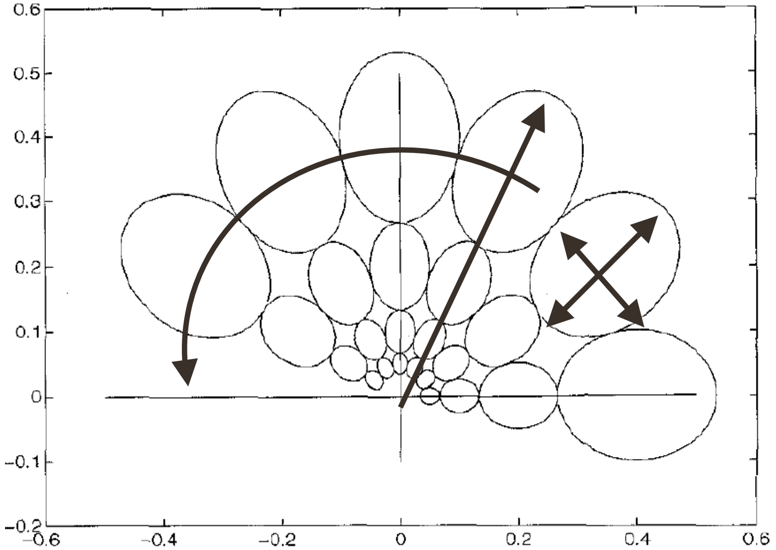
Four main factors to be considered

- The number of free parameters should be small.
- The whole spectrum should be covered
- The overlap between neighboring channels should be minimized.
- The characteristics of visual perception should be considered.

There are some parameters to determine a Gabor filter bank.

- Scales and orientations
- Scaling factor between successive filters.
- The std of the Gaussian in each scale and orientation

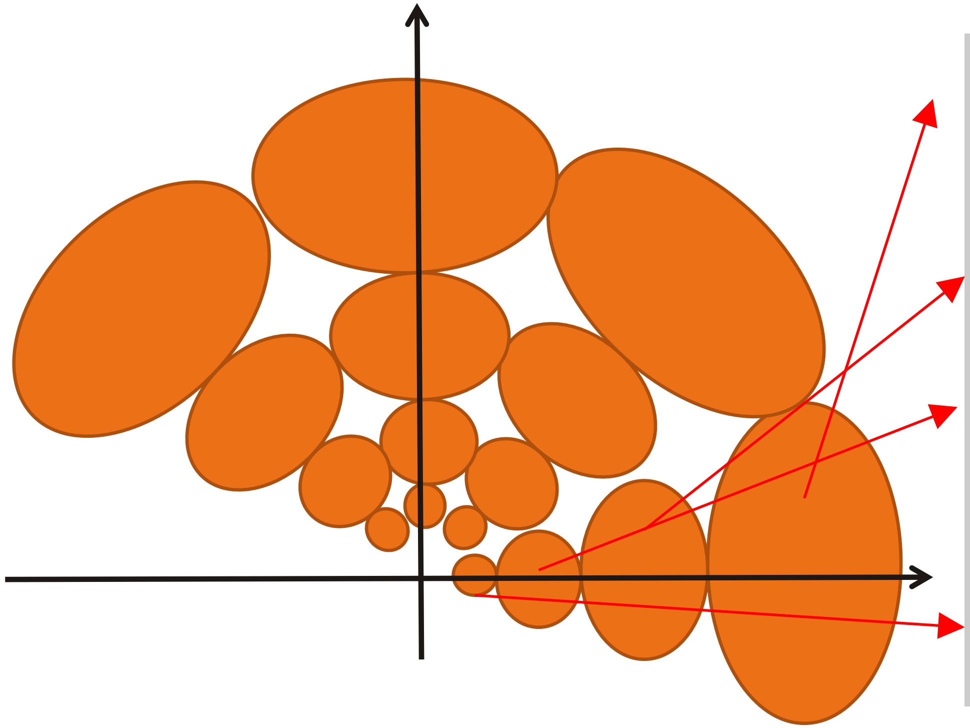
Why not like this?



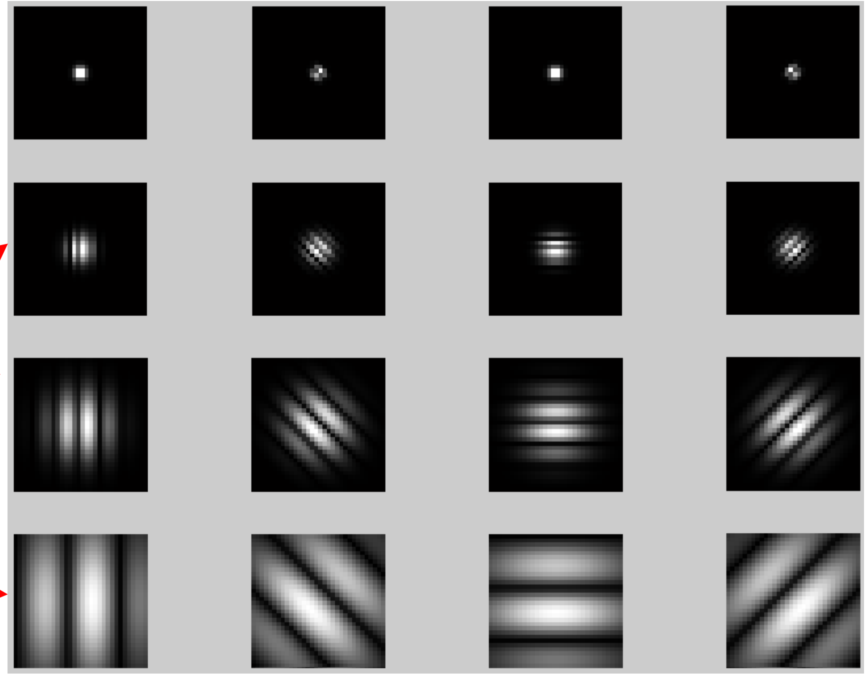
Frequency representation of the Gabor filter design

S. Manjunath and W.Y. Ma, "Texture features for browsing and retrieval of image data", IEEE Trans. on Pattern Analysis and Machine Intelligence (PAMI), vol.18, no.8, pp.837-42, Aug 1996.

Gabor Filter Kernels (Magnitude): 4 Scales and 4 Orientations

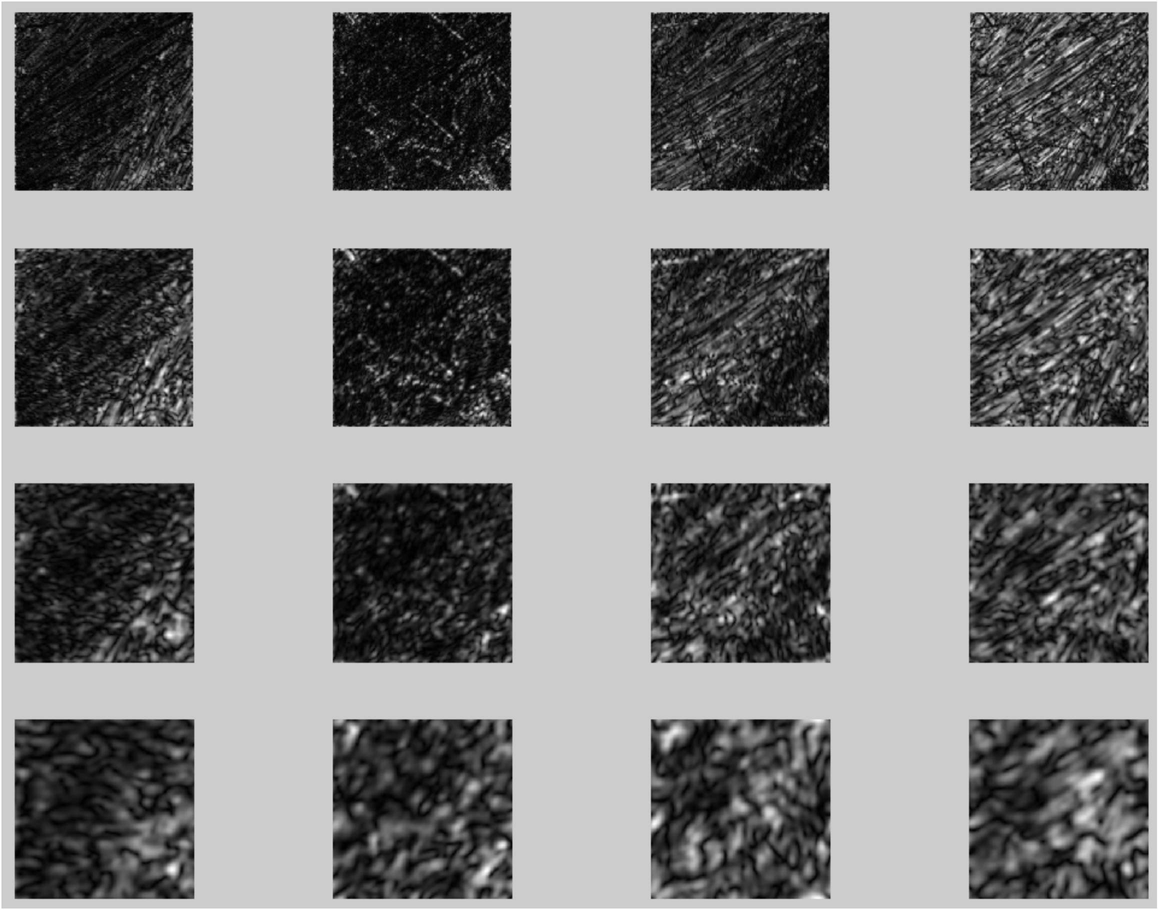


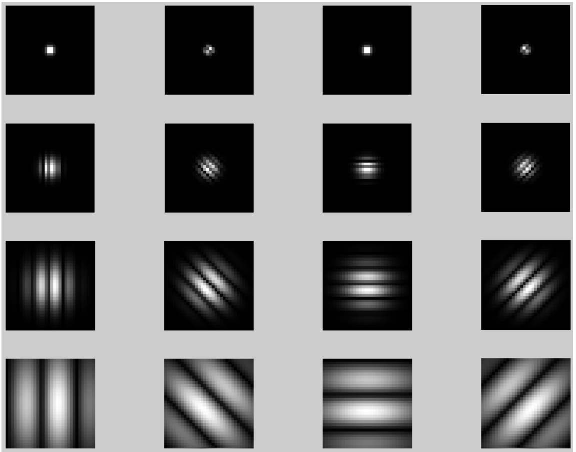
Frequency representation
of the Gabor filter design



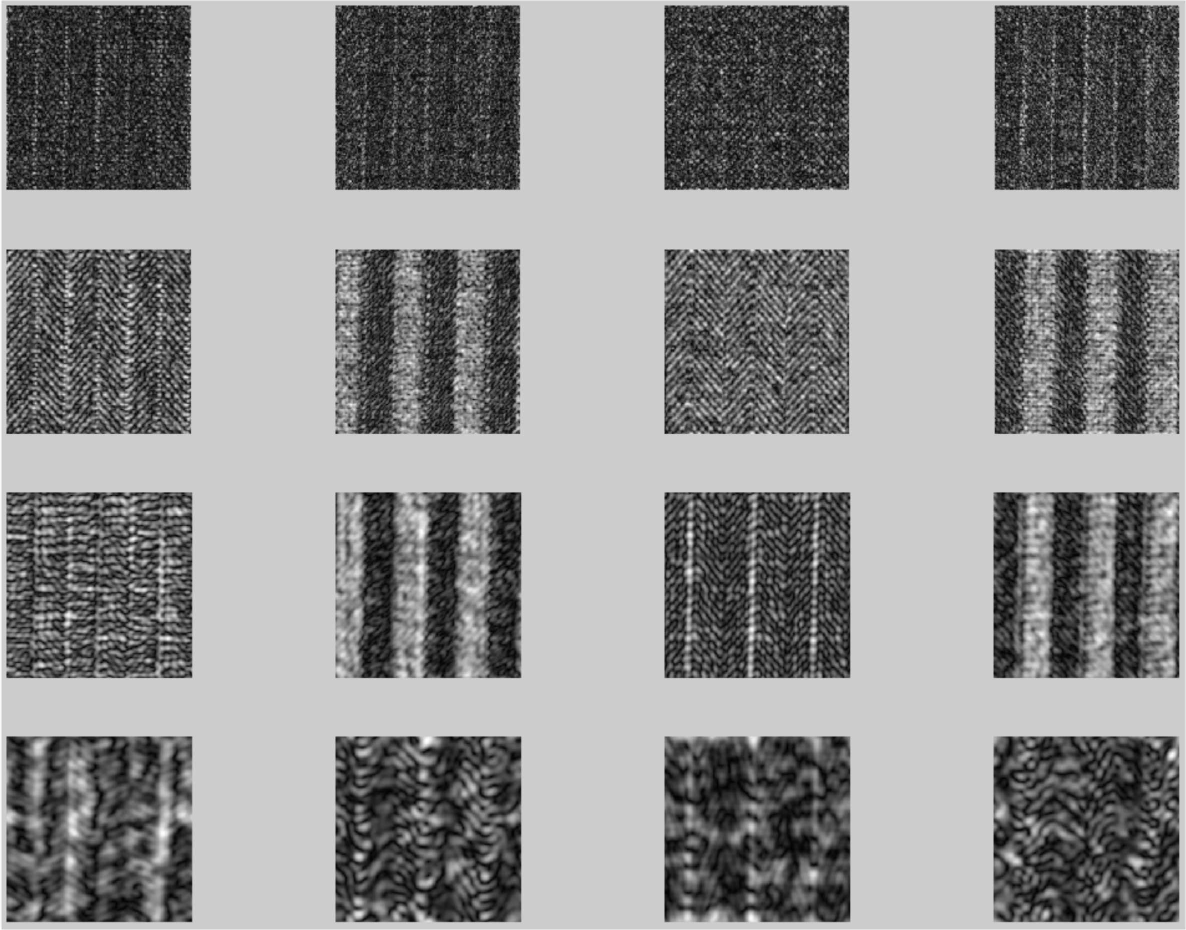
Spatial representation of
of Gabor filter kernels

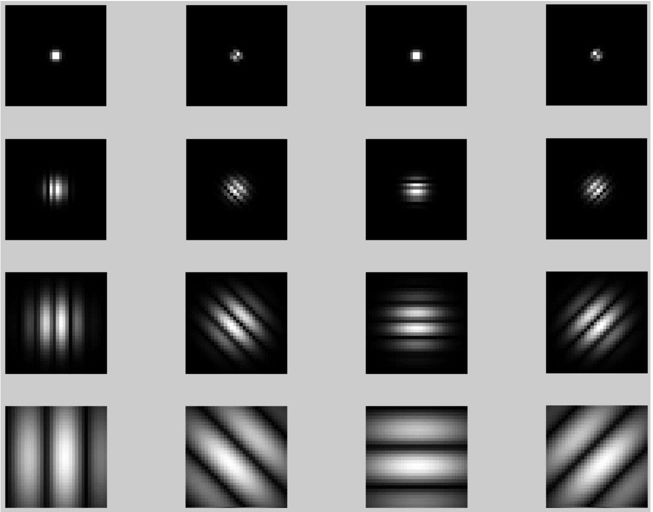
Gabor Filtering of Brodaz Texture D15

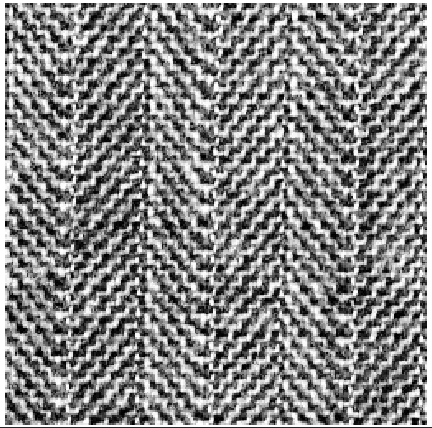




Gabor Filtering of Brodaz Texture D16







Gabor Filtering of Brodaz Texture D84