MVC Notes

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1 Vectors

$$u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = i \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - j \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} + k \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

$$= i(u_2v_3 - u_3v_2) - j(u_1v_3 - u_3v_1) + k(u_1v_2 - u_2v_1)$$

$$\text{vector projection: } proj_v u = \frac{u \cdot v}{|v|}\hat{v}$$

$$\text{scalar projection: } |proj_v u| = \frac{u \cdot v}{|v|}$$

2 3D graphs

The distance d between a point Q and a line $r = r_0 + tv$

$$d = \frac{|v \times \vec{PQ}|}{|v|}$$

where P is any point on the line and v is a vector parallel to the line. This makes sense since $d = |\vec{PQ}| \sin\theta$

For a plane $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ the normal vector $n = (a, b, c)$

2.1 Quadric Surfaces

$$\text{Ellipsoid: } \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\text{Elliptic paraboloid: } z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$\text{Hyperboloid of one sheet: } \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\text{Hyperboloid of two sheets: } -\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\text{Elliptic cone } \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$

$$\text{Hyperbolic paraboloid: } z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

3 Vector Valued Functions

$$\text{unit tangent vector: } \vec{T} = \frac{\vec{v}}{|\vec{v}|}$$

$$\text{principle unit vector: } \vec{N} = \frac{d\vec{T}/dt}{|d\vec{T}/dt|}$$

The principle unit vector is always perpendicular to the unit tangent vector, which makes sense since it points in the rate of change of the unit tangent vector, which can only change in direction, not length

$$\text{curavture: } \kappa = \left|\frac{d\vec{T}}{ds}\right| = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right| = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3}$$

$$\text{components of acceleration: } \vec{a} = a_N\vec{N} + a_T\vec{T}$$

$$a_n = \kappa|\vec{v}|^2 = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|}$$

$$a_T = \frac{d^2s}{dt^2} = \frac{\vec{v} \cdot \vec{a}}{|\vec{v}|}$$

$$\text{unit binormal vector: } \vec{B} = \vec{T} \times \vec{N} = \frac{\vec{v} \times \vec{a}}{|\vec{v} \times \vec{a}|}$$

$$\text{torsion: } \tau = -\frac{d\vec{B}}{ds} \cdot \vec{N} = \frac{(\vec{v} \times \vec{a}) \cdot \vec{a}'}{|\vec{v} \times \vec{a}|^2} = \frac{(\vec{r}' \times \vec{r}'') \cdot \vec{r}'''}{|\vec{r}' \times \vec{r}''|^2}$$

4 Derivatives

Chain Rule for one independent variable:

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

Chain Rule for two independent variables:

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

Implicit Differentiation:

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$