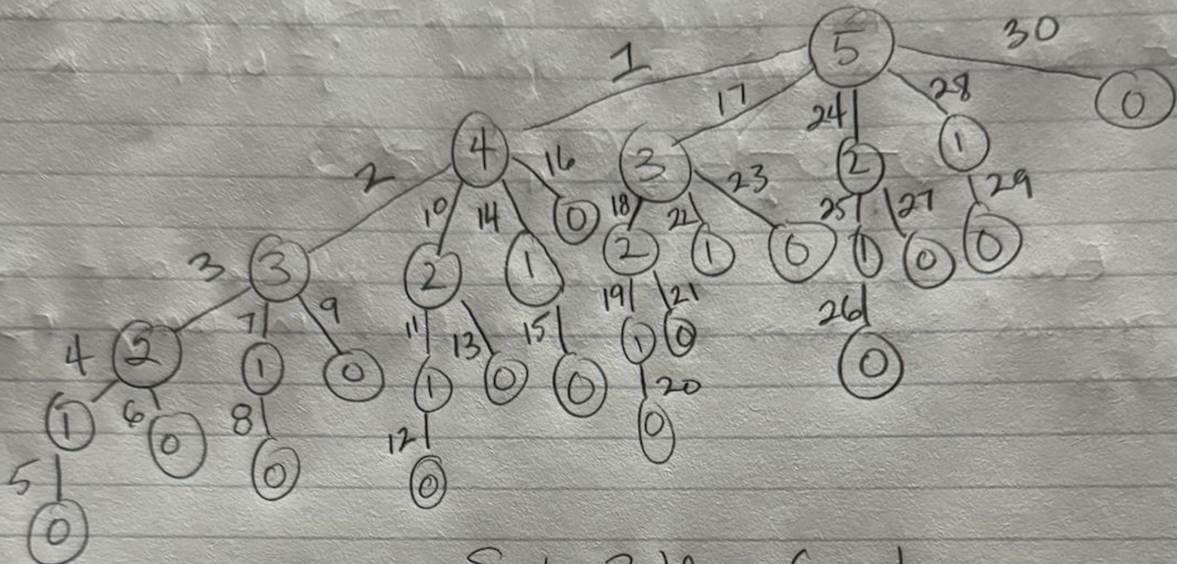
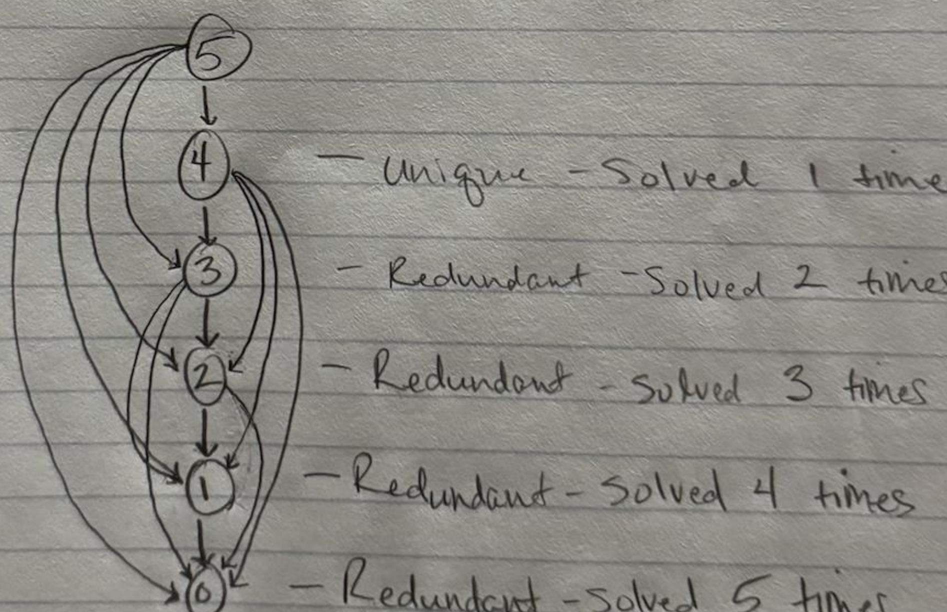
① n=5 CUT-Rod(p,n)



Sub-Problem Graph



(1) $n=5$ Bottom-UP-CUT-Rod (p, n)

No recursion graph - Solves each Subproblem of size j in increasing order $j = 0, 1, \dots n$

$$0 \xrightarrow{1} 1 \xrightarrow{2} 2 \xrightarrow{3} 3 \xrightarrow{4} 4 \xrightarrow{5} 5$$

Every Solved Subproblem is unique as it stores previous solutions to Subproblems in array $r[n]$

(2.) d = cost of each cut
No cuts = no cost

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Extended-Bottom-up-cut-Rod(p, n)
  r[0] = 0
  for j = 1 to n:
      q = -∞
    for i = 1 to j:
        if q < p[i] + r[j-i]
            if i == j:        ——> if no cuts made do not add cost
              q = p[i] + r[j-i]
            else:
              q = p[i] + r[j-i] - d     - Subtract cost of cut
          S[j] = i
    r[j] = q
return r and S
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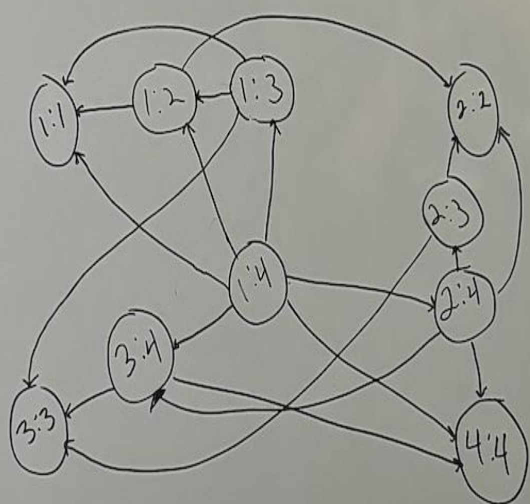
3.

$\langle A_1, A_2, A_3, A_4 \rangle$ $G=(V,E)$ $V = m_{ij}$ where $i \leq j$

$E = (m_{ii}, m_{ik})$ \$ $(m_{ij}, m_{k+1,j})$

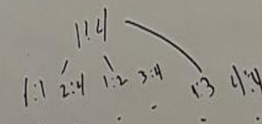
$\frac{4(4+1)}{2} = \frac{4(5)}{2} = 10$ vertices



- 1:1 — no edges
- 1:2
- 1:3
- 1:4
- 2:2 — No edges
- 2:3
- 2:4
- 3:3 — No edges
- 3:4
- 4:4 — No edges

1:4 = 6 combos

$\frac{4!}{2!(4-2)!} = 6$



d = dimensions
(P in slides)

d_0 d_1 d_2 d_3 d_4

#4.) 4 matrices $\langle 3, 2, 4, 1, 2 \rangle$

$$m[i,j] = \begin{cases} 0 & \text{if } i = j \\ \min\limits_{i \le k < j} \left(m[i,k] + m[k+1,j] + d_{i-1} \times d_k \times d_j \right) \end{cases}$$

j →

m	1	2	3	4
1	0	24	14	20
2		0	8	12
3			0	8
4				0

i ↓

Because if $i = j$ we get 0 we can place 0 along diagonal where $i = j$ i.e $m[1,1] = 0$

s	1	2	3	4
1		1	1	3
2			2	3
3				3
4				

→ Put first paren at A_3 for 1:4 then 1:3 should be at A1

b.) Optimal value for $m[1,4] = 20$

$((A_1)(A_2 \; A_3)) \; A_4$

$$\langle 3, 2, 4, 1, 2 \rangle$$
$$d_0 \quad d_1 \quad d_2 \quad d_3 \quad d_4$$

$$m[1,4] = \min_{1 \le k < 4} \begin{cases} k=1 & m[1,1] + m[2,4] + d_0 \times d_1 \times d_4 \\ k=2 & m[1,2] + m[3,4] + d_0 \times d_2 \times d_4 \\ k=3 & m[1,3] + m[4,4] + d_0 \times d_3 \times d_4 \end{cases}$$

$$m[1,2] = \min_{1 \le k < 2} \quad k=1 \begin{cases} m[1,1] + m[2,2] + d_0 \times d_1 \times d_2 \\ 0 + 0 + 3 \times 2 \times 4 = 24 \end{cases}$$

$$m[2,3] = \min_{2 \le k < 3} \quad k=2 \begin{cases} m[2,2] + m[3,3] + d_1 \times d_2 \times d_3 \\ 0 + 0 + 2 \times 4 \times 1 = 8 \end{cases}$$

$$m[3,4] = \min_{3 \le k < 4} \quad k=3 \begin{cases} m[3,3] + m[4,4] + d_2 \times d_3 \times d_4 \\ 0 + 0 + 4 \times 1 \times 2 = 8 \end{cases}$$

$$m[1,3] = \min_{1 \le k < 3} \begin{cases} k=1 & m[1,1] + m[2,3] + d_0 \times d_1 \times d_3 \\ & 0 + 8 + 3 \times 2 \times 1 = 8 + 6 = 14 \\ k=2 & m[1,2] + m[3,3] + d_0 \times d_2 \times d_3 \\ & 24 \text{ - Already too large} \end{cases}$$

$$m[2,4] = \min_{2 \le k < 4} \begin{cases} k=2 & m[2,2] + m[3,4] + d_1 \times d_2 \times d_4 \\ & 0 + 8 + 2 \times 4 \times 2 = 24 \\ k=3 & m[2,3] + m[4,4] + d_1 \times d_3 \times d_4 \\ & 8 + 0 + 2 \times 1 \times 2 = 8 + 4 = 12 \end{cases}$$

$$m[1,4] = \min_{1 \le k < 4} \begin{cases} k=1 & m[1,1] + m[2,4] + d_0 \times d_1 \times d_4 \\ & 0 + 12 + 3 \times 2 \times 2 = 12 + 12 = 24 \\ k=2 & m[1,2] + m[3,4] + d_0 \times d_2 \times d_4 \\ & 24 + 8 \\ k=3 & m[1,3] + m[4,4] + d_0 \times d_3 \times d_4 \\ & 14 + 0 + 3 \times 1 \times 2 \\ & 14 + 6 = 20 \end{cases}$$