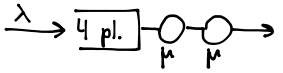
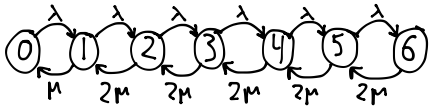
Tenta 27/5 2013

1. 

a. 

b. $\lambda p_0 = \mu p_1 \Rightarrow p_1 = \frac{\lambda}{\mu} p_0 = \rho p_0$

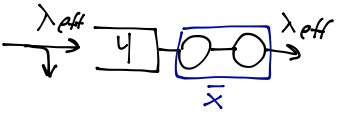
$\lambda p_1 = 2\mu p_2 \Rightarrow p_2 = \frac{\lambda}{2\mu} p_1 = \frac{\rho}{2} p_1 = \frac{\rho^2}{2} p_0$

$p_3 = \frac{\lambda}{2\mu} p_2 = \frac{\rho}{2} p_2 = \frac{\rho^3}{4} p_0$

$\vdots$

$p_6 = \frac{\rho^6}{32} p_0$

$\sum p_k = 1 \Rightarrow p_0 \left(1 - \rho + \frac{\rho^2}{2} + \frac{\rho^3}{4} + \frac{\rho^4}{8} + \frac{\rho^5}{16} + \frac{\rho^6}{32}\right) = 1$

c. 

$E(\text{upptagen betjänare}) = \lambda_{eff} \cdot \bar{x} = \lambda_{eff} \cdot \frac{1}{\mu}$

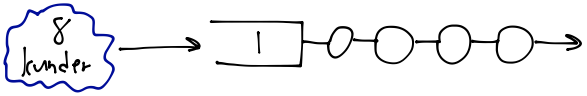
$= 0 \cdot P(0 \text{ upptagna}) + 1 \cdot P(1 \text{ upptagen}) + 2 \cdot P(2 \text{ upptagna})$

$= 1p_1 + 2(p_2 + p_3 + p_4 + p_5 + p_6)$

$\lambda_{eff} = \mu(p_1 + 2(p_2 + p_3 + p_4 + p_5 + p_6))$

Alternativt: $\lambda_{eff} = \lambda(1 - p_6)$

d. $N = \lambda_{eff} \cdot T \Rightarrow T = \frac{N}{\lambda_{eff}}$ $\quad W = T - \bar{x} = \frac{N}{\lambda_{eff}} - \frac{1}{\mu} = \frac{\sum_{k=1}^{6} k p_k}{\lambda_{eff}} - \frac{1}{\mu}$

2. 

$\beta = 1\ s^{-1}$ (intensitet per kund)

$\bar{x} = 0{,}1\ s \Rightarrow \mu = 10\ s^{-1}$

a. 

Snittmetoden

$8p_0 = 10p_1 \Rightarrow p_1 = \frac{8}{10}p_0$

$7p_1 = 20p_2 \Rightarrow p_2 = \frac{7}{20}p_1 = \frac{7}{20}\frac{8}{10}p_0$

$6p_3 = 30p_2 \Rightarrow p_3 = \frac{6}{30}p_2 = \frac{6}{30}\frac{7}{20}\frac{8}{10}p_0$

$\vdots$

Svar: p_1

b. Anropsspärr: $\frac{\lambda_5 p_5}{\sum_{i=0}^{5} \lambda_i p_i}$

Antal spärrade per minut: $60\lambda_5 p_5$

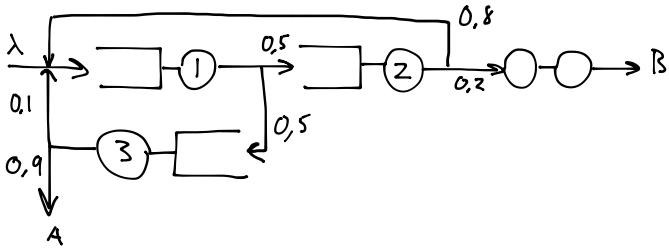
c. $N = \lambda_{eff} \cdot T \Rightarrow T = \frac{N}{\lambda_{eff}} = \frac{\sum_{i=1}^{5} i p_i}{\sum_{k=0}^{4} \lambda_k p_k}$

$W = T - \bar{x} = T - \frac{1}{\mu}$

d. Def: $\sum_{k=0}^{4} k P(k \text{ upptagna}) = 1p_1 + 2p_2 + 3p_3 + 4(p_4 + p_5)$

$N_S = \lambda_{eff} \cdot \frac{1}{\mu}$

3a.



$$\begin{cases} \lambda_1 = \lambda + 0,8\lambda_2 + 0,1\lambda_3 \\ \lambda_2 = 0,5\lambda_1 \\ \lambda_3 = 0,5\lambda_1 \\ \lambda_4 = 0,2\lambda_2 \end{cases} \Rightarrow \quad \lambda_1 = 2 + 0,4\lambda_1 + 0,05\lambda_1$$

$$\Rightarrow \lambda_1(1 - 0,4 - 0,05) = 2$$

$$\Rightarrow \lambda_1 = \frac{2}{1 - 0,4 - 0,05} = \frac{2}{0,55}$$

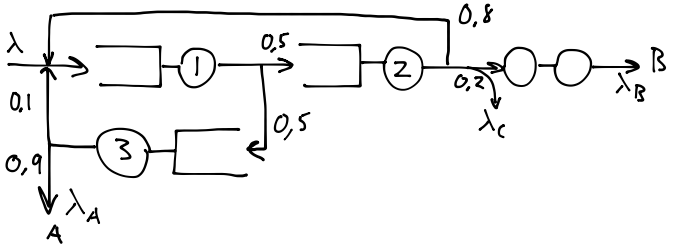
$$\Rightarrow \begin{cases} \lambda_1 = \frac{2}{0,55} \\ \lambda_2 = \frac{1}{0,55} \\ \lambda_3 = \frac{1}{0,55} \\ \lambda_4 = \frac{0,2}{0,55} \end{cases} \Rightarrow \begin{cases} \rho_1 = \frac{\lambda_1}{\mu_1} = \frac{2}{6 \cdot 0,55} \\ \rho_2 = \frac{1}{4 \cdot 0,55} \\ \rho_3 = \frac{1}{4 \cdot 0,55} \\ \rho_4 = \frac{0,2}{5 \cdot 0,55} \end{cases}$$

$$N_4 = \rho_4 (1 - E_2(\rho_4))$$

Härledning:

$$N = \lambda_{eff} \frac{1}{\mu} = \lambda \cdot \underbrace{(1 - E_2(\rho))}_{P(\text{inte fullt})} \frac{1}{\mu} = \rho(1 - E_2(\rho))$$

3b.



$\lambda = \lambda_A + \lambda_B + \lambda_C$

$\lambda_A = \lambda_3 \cdot 0,9$

$\lambda_B = \lambda_4 (1 - E_2(\rho_4))$

$\lambda_C = \lambda_4 E_2(\rho_4))$

$\frac{\lambda_B + \lambda_A}{\lambda}$

c. $N_{q1} = N_1 - \rho_1$ (medelantal i betjänaren)

$N_q = \sum N_{qi}$

$N_{q2} = N_2 - \rho_2$

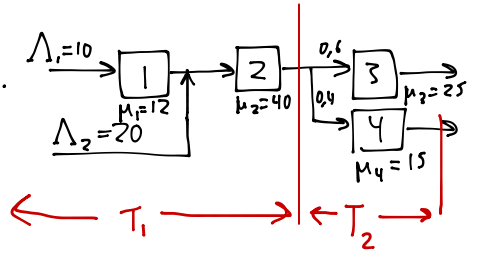
$N_{q3} = N_3 - \rho_3$

Svar: $\frac{N_q}{\lambda}$?

d. $\lambda_1 = \frac{2}{0,55}$, $\lambda = 2$

$\frac{\lambda_1}{\lambda} = \frac{1}{0,55} \approx 1,8 \Rightarrow$ Varje kund besöker nod 1 i snitt 1,8 gånger

4.



a. $\begin{cases} \lambda_1 = 10 \\ \lambda_2 = 30 \\ \lambda_3 = 18 \\ \lambda_4 = 12 \end{cases} \Rightarrow \rho_1 = \frac{\lambda_1}{\mu_1} = \frac{10}{12} \Rightarrow N_1 = \frac{\rho_1}{1-\rho_1} = \frac{10/12}{1-10/12} = \frac{10}{12-10} = 5$

$N_2 = \dots$

$\vdots$

N_4

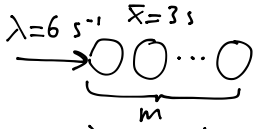
b. $T_1 = \frac{N_1+N_2}{\Lambda_1+\Lambda_2}$ Svar: T_1+T_2

$T_2 = \frac{N_4}{\lambda_4}$

c. Ej överbelastning $\Rightarrow \Lambda_1+\Lambda_2 = 30\ s^{-1}$

d. $\frac{N_1}{\lambda_1} + \frac{N_2}{\lambda_2} + \frac{N_3+N_4}{\lambda_2}$

$\frac{N_1}{\lambda_1}$: tid i 1 — $\frac{N_2}{\lambda_2}$: tid i 2

5. 

a. $\rho = \frac{\lambda}{\mu} = 6 \cdot \frac{1}{1/3} = 18$

$E_m(18) < 0{,}01 \Rightarrow$ m är minst 28 (enligt tabell)

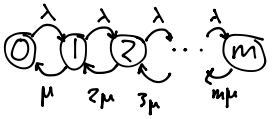
b. Erbjuden trafik: $\rho = 18$

Avverkad trafik = E(upptagna betjänare):

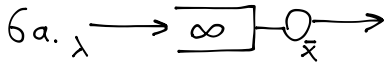
$\lambda_{eff} \cdot \bar{x} = \lambda(1 - E_m(\rho))\bar{x} = \rho(1 - E_m(\rho))$, $\bar{x} = \frac{1}{\mu}$

Spärrad trafik = erbjuden - avverkad

c. Avverkad trafik, se b.

d. 

Snittmetoden, beräkna p_k.

6a. 

$E(\text{antal kunder}) = \lambda \bar{x} = \rho$

b. $N_q = \frac{\lambda^2 E(x^2)}{2(1-\rho)}$ $\qquad W = \frac{N_q}{\lambda} = \frac{\lambda E(x^2)}{2(1-\rho)} = \frac{5E(x^2)}{2(1-0,5)} = 5E(x^2)$

$W < 0,5 \Rightarrow 5E(x^2) < 0,5 \Rightarrow E(x^2) < 0,1$

$V(X) = E(X^2) - E^2(X) = E(X^2) - 0,1^2$

$V(X) = 0,1 - 0,1^2 = 0,09$

c. $V(X) = 5 \Rightarrow E(X^2) = V(X) + E^2(X) = 5 + 0,1^2 = 5,01$

$W + x = 5 \cdot E(X^2) + 0,1 = 5 \cdot 5,01 + 0,1 = 25,15$

d. Konstanta

Lösning av Christian Nyberg.

Antecknat på föreläsning den 20 maj 2014
av Anton Eliasson, D11.

Lycka till med tentaplugget!