

Math Pset #2: Inner Product Spaces

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Problem 1.

WTS: $\langle x, y \rangle = \frac{1}{4}(\|x + y\|^2 - \|x - y\|^2)$

PF:

$$\begin{aligned} &= \frac{1}{4}(\langle x + y, x + y \rangle - \langle x - y, x - y \rangle) \\ &= \frac{1}{4}(\langle x, x + y \rangle + \langle y, x + y \rangle - \langle x, x - y \rangle + \langle y, x - y \rangle) \\ &= \frac{1}{4}(\langle x, x + y + y - x \rangle + \langle y, x + y + x - y \rangle) \\ &= \frac{1}{4}(\langle x, 2y \rangle + \langle y, 2x \rangle) \\ &= \frac{1}{4}(2\langle x, y \rangle + 2\langle y, x \rangle) \\ &= \frac{1}{4}(2\langle x, y \rangle + 2\overline{\langle x, y \rangle}) \end{aligned}$$

As we are in \mathbb{R} ,

$$= \langle x, y \rangle$$