Problem Set #1

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Problem 1

Consider the problem of the owner of an oil field. The owner has B barrels of oil. She can sell these barrels at price pt at time t. Her objective is to maximize the discounted present value of sales of oil - well assume there are no extraction costs. The owner discounts the future at a rate given by 1/1+r (where r is the real interest rate and assumed to be constant). Answer the following:

1. What are the state variables?
2. What are the control variables?
3. What does the transition equation look like?
4. Write down the sequence problem of the owner. Write down the Bellman equation.
5. What does the owners Euler equation like?
6. What would the solution of the problem look like if pt+1 = pt for all t? What would the solution look like if pt+1 ¿ (1 + r)pt for all t? What is the condition on the path of prices necessary for an interior solution (where the owner will extract some, but not all, of the oil)?

Solution 1.

1. The state variable is B. The prices r,p_t are given exogenously.
2. Control variables are B' (oil to save) and b (oil to sell today)
3. Transition equation is B' = B - b
4. Sequence problem is:
$$\max \sum_{i=0}^{\infty} \frac{b_t p_t}{(1+r)^t}$$
   Bellman equation is:
$$V(B) = p_t b_t + \quad \frac{1}{1+r} V(B')$$
5. Euler equation is:
$$p_t = \frac{1}{1+r} p_{t+1}$$
6.
   - If $p_{t+1} = p_t = p \quad \forall t$ then the owner will sell all oil today.
   - If $p_{t+1} > (1+r)p_t \quad \forall t$ then she always saves all B til tomorrow.
   - $p_t = \frac{1}{1+r} p_{t+1}$ is the necessary condition for the owner to be indifferent between extracting today and extracting tomorrow.

Problem 2
The Neoclassical Growth Model is a workhorse model in macroeconomics. The problem for the social planner is to maximize the discounted expected utility for agents in the economy: maxE tu(c) (1) c0 t t=0 t=0 1 The resource constraint is given as: yt = ct + it (2) The law of motion for the capital stock is: kt+1 =(1)kt +it (3) Output is determined by the aggregate production function: yt = ztkt (4) Assume that zt is stochastic. In particular, it is an i.i.d. process distributed as ln(z) N(0,z).

Solution 2.

1. State variable is k_t
2. Control variables are c_t, i_t
3. $V(k_t) = u(c_t) + \beta E_t[V(k_{t+1})]$
   s.t.

$$y_t = c_t + i_t \tag{1}$$
$$k_{t+1} = (1-\delta)k_t + i_t \tag{2}$$
$$y_t = z_t k_t^\alpha \tag{3}$$