Dynamic Programming Exercises

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OSM Lab 2018
Due Monday, June 25, 8:00 a.m.

Exercise 1.

Consider the problem of the owner of an oil field. The owner has B barrels of oil. She can sell these barrels at price p_t at time t. Her objective is to maximize the discounted present value of sales of oil - we'll assume there are no extraction costs. The owner discounts the future at a rate given by $\frac{1}{1+r}$ (where r is the real interest rate and assumed to be constant).

Answer the following:

1. *What are the state variables?*
2. *What are the control variables?*
3. *What does the transition equation look like?*
4. *Write down the sequence problem of the owner. Write down the Bellman equation.*
5. *What does the owner's Euler equation like?*
6. *What would the solution of the problem look like if $p_{t+1} = p_t$ for all t? What would the solution look like if $p_{t+1} > (1+r)p_t$ for all t? What is the condition on the path of prices necessary for an interior solution (where the owner will extract some, but not all, of the oil)?*

Tips:

1. *No need to use a computer here - this question wants you to apply your theory of dynamic programming.*
2. *Pay attention to binding constraints.*

Exercise 2. *The Neoclassical Growth Model is a workhorse model in macroeconomics. The problem for the social planner is to maximize the discounted expected utility for agents in the economy:*

$$\max_{\{c\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

The resource constraint is given as:

$$y_t = c_t + i_t \tag{2}$$

The law of motion for the capital stock is:

$$k_{t+1} = (1-\delta)k_t + i_t \tag{3}$$

Output is determined by the aggregate production function:

$$y_t = z_t k_t^\alpha \tag{4}$$

Assume that z_t is stochastic. In particular, it is an i.i.d. process distributed as $ln(z) \sim N(0, \sigma_z)$.

1. *What is (are) the state variable(s)?*
2. *What are the control variables?*
3. *Write down the Bellman Equation that represents this sequence problem.*
4. *Solve the growth model given the following parameterization (you may use VFI or PFI):*

Table 1: Parameterization

Parameter	Description	Value
$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$	CRRA utility	
γ	Coefficient of Relative Risk Aversion	0.5
β	Discount factor	0.96
δ	Rate of physical depreciation	0.05
α	Curvature of production function	0.4
σ_z	Standard deviation of productivity shocks	0.2

- *Plot the value function*
- *Plot the policy function for the choice of consumption*
- *Plot the policy function for the choice of capital next period*

Tips:

1. *The fact that the shocks are i.i.d. makes the computation simpler. Consider integrating over a Monte Carlo simulation of the shocks to find the expected values needed.*
2. *Write functions for the utility function and the production function - and depending on your solution method, variants on these as well such as the marginal utility function.*
3. *Be careful when it's possible that infeasible values of c_t of k_t may be chosen in your solution method.*

Exercise 3. *Use the same neoclassical growth model as above, but consider the case where there is serial correlation in the productivity shock. In particular, assume that z_t is given by:*

$$ln(z_t) = \rho ln(z_{t-1}) + \upsilon_t \tag{5}$$

where $\upsilon_t \sim N(0, \sigma_\upsilon)$. Let $\rho = 0.8$ and $\upsilon = 0.1$

1. *Write down the Bellman Equation that represents the planner's problem in this case.*

2. *Approximate the AR(1) process with a Markov chain and solve the model:*

   - *Plot the value function for at least 3 values of the productivity shock.*
   - *Plot the policy function for the choice of consumption for at least 3 values of the productivity shock.*
   - *Plot the policy function for the choice of capital next period for at least 3 values of the productivity shock.*

Tips:

1. *Use `quantecon.markov.approximate` or the `ar1_approximate.py` module to approximate the AR(1) process.*

Exercise 4. *The search and matching model of labor markets is a key model in the macro-labor literature. In one version of this model, potential workers receive wage offers from a distribution of wages each period. Potential workers must decide whether to accept and begin work at this wage (and work at this wage forever) or decline the offer and continue to "search" (i.e., receive wage offers from some exogenous distribution).*

The potential workers seek to maximize the expected, discounted sum of earnings:

$$E_0 \sum_{t=0}^{\infty} \beta^t y_t \tag{6}$$

Income, y_t, is equal to w_t if employed. If unemployed, agents receive unemployment benefits b.

Assume that wage offers are distributed as $ln(w_t) \sim N(\mu, \sigma)$.

1. *Write down the Bellman equation representing this optimal stopping problem.*

2. *Solve this model, using the following parameterization:*

Table 2: Parameterization

Parameter	Description	Value
β	Rate of time preference	0.96
b	Unemployment benefits	0.05
μ	Mean of log wages	0.0
σ	Standard deviation of wage draws	0.15

- *Plot the value function*
- *Find the "reservation wage" for the unemployed worker (i.e., the wage that makes her indifferent between accepting the job offer and not).*
- *Vary b from 0.5 to 1.0 and plot the reservation wage for each value of b. How do unemployment benefits affect the reservation wage?*