MATH PSET 3:
SPECTRAL THEORY

ALEX WEINBERG
OSM BOOT CAMP 2018

Problem 2. Find the eigenvalues of D[p](x) = p'(x)

Proof.

$$D = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

So,

$$p_A(\lambda) = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -2 \\ 0 & 0 & \lambda \end{bmatrix}$$

So,

$$det(p_D(\lambda)) = \lambda^3 = 0 \iff \lambda = 0$$

□

Problem 4. *Proof.* Eigenvalues in 2x2 matrix are roots to

$$p_A(\lambda) = \lambda^2 + \lambda(a+d) + (ad - bc) = 0$$

Quadratic formula gives us: □