Problem 6.6. Find the critical points of $f(x, y) = 3x^2y + 4xy^2 + xy$

Proof. We want $f' = 0$

$$\Delta f = (6xy + 4y^2 + y, 8xy + 3x^2 + x)$$

This becomes a system of linear equations

$$\begin{cases} +\,0 = 6xy + 4y^2 + y \\ +\,0 = 8xy + 3x^2 + x \end{cases}$$

Case 1: x=0

$$\begin{aligned} &\implies 4y^2 + y = 0 \\ &\implies y(4y+1) = 0 \\ &\implies y = 0, \frac{-1}{4} \end{aligned}$$

Case 2: y=0

$$\begin{aligned} &\implies 3x^2 + x = 0 \\ &\implies x(3x+1) = 0 \\ &\implies x = 0, \frac{-1}{3} \end{aligned}$$

Case 3: $y \neq 0, \quad x \neq 0$

$$\begin{cases} +\,0 = y(6x + 4y + 1) \\ +\,0 = x(3x + 8y + 1) \end{cases} \implies \begin{cases} +\,0 = 6x + 4y + 1 \\ +\,0 = 3x + 8y + 1 \end{cases}$$

$$\begin{aligned} &\implies -12y - 1 = 0 \\ &\implies y = \frac{-1}{12} \\ &\implies x = \frac{-1}{9} \end{aligned}$$

x	y	Type
0	0	
0	-1/4	
-1/3	0	
-1/9	-1/12	

$\square$

Problem 6.11. $f(x) = ax^2 + bx + c$. Show that one iteration of newton's method will give you a unique solution.

Proof.

$$x_1 := x_0 - \frac{f'(x_0)}{f"(x_0)}$$
$$x_1 = x_0 - \frac{2ax_0 + b}{2a}$$

□

Problem 7.1. **WTS:** if $S \subset V, s \neq \emptyset \quad \text{then} \quad conv(S)$ is convex.

Proof. **WTS:**

$$\lambda x + (1-\lambda)y \in conv(S)$$
$$\lambda a_1 x_1 + \ldots + \lambda a_k x_k + (1-\lambda)b_1 y_1 + \ldots + (1-\lambda)b_k y_k$$

As $0 \leq \lambda \leq 1$,

$$\lambda \sum a_i + (1-\lambda) \sum b_i = \lambda + (1-\lambda) = 1$$

□