Problem Set #1

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Solution 1. (1.3)

1. Suppose $s \in \mathbb{R}$ open. So $s \in \mathbb{G}_1$ by definition. Take $s^c \in \mathbb{R}$ is close by properties of open/closed sets.
   $\Rightarrow s^c \notin \mathbb{G}_1$ by definition of $\mathbb{G}_1$
   $\Rightarrow$ not closed under complements. So G1 is not a sigma-algebra.

2. **WTS:** G2 algebra

   - $\varnothing \in (a, b] \Rightarrow \varnothing \in \mathbb{G}_2$
   - Suppose $A_j = \cup_{i=1}^{N_j}((a_i, b_i] \cup (-\infty, b] \cup (a, \infty))$
     So then $\cup_j^M A_j = \cup_j^M(\cup_{i=1}^{N_j}((a_i, b_i] \cup (-\infty, b] \cup (a, \infty))) \in \mathbb{G}_2 \quad \square$

   So G2 is an algebra.

3. **WTS:** G3 sig-alg

   - $\varnothing \in (a, b] \Rightarrow \varnothing \in \mathbb{G}_3$
   - Suppose $A_j = \cup_{i=1}^{N_j}((a_i, b_i] \cup (-\infty, b] \cup (a, \infty))$
     So then $\cup_j^\infty A_j = \cup_j^\infty(\cup_{i=1}^{\infty}((a_i, b_i] \cup (-\infty, b] \cup (a, \infty))) \in \mathbb{G}_3 \quad \square$

   So, G3 is a sigma algebra

Solution 2. (1.7)

Suppose $\mathbb{A}$ is an algebra. **WTS:** $\varnothing, X \subset \mathcal{A} \subset \mathcal{P}(X)$

[1]worked with Arpan Chakrabarti and Arushi Saksena