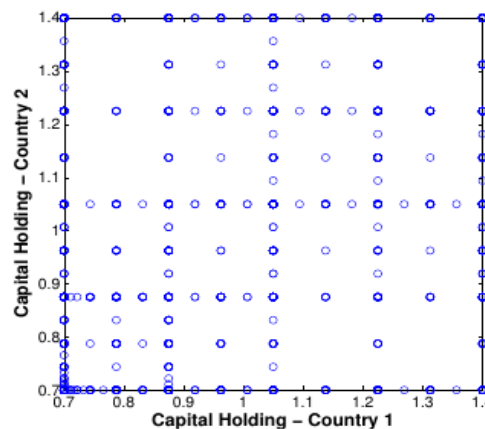


Exercises to familiarize with Sparse Grids

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**KEEP
CALM
AND
DO YOUR
HOMEWORK**

1. Analytical examples

Create sparse grids based on different analytical test functions, e.g. Genz (1984).

→ different test functions can be obtained by varying $c = (c_1, \dots, c_d)$ ($c > 0$) and $w = (w_1, \dots, w_d)$

→ difficulty of functions is monotonically increasing with c .

→ randomly generate 1,000 test points and compute error(s): $e = \max_{i=1, \dots, 1000} |f(\vec{x}_i) - u(\vec{x}_i)|$.

→ **play with adaptive/non-adaptive sparse grids/refinement level and criterion.**

→ generate convergence plots (number of points versus error – as done above).

Genz (1984) test functions

1. OSCILLATORY: $f_1(x) = \cos \left(2\pi w_1 + \sum_{i=1}^d c_i x_i \right),$
2. PRODUCT PEAK: $f_2(x) = \prod_{i=1}^d (c_i^{-2} + (x_i - w_i)^2)^{-1},$
3. CORNER PEAK: $f_3(x) = \left(1 + \sum_{i=1}^d c_i x_i \right)^{-(d+1)},$
4. GAUSSIAN: $f_4(x) = \exp \left(- \sum_{i=1}^d c_i^2 t (x_i - w_i)^2 \right),$
5. CONTINUOUS: $f_5(x) = \exp \left(- \sum_{i=1}^d c_i |x_i - w_i| \right),$
6. DISCONTINUOUS: $f_6(x) = \begin{cases} 0, & \text{if } x_1 > w_1 \text{ or } x_2 > w_2, \\ \exp \left(\sum_{i=1}^d c_i x_i \right), & \text{otherwise.} \end{cases}$

2. Growth model – Homework (I)

- I implemented you the model in Python (TASMANIAN)
 - OSM2018/day1_SparseGrid/SparseGridCode/growth_model

II) Familiarize with the code (it will show up again :))

- a) run the model with different settings
 - vary the dimensionality of the problem
 - vary the refinement level of the problem
 - compute the average and maximum errors (“contraction mapping”)
- b) Add adaptivity to the code (cf. the analytical examples)

2. Growth model – Homework (II)

→ Add stochastic production to the model

$$f(k_i, l_i, \theta_i) = \theta_i A k_i^\psi l_i^{1-\psi}$$

→ Here we assume 5 possible values of
 $\Theta_i = \{0.9, 0.95, 1.00, 1.05, 1.10\}$

→ for simplicity, we assume **$\Pi(*,*) = 1/5$**

→ solve
$$V_t(k, \theta) = \max_{c, l, I} u(c, l) + \beta \mathbb{E} \{ V_{t+1}(k^+, \theta^+) \mid \theta \}$$