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Topic: Determinant

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Questions, answers: by email [agrinikh@hse.ru]

Tasks:

1. Calculate the determinants:

a. $\begin{vmatrix} 6 & 3 \\ -2 & 2 \end{vmatrix}$ b. $\begin{vmatrix} a & ab \\ b & b^2 \end{vmatrix}$

2. Calculate the determinants:

a. $\begin{vmatrix} 1 & 3 & 5 \\ -6 & -2 & 1 \\ 2 & 2 & 1 \end{vmatrix}$ b. $\begin{vmatrix} -1 & 3 & 1 \\ 3 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$ c. $\begin{vmatrix} -6 & 3 & 4 \\ 0 & 3 & 1 \\ 2 & 1 & -3 \end{vmatrix}$ d. $\begin{vmatrix} 1 & 1 & 4 \\ -1 & -2 & -3 \\ 2 & 2 & 10 \end{vmatrix}$

e. $\begin{vmatrix} 0 & 0 & 0 \\ 0 & 3 & 1 \\ 2 & 1 & -3 \end{vmatrix}$ f. $\begin{vmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 30 \end{vmatrix}$

3. Solve the equations (find x):

a. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 2-x \end{vmatrix} = 0$ b. $\begin{vmatrix} 1 & x & x^2 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix} = 0$

4. Calculate the determinants using either row or column expansion and possibly with the help of other determinant properties:

a. $\begin{vmatrix} 1 & 0 & 0 \\ -5 & -2 & 1 \\ 7 & 3 & 32 \end{vmatrix}$ b. $\begin{vmatrix} 1 & 2 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ -3 & 3 & 6 & 9 \\ 0 & -1 & 1 & 2 \end{vmatrix}$

5. Please, find BC and CB if:

a. $B = \begin{bmatrix} 0 & 1 \\ 3 & -1 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 2 & -4 \\ 3 & -1 & 1 \end{bmatrix}$

b. $B = \begin{bmatrix} 5 & 0 \\ 3 & -2 \\ -1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 & 5 \\ 2 & -3 & 1 \end{bmatrix}$

6. For given matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 1 & 3 & 2 \end{bmatrix}$ find A^T, $\left(A^T\right)^2$ and A^2.

Good luck!