

(*

Задача 2. Решете задачата на линейното оптимиране

$$F(x, y, z) = x + y + 3z \rightarrow \max$$

Ω е областта съставена от

$$|4x - 4y + 3z - 16 \leq 0$$

$$|4x + 4y + 3z - 24 \leq 0$$

$$|2x + 4y + 3z - 20 \leq 0$$

$$|x + y + 3z - 15 \leq 0$$

$$|-x + y - z + 5 \geq 0$$

$$|x \geq 0$$

$$|y \geq 0$$

$$|z \geq 0$$

*)

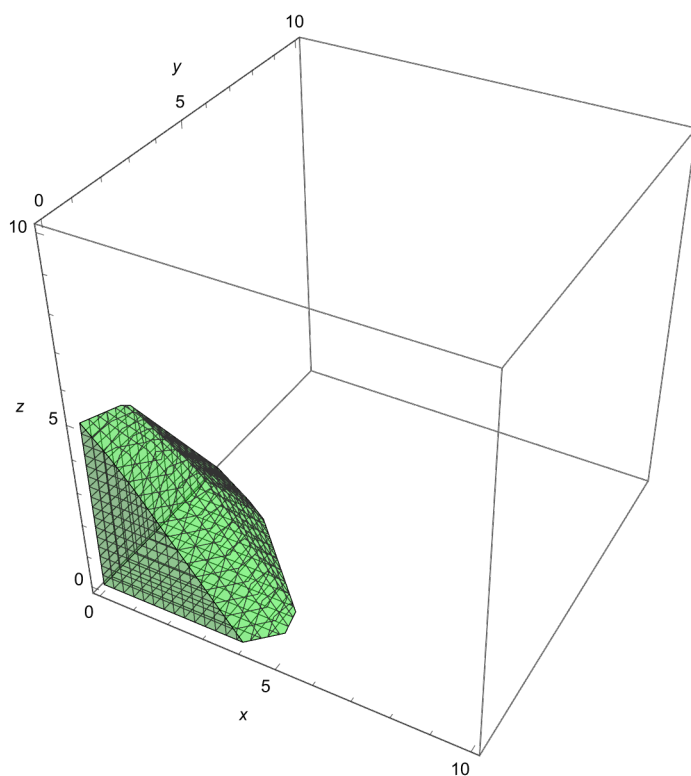
(* 1. Скицирайте областта Ω и декартовата координатна система. *)

```

In[ ]:= p = RegionPlot3D[
  4 * x - 4 * y + 3 * z - 16 ≤ 0 &&
  4 * x + 4 * y + 3 * z - 24 ≤ 0 &&
  2 * x + 4 * y + 3 * z - 20 ≤ 0 &&
  x + y + 3 * z - 15 ≤ 0 &&
  -x + y - z + 5 ≥ 0 &&
  x ≥ 0 &&
  y ≥ 0 &&
  z ≥ 0,
  {x, 0, 10},
  {y, 0, 10},
  {z, 0, 10},
  Axes → True,
  AxesLabel → {x, y, z},
  PlotStyle → {Directive[Green, Opacity[0.25]]}
]

```

Out[]:=



(* 2. Направете анимация за откриване на оптималните решения. *)

```

In[ ]:= F[x_, y_, z_] = x + y + 3 z

```

Out[]:=

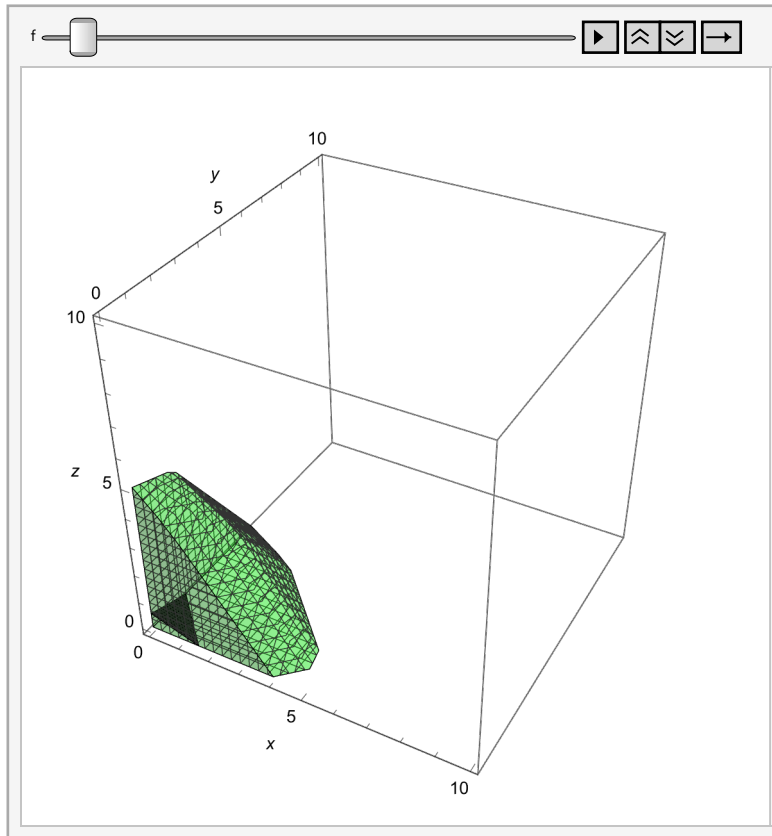
$x + y + 3 z$

```

In[ ]:= Animate[
  Show[
    RegionPlot3D[
      4 * x - 4 * y + 3 * z - 16 ≤ 0 &&
      4 * x + 4 * y + 3 * z - 24 ≤ 0 &&
      2 * x + 4 * y + 3 * z - 20 ≤ 0 &&
      x + y + 3 * z - 15 ≤ 0 &&
      -x + y - z + 5 ≥ 0 &&
      x ≥ 0 &&
      y ≥ 0 &&
      z ≥ 0,
      {x, 0, 10},
      {y, 0, 10},
      {z, 0, 10},
      Axes → True,
      AxesLabel → {x, y, z},
      PlotStyle → {Directive[Green, Opacity[0.25]]}
    ],
    ContourPlot3D[
      x + y + 3 * z == f,
      {x, 0, 10},
      {y, 0, 10},
      {z, 0, 10},
      ContourStyle → Directive[Black, Thin],
      Axes → True,
      AxesLabel → {x, y, z}
    ]
  ],
  {f, 0, 50},
  AnimationRunning → False
]

```

Out[*]=



(* 3. Намерете едно решение с функцията LinearProgramming[...]. *)

```
(*
| 4x-4y+3z≤16
| 4x+4y+3z≤24
| 2x+4y+3z≤20
| x+y+3z≤15
| x-y+z≤5
| x≥0
| y≥0
| z≥0
*)
```

```
In[*]:= F[x_, y_, z_] = x + y + 3 z
```

Out[*]=

```
x + y + 3 z
```

```
In[*]:= c = {1, 1, 3}
```

Out[*]=

```
{1, 1, 3}
```

```
In[*]:= m =  $\begin{pmatrix} 4 & -4 & 3 \\ 4 & 4 & 3 \\ 2 & 4 & 3 \\ 1 & 1 & 3 \\ 1 & -1 & 1 \end{pmatrix}$ 
```

```
Out[*]= {{4, -4, 3}, {4, 4, 3}, {2, 4, 3}, {1, 1, 3}, {1, -1, 1}}
```

```
In[*]:= b = {16, 24, 20, 15, 5}
```

```
Out[*]= {16, 24, 20, 15, 5}
```

```
In[*]:= v = LinearProgramming[c, m, b]
```

```
Out[*]= {2, 1, 4}
```

```
In[*]:= F[2, 1, 4]
```

```
Out[*]= 15
```

```
In[*]:= {2, 1, 4}
```

```
Out[*]= {2, 1, 4}
```

```
In[*]:= Maximize[{F[x, y, z], 4 * x - 4 * y + 3 * z - 16 ≤ 0 &&
  4 * x + 4 * y + 3 * z - 24 ≤ 0 &&
  2 * x + 4 * y + 3 * z - 20 ≤ 0 &&
  x + y + 3 * z - 15 ≤ 0 &&
  -x + y - z + 5 ≥ 0 &&
  x ≥ 0 &&
  y ≥ 0 &&
  z ≥ 0}, {x, y, z}]
```

```
Out[*]= {15, {x → 0, y → 0, z → 5}}
```

In[*]:= (* При известен максимум търсим всички оптимални планове с функцията Reduce *)

```
Reduce[
  x + y + 3 z == 15 &&
  4 * x - 4 * y + 3 * z - 16 ≤ 0 &&
  4 * x + 4 * y + 3 * z - 24 ≤ 0 &&
  2 * x + 4 * y + 3 * z - 20 ≤ 0 &&
  x + y + 3 * z - 15 ≤ 0 &&
  -x + y - z + 5 ≥ 0 &&
  x ≥ 0 &&
  y ≥ 0 &&
  z ≥ 0,
  {x, y, z}
]
```

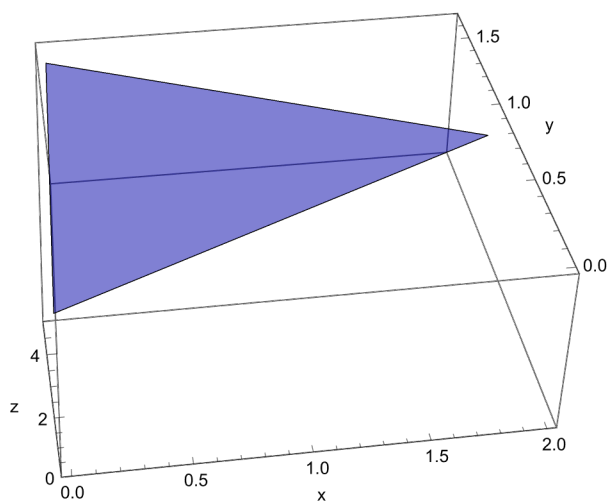
Out[*]=

$$\left(\left(0 \leq x < 2 \ \&\& \ \frac{x}{2} \leq y \leq \frac{5-x}{3} \right) \ || \ (x == 2 \ \&\& \ y == 1) \right) \ \&\& \ z == \frac{1}{3} (15 - x - y)$$

(* Получаваме всички решения *)

```
In[*]:= region = Plot3D[(1/3) (15 - x - y), {x, 0, 2}, {y, x/2, (5-x)/3},
  PlotRange -> {Automatic, Automatic, {0, 5}}, Mesh -> None, Axes -> True,
  AxesLabel -> {"x", "y", "z"}, PlotStyle -> Directive[Blue, Opacity[0.5]]]
```

Out[*]=



```
In[ ]:= Show[p, region]
```

Out[]=

