Nombre ____

DEBER No.5 Límites infinitos y límites al infinito

1. *Calcular los siguientes límites:*

$$\lim_{x\to\infty} \frac{(x+1)^2+(x+2)^2+(x+3)^2}{3x^2+8x-9}$$

$$\lim_{x\to\infty} \frac{(x+1)^2+(x+2)^2+(x+3)^2}{3x^2+8x-9}$$

$$\lim_{x\to\infty} \frac{(\infty+1)^2+(\infty+2)^2+(\infty+3)^2}{3(\infty)^2+8(\infty)-9} = \frac{\infty}{\infty} \text{ indeterminación}$$

$$\lim_{x\to\infty} \frac{(x+1)^2+(x+2)^2+(x+3)^2}{3x^2+8x-9} = \frac{\cancel{x^2}+\cancel{2x}+1+\cancel{x^2}+\cancel{4x}+4+\cancel{x^3}+\cancel{6x}+9}{3x^2+8x-9} =$$

$$\lim_{x\to\infty} = \frac{3x^2+12x+1+4+9}{3x^2+8x-9} = \frac{3\cancel{x^2}}{3\cancel{x^2}} = \frac{3}{3} = 1 \;//$$

$$\lim_{x\to\infty} \frac{(2x+1)^2+(3x+2)^2+(4x+3)^2}{(x-1)^2+(x-2)^2+(x-3)^2}$$

2)

$$\lim_{x\to\infty} \frac{(2x+1)^2+(3x+2)^2+(4x+3)^2}{(x-1)^2+(x-2)^2+(x-3)^2}$$

$$\lim_{x\to\infty} \frac{(2(\infty)+1)^2+(3(\infty)+2)^2+(4(\infty)+3)^2}{(\infty+1)^2+(\infty-2)^2+(\infty-3)^2} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{(2x+1)^2+(3x+2)^2+(4x+3)^2}{(x-1)^2+(x-2)^2+(x-3)^2} =$$

$$\lim_{x\to\infty} \frac{2x^2+2(2x)(1)+1^2+3x^2+2(3x)(2)+2^2+4x^2+2(4x)(2)+3^2}{x^2+2(x)(1)+1^2+x^2+2(x)(2)+2^2+x^2+2(x)(3)+3^2}$$

$$\lim_{x\to\infty} \frac{4x^2+4x+1+9x^2+12x+4+16x^2+24x+9}{3x^2-12x+14}$$

$$\lim_{x\to\infty} \frac{29x^2+40x+14}{3x^2-12x+14} = \frac{29\cancel{x^2}}{3\cancel{x^2}} = \frac{29}{3} \;//$$

$$\lim_{x\to\infty} \frac{(3x-1)^3-(4x+1)^2+(2x+4)^2}{(4x-3)^3+(3x-4)^2+(2x-3)^2}$$

3) $\lim\limits_{x\to\infty} \dfrac{(3x-1)^3-(4x+1)^2+(2x+4)^2}{(4x-3)^3+(3x-4)^2+(2x-3)^2}$

$\lim\limits_{x\to\infty} \dfrac{(3(\infty)-1)^3-(4(\infty)+1)^2+(2(\infty)+4)^2}{(4(\infty)-3)^3-(3(\infty)-4)^2+(2(\infty)-3)^2} = \dfrac{\infty}{\infty}$

$\lim\limits_{x\to\infty} \dfrac{27x^3-27x^2+9x-1-16x^2-8x-1+4x^2+16+16}{64x^3-144x^2+108x-27+9x^2-24x+16+4x^2-12x+9}$

$\lim\limits_{x\to\infty} \dfrac{27x^3-39x^2+17x+14}{64x^3-131x^2+72x-2} = \dfrac{27\cancel{x^3}}{64\cancel{x^3}} = \dfrac{27}{64}$ //

$$\lim_{x\to 0} \frac{\cos x}{1-\cos x}$$

4) $\lim\limits_{x\to 0} \dfrac{\cos x}{1-\cos x}$

$\lim\limits_{x\to 0} \dfrac{\cos(0)}{1-\cos(0)} = \dfrac{1}{0}$ Indeterminacion $\dfrac{K}{0}$

$\lim\limits_{x\to 0} \dfrac{\cos x}{1-\cos x} = \dfrac{\cos(x)}{1-\cos(x)} \cdot \dfrac{\cos(x)}{1-\cos(x)} = \dfrac{1}{1-\cos(x)} \cdot \dfrac{1}{\cos(x)}$

$\lim\limits_{x\to 0} \dfrac{1}{1-1} \cdot \dfrac{1}{0} = \infty$ //

$$\lim_{x\to\infty} \frac{3^{x+1}}{1-2^{2x}}$$

5) $\lim\limits_{x\to\infty} \dfrac{3^{x+1}}{1-2^{x}}$

$\lim\limits_{x\to\infty} \dfrac{3^{\infty+1}}{1-2^{\infty}} = \dfrac{\infty}{\infty}$

$\lim\limits_{x\to\infty} \dfrac{3^{x+1}}{1-2^{x}} = \dfrac{3^x \cdot 3}{1-2^x \cdot 2} = \dfrac{\left(\lim\limits_{x\to\infty} 3^x\right) / \left(\lim\limits_{x\to\infty} 3\right)}{\left(\lim\limits_{x\to\infty} 1\right) - \left(\lim\limits_{x\to\infty} 2^x\right)\left(\lim\limits_{x\to\infty} 2\right)}$

$\lim\limits_{x\to\infty} \dfrac{0}{1-0} = \dfrac{0}{1} = 0$ //

$$\lim_{x\to -\infty} \frac{x^{x+2}}{x + x^{2x}}$$

6) $\lim\limits_{x\to -\infty} \dfrac{x^{x+2}}{x+x^{2x}}$

$\lim\limits_{x\to -\infty} \dfrac{-\infty^{(-\infty)+2}}{-\infty+(-\infty)^{2(-\infty)}} = \dfrac{\infty}{\infty}$

$\lim\limits_{x\to -\infty} \dfrac{x^x \cdot x^2}{x+(x^2)^x} = \dfrac{\frac{x^x}{x^x}\cdot\frac{x^2}{x^x}}{\frac{x}{x^x}+\frac{x^{2x}}{x^x}} =$

$\lim\limits_{x\to -\infty} \dfrac{1\cdot\left(\frac{x^2}{x}\right)^{0}}{\left(\frac{x}{x}\right)+\left(\frac{x^2}{x}\right)} = \dfrac{1\cdot 0}{0+0} = \dfrac{0}{0} = -\infty$ //

$$\lim_{x\to\infty} x \operatorname{sen}\frac{1}{x}$$

7) $\lim\limits_{x\to\infty} x \operatorname{sen}\dfrac{1}{x}$

$\lim\limits_{x\to\infty} (\infty)\operatorname{sen}\dfrac{1}{\infty} = \infty \operatorname{sen} 0 = \infty\cdot 0$ Indeterminada

$u = \dfrac{1}{x}$ $\quad u \to \dfrac{1}{\infty}$ $\quad$ Cambio de variable $x = u$

$x\cdot u = 1$ $\quad u \to 0$

$x = \dfrac{1}{u}$

$\lim\limits_{x\to 0^+} \dfrac{1}{u}\operatorname{sen} u = 1 \lim\limits_{x\to 0^+} \dfrac{\operatorname{sen} u}{u} = 1(1) = 1$ //

$$\lim_{x\to\infty}\left[\frac{(5x+3)^2+3(2x-1)^2+4(x-2)^2}{5x^2+3x-7}\right]$$

8) $\lim\limits_{x\to\infty}\left[\dfrac{(5x+3)^2+3(2x-1)^2+4(x-2)^2}{5x^2+3x-7}\right]$

$\lim\limits_{x\to\infty}\left[\dfrac{(5(\infty)+3)^2+3(2(\infty)-1)^2+4(\infty-2)^2}{5(\infty)^2+3(\infty)-7}\right] = \dfrac{\infty}{\infty}$

$\lim\limits_{x\to\infty}\left[\dfrac{25x^2+30x+9+12x^2-12x+3+4x^2-16x+16}{5x^2+3x-7}\right]$

$\lim\limits_{x\to\infty} \dfrac{41x^2+2x+28}{5x^2+3x-7} = \lim\limits_{x\to\infty} \dfrac{41\cancel{x^2}}{5\cancel{x^2}} = \dfrac{41}{5}$ //

$$\lim_{x\to\infty}\left[\frac{3(2x+3)^2-4(2x+4)^2-5(2x+5)^2}{(3x-1)^2+2(3x-2)^2+3(3x-3)^2}\right]$$

9)

$$\lim_{x\to\infty}\left[\frac{3(2x+3)^2-4(2x+4)^2-5(2x+5)^2}{(3x-1)^2+2(3x-2)^2+3(3x-3)^2}\right]$$

$$\lim_{x\to\infty}\left(\frac{3(2(\infty)+3)^2-4(2(\infty)+4)^2-5(2(\infty)+5)^2}{(3(\infty-1)^2+2(3(\infty)-2)^2+3(3(\infty)-3)^2}\right)=\frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{12x^2+36x+27-16x^2-64x-64-20x^2-100x-125}{9x^2-6x+1+18x^2-24x+8+27x^2-54x+27}$$

$$\lim_{x\to\infty}\frac{-24x^2-128x-162}{54x^2-84x+36}=\frac{-24\cancel{x^2}}{54\cancel{x^2}}=\frac{12}{27}=\frac{4}{9}//$$

$$\lim_{n\to\infty}\left(\sqrt{n+1}\right)-$$

10-

$$\lim_{n\to\infty}\left(\sqrt{n+1}\right)-\sqrt{n}$$

$$\lim_{n\to\infty}\left(\sqrt{\infty+1}\right)-\sqrt{\infty}=\infty$$

$$\lim_{n\to\infty}\frac{\sqrt{n+1}-\sqrt{n}}{1}\cdot\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+1}-\sqrt{n}}$$

$$\lim_{n\to\infty}\frac{\cancel{n}+1-\cancel{n}}{\sqrt{n+1}-\sqrt{n}}=\frac{1}{\infty+\infty}=\frac{1}{\infty}=0//$$

$$\lim_{n\to\infty}\left(\frac{1}{n^2+1}\cdot\frac{2n^3+1}{n-5}\right)$$

11)

$$\lim_{n\to\infty}\left(\frac{1}{n^2+1}\cdot\frac{2n^3+1}{n-5}\right)$$

$$\lim_{n\to\infty}\left(\frac{1}{\infty^2+1}\cdot\frac{2(\infty)^3+1}{\infty-5}\right)=\frac{0}{\infty}$$

$$\lim_{n\to\infty}\left(\frac{1}{n^2+1}\cdot\frac{2n^3+1}{n-5}\right)=\frac{2n^3+1}{n^3-5n^2+n-5}$$

$$\lim_{n\to\infty}\left(\frac{n^3 x\left(2+\frac{1}{n^3}\right)}{n^3 x\left(1-\frac{5}{n}+\frac{1}{n^2}-\frac{5}{n^3}\right)}\right)$$

$$\lim_{n\to+\infty}\left(\frac{2+\frac{1}{n^3}^{0}}{1-\frac{5}{\cancel{n}}^{0}+\frac{1}{\cancel{n^2}}^{0}-\frac{5}{n^3}^{0}}\right)=\frac{2+0}{1}=\frac{2}{1}=2//$$

$$\left(\sqrt{n}\right)$$

$$\lim_{n\to\infty}\left(\quad n \right)$$

12)

$$\lim_{n\to\infty}\left(\frac{\sqrt{n}}{n}\right)$$

$$\lim_{n\to\infty}\left(\frac{\sqrt{\infty}}{\infty}\right)=\frac{\infty}{\infty}$$

$$\lim_{n\to\infty}\frac{\sqrt{n}}{n}\cdot\frac{\sqrt{n}}{\sqrt{n}}=\sqrt{n^2\times\frac{1}{n}}=\sqrt{n^2}\sqrt{\frac{1}{n}}$$

$$\lim_{n\to\infty}\left(\frac{\cancel{n}\sqrt{\frac{1}{n}}}{\cancel{n}}\right)=\sqrt{\frac{1}{n}}=\sqrt{0}=0\,//$$

$$\lim_{n\to\infty}\left(\frac{x^7-x^2+1}{2x^7+x^3+300}\right)$$

13)

$$\lim_{x\to\infty}\left(\frac{x^7-x^2+1}{2x^7+x^3+300}\right)$$

$$\lim_{x\to\infty}\left(\frac{\infty^7-\infty^2+1}{2(\infty)^7+\infty^3+300}\right)=\frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{x^7-x^2+1}{2x^7+x^3+300}=\frac{1\cancel{x^7}}{2\cancel{x^7}}=\frac{1}{2}\,//$$

Metodo de simplificación

$$\lim_{n\to\infty}\left(\frac{(n+1)^{100}}{(2x+50)^{100}}\right)$$

14)

$$\lim_{x\to\infty}\left(\frac{(n+1)^{100}}{(2x+50)^{100}}\right)$$

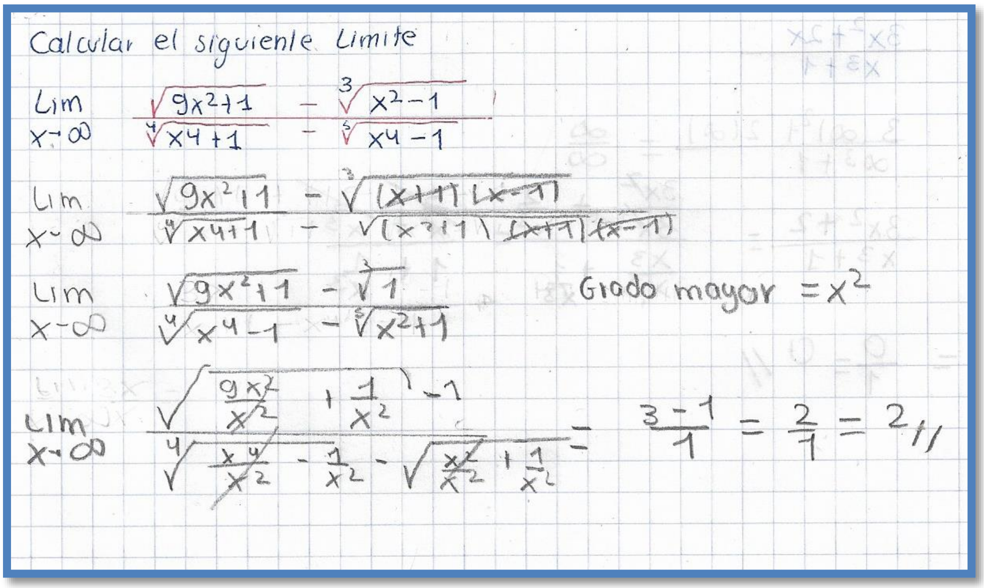
$$\frac{1}{(2x+50)^{100}}\cdot\lim_{n\to\infty}\left((n+1)^{100}\right)$$

$$\frac{1}{(2x+50)^{100}}\cdot\left(\lim_{n\to\infty}(n+1)^{100}\right)\rightarrow$$ Propiedades de Limites

$$=\frac{1}{(2x+50)^{100}}\cdot\infty^{1000}$$

$$=\infty\,//$$

2. Calcular el siguiente límite: $\lim\limits_{x\to\infty} \dfrac{\sqrt{9x^2+1} - \sqrt[3]{x^2-1}}{\sqrt[4]{x^4+1} - \sqrt[5]{x^4-1}}$



Calcular el siguiente limite

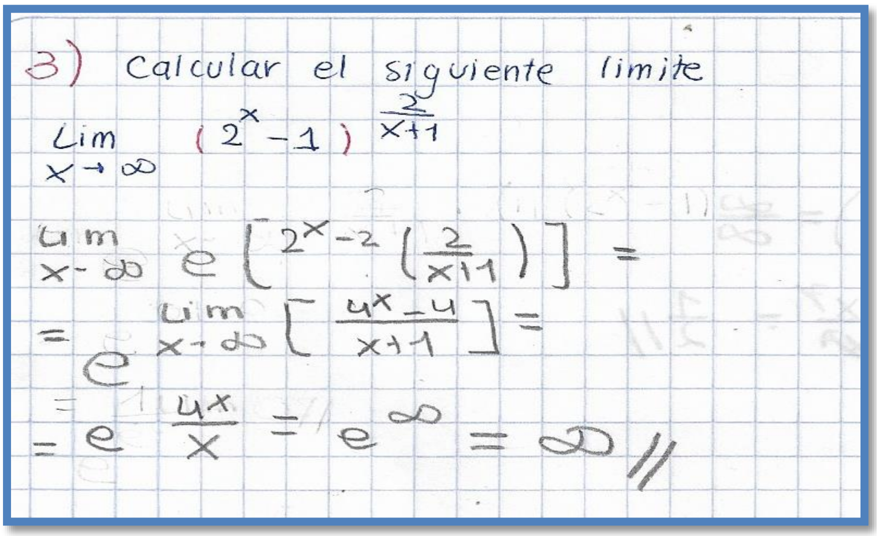
$$\lim_{x\to\infty} \frac{\sqrt{9x^2+1} - \sqrt[3]{x^2-1}}{\sqrt[4]{x^4+1} - \sqrt[5]{x^4-1}}$$

$$\lim_{x\to\infty} \frac{\sqrt{9x^2+1} - \sqrt[3]{\cancel{(x+1)}\cancel{(x-1)}}}{\sqrt[4]{x^4+1} - \sqrt{(x^2+1)\cancel{(x+1)}\cancel{(x-1)}}}$$

$$\lim_{x\to\infty} \frac{\sqrt{9x^2+1} - \sqrt[3]{1}}{\sqrt[4]{x^4-1} - \sqrt[5]{x^2+1}}$$ Grado mayor $= x^2$

$$\lim_{x\to\infty} \frac{\sqrt{\frac{9\cancel{x^2}}{\cancel{x^2}}+\frac{1}{x^2}}-1}{\sqrt[4]{\frac{x^4}{x^2}-\frac{1}{x^2}} - \sqrt{\frac{\cancel{x^2}}{\cancel{x^2}}+\frac{1}{x^2}}} = \frac{3-1}{1} = \frac{2}{1} = 2 \,//$$

3. Calcular el siguiente límite: $\lim\limits_{x\to\infty} \left(2^x-1\right)^{\frac{2}{x+1}}$



3) Calcular el siguiente limite

$$\lim_{x\to\infty} (2^x-1)^{\frac{2}{x+1}}$$

$$\lim_{x\to\infty} e\left[2^x-2\left(\frac{2}{x+1}\right)\right] =$$

$$= e^{\lim\limits_{x\to\infty}\left[\frac{4^x-4}{x+1}\right]} =$$

$$= e^{\frac{4x}{x}} = e^{\infty} = \infty \,//$$

4. Calcular los siguientes límites:

$$\lim_{x\to\infty} \frac{3x^5}{2x^5+3}$$

4) Calcular los siguientes límites

$$\lim_{x \to \infty} \frac{3x^5}{2x^5+3}$$

$$\lim_{x \to \infty} \frac{3(\infty)^5}{2(\infty)^5+3} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{3x^5}{2x^5+3} \Rightarrow \text{simplificación}$$

$$\lim_{x \to \infty} \frac{3\cancel{x^5}}{2\cancel{x^5}} = \frac{3}{2} \;//$$

$$\underset{x \to \infty}{\text{lím}} \frac{3x^2+2x}{x^3+1}$$

$$\lim_{x \to \infty} \frac{3x^2+2x}{x^3+1}$$

$$\lim_{x \to \infty} \frac{3(\infty)^2+2(\infty)}{\infty^3+1} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{3x^2+2}{x^3+1} = \frac{\frac{3x^2}{3x^3}+\frac{2}{x^3}}{\frac{x^3}{x^3}+\frac{1}{x^3}} = \frac{\frac{3}{x}+\frac{2}{x^3}}{1+\frac{1}{x^3}}$$

$$\lim_{x \to \infty} = \frac{0}{1} = 0 \;//$$

$$\underset{x \to \infty}{\text{lím}} \frac{x^4}{3x^3}$$

$$\lim_{x \to \infty} \frac{x^4}{3x^3}$$

$$\lim_{x \to \infty} \frac{\infty^4}{3(\infty)^3} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{x^4}{3x^3} = \frac{\frac{\cancel{x^4}}{\cancel{x^4}}}{\frac{3\cancel{x^3}}{\cancel{x^4}}} = \left(\frac{1}{\frac{3}{x}}\right) = \frac{3}{x} = \frac{3}{\infty} = +\infty \;//$$