# ESCUELA POLITÉCNICA NACIONAL
# ESCUELA DE FORMACIÓN DE TECNÓLOGOS

**CALCULO DIFERENCIAL E INTEGRAL**

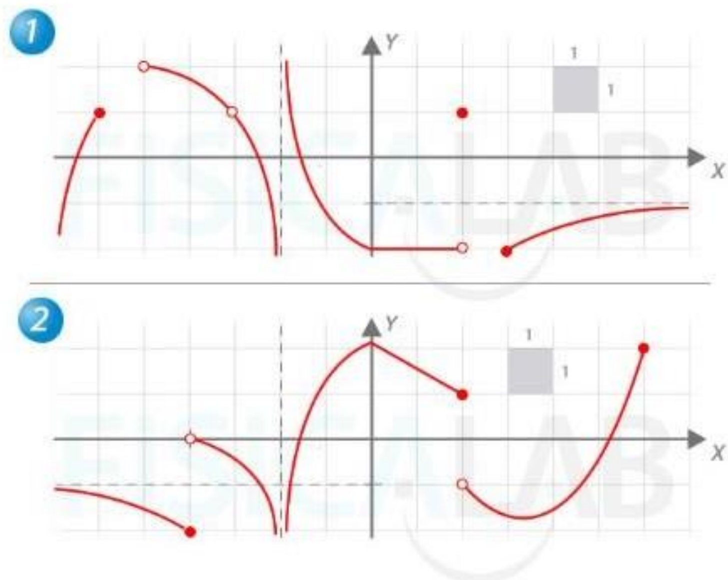
**GR2**

**SEMESTRE: 2020-A**

**HOJA DE TRABAJO N° 2**

**TEMA:** Propiedades de los límites, límites laterales e indeterminaciones

Las siguientes gráficas corresponden a las funciones *f(x)*, en 1, y *g(x)*, en 2.




**FECHA DE ENTREGA: 19/12/202**

Determinar

1

1) $\lim_{x \to -6^-} f(x) \Rightarrow 1$

2) $\lim_{x \to 6^+} f(x) \Rightarrow$ No existe

3) $\lim_{x \to -5^-} f(x) \Rightarrow$ No existe

4) $\lim_{x \to -5^+} f(x) \Rightarrow 2$

5) $\lim_{x \to -3^-} f(x) \Rightarrow 1$

6) $\lim_{x \to -3^+} f(x) \Rightarrow 1$

7) $\lim_{x \to -2^-} f(x) \Rightarrow -\infty$

8) $\lim_{x \to +2} f(x) \Rightarrow +\infty$

9) $\lim_{x \to 0^-} f(x) \Rightarrow -2$

10) $\lim_{x \to 0^+} f(x) \Rightarrow -2$

11) $\lim_{x \to 2^-} f(x) \Rightarrow$ No existe

12) $\lim_{x \to 2^+} f(x) \Rightarrow -2$

13) $\lim_{x \to -4^-} f(x) \Rightarrow -2$

14) $\lim_{x \to -4^+} f(x) \Rightarrow 0$

15) $\lim_{x \to -2^-} f(x) \Rightarrow -\infty$

16) $\lim_{x \to -2^+} f(x) \Rightarrow +\infty$

17) $\lim_{x \to 0^-} g(x) \Rightarrow 2$

18) $\lim_{x \to 0^+} g(x) \Rightarrow 2$

19) $\lim_{x \to 2^+} g(x) \Rightarrow 1$

20) $\lim_{x \to 0} (g(x) - f(x)) \Rightarrow (2-2) = 0$ //

**Calcule los siguientes límites laterales de manera algebraica**

$$\lim_{x\to 0^-} f(x) \text{ con } f(x)=\begin{cases} 3x+2 & \text{si} \quad x<0 \\ \frac{1}{x+2} & \text{si} \quad x\geq 0 \end{cases}$$

$$\lim_{x\to -2^-} f(x) \text{ con } f(x)=\begin{cases} 3x+2 & \text{si} \quad x<0 \\ \frac{1}{x+2} & \text{si} \quad x\geq 0 \end{cases}$$

$\lim_{x\to 0^-} 3x+2$

$\lim_{x\to 0} 3(0)+2=\boxed{2}$ Por izquierda $\therefore \lim_{x\to 0} f(x)=2$,

$\lim_{x\to 0^+} \frac{1}{x+2}=\frac{1}{0+2}=\frac{1}{2}$ No tiene limite

$\lim_{x\to 2^+} = 3x+2=3(2)+2=8$

$\lim_{x\to 2} \frac{1}{x+2}=\frac{1}{2+2}=\frac{1}{4}$

$f(0)=2$

185)

$$\lim_{x\to\infty} \frac{(2x+3)^3\cdot(2x-2)^2}{x^5+5}=\frac{(2(\infty)+3)^3\cdot(2(\infty)-2)^2}{(\infty)^5+5}=\frac{\infty}{\infty} \text{ IND}$$

$$\lim_{x\to\infty} \frac{(2x+3)^3(2x-2)^2}{x^5+5}=\frac{\frac{(2x+3)^3\cdot(2x-2)^2}{x^5}}{\frac{\cancel{x^5}}{\cancel{x^5}}+\frac{5}{x^5}}=\frac{0}{1}=0 //$$

186)

$$\lim_{x\to\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}}=\frac{2(\infty)^2-3(\infty)-4}{\sqrt{\infty^4+1}}=\frac{\infty}{\infty} \text{ IND}$$

$$\lim_{x\to\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}}=\frac{\frac{2\cancel{x^2}}{\cancel{x^2}}-\frac{3x}{x^2}-\frac{4}{x^2}}{\sqrt{\frac{x^4}{x^4}+\frac{1}{x^4}}}=\frac{2}{1}=2 //$$

187)

$$\lim_{x\to\infty} \frac{2x+3}{x+\sqrt[3]{x}}=\frac{2(\infty)+3}{\infty+\sqrt[3]{\infty}}=\frac{\infty}{\infty} \text{ IND}$$

$$\lim_{x\to\infty} \frac{2x+3}{x+\sqrt[3]{x}}=\left(\frac{\frac{2\cancel{x}}{\cancel{x}}+\cancel{\frac{3}{x}}}{\frac{\cancel{x}}{\cancel{x}}+\cancel{\frac{\sqrt[3]{x}}{x}}}\right)=\frac{2}{1}=2 //$$

188)

$$\lim_{x\to\infty} \frac{x^2}{10+x\sqrt{x}}=\frac{\infty^2}{10+\infty\sqrt{\infty}}=\frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{x^2}{10+x\sqrt{x}}=\frac{x^2 x(10-x\sqrt{x})}{(10+x\sqrt{x})x(10-x\sqrt{x})}=$$

$$\lim_{x\to\infty} \frac{10x^2-x^3\sqrt{x}}{100-x^2\cdot x}=\frac{10x^2-x^3\sqrt{x}}{100-x^3} //$$

# ESCUELA POLITÉCNICA NACIONAL
# ESCUELA DE FORMACIÓN DE TECNÓLOGOS

**CALCULO DIFERENCIAL E INTEGRAL**

**GR2**

**SEMESTRE: 2020-A**

189)

$$\lim_{x\to\infty} \frac{\sqrt[3]{x^2+1}}{x+1} = \frac{\sqrt[3]{\infty^2+1}}{\infty+1} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} = \frac{\sqrt[3]{\frac{x^2}{x}+\frac{1}{x}}}{\frac{x}{x}+1} = \frac{1}{1} = 1 //$$

190)

$$\lim_{x\to+\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$$

$$\lim_{x\to\infty} = \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} = \frac{\sqrt{\frac{x}{x}}}{\sqrt{\frac{x}{x}+\frac{\sqrt{x}}{x}+\frac{\sqrt{x}}{x}}} =$$

$$= \lim_{x\to\infty} = \frac{\sqrt{1}}{1+1+1} = \frac{1}{3} //$$

191)

$$\lim_{x\to 2} \frac{x^2+1}{x^2+1} =$$

$$\lim_{x\to -1} \frac{x^2+1}{x^2+1} = \lim_{x\to -1} \frac{(x^2+1)(x-1)(x+1)}{(x-1)(x+1)} =$$

$$\lim_{x\to -1} (1^2-1) = 0 //$$

192)

$$\lim_{x\to 5} \frac{x^2-5x+10}{x^2-25}$$

$$\lim_{x\to 5} \frac{x^2-5x+10}{x^2-25} = \frac{5^2-5(5)+10}{5^2-25} = \frac{10}{0} = \frac{K}{0}$$

$$\lim_{x\to 5^-} = \frac{x^2-5x+10}{x^2-25} = -\infty$$

$$\lim_{x\to 5^+} = \frac{x^2-5x+10}{x^2-25} = +\infty$$

193)

$$\lim_{x\to -1} \frac{x^2-1}{x^2+3x+2} = \frac{(-1)^2-1}{(-1)^2+3(-1)+2} = \frac{0}{0} \quad [ID]$$

$$\lim_{x\to -1} \frac{x^2-1}{x^2+3x+2} = \frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(x+1)} = \frac{(x-1)}{(x+1)} = \frac{-1-1}{-1+1} = \frac{-2}{0} \; (ID)$$

$$\lim_{x\to -1^-} = \frac{x^2-1}{x^2+3x+2} = -\infty$$

$$\lim_{x\to -1^+} \frac{x^2-1}{x^2+3x+1} = +\infty$$

194)

$$\lim_{x\to 2} \frac{x^2-2x}{x^2-4x+4} = \frac{2^2-2(2)}{2^2-4(2)+4} = \frac{0}{0}$$

$$\lim_{x\to} \frac{x^2-2x}{x^2-4x+4} = \left(\frac{x\cdot\cancel{(x-2)}}{(x-2)^{\cancel{2}}}\right) = \left(\frac{x}{(x-2)}\right)$$

$$\lim_{x\to 2^-} \frac{x}{x-2} = -\infty // \qquad \lim_{x\to 2^+} = \frac{x}{x-2} = +\infty //$$

No existe

195)

$$\lim_{x\to 1} \frac{x^3-3x+2}{x^4-4x+3} = \frac{1^3-3(1)+2}{1^4-4(1)+3} = \frac{0}{0}$$

$$\lim_{x\to 1} \frac{x^3-3x+2}{x^4-4x+3} = \frac{(x+2)\cancel{(x-1)^2}}{\cancel{(x-1)^2}(x^2-2x+3)} = \frac{(x+2)}{x^2-(x+3)} = \frac{1+2}{1^2-(1+3)}$$

$$\lim_{x\to 1} \frac{3}{2}$$

196)

$$\lim_{x\to a} \frac{x^2-(a+1)x+a}{x^3-a^3} = \frac{a^2-(a+1)a+a}{a^3-a^3} = \frac{0}{0} \text{ IND}$$

$$\lim_{x\to a^-} \frac{x^2-(a+1)x+a}{x^3-a^3} = -\infty$$

$$\lim_{x\to a^+} \frac{x^2-(a+1)x+a}{x^3-a^3} = +\infty$$

197)

$$\lim_{h\to 0} \frac{(x+h)^3-x^3}{h} = \frac{(x+0)^3-x^3}{0} = \frac{0}{0} \text{ (IND)}$$

$$\lim_{h\to 0} \frac{(x+h)^3-x^3}{h} = \frac{\cancel{h}(3x^2+3hx+h^2)}{\cancel{h}} = 3x^2+3hx+h^2 = 3x^2$$

198)

$$\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right) = \left(\frac{1}{1-1} - \frac{3}{1-1^3}\right) = \frac{1}{0} - \frac{3}{0} = \frac{1-3}{0} = \text{IND}$$

$$\lim_{x\to 1} \left(\frac{1}{1-x}\right) - \lim_{x\to 1} \left(\frac{3}{1-x^3}\right)$$

$$\lim_{x\to 1^-} \left(\frac{1}{1-x}\right) = \lim_{x\to 1^-} \left(\frac{1}{1-x}\right) = -\infty \qquad \lim_{x\to 1^+} \left(\frac{3}{1-x^3}\right) = +\infty$$

$$\lim_{x\to 1^+} \left(\frac{1}{1-1}\right) = +\infty \qquad \lim_{x\to 1^+} \left(\frac{3}{1-x^3}\right) = -\infty$$

# ESCUELA POLITÉCNICA NACIONAL
# ESCUELA DE FORMACIÓN DE TECNÓLOGOS

**CALCULO DIFERENCIAL E INTEGRAL**

**GR2**

**SEMESTRE: 2020-A**

199)

$$\lim_{x\to1}\frac{\sqrt{x}-1}{x-1}=\frac{\sqrt{1}-1}{1-1}=\frac{0}{0}=\text{IND}$$

$$\lim_{x\to1}\frac{\sqrt{x}-1}{x-1}=\frac{1}{\sqrt{x}-1}=\lim_{x\to1}\left(\frac{1}{\sqrt{x}+1}\right)=$$

$$\lim_{x\to1}\frac{1}{\sqrt{1}+1}=\frac{1}{2}$$

200)

$$\lim_{x\to64}\frac{\sqrt{x}-8}{\sqrt[3]{x}-4}=\frac{\sqrt{64}-8}{\sqrt[3]{64}-4}=\frac{8-8}{4-4}=\frac{0}{0}$$

$$\lim_{x\to64}\frac{\sqrt{x}-8}{\sqrt[3]{x}-4}=\lim_{x\to64}\left(\frac{\frac{1}{2\sqrt{x}}}{\frac{1}{3x^{2/3}}}\right)=\lim_{x\to64}\left(\frac{3}{2}\sqrt[6]{x}\right)=\frac{3\sqrt[6]{64}}{2}=3 \text{ //}$$

201)

$$\lim_{x\to1}\frac{\sqrt[5]{x}-1}{\sqrt[4]{x}-4}=\frac{1-1}{1-1}=\frac{0}{0}$$

$$\lim_{x\to1}\frac{\sqrt[5]{x}-1}{\sqrt[4]{x}-1}=\lim_{x\to}\frac{y^5-1}{y^4-1}=\lim_{x\to1}\frac{x+1}{x+1}=\frac{1+1}{1+1}=\frac{2}{2}=1$$

202)

$$\lim_{x\to1}\frac{\sqrt[3]{x^2}-2\sqrt{x}+1}{(x-1)2}=\frac{\sqrt[3]{1^2}-2\sqrt{1}+1}{(1-1)^2}=\frac{0}{0}\ \text{IND}$$

$$\lim_{x\to1}\frac{\sqrt[3]{x^2}-2\sqrt{x}+1}{(x-1)^2}=\frac{y^2-1}{y^2-1}=\frac{x^2-2x+1}{(x-1)(x-1)}=\frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}(x-1}$$

$$\lim_{x\to1}\frac{2}{2}=1 \text{ //}$$

**ESCUELA POLITÉCNICA NACIONAL**

**ESCUELA DE FORMACIÓN DE TECNÓLOGOS**

ESFOT ESCUELA DE FORMACION DE TECNOLOGOS

**CALCULO DIFERENCIAL E INTEGRAL**

**GR2**

**SEMESTRE: 2020-A**

203)

$$\lim_{x\to 7} \frac{2-\sqrt{x-3}}{x^2-49} = \frac{2-\sqrt{\infty-3}}{\infty^2-49}$$

$$\lim_{x\to 7} \frac{2-\sqrt{x-3}}{(x+7)(x-7)} = \frac{2+\sqrt{x+3}}{2+\sqrt{x-3}} = \lim_{x\to 7} \frac{2^2-(\sqrt{x-3})^2}{(x+7)(x-7)(2+\sqrt{x-3})}$$

$$\lim_{x\to 7} \frac{4-(x-3)}{(x+7)(x-7)(2+\sqrt{x-3})} =$$

$$\lim_{x\to 7} \frac{-(x-7)}{(x+7)(x-7)(2+\sqrt{x-3})} = \frac{-1}{(7+7)(2+\sqrt{7-3})} = -\frac{1}{56}//$$

204)

$$\lim_{x\to 8} \frac{x-8}{\sqrt[3]{x}-2} = \lim_{x\to 8} \frac{8-8}{\sqrt[3]{8-2}} = \frac{0}{\sqrt[3]{6}} = \infty$$

205) $$\lim_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1} = \frac{\sqrt{1}-1}{\sqrt[3]{1}-1} = \frac{0}{0}$$ Indeterminoción

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \frac{x-1 \quad (x-1)}{\sqrt[6]{x^3(x-1)^2}+\sqrt[3]{x}-1}$$

$$\lim_{x\to 1} = 0//$$

206)

$$\lim_{x\to} \frac{3-\sqrt{5+x}}{1-+\sqrt{5-x}} = \quad -$$

$$\lim_{x\to 4} \frac{3-\sqrt{5+x}}{1-\sqrt{5-x}} = \frac{3-\sqrt{5+4}}{1-\sqrt{5-4})} = \frac{0}{0}\Big\}$$ Indeterminocion

$$\lim_{x\to 4} \frac{3-\sqrt{5+x}}{1-\sqrt{5-x}} \cdot \frac{1+\sqrt{5-x}}{1+\sqrt{5-x}} = \lim_{x\to 4} \frac{(3-\sqrt{5+x})(1+\sqrt{5-x}}{1-5-x}$$

$$\lim_{x\to 4} = 0$$ No esto definido

207)

$$\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x} = \frac{\sqrt{1+0}-\sqrt{1-0}}{0} = \frac{0}{0} //$$

$$\lim_{x\to 0} \frac{1+x-1+x}{x(\sqrt{1+x}+\sqrt{1-x})} = \lim_{x\to 0} = 0$$

208)

$$\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$$

$$\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \lim_{h\to 0} \frac{x+h-x+h+x}{h(\sqrt{x+h}+\sqrt{x})}$$

Limites Laterales

$$\lim_{h\to 0^+} \frac{2h+x}{0} = \frac{2(0)+x}{0} = \frac{x}{0} = +\infty$$

$$\lim_{h\to 0} \frac{2h+x}{0} = \frac{2(0)+x}{0} = \frac{x}{0} = -\infty //$$

209)

$$\lim_{h\to 0} \frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h} = \frac{0}{0} \text{ Indeterminac..}$$

$$\lim_{h\to 0} \frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h} = -\infty$$

$$\lim_{h\to 0^+} \frac{\sqrt{x+h}-\sqrt{x}}{h} = +\infty$$

210) $$\lim_{x\to 3} \frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3}$$

$$\lim_{x\to 3} = \frac{\sqrt{3^2-2(3)+6}-\sqrt{3^2+2(3)-6}}{3^2-4(3)+3} = \frac{0}{0}$$

$$\lim_{x\to 3} \frac{x^2-2x+6-(x^2+3x-6)}{(x^2-4x+3)(\sqrt{x^2-2x+6}+\sqrt{x^2+3x-6})}$$

$$\lim_{x\to 3} \frac{-4}{(3-1)(\sqrt{3^2-2(3)+6}+\sqrt{3^2+3(3)-6}} = -\frac{1}{3} //$$

# ESCUELA POLITÉCNICA NACIONAL
# ESCUELA DE FORMACIÓN DE TECNÓLOGOS

**CALCULO DIFERENCIAL E INTEGRAL**

**GR2**

**SEMESTRE: 2020-A**

211)

$$\lim_{x \to +\infty} \left(\sqrt{x+a} - \sqrt{x}\right) = \sqrt{\infty + a} - \sqrt{\infty} = \infty - \infty$$

$$\lim_{x \to \infty} \sqrt{x+a} - \sqrt{x} = \frac{\sqrt{x+a} + \sqrt{x}}{\sqrt{x+a} + \sqrt{x}} = \frac{(\sqrt{x+a})^2 - (\sqrt{x})^2}{\sqrt{x+a} + \sqrt{x}}$$

$$\lim_{x \to \infty} = \frac{(x+a) - (x)}{\sqrt{x+a} + \sqrt{x}} = \frac{a}{\sqrt{x+a} + \sqrt{x}} = \frac{a}{+\infty} = 0 //$$

212)

$$\lim_{x \to \infty} \left[\sqrt{x(x+a)} - x\right] = \infty - \infty$$

$$\lim_{x \to \infty} \sqrt{x(x+a)} - x \cdot \frac{\sqrt{x(x+a)} + x}{\sqrt{x(x+a)} + x}$$

$$\lim_{x \to +\infty} \frac{x(x+a) - x^2}{\sqrt{x(x+a)} + x} = \frac{x^2 + ax - x^2}{\sqrt{x^2 + ax} + x} = \frac{\infty}{\infty} \text{ IND}$$

$$\lim_{x \to \infty} \frac{ax}{\sqrt{x^2 + ax} + x} = \infty //$$

213)

$$\lim_{x \to +\infty} \left(\sqrt{x^2 - 5x + 6} - x\right) = \infty - \infty$$

$$\lim_{x \to \infty} \left(\sqrt{x^2 - 5x + 6} - x\right) \cdot \frac{\left(\sqrt{x^2 - 5x + 6} + x\right.}{\sqrt{x^2 - 5x + 6} + x}$$

$$\lim_{x \to \infty} \frac{(x^2 - 5x + 6) - x^2}{\sqrt{x^2 - 5x + 6} + x} = \frac{-5x + 6}{\sqrt{x^2 - 5x + 6} + x}$$

$$\lim_{x \to \infty} -\frac{5}{2} //$$

214)

$$\lim_{x \to +\infty} x\left(\sqrt{x^2+1} - x\right) = \infty\left(\sqrt{\infty^2 + 1} - \infty\right) = \infty - \infty$$

$$\lim_{x \to \infty} x\left(\sqrt{x^2+1} - x\right) \lim_{x \to \infty} = \frac{\left(x\sqrt{x^2+1} - x\right)\left(x\sqrt{x+1} + x\right)}{\left(x\sqrt{x^2+1} + x\right)} =$$

$$\lim_{x \to \infty} = \frac{1}{x} = 1$$

215) $\lim_{x \to \infty} \left(x + \sqrt[3]{1 - x^3}\right.$

$$\lim_{x \to \infty} \left(x + \sqrt[3]{1 - x^3}\right) = \infty + \sqrt[3]{1 - \infty^3} = \infty \cdot +\infty$$

$$\lim_{x \to \infty} x + \sqrt[3]{1 - x^3} \cdot \frac{x\sqrt[3]{1 - x^3}}{x\sqrt[3]{1 - x^3}} = \frac{x^2}{x^2\left(\sqrt{1 + \frac{1}{x^2}} + 1\right.} = \frac{1}{\infty} = 0 //$$

$$\lim_{x \to \infty} \frac{x^2 - \sqrt[3]{(1 - x^3)^2}}{x - \sqrt[3]{2 - x^3}} = 0 //$$