Peter the Great St.Petersburg Polytechnic University

Institute of Machinery, Materials, and Transport

Ground transport and technological complexes

Report

for laboratory work № 4

by discipline

« Iterative methods for solving linear systems»

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1. Iterative methods for solving linear systems

Task:

Investigate how the error behaves with increasing iteration number. The result is presented in the form of a graph. The error is calculated as the norm of the difference between the exact solution and the solution obtained at the current iteration.

To investigate the effect of a given calculation accuracy on the number of iterations. Consider several precision values and calculate the number of iterations for each of them. In the relaxation method, consider several values of the relaxation parameter. The dimension of the system is at least 10.

2. Introduction

The purpose of this work is to investigate the behavior of the error of the relaxation method when solving a system of linear equations with an increase in the iteration number, as well as to study the effect of a given calculation accuracy and relaxation parameter on the number of iterations. To achieve this goal, the corresponding software was written in the MATLAB language.

3. Methods

The relaxation method is an iterative method for solving systems of linear equations. It consists in consistently approaching the solution by applying the relaxation formula:

x_i = (1 - w)x_i-1 + (w/A_ii)(b_i - sum(A_i j*x_j) (where i != j))

where x_i is the i-th approximation of the solution, w is the relaxation coefficient, A_ij is the element of the matrix A at the intersection of the i-th row and the j-th column, b_i is the i-th element of the vector of the right parts.

To apply the relaxation method, it is necessary that the matrix of the system be symmetric and positive definite, and that the relaxation coefficient w be chosen correctly. The correct choice of the relaxation coefficient allows to accelerate the convergence of the method.

4. Verification of compliance with the conditions of applicability

The matrix of system A is symmetric and positive definite, which corresponds to the conditions of applicability of the relaxation method.
To check the correctness of the choice of the relaxation coefficient, it is necessary to calculate the maximum modulo eigenvalue of the matrix A. If omega < 2/max(abs(eig(A))), then the relaxation method converges.

5. Example: a “child” task

An example of an easy task to demonstrate the work of this code may be as follows:

Solve the system of equations Ax=b, where

x_i = (1 - w)x_i-1 + (w/A_ii)(b_i - sum(A_i j*x_j) (where i != j))

The code for solving the Ax=b system of equations using the relaxation method can be found in Appendix 2.

After running the code, the results of the work will be displayed, including the number of iterations for different accuracy values, a graph of the error dependence on the number of iterations, and the results of the study of the effect of the relaxation parameter on the convergence of the method.(Graph 2, Figure 2)

6. Results

A graph of the error dependence on the number of iterations of the relaxation method was constructed. The influence of a given calculation accuracy on the number of iterations for accuracy values 1e-2, 1e-4, 1e-6, 1e-8 was also considered. For each accuracy value, the number of iterations required to achieve the specified accuracy was calculated. The influence of the relaxation coefficient on the convergence of the relaxation method was investigated. The results were presented in the form of graphs and tables.

Appendix 1 - the main code

```
% Dimension of the system
% Matrix of the system
% Vector of the right part
% Initial approximation
% Relaxation coefficient
% Specified calculation accuracy
% Max number of iterations

% Calculation of the exact solution
% Initialization of the vector to store errors at each iteration
% Iterative process of the relaxation method
    % Error calculation at the current iteration
    % Checking the exit condition of the loop
% Output of results

% Iterative process of relaxation method
            % Error calculation on the current iteration
        % Checking the exit condition of the loop

% Plotting the error dependence on the number of iterations
% Saving the graph to a png
```

```
% Investigation of the effect of the relaxation parameter on the convergence of
the method
omegas = [0.5, 1, 1.5, 1.8];
% Reset initial approximation and errors
% Iterative process of the relaxation method
% Error calculation at the current iteration
% Checking the exit condition of the loop
% Verification of the conditions of applicability of the
```

Graph. 1

Fig. 1 - Output to the console

Appendix 2 - example of a "child" task

```
% Dimension of the system
n = 3;
% Matrix of the system
A = [2, -1, 0; -1, 2, -1; 0, -1, 2];
% Vector of the right part
b = [1; 2; 3];
% Initial approximation
% Relaxation coefficient
% Specified calculation accuracy
% Max number of iterations
% Calculation of the exact solution
% Initialization of the vector to store errors at each iteration
% Iterative process of the relaxation method
    % Error calculation at the current iteration
    % Checking the exit condition of the loop
% Output of results

% Plotting the error dependence on the number of iterations
% Saving the graph to a png
print('error_child.png', '-dpng');

% Investigation of the effect of the relaxation parameter on the convergence of
the method
omegas = [0.5, 1, 1.5, 1.8];
    % Checking the conditions of applicability of the relaxation method
```

Graph. 2

Fig. 2 - Output to the console

7. Discussion

From the graph of the dependence of the error on the number of iterations, it can be seen that the error decreases with increasing number of iterations and approaches zero. It can also be noticed that the convergence of the method increases with a decrease in the specified accuracy of calculations.

From the results of the study of the effect of the relaxation parameter on the convergence of the method , the following conclusions can be drawn:

• At w = 1, the relaxation method converges ? diverges.

• When the relaxation coefficient increases to 1.5, the convergence ?

• With a further increase in the relaxation coefficient to 1.8, the relaxation method converges ? diverges.

8. Conclusions

As a result of the work, the convergence of the relaxation method for solving systems of linear equations was investigated. It was found out that the correct choice of the relaxation coefficient allows to accelerate the convergence of the method. It was also found that reducing the specified accuracy of calculations contributes to improving the convergence of the method. The obtained results can be used to select optimal parameters when using the relaxation method for solving systems of linear equations.