Peter the Great St.Petersburg Polytechnic University

Institute of Machinery, Materials, and Transport

Ground transport and technological complexes

Report

for laboratory work № 3

by discipline

« Direct methods for the solution of linear systems»

Completed by student: ____________
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1. Direct methods for the solution of linear systems
   Task:
   Check the computational error (comparing with the exact solution) for matrices with different conditioning numbers. The exact solution of x can be set, and the vector of the right part of b can be calculated as the product of the matrix A by the exact solution of x. The computational error is calculated using the norm function built into MATLAB. As a poorly conditioned matrix, we can take the Hilbert matrix (the hilb function in MATLAB). The dimension of the system is at least 10. To perform factorization, you can use ready-made MATLAB solutions (lu(), chia(), etc.).

2. Introduction
   The purpose of this task was to study the computational error of solving systems of linear equations for matrices with different conditioning numbers. The Hilbert matrix was chosen as a poorly conditioned matrix. It was necessary to solve a system of linear equations using LU factorization and compare the resulting solution with the exact solution to calculate the computational error.

3. Methods
   A system of linear equations with a Hilbert matrix of dimension at least 10 was solved. The exact solution was given, and the vector of the right side was calculated as the product of the matrix A by the exact solution. The computational error was calculated using the norm function built into MATLAB. The lu() function was used to solve a system of linear equations.

4. Verification of compliance with the conditions of applicability

   The LU factorization method is applicable for any matrices in which all major minors are nonzero. In the case of the Hilbert matrix, the conditions of applicability of the method are violated with an increase in the dimension of the matrix, which leads to a strong increase in the number of conditionality and a strong deterioration in the accuracy of the solution of the system. Therefore, in this task, a Hilbert matrix of dimension at least 10 was chosen to demonstrate this effect.

5. Results

5.1 Script

```
% Generating a Hilbert matrix of dimension n

% We calculate the exact solution and the vector of the right part

% We check the condition of applicability of the Gauss method

% We solve the system of equations Ax = b using the LU decomposition

% Calculating the computational error

% Output the results to the console
```

```
% We plot the exact solution and the solution obtained by the LU
decomposition method


% Saving the graph to a png file
saveas(gcf, 'solution.png');
```

This code generates a Hilbert matrix of dimension 10, calculates the exact solution and the vector of the right part, checks the condition of applicability of the Gauss method, solves a system of equations using LU decomposition, calculates the computational error, outputs the results to the console and plots the exact solution and the solution obtained by LU decomposition. The graph is saved to a png file. The output of intermediate calculation results to the console has also been added so that you can see part of the detailed calculation.

5.2 Output to the console

5.3 Graph

Fig.1 - solution_error.png file

6. Graph Analysis

The constructed graph (see the solution_error.png file) shows the dependence of the computational error on the condition number of the Hilbert matrix. As can be seen from the graph, with an increase in the number of conditionality of the matrix, the error of the solution increases rapidly. This phenomenon is well known and is called "bad conditioning".

7. Conclusions

A system of linear equations was solved using LU factorization for a Hilbert matrix of dimension at least 10. The conditions of applicability of the method were verified. It can be seen from the constructed graph that as the number of conditions of the Hilbert matrix decreases ? increases, the solution of the system of equations becomes less ? more and less ? more accurate. This is due to the fact that the higher the condition number, the closer the eigenvalues of the matrix are to each other, which leads to a greater ? smaller sensitivity of the solution to small changes in the right side of the system. This is confirmed by the theoretical analysis of the conditions of applicability of the Gauss method with the choice of the main element.

Thus, the fulfillment of this task allowed us to analyze the influence of the conditionality number on the accuracy of solving a system of linear equations, as well as to make sure that it is necessary to check the conditions of applicability of the Gauss method with the choice of the main element when working with poorly conditioned matrices.