UPPSALA UNIVERSITY
Dept of Mathematics
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Matematical statistics
Analysis of Time Series 1MS014
2020–03–27

Hand-in assignment 1

There are six non-compulsory home assignments. By the end of the course, I will sum your points on these to decide how much bonus you will obtain for the final exam (no bonus on re-exams).

1. Let $\{w_t\}$, $t = 0, \pm 1, \pm 2, ...$, be a white noise process where the w_t are simultaneously independent with variance $\sigma_w^2 = 3$, and let

$$x_t = 2t + w_t w_{t-1} + 0.6 w_{t-3}.$$

   (a) Calculate the mean and autocovariance function of x_t and state whether it is stationary. (2p)

   (b) Do the same for the differenced series, ∇x_t. (2p)

2. Consider the series

$$x_t = \sin(2\pi U t),$$

   for $t = 1, 2, ...$, where U is uniformly distributed at the interval $(0, 1)$. (Observe that it is the same U for all t.)

   (a) Prove that x_t is weakly stationary. (2p)

   (b) Prove that x_t is not strictly stationary. (4p)

   Hint:

   i. It is enough to prove that, for some constants c_1, c_2,

$$P(x_1 > c_1, x_2 > c_2) \neq P(x_2 > c_1, x_3 > c_2), \tag{1}$$

   why?

   ii. Take $c_1 = c_2 = c = \sin(\pi/3) = \sqrt{3}/2$.
   For $n = 1, 2, 3$, define a set of intervals $A_n = \{u : \sin(2\pi n u) > c\}$. Show that

$$P(x_1 > c, x_2 > c) = l(A_1 \cap A_2),$$
$$P(x_2 > c, x_3 > c) = l(A_2 \cap A_3),$$

   where l denotes interval length and $\cap$ is intersection.

   iii. Finally, by e.g. drawing a figure, show that $l(A_1 \cap A_2) = 0$ and $l(A_2 \cap A_3) \neq 0$. Hence, we have that (1) holds, and we are done.