Analysis of Time Series, L1

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Course overview

- Shumway and Stoffer:
  Time Series Analysis and its Applications, with R examples, 4th ed.
  Springer 2017.
- Chapters:
  1. Characteristics of Time Series (L1-2)
  2. Time Series Regression and Explanatory Data Analysis (L2)
  3. ARIMA Models (L3-9)
  4. Spectral Analysis and Filtering (L10-12)
  5. Additional Time Domain Topics (L13-16)
  6. State-Space Models (L17-18)

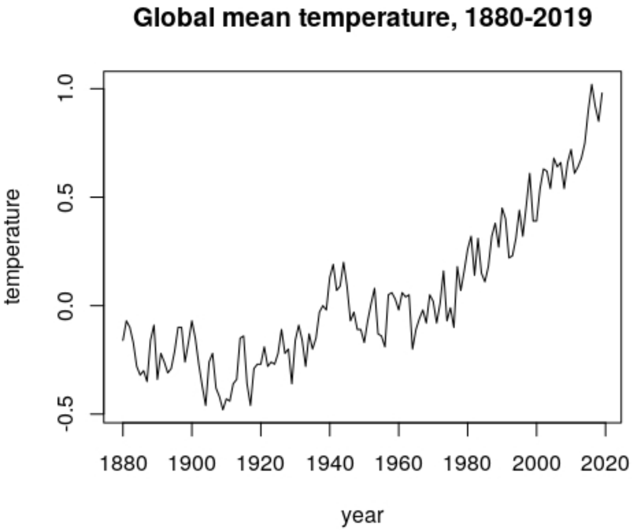
Course overview

- 18 theory lectures with comments on studentportalen.
  Also some videos.
- Written exam (with book and/or homemade notes).
- Six hand in assignments, not compulsory but give bonus points.
- Project: Analyse your own time series. (Compulsory.)
- Please check "Studentportalen" for further information!
  (The file "kursinfo", including a list of recommended exercises.)

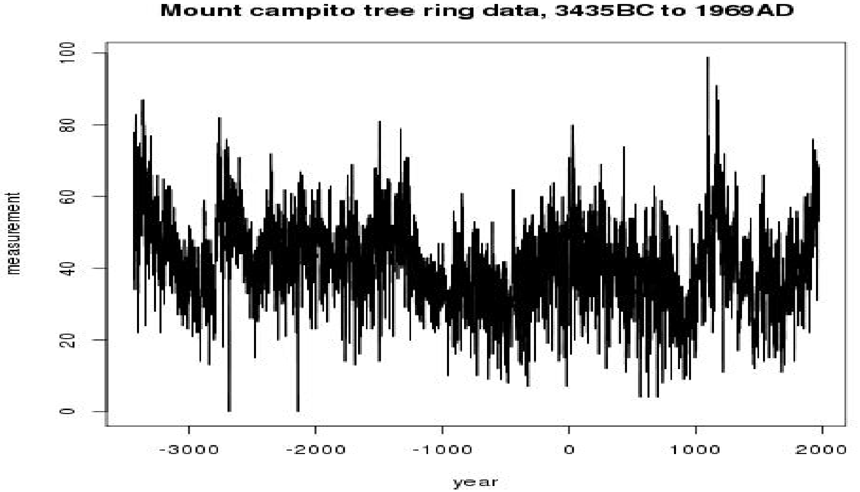
Today

- 1.1-2: Introduction, examples
- 1.3: Statistical models
- 1.4: Measures of dependence

Introduction, examples

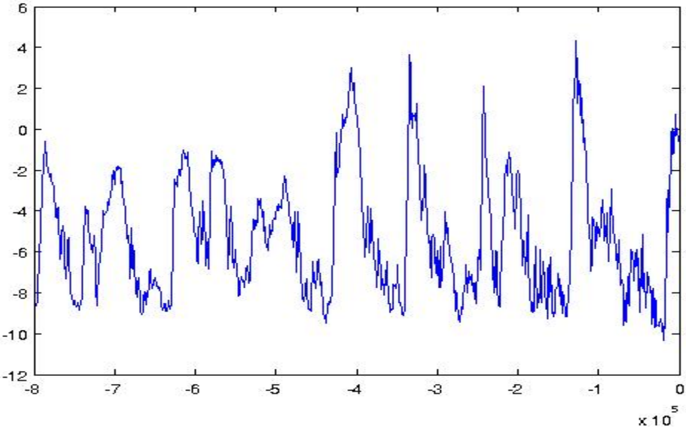


Introduction, examples

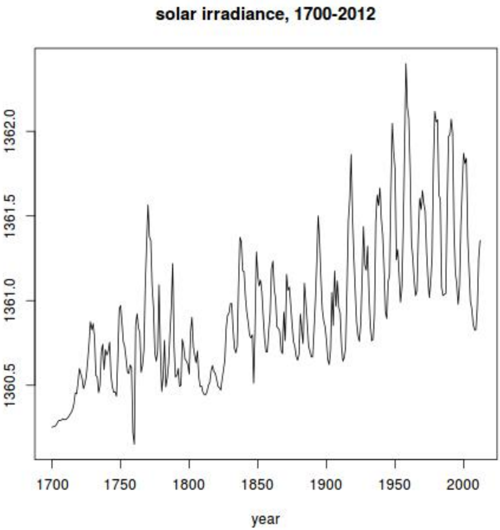


Introduction, examples

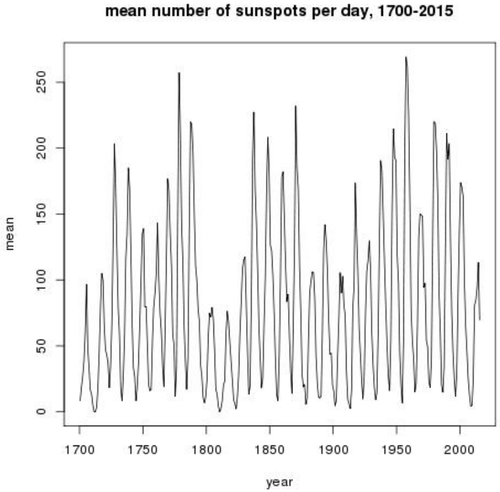
Antarctic ice core temperature proxies from about 800 000 BC to now.



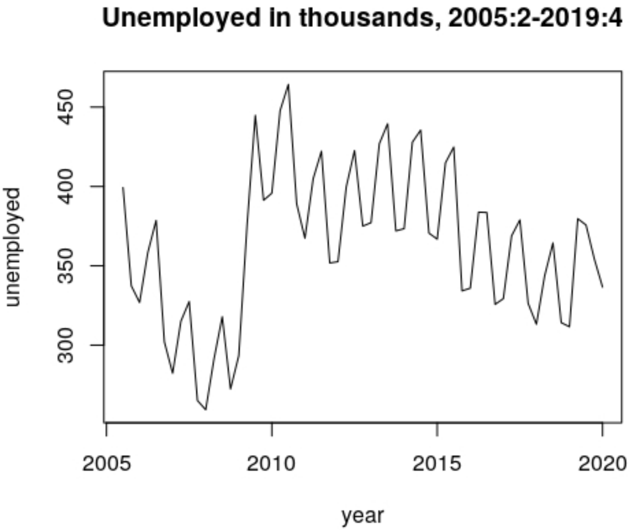
Introduction, examples



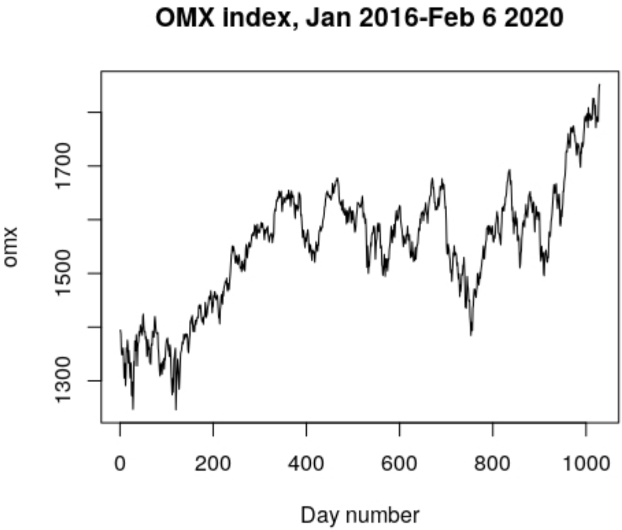
Introduction, examples



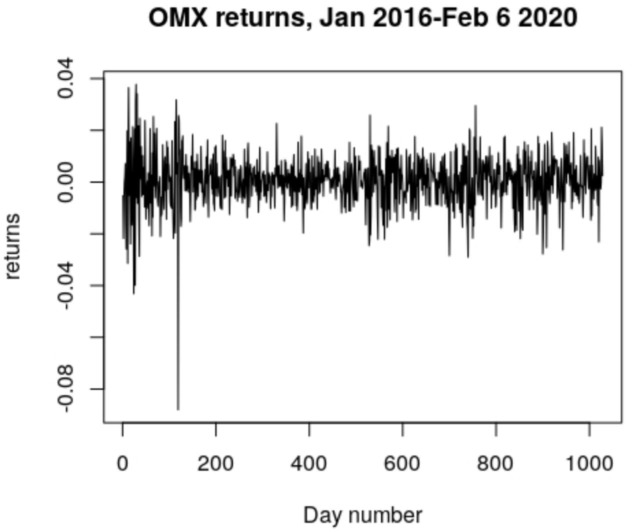
Introduction, examples



Introduction, examples



Introduction, examples



Statistical models

Definition (Stochastic process)

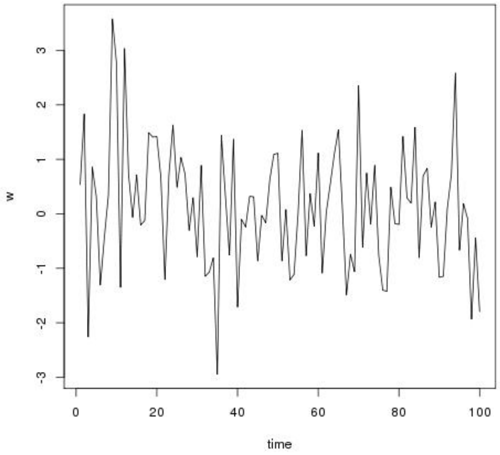
A collection of random variables $\{x_t\}$ where t ranges over a set of integers, is called a *stochastic process in discrete time* (time series).

Definition (Realization)

A collection of observed values of a stochastic process is called a *realization*.

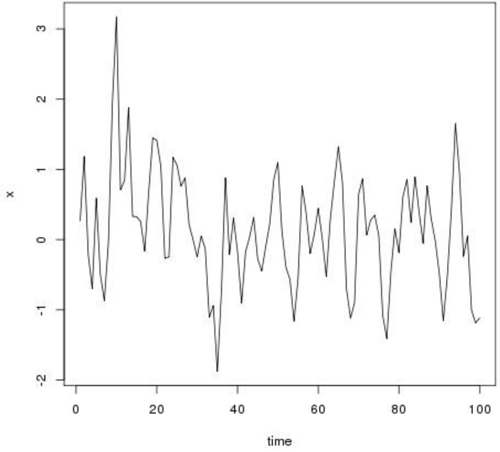
Statistical models

Example 1: White noise, $w_t \sim N(0, \sigma_w^2)$, independent



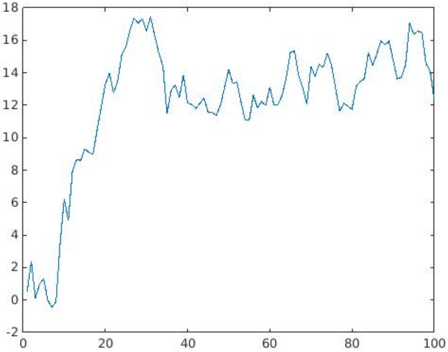
Statistical models

Example 2: Moving average, $x_t = \frac{1}{2}(w_t + w_{t-1})$



Statistical models

Example 3: Random walk, $x_t = x_{t-1} + w_t$



Measures of dependence

Definition (1.1)

The *mean function* of a stochastic process $\{x_t\}$ is defined as

$$\mu_t = E(x_t).$$

Definition (1.2)

The *autocovariance function* of a stochastic process $\{x_t\}$ is defined as

$$\gamma(s,t) = \mathrm{cov}(x_s, x_t).$$

Definition (1.3)

The *autocorrelation function* of a stochastic process $\{x_t\}$ is defined as

$$\rho(s,t) = \mathrm{corr}(x_s, x_t) = \frac{\gamma(s,t)}{\sqrt{\gamma(s,s)\gamma(t,t)}}.$$

Measures of dependence

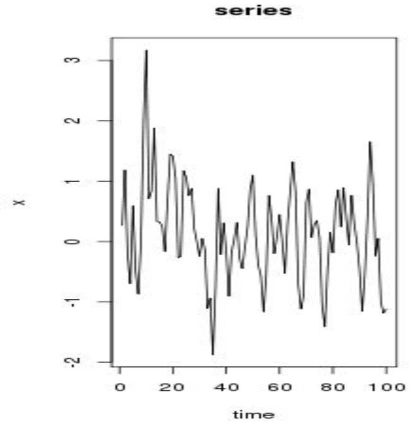
Calculate μ_t, $\gamma(s,t)$ and $\rho(s,t)$ for

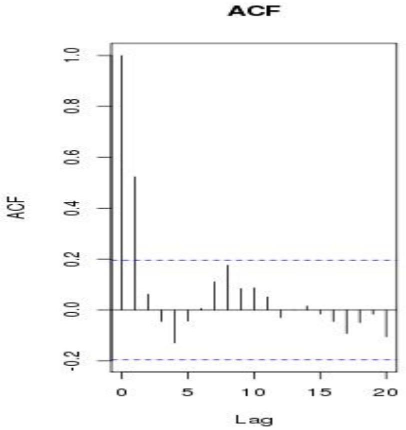
1. the white noise process w_t.
2. the moving average process $x_t = \frac{1}{2}(w_t + w_{t-1})$.
3. the random walk process $x_t = x_{t-1} + w_t$ where $x_0 = 0$.

In general, is it true that $\gamma(s,t) = \gamma(t,s)$ and $\rho(s,t) = \rho(t,s)$?

Measures of dependence

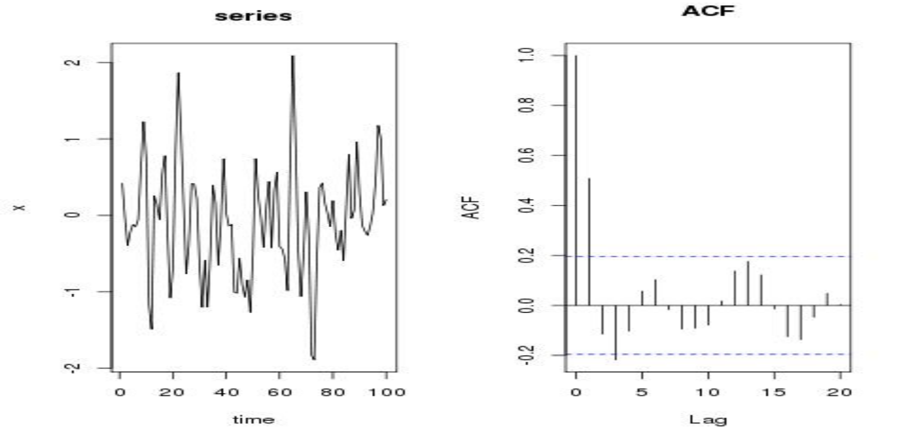
Simulation one of $x_t = \frac{1}{2}(w_t + w_{t-1})$





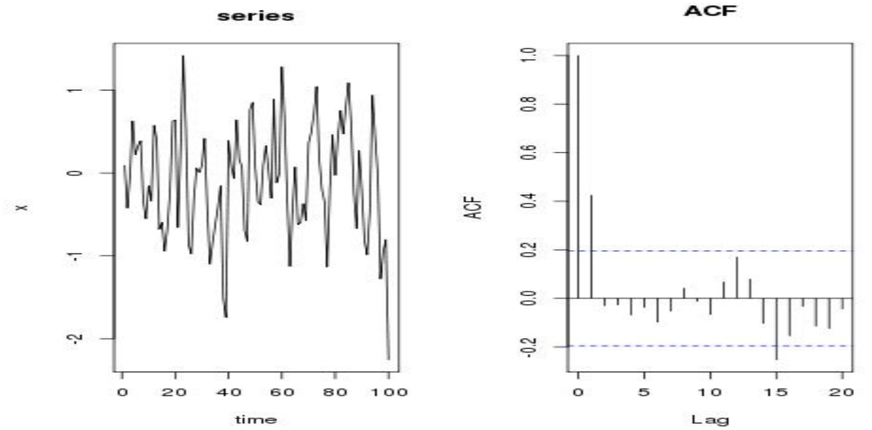
Measures of dependence

Simulation two of $x_t = \frac{1}{2}(w_t + w_{t-1})$



Measures of dependence

Simulation three of $x_t = \frac{1}{2}(w_t + w_{t-1})$



Measures of dependence

Definition (1.4)

The *cross-covariance function* between two series $\{x_t\}$ and $\{y_t\}$ is defined as

$$\gamma_{xy}(s,t) = \mathrm{cov}(x_s, y_t).$$

Definition (1.5)

The *cross-correlation function* between two series $\{x_t\}$ and $\{y_t\}$ is defined as

$$\rho_{xy}(s,t) = \mathrm{corr}(x_s, y_t) = \frac{\gamma_{xy}(s,t)}{\sqrt{\gamma_x(s,s)\gamma_y(t,t)}}.$$

Measures of dependence

Let $x_t = \frac{1}{2}(w_t + w_{t-1})$ and $y_t = w_t$.

1. Calculate $\gamma_{xy}(s,t)$ and $\rho_{xy}(s,t)$.
2. Calculate $\gamma_{yx}(s,t)$ and $\rho_{yx}(s,t)$.

In general:

1. Is it true that $\gamma_{xy}(s,t) = \gamma_{xy}(t,s)$ and $\rho_{xy}(s,t) = \rho_{xy}(t,s)$?
2. Is it true that $\gamma_{xy}(s,t) = \gamma_{yx}(t,s)$ and $\rho_{xy}(s,t) = \rho_{yx}(t,s)$?

News of today

Definitions of

- a discrete time stochastic process (time series)
- the mean function
- the autocovariance function
- the autocorrelation function
- the cross-covariance function
- the cross-correlation function