NE 155 Spring 2017
Homework #3 Due Feb. 22, 2017

SSID:

On homework:

- If you work with anyone else, document what you worked on together.
- Show your work.
- Always clearly label plots (axis labels, a title, and a legend if applicable).
- Homework should be done "by hand" (i.e. not with a numerical program such as MATLAB, Python, or Wolfram Alpha) unless otherwise specified. You may use a numerical program to check your work.
- If you use a numerical program to solve a problem, submit the associated code, input, and output (email submission is fine).
- If using Python, be aware of `copy` vs. `deep copy`: https://docs.python.org/2/library/copy.html

1. (10 points) Using Taylor's theorem and the points $(x_i - 2h), (x_i - h)$, and x_i derive the $O(h^2)$ backward difference formula for $f''(x_i)$. This means find a, b, c, and an expression for the truncation error E such that

$$f''(x_i) = af(x_i - 2h) + bf(x_i - h) + cf(x_i) + E\ .$$

2. (15 points) Consider the following four equally-spaced points on the interval $[x_0, x_3]$:

$$x_j = x_0 + jh \qquad j = 0, 1, 2, 3 \qquad h = (x_3 - x_0)/3\ .$$

Using the formula for the interpolating polynomial $P_3(x)$ you used in Question 1b of Homework 2, integrate $P_3(x)$ to derive the following Newton-Cotes formula, often referred to as **Simpson's three-eights rule**:

$$I(f(x)) \approx I_3(f(x)) = \frac{3h}{8}\Big[f(x_0) + 3f(x_0 + h) + 3f(x_0 + 2h) + f(x_0 + 3h)\Big]$$

Note that you can substitute h in for the xs in P_3 because we are explicitly stating that the points are equally spaced.
For this problem, don't worry about an error term.

3. (20 points) Using a general interpolant $I_3(f(x))$ (that is, there is no need for equally spaced points as this is not a fundamental part of the derivation):
   (a) (5 points) Compute an *expression* for the error term, $E_3(x) = I(f(x)) - I_3(f(x))$. Recall, this requires constructing $R_3(x)$ (what we called $err(x)$ in the last homework; this can stay in integral form).
   (b) (7 points) Given
   $$f(x) = \sin(\frac{\pi}{2}x) + \frac{x^2}{4} \ ,$$
   use information about the function to bound the expression for $E_3(x)$ (you may use the result you got in Homework 2 to help you).
   You may use a mathematical package for help with the integration.
   (c) (2 points) Use the values $x_0 = 0, x_1 = 2, x_2 = 3$, and $x_3 = 4$ to get a bounded value for $E_3(x)$ over this interval.
   (d) (2 points) What is the maximum value of R_3 at $x = 1$? What about E_3?
   (e) (4 points) What is the maximum value of R_3 at $x = 5$?
   How does that compare to $x = 1$ and what insight can you draw from that?
   Can you make any comments about E_3 in this case?

4. (20 points) Consider the following integral
   $$I = \int_2^4 \frac{x}{\sqrt{x^2-1}} dx \ .$$
   (a) (6 points) Compute the integral exactly by hand.
   (b) (8 points) Write a code that performs Composite Simpson's 3/8 rule to compute the integral. I advise something like
   `I = CompSimp38(a,b,n)`
   where `a` and `b` are the endpoints and `n` is the number of points to use in the integration (which must be divisible by 3!).
   (c) (6 points) Experimentally determine the rate of convergence as a function of `h`.

BONUS: submit your code by providing read/clone access to an online version control repository where your code is stored (e.g. github or bitbucket). If you don't know what that means and want to learn about it, come talk to me or check out resources here: http://software-carpentry.org/lessons.html.