NE 155 Spring 2017
Homework #6 Due April 10, 2017

SSID:

On homework:

- If you work with anyone else, document what you worked on together.
- Show your work.
- Always clearly label plots (axis labels, a title, and a legend if applicable).
- Homework should be done "by hand" (i.e. not with a numerical program such as MATLAB, Python, or Wolfram Alpha) unless otherwise specified. You may use a numerical program to check your work.
- If you use a numerical program to solve a problem, submit the associated code, input, and output (email submission is fine).
- If using Python, be aware of `copy` vs. `deep copy`:
  https://docs.python.org/2/library/copy.html

1. (30 points) Harness your knowledge from your differential equations class to analytically solve the fixed-source diffusion equation (assuming D and Σ_a are constant):

$$-D\frac{d}{dx}\frac{d\phi(x)}{dx} + \Sigma_a\phi(x) = S(x)$$
$$\text{Boundary Conditions: } \phi(\pm a) = 0$$

in the following three situations:

a. $S(x) = 0$ for $x \in [-a, a]$
b. $S(x) = S_0$ (a constant) for $x \in [-a, a]$
c. $S(x) = \cos(x)$ for $x \in [-a, a]$

Hint: for b and c you can deduce an additional boundary condition that may make things slightly simpler.

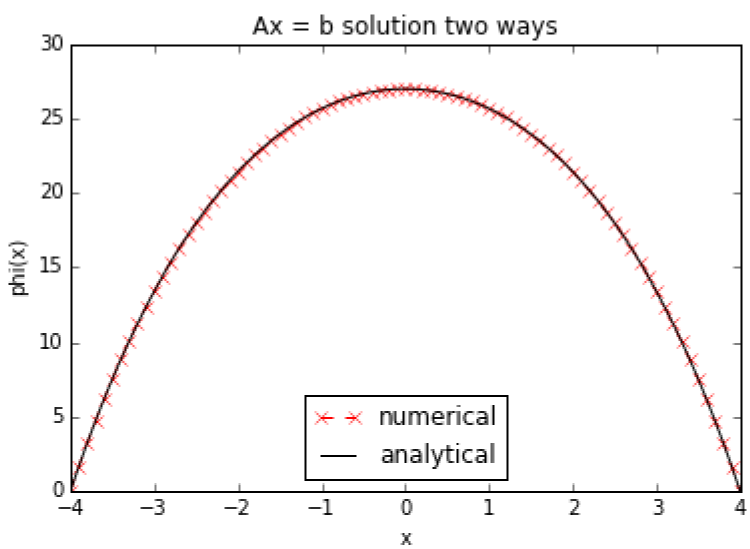
** Note: If you are unsure if you did this correctly (you need it for comparison in the next two questions), feel free to contact me. **

2. (20 points) Numerically solve the fixed-source diffusion equation as described in Question 1 using the finite *difference method* for discretization of the spatial variable and *Gaussian elimination* (a.k.a. the Thomas algorithm; note that you will have a tridiagonal system to solve) for solving the system of linear algebraic equations.

   Use the following parameters:

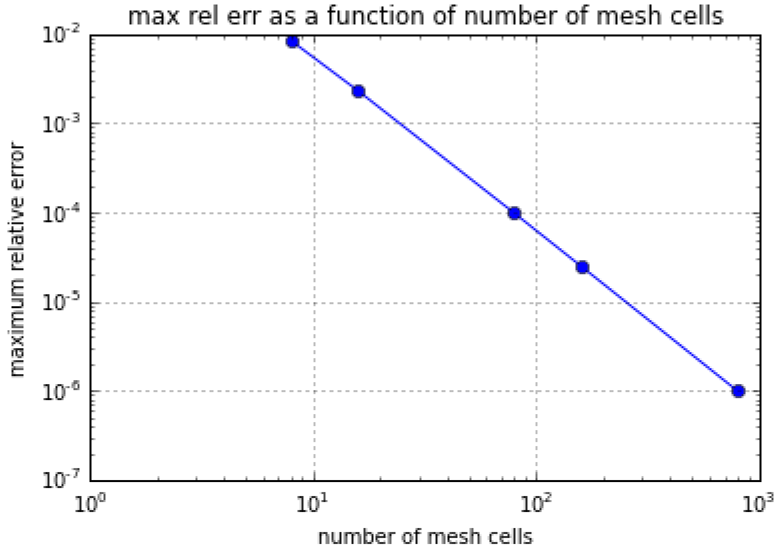
   - $a = 4$ cm,
   - $D = 1$ cm,
   - $\Sigma_a = 0.2$ cm^{-1},
   - $S = 8$ n/(cm^3 s), and
   - $h = 0.1$ cm.

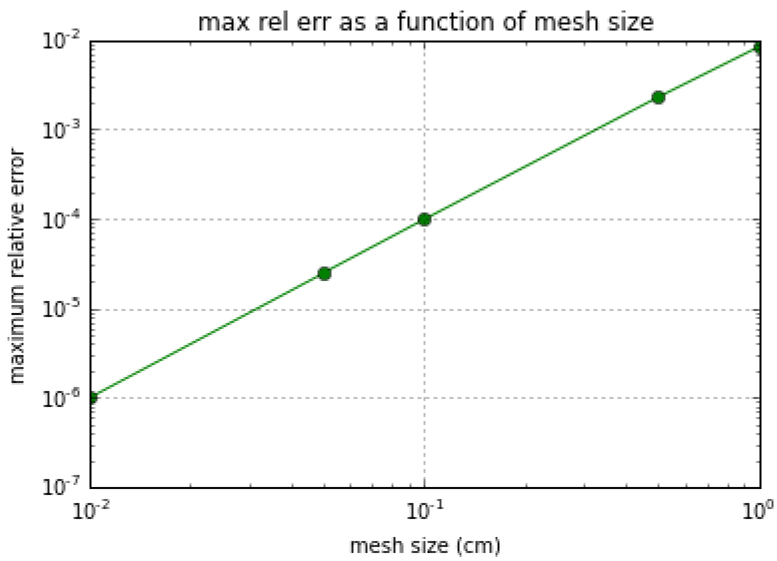
   Plot the solution from $x = -a$ to $x = a$. Compare your answer to one of your solutions from Question 1.



3. (15 points) Investigate how well your numerical solution approximates the analytical solution by computing ϕ_i for various constant mesh sizes: h = *1 cm, 0.5 cm, 0.1 cm, 0.05 cm, 0.01 cm.*

   For each mesh length calculate the relative error between your numerical and analytical solutions. Plot the maximum relative error as a function of total number of meshes for each case. What can you conclude about the relationship between the maximum error and the total number of meshes?





4. (35 points) Numerically solve the eigenvalue form of the diffusion equation:

$$-\frac{d}{dx}D(x)\frac{d\phi(x)}{dx} + \Sigma_a(x)\phi(x) = \frac{1}{k}\nu\Sigma_f(x)\phi(x)$$
$$\text{Boundary conditions} \quad \phi(\pm a) = 0$$

Assume D, Σ_a, and $\nu\Sigma_f$ are constant in space.

Use the same parameters as above with the $h = 0.1$ cm case, except:

- $\Sigma_a = 0.7$ cm^{-1}, and
- $\nu\Sigma_f = 0.6$ cm^{-1}.

To solve

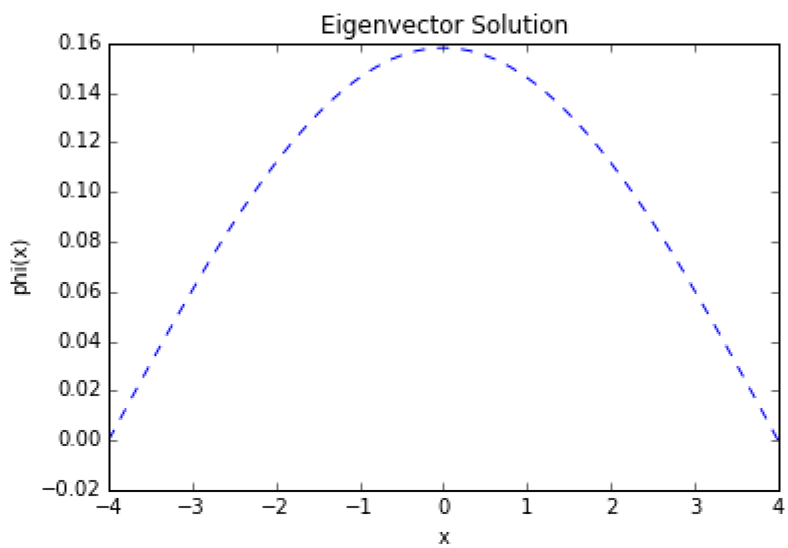
- Use the *finite difference method* for discretization of the spatial variable (as above).
- Use the *Power Iteration* algorithm we went over in class to find the dominant eigenvalue and corresponding eigenvector. Note that you may find you need to normalize the the solution vector; I suggest at least normalizing the initial guess.
- Use the SOR or GS method that you wrote to complete the

$$\mathbf{A}\vec{\phi}^{(m)} = \frac{1}{k^{(m-1)}}\vec{Q}_f^{(m-1)}$$

portion of the algorithm.

Use an absolute error tolerance to check for convergence. Use $\epsilon = 10^{-4}$ for k and $\epsilon = 10^{-4}$ and the 2-norm for $\vec{\phi}$.

Plot the eigenvector, ϕ, from $x = -a$ to $x = a$. Report the eigenvalue, k, and the number of power iterations required for convergence. (Note: this calculation may be somewhat time consuming.)



BONUS: submit your code by providing read/clone access to an online version control repository where your code is stored (e.g. github or bitbucket). If you don't know what that means and want to learn about it, come talk to me or check out resources here: http://software-carpentry.org/lessons.html.