NE 155 Spring 2017
Homework #4 Due March 3, 2017

SSID:

On homework:

- If you work with anyone else, document what you worked on together.
- Show your work.
- Always clearly label plots (axis labels, a title, and a legend if applicable).
- Homework should be done "by hand" (i.e. not with a numerical program such as MATLAB, Python, or Wolfram Alpha) unless otherwise specified. You may use a numerical program to check your work.
- If you use a numerical program to solve a problem, submit the associated code, input, and output (email submission is fine).
- If using Python, be aware of `copy` vs. `deep copy`: https://docs.python.org/2/library/copy.html

1. (10 points) Determine which of the following matrices are non-singular and compute the inverse of these matrices:

$$\text{a. } \begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \qquad \text{b. } \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & 2 & -4 & -2 \\ 2 & 1 & 1 & 5 \\ -1 & 0 & -2 & -4 \end{pmatrix}$$

2. (10 points) Determine the eigenvalues and associated eigenvectors of the following matrices; also indicate what the spectral radius is:

$$\text{a. } \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix} \qquad \text{b. } \begin{pmatrix} 3 & 2 & -1 \\ 1 & -2 & 3 \\ 2 & 0 & 4 \end{pmatrix}$$

3. (15 points) We have three systems of linear equations that are similar but different. Of them, <u>one</u> has an exact solution, <u>one</u> has infinitely many solutions, and <u>one</u> has no solution.

   (a) (3 points) Determine which system is which.

(b) (4 points) Discuss the approach(es) you would use to solve these systems by hand.

(c) (8 points) Find the solutions (as applicable). You may use a numerical program to solve these systems; submit the code and output that you use.

1.

$$\begin{aligned} 4x_1 + -1x_2 + 2x_3 + 3x_4 &= 10 \\ -2x_2 + 7x_3 + -4x_4 &= -7 \\ 6x_3 + 5x_4 &= 4 \\ 3x_4 &= 6 \end{aligned}$$

2.

$$\begin{aligned} 4x_1 + -1x_2 + 2x_3 + 3x_4 &= 10 \\ 0x_2 + 7x_3 + -4x_4 &= -7 \\ 6x_3 + 5x_4 &= 4 \\ 3x_4 &= 6 \end{aligned}$$

3.

$$\begin{aligned} 4x_1 + -1x_2 + 2x_3 + 3x_4 &= 10 \\ 0x_2 + 7x_3 + 0x_4 &= -7 \\ 6x_3 + 5x_4 &= 4 \\ 3x_4 &= 6 \end{aligned}$$

4. (10 points) Find the parabola

$$y = a + bx + cx^2$$

that passes through the points $(1, 1)$, $(2, -4)$, and $(3, 1)$.

Use Gaussian elimination and backward substitution as your solution technique.

5. (15 points) Find the LU Decomposition of $\mathbf{A}$ using Gaussian elimination and use it to solve $\mathbf{A}\vec{x} = \vec{b}$.

$$\mathbf{A} = \begin{pmatrix} 10 & -7 & 0 \\ -3 & 2 & 6 \\ 1 & -1 & 5 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 7 \\ 4 \\ 6 \end{pmatrix}$$