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A long-period vertical vibration isolator for gravitational wave detection

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Abstract

A novel design for a long-period passive vertical suspension system based on a torsion crank linkage is presented together with proof-of-concept measurements. A working range of centimetres and periods in excess of 20 s were obtained in agreement with theory. The design is cascadable with a horizontal isolator for 3D isolation. © 1998 Elsevier Science B.V.

1. Introduction

The sensitivity of terrestrial gravitational wave detectors is limited at low frequencies by the degree of seismic isolation obtainable and by gravity gradient noise. Efforts to improve low-frequency seismic isolation of test masses have taken two main approaches. One is to use sensitive low-frequency accelerometer sensing and feedback to actively suppress seismic noise [1,2]. The other approach is passive isolation. Both methods require a test mass to be suspended with as low a resonant frequency as possible (either the small test mass within the accelerometer or the entire large test mass assembly). Suspending the entire assembly has some advantages in that low resonant frequencies are easiest to obtain using large dimensions and heavy masses. Also the entire structure may then be readily translated useful distances with low power actuation. For these reasons several designs for ultra-low frequency (ULF) horizontal pre-isolators have appeared in the literature recently [3-5].

While interferometer based gravitational wave detectors are primarily affected by horizontal motion, they are also affected by vertical seismic noise – through mechanical cross-coupling and also due to the local vertical not being exactly perpendicular to the long baseline laser beam. For this reason vertical isolation is also essential – and presents quite different problems from the horizontal. ULF horizontal isolators must be carefully aligned to cancel the effect of gravity and may be designed so that minimal energy is stored in elastic elements. Vertical isolators, however, have to support the entire weight against the force of gravity, and large energy storage in elastic elements is almost unavoidable.

The resonant frequency of a mass suspended by a linear elastic element can be given as $(g/l)^{1/2}$ where g is the acceleration due to gravity and l is the length extension under load. In order to obtain a low resonant frequency from a passive system without requiring extravagant vertical extension or pre-stress, it is possi-

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ble to arrange a non-linear force–displacement characteristic with a flat region near the operating point. If this is done then the resonant frequency is better expressed as $(k/m)^{1/2}$ where $k = \partial F/\partial y$ (F is the force, y the vertical displacement) is the effective spring constant at the operating point and m is the suspended mass. Significant effort has been applied to obtaining low vertical resonant frequencies in seismometer and gravimeter designs. One well-known approach is the LaCoste pendulum which makes use of a helical tension spring acting at 45° on a horizontal pendulum [6]. In order to obtain good cancellation of the non-linear effects, LaCoste innovated the use of a "zero-length" spring and obtained a very constant period with significant displacement (i.e. 37 ± 1 s for $\pm 9°$ pendulum deflection). Another technique which does not provide nearly such good cancellation is to sum a magnetic anti-spring with a mechanical spring. The Benioff variable-reluctance seismometer utilises this principle and while not aimed at long periods, obtains a spring constant cancellation of $\sim 90\%$ with an operating range of a few millimetres [7]. The VIRGO group have investigated this technique thoroughly on their super-attenuators (SA) using blade springs and permanent magnets [8,9]. A standard SA stage uses blade springs 354 mm long and the resonant period remains between 5 and 10 s for an operating range of ± 1.3 mm. In a special prototype SA with slightly more than double length blade springs and magnet array height they obtained an operating range of ± 5.2 mm for the same resonant period limits [10].

These results show that it is difficult to obtain good spring constant neutralisation over a large operating range. Another disadvantage of the magnetic anti-springs is the large temperature coefficient of the ferrite magnets (ferrite was chosen for minimum eddy current losses). In this Letter we present a theoretical analysis and experimental confirmation of a new ULF vertical stage – the torsion crank linkage (TCL). It uses only elastic material to produce the spring constant and modifies it with geometry to obtain the required long periods. We avoid the use of helical tension springs – in particular the problem of manufacturing powerful zero-length springs. Our geometry does not produce as unvarying a spring constant as the more ideal LaCoste geometry, but its operating range appears to be almost fifty times larger than a standard VIRGO SA stage for comparable parameters.

2. Design of torsion crank linkage

The TCL makes use of the non-linearity produced by a torsion-sprung crank arm connected to a suspension link, the loaded end of which is constrained to move in a vertical line. Fig. 1 shows the concept of the isolator. The simplest way of achieving the vertical motion constraint is to use a pair of cranks in a symmetrical arrangement as shown. The torsion rods are pre-stressed to provide an upward acting torque on the crank arms and the mass is supported by links with flexible joints at each end.

If we normalise the geometry variations by fixing the crank-arm length to a constant r, then there are three other parameters which may be changed to adjust the force–displacement characteristic: (i) the length l of the supporting link, (ii) the offset distance x from crank-arm centre to the vertical path followed by the end of the link, and (iii) the amount of pre-stress ϕ in the torsion spring. We define this pre-stress angle ϕ as the angle through which the crank arm must be turned (in the loading direction) from being unstressed to being horizontal.

In Fig. 2 the force–displacement characteristic is shown for the case where the suspension link and offset are the same length as the crank arm ($l = x = r$) and the pre-stress is varied in steps of 90°. It is apparent that as the pre-stress is increased, so the non-linearity in $F(y)$ becomes more pronounced until $\partial F/\partial y$ becomes zero and then negative (in the region where the crank-arm angle α is about 20°). This is

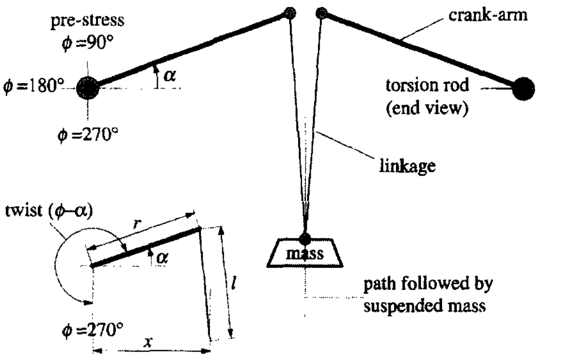

Fig. 1. Geometry schematic. A balanced pair of torsion-sprung crank arms connected to a single mass provides a non-linear single axis suspension.

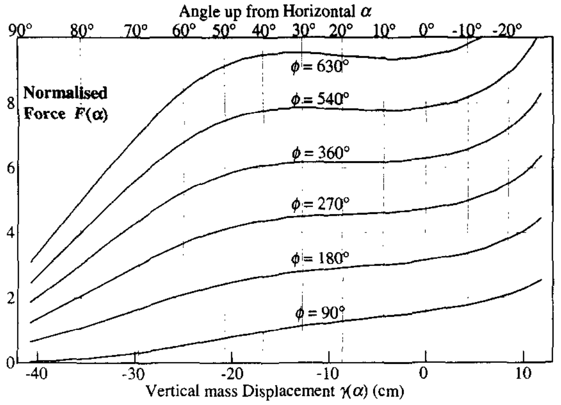

Fig. 2. Plot of force versus height. These curves are for equal length crank, link, and offset. The non-linearity in the characteristic increases with pre-stress and the slope changes from positive to negative for angles near 20°.

the desired effect. We wish to operate with the spring constant $\partial F/\partial y$ almost zero but remaining slightly positive, thus giving a very long vertical resonant period for a suspended mass. Typical resonant period versus vertical position curves for the same geometry ($l = x = r = 25$ cm) are shown in Fig. 3. This figure shows that as the pre-stress is increased towards 345°, the resonant period increases rapidly – as the force–displacement characteristic approaches zero slope.

We believe this is the simplest torsion based geometry capable of providing an almost flat $\partial F/\partial y$ together with inflection at the same point ($\partial^2 F/\partial y^2 = 0$). (The geometry can be arranged to also give $\partial^3 F/\partial y^3 = 0$ at the same point, but we show that this is not particularly advantageous for our application.) This design

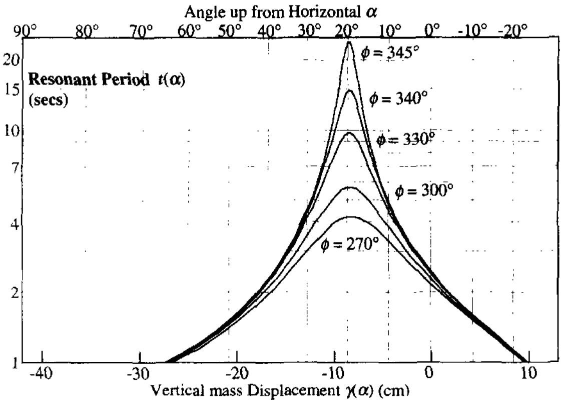

Fig. 3. Resonant period versus height. As the pre-stress is increased towards 345°, the force–displacement slope approaches zero and the resonant period increases rapidly.

has advantages over other schemes in compactness, ease of manufacture and adjustment, and application to arbitrarily large masses.

3. Geometry analysis

Consider a torsion lever arm (length r) at an angle α (anticlockwise and unwinding the torsion rod) from horizontal, and with pre-stress to horizontal ϕ as shown in Fig. 1. A link of length l is joined to it and the lower end constrained to move in a vertical line at a distance x from the torsion-rod centre. The energy E stored in the torsion rod as a function of angle will be

$$E(\alpha) = \tfrac{1}{2}k_a(\phi - \alpha)^2, \qquad (1)$$

where k_a is the angular spring constant of the torsion rod. If we neglect other energy storage mechanisms such as the finite flexibility of the joints etc, then the force acting in the vertical direction as a function of vertical position is given by the rate of change of this energy with position $\partial E/\partial y = (\partial E/\partial \alpha)(\partial \alpha/\partial y)$;

$$F(\alpha) = \frac{k_a(\phi - \alpha)}{y'(\alpha)}. \qquad (2)$$

It is the non-linearity of the derivative $y'(\alpha)$ that allows the force $F(\alpha)$ to remain almost constant with displacement in the region of a particular operating angle. The relationship between the vertical position y of the mass and the angle α is given by the geometry,

$$y(\alpha) = \sqrt{l^2 - (x - r\cos\alpha)^2} - r\sin\alpha + \text{const.} \qquad (3)$$

Using $t = (m/k)^{1/2}$ where k is the effective vertical spring constant $\partial F/\partial y = (\partial F/\partial \alpha)(\partial \alpha/\partial y)$ and $m = F(\alpha)/g$, it follows that

$$t(\alpha) = 2\pi\sqrt{\frac{F(\alpha)y'(\alpha)}{gF'(\alpha)}}. \qquad (4)$$

Since both $F(\alpha)$ and $F'(\alpha)$ are proportional to the torsional spring constant k_a this expression for the period $t(\alpha)$ is independent of this spring constant and only depends on the geometry function $y(\alpha)$ and the gravitational acceleration g.

In order to obtain a constant resonant period for small vertical motion, it is necessary to choose an operating point where the resonant period is at a turning point – i.e. $\partial t/\partial y = (\partial t/\partial \alpha)(\partial \alpha/\partial y = 0)$. Since

$\partial\alpha/\partial y$ is slowly varying and non-zero near the operating point, we may simply solve for $\partial t/\partial\alpha = 0$ deriving it from Eq. (4). Keeping only the variable terms from the numerator gives the turning point condition,

$$F'(\alpha)y'(\alpha) - F(\alpha)F''(\alpha)y'(\alpha) + F(\alpha)F'(\alpha)y''(\alpha) = 0. \quad (5)$$

Eqs. (4) and (5) become rather large once the substitutions from Eqs. (2) and (3) are made due to the multiple derivatives present. However, they are readily solved numerically.

The solution of Eq. (4) to obtain a particular period (we chose 20 s) together with Eq. (5) with fixed r (we chose 25 cm), is sufficient to fix only one of the three parameters l, x and ϕ. Thus, there is a two-dimensional space of solutions which needs to be searched for an optimum.

4. Optimising the geometry

We have searched the space of solutions of Eqs. (4) and (5) in an attempt to find an optimum geometry for which the resonant frequency is minimised over the largest possible vertical range. We determined that the special geometry giving a line of solutions with $\partial^3 F/\partial y^3 = 0$ gives rise to a false optimum because it is associated with a horizontal instability over part of the operating region. (The symmetric opposing cranks become horizontally unstable over approximately the upper half of the vertical range, reducing the useful range to about the same as that of a simple maximum. Horizontal motion occurs when both crank arms turn the same way allowing one arm to rise and the other to fall.) This horizontal instability can be overcome by adding constraints to limit motion to vertical translation only. The VIRGO magnetic anti-spring solves a similar problem by the use of horizontal centering wires, at the expense of significant complexity.

Other than this special line of solutions requiring horizontal stabilisation we find that there is no strongly optimum geometry. Over an area spanned by x (horizontal offset) varying from 10 cm to 40 cm and l (link length) varying from 10 cm to 50 cm using a crank-arm radius of 25 cm, approximately one third of the area gives roughly equally acceptable solutions (another third of the area is not so useful, and the remaining area has no realisable solutions). The tuning curves shown in Fig. 3 are typical of the useful solutions. The geometry is so flexible that it can be chosen depending on the available space for mounting a practical structure and the amount of pre-stress that can be applied to a particular torsion element. However, for dynamic balancing reasons (see Section 5) there is some advantage in choosing a geometry which has an operating angle close to 90° between the crank arm and link. Detailed graphical results of the parameter space analysis will be published elsewhere.

5. Functional test

We used a pair of high-tensile steel torsion rods 2.3 mm in diameter and 600 mm long, operated at a wind-up angle of just over 3/4 of a revolution. The crank-arm length r was 25 cm and the pivots were knife edges. The link length l was 29 cm and we suspended an adjustable mass of the order of a kilogram from the system. Adjustment is simple. If the resonant period is too short, then more mass is added and compensated by increased pre-stress to maintain a constant operating point (increasing ϕ in Figs. 2 and 3). If the motion becomes bistable with vertical instability in between two stable points, then both pre-stress and mass are reduced. In this way the system is readily tuned for long periods.

The fine wires forming the links were terminated by winding them around a 4 mm diameter rod at the ends of the pivot arms and at the mass. This avoided the significant energy storage of strong flexures and the anelastic bending problems of thin flexures for the large flex angles under test. However, this termination geometry is slightly inconsistent with the mathematical analysis (e.g. l is slightly dependent on α due to the 2 mm winding radius). Regardless of this, the general agreement between theory and experiment was remarkable, as shown in Fig. 4. As predicted, the resonant period remained greater than 10 s over a vertical range of almost ±2 cm. Scaling the crank arm up to match the 354 mm length of the VIRGO SA blade springs and choosing the same period limits (5 to 10 s) would give a vertical range of almost ±6 cm. Periods greater than 20 s were readily and repeatably achievable with a Q-factor of about 2.

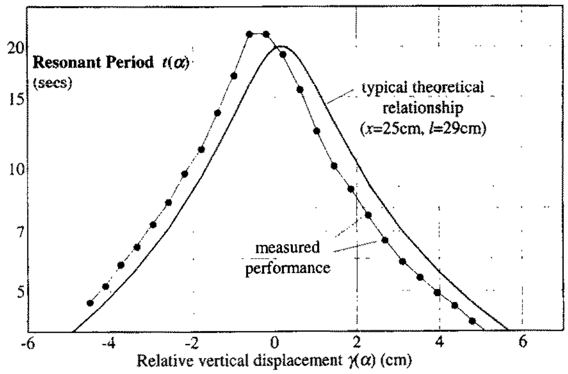

Fig. 4. Measured resonant period tuning curve. Points show the resonant period measured as a function of vertical position (lower axis) or angle (upper axis). The curve is a theoretical resonant period curve assuming simple pivots (as opposed to the wrapped flexures used).

6. A practical implementation

Fig. 5 shows a full scale conceptual implementation of a TCL vertical isolation stage, cascaded after a Scott–Russel horizontal isolator stage [3]. A passive ULF horizontal stage always requires rigid alignment with gravity and so needs to precede the vertical stage. The diagram shows a single beam Scott–Russel stage in the centre with torsion cylinders placed on either side of it to make good use of the space.

The torsion cylinder can be made from a coil spring or multiple concentric coil springs (used in torsion mode rather than tension). Alternatively many thin torsion rods may be summed together in a cylindrical arrangement to allow for support of arbitrary loads in a compact structure. It should be noted for practical purposes that in order to obtain a whole revolution of the wind-up angle in a cylindrical rod without exceeding the yield point in a typical spring steel (e.g. SAE1095 with torsional elastic limit 0.69 GPa) a ratio of length to diameter in excess of 360 : 1 is needed. In addition a helical instability occurs for wind-up angles greater than about 7/8ths of a turn unless the rod (or coil spring) is held in tension [11].

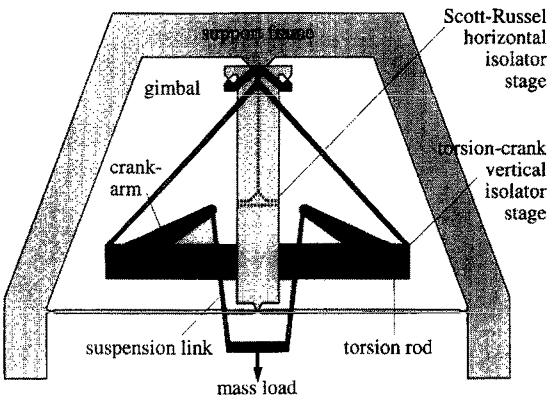

Fig. 5. Schematic of a practical implementation. A torsion crank vertical isolator with dual torsion cylinders is shown suspended from a single beam Scott-Russel horizontal isolator. This provides full three-dimensional isolation to very low frequencies.

In order to achieve good isolation at frequencies a decade or so above the resonant frequency the crank arm must be counter-weighted so that no vertical acceleration (to first order) appears on the load as a reaction to sudden vertical accelerations applied at the suspension point. In order to achieve this it is necessary to ensure that the instantaneous rotation centre of the crank-arm structure in response to vertical acceleration applied at the torsion-rod centre, occurs at the point where the link crosses an imaginary horizontal line through the torsion-rod centre. If the crank arm is considered as a rigid massive beam, then it is necessary to simply extend one end of the beam with mass beyond the torsion-rod/link attachment points, to place its centre of percussion to line up (perpendicular to the beam) with the required instantaneous rotation centre. This dynamic motion null will remain effective over a larger vertical range for geometries which have close to 90° operating angle between the crank arm and link.

Temperature compensation can be built into a design of this type by arranging that the differential expansion rates of say aluminium and steel cause additional torque to be applied to counteract the normal reduction in modulus of elasticity with temperature. However, it may be easier to control the temperature and even to use temperature as the means of level adjustment control.

7. Conclusion

We have introduced and demonstrated a torsion crank suspension system suitable for gravitational wave detectors. The TCL has a large vertical operating range and can provide exceptional vertical seismic isolation at very low frequencies. Cascaded with a

Scott–Russel ultra-low frequency two-dimensional horizontal stage, this structure offers full three-dimensional isolation to very low frequencies. Such isolation is expected to dramatically improve interferometer performance by greatly reducing the control forces required to maintain dark fringe locking.

Acknowledgement

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