

STAT 453: Introduction to Deep Learning and Generative Models

Ben Lengerich

Lecture 12: Optimization / Learning Rates

October 13, 2025



Midterm

- In-class next Wednesday
- Open-note, no electronics/Al
- Review session next Monday
- Study guide up today

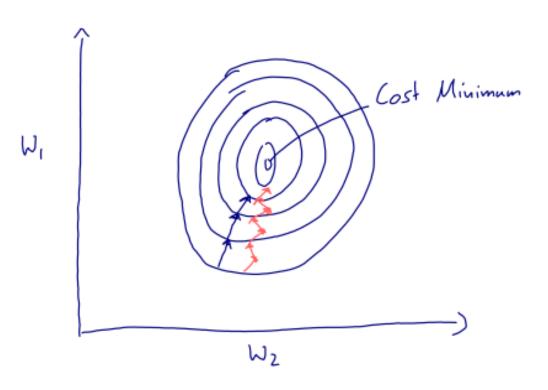


Today: Improving Optimization

- 1. Learning Rate Decay
- 2. Learning Rate Schedulers in PyTorch
- 3. Training with "momentum"
- 4. ADAM: Adaptive Learning Rates
- 5. Optimization algorithms in PyTorch



Minibatch Training Recap



- Minibatch learning is a form of stochastic gradient descent
- Each minibatch can be considered a sample drawn from the training set (where the training set is in turn a sample drawn from the population)
- Hence, the gradient is noisier

A **noisy** gradient can be:

- good: chance to escape local minima
- bad: can lead to extensive oscillation



Minibatch Training: Practical Tip

- Typical minibatch sizes: 32, 64, 128, 256, 512, 1024
 - why powers of 2?
- Usually, pick the biggest that is allowed by your GPU memory
- Practical tip: also want to make the batch size proportional to the number of classes in the dataset

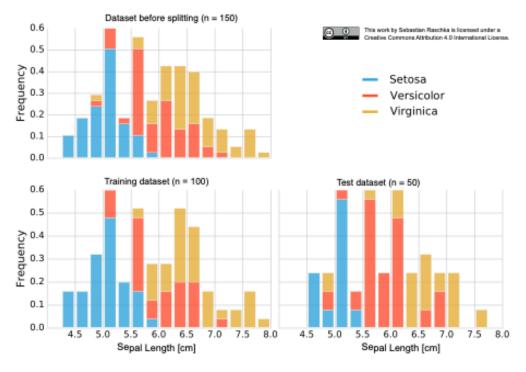


Figure 1: Distribution of Iris flower classes upon random subsampling into training and test sets.

Raschka, S. (2018). Model evaluation, model selection, and algorithm selection in machine learning.

https://arxiv.org/abs/1811.12808

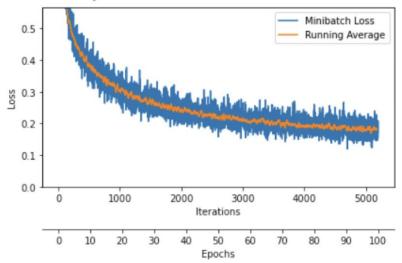
batchsize-1024.ipynb

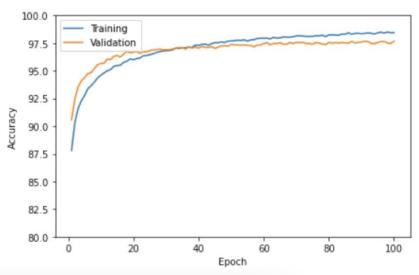
Epoch: 100/100 | Train: 98.45% | Validation: 97.67%

Time elapsed: 4.38 min

Total Training Time: 4.38 min

Test accuracy 97.08%





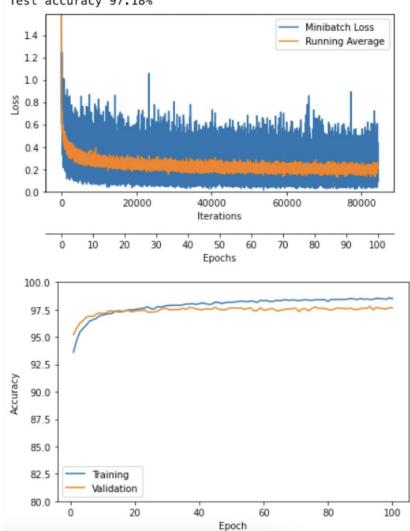
batchsize-64.ipynb

Epoch: 100/100 | Train: 98.50% | Validation: 97.65%

Time elapsed: 5.59 min

Total Training Time: 5.59 min

Test accuracy 97.18%







- Batch effects -- minibatches are samples of the training set, hence minibatch loss and gradients are approximations
- Hence, we usually get oscillations

To dampen oscillations towards the end of the training, we can

decay the learning rate

Danger of learning rate is to decrease the learning rate too early

Practical tip: try to **train the model without learning rate decay first**, then add it later \(\lambda_0 \) \(\lambda_0 \) \(\lambda_0 \)

You can also use the validation performance (e.g., accuracy) to judge whether Ir decay is useful (as opposed to using the training loss)

exponentially weighted average or Whole-training set loss minibator loss

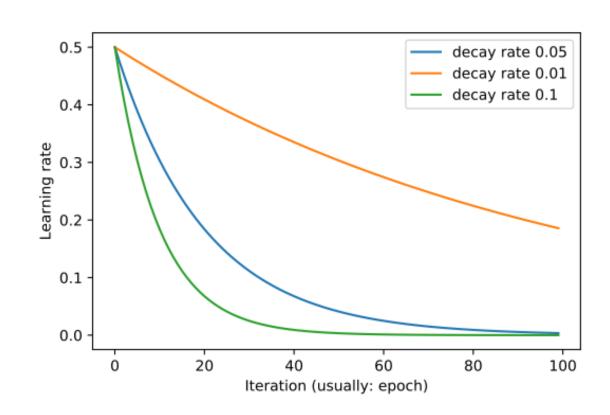


Most common variants for Ir decay:

1. Exponential Decay:

where k is the decay rate

$$\eta_t \coloneqq \eta_0 \mathrm{e}^{-\mathrm{k} \cdot t}$$





Most common variants for Ir decay:

1. Exponential Decay:

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where k is the decay rate

2. Halving the learning rate:

$$\eta_t \coloneqq \eta_{t-1}/2$$

where t is a multiple of T_0 (e.g. $T_0 = 100$)

3. Inverse decay:

$$\eta_t \coloneqq \frac{\eta_0}{1 + k \cdot t}$$



Many, many more!

E.g., Cyclical Learning Rate

Smith, Leslie N. "Cyclical learning rates for training neural networks." Applications of Computer Vision (WACV), 2017 IEEE Winter Conference on. IEEE, 2017.

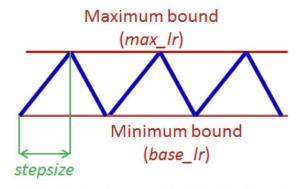
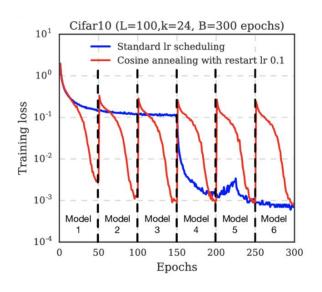


Figure 2. Triangular learning rate policy. The blue lines represent learning rate values changing between bounds. The input parameter *stepsize* is the number of iterations in half a cycle.

Cosine Learning rate decay

SGDR: STOCHASTIC GRADIENT
DESCENT WITH WARM RESTARTS,
ICLR 2017





Relationship between Learning rate and Batch Size

DON'T DECAY THE LEARNING RATE, INCREASE THE BATCH SIZE

Samuel L. Smith*, Pieter-Jan Kindermans*, Chris Ying & Quoc V. Le Google Brain {slsmith, pikinder, chrisying, qvl}@google.com

ABSTRACT

It is common practice to decay the learning rate. Here we show one can usually obtain the same learning curve on both training and test sets by instead increasing the batch size during training. This procedure is successful for stochastic gradient descent (SGD), SGD with momentum, Nesterov momentum, and Adam. It reaches equivalent test accuracies after the same number of training epochs, but with fewer parameter updates, leading to greater parallelism and shorter training times. We can further reduce the number of parameter updates by increasing the learning rate ϵ and scaling the batch size $B \propto \epsilon$. Finally, one can increase the momentum coefficient m and scale $B \propto 1/(1-m)$, although this tends to slightly reduce the test accuracy. Crucially, our techniques allow us to repurpose existing training schedules for large batch training with no hyper-parameter tuning. We train ResNet-50 on ImageNet to 76.1% validation accuracy in under 30 minutes.

Smith, S. L., Kindermans, P. J., Ying, C., & Le, Q. V. (2017). Don't decay the learning rate, increase the batch size. arXiv preprint arXiv:1711.00489.



Relationship between Learning rate and Batch Size

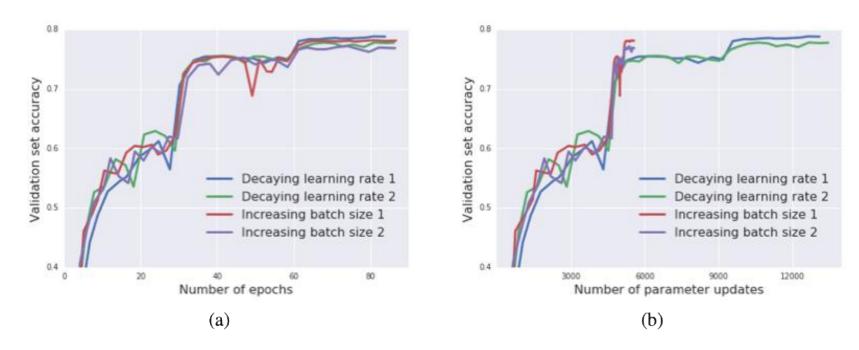


Figure 6: Inception-ResNet-V2 on ImageNet. Increasing the batch size during training achieves similar results to decaying the learning rate, but it reduces the number of parameter updates from just over 14000 to below 6000. We run each experiment twice to illustrate the variance.



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Learning Rate Schedulers in PyTorch

Option 1. Just call your own function at the end of each epoch:

```
def adjust_learning_rate(optimizer, epoch, initial_lr, decay_rate):
    """Exponential decay every 10 epochs"""
    if not epoch % 10:
        lr = initial_lr * torch.exp(-decay_rate*epoch)
        for param_group in optimizer.param_groups:
             param_group['lr'] = lr
```



Learning Rate Schedulers in PyTorch

Option 2. Use one of the built-in tools in PyTorch: (many more available)

(Here, the most generic version.)

```
CLASS torch.optim.lr_scheduler.LambdaLR(optimizer, lr_lambda, last_epoch=-1) [SOURCE]
```

Sets the learning rate of each parameter group to the initial Ir times a given function. When last_epoch=-1, sets initial Ir as Ir.

Parameters:

- optimizer (Optimizer) Wrapped optimizer.
- Ir_lambda (function or list) A function which computes a multiplicative factor given an integer
 parameter epoch, or a list of such functions, one for each group in optimizer.param_groups.
- last_epoch (int) The index of last epoch. Default: -1.

Example

```
>>> # Assuming optimizer has two groups.
>>> lambda1 = lambda epoch: epoch // 30
>>> lambda2 = lambda epoch: 0.95 ** epoch
>>> scheduler = LambdaLR(optimizer, lr_lambda=[lambda1, lambda2])
>>> for epoch in range(100):
>>> scheduler.step()
>>> train(...)
>>> validate(...)
```

Source: https://pytorch.org/docs/stable/optim.html



Learning Rate Schedulers in PyTorch

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Reproducibility in DL is tough

Need to report all training details – they all matter.

standard color augmentation in [21] is used. We adopt batch normalization (BN) [16] right after each convolution and before activation, following [16]. We initialize the weights as in [12] and train all plain/residual nets from scratch. We use SGD with a mini-batch size of 256. The learning rate starts from 0.1 and is divided by 10 when the error plateaus, and the models are trained for up to 60×10^4 iterations. We use a weight decay of 0.0001 and a momentum of 0.9. We do not use dropout [13], following the practice in [16].

http://openaccess.thecvf.com/content_cvpr_2016/html/He_Deep_Residual_Learning_CVPR_2016_paper.html

He K, Zhang X, Ren S, Sun J. Deep residual learning for image recognition. Proceedings of the IEEE conference on computer vision and pattern recognition 2016 (pp. 770-778).



Saving Models in PyTorch

Save Model

```
model.to(torch.device('cpu'))
torch.save(model.state_dict(), './my_model_2epochs.pt')
torch.save(optimizer.state_dict(), './my_optimizer_2epochs.pt')
torch.save(scheduler.state_dict(), './my_scheduler_2epochs.pt')
```

Load Model

Learning rate schedulers have the advantage that we can also simply save their state for reuse (e.g., saving and continuing training later)



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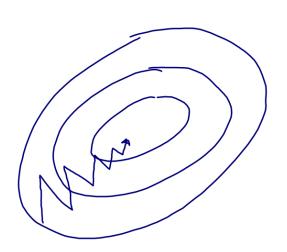
• Main idea: Let's dampen oscillations by using "velocity" (the speed of the "movement" from previous updates)



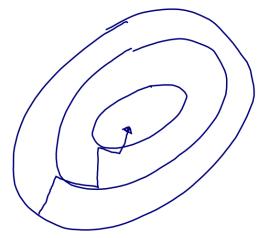
https://www.asherworldturns.com/zorbing-new-zealand/



 Main idea: Let's dampen oscillations by using "velocity" (the speed of the "movement" from previous updates)





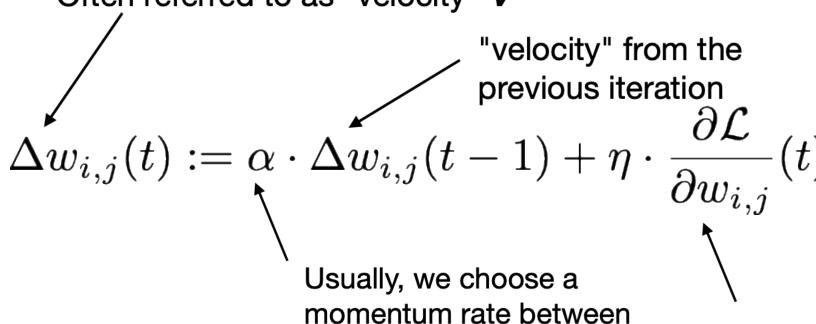


With momentum

Key take-away: Not only move in the (opposite) direction of the gradient, but also move in the "weighted averaged" direction of the last few updates



Often referred to as "velocity" V



0.9 and 0.999; you can

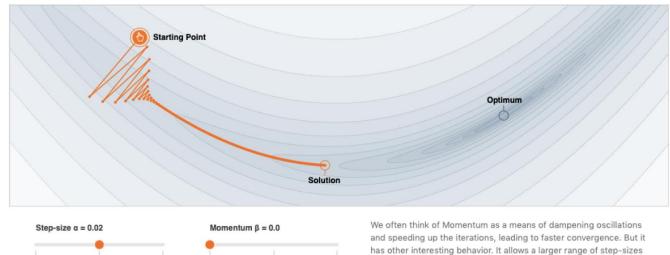
think of it as a "friction" or

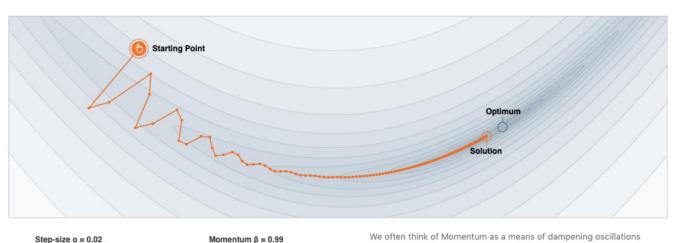
"dampening" parameter

Qian, N. (1999). On the momentum term in gradient descent learning algorithms. Neural Networks: The Official Journal of the International Neural Network Society, 12(1), 145–151. http://doi.org/10.1016/S0893-6080(98)00116-6

Regular partial derivative/ gradient multiplied by learning rate at current time step *t*







Source: https://distill.pub/2017/momentum/

and speeding up the iterations, leading to faster convergence. But it has other interesting behavior. It allows a larger range of step-sizes to be used, and creates its own oscillations. What is going on?

to be used, and creates its own oscillations. What is going on?



Momentum in PyTorch

CLASS torch.optim.SGD(params, lr=<required parameter>, momentum=0, dampening=0, weight_decay=0, [SOURCE nesterov=False) Implements stochastic gradient descent (optionally with momentum). Nesterov momentum is based on the formula from On the importance of initialization and momentum in deep learning. Parameters: • params (iterable) – iterable of parameters to optimize or dicts defining parameter groups Ir (float) – learning rate momentum (float, optional) – momentum factor (default: 0) weight_decay (float, optional) – weight decay (L2 penalty) (default: 0) dampening (float, optional) – dampening for momentum (default: 0) **nesterov** (bool, optional) – enables Nesterov momentum (default: False) Example

Source: https://pytorch.org/docs/stable/optim.html



Nesterov: A Better Momentum

We already know where the momentum part will push us in this step. Let's calculate the **new gradient** with that update in mind:

Before:

$$\Delta \mathbf{w}_t := \alpha \cdot \Delta \mathbf{w}_{t-1} + \eta \cdot \nabla_{\mathbf{w}} \mathcal{L}(\mathbf{w}_t)$$

$$\mathbf{w}_{t+1} := \mathbf{w}_t - \Delta \mathbf{w}_t$$

Nesterov:

$$\Delta \mathbf{w}_{t} := \alpha \cdot \Delta \mathbf{w}_{t-1} + \eta \cdot \nabla_{\mathbf{w}} \mathcal{L}(\mathbf{w}_{t} - \alpha \cdot \Delta \mathbf{w}_{t-1})$$

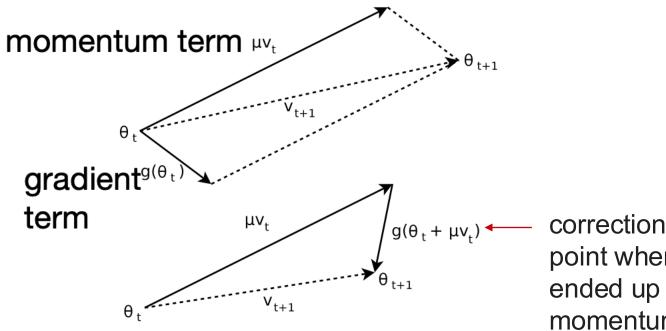
$$\mathbf{w}_{t+1} := \mathbf{w}_{t} - \Delta \mathbf{w}_{t}$$

Nesterov, Y. (1983). A method for unconstrained convex minimization problem with the rate of convergence o(1/k2). Doklady ANSSSR (translated as Soviet.Math.Docl.), vol. 269, pp. 543–547.

Sutskever, I., Martens, J., Dahl, G. E., & Hinton, G. E. (2013). On the importance of initialization and momentum in deep learning. ICML (3), 28(1139-1147), 5.



Nesterov: A Better Momentum



correction term (gradient of the point where you would have ended up via the standard momentum method)

Figure 1. (Top) Classical Momentum (Bottom) Nesterov Accelerated Gradient

Sutskever, I., Martens, J., Dahl, G. E., & Hinton, G. E. (2013). On the importance of initialization and momentum in deep learning. ICML (3), 28(1139-1147), 5.



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Adaptive Learning Rates

Many different flavors of adapting the learning rate

Rule of thumb:

- 1. decrease learning if the gradient changes its direction
- 2. increase learning if the gradient stays consistent



Adaptive Learning Rates

Rule of thumb:

- 1. decrease learning if the gradient changes its direction
- 2. increase learning if the gradient stays consistent

Step 1: Define a local gain (g) for each weight (initialized with g=1)

$$\Delta w_{i,j} := \eta \cdot g_{i,j} \cdot \frac{\partial \mathcal{L}}{\partial w_{i,j}}$$



Adaptive Learning Rates

Rule of thumb:

- 1. decrease learning if the gradient changes its direction
- 2. increase learning if the gradient stays consistent

Step 1: Define a local gain (g) for each weight (initialized with g=1)

$$\Delta w_{i,j} := \eta \cdot g_{i,j} \cdot \frac{\partial \mathcal{L}}{\partial w_{i,j}}$$

Step 2: Define a local gain (g) for each weight (initialized with g=1)

If gradient is consistent:

$$g_{i,j}(t) := g_{i,j}(t-1) + \beta$$

Else:

$$g_{i,j}(t) := g_{i,j}(t-1) \cdot (1-\beta)$$



RMSProp

- Unpublished (but very popular) algorithm by Geoff Hinton
- Based on Rprop [1]
- Very similar to another concept called AdaDelta
- Main idea: divide learning rate by an exponentially decreasing moving average of the squared gradients
 - RMS = "Root Mean Squared"
 - Takes into account that gradients can vary widely in magnitude
 - Damps oscillations like momentum (in practice, works better)

[1] Igel, Christian, and Michael Hüsken. "Improving the Rprop learning algorithm." Proceedings of the Second International ICSC Symposium on Neural Computation (NC 2000). Vol. 2000. ICSC Academic Press, 2000.



ADAM (Adaptive Moment Estimation)

- Probably the most widely used optimization algorithm in DL
- Combination of momentum + RMSProp

Momentum-like term:

$$\frac{\Delta w_{i,j}(t) := \alpha \cdot \Delta w_{i,j}(t-1) + \eta \cdot \frac{\partial \mathcal{L}}{\partial w_{i,j}}(t)}{\partial w_{i,j}(t)}$$

$$m_t := \alpha \cdot m_{t-1} + (1 - \alpha) \cdot \frac{\partial \mathcal{L}}{\partial w_{i,j}}(t)$$

RMSProp term:

$$r := \beta \cdot MeanSquare(w_{i,j}, t-1) + (1-\beta) \left(\frac{\partial \mathcal{L}}{\partial w_{i,j}(t)} \right)^2$$

ADAM update:

$$\overline{w_{i,j}} := w_{i,j} - \eta \frac{m_t}{\sqrt{r} + \epsilon}$$

Kingma, D. P., & Ba, J. (2014). Adam: A method for stochastic optimization. arXiv preprint arXiv:1412.6980.



ADAM (Adaptive Moment Estimation)

Algorithm 1: Adam, our proposed algorithm for stochastic optimization. See section 2 for details, and for a slightly more efficient (but less clear) order of computation. g_t^2 indicates the elementwise square $g_t \odot g_t$. Good default settings for the tested machine learning problems are $\alpha = 0.001$, $\beta_1 = 0.9$, $\beta_2 = 0.999$ and $\epsilon = 10^{-8}$. All operations on vectors are element-wise. With β_1^t and β_2^t we denote β_1 and β_2 to the power t.

```
Require: \alpha: Stepsize
Require: \beta_1, \beta_2 \in [0, 1): Exponential decay rates for the moment estimates
Require: f(\theta): Stochastic objective function with parameters \theta
Require: \theta_0: Initial parameter vector
   m_0 \leftarrow 0 (Initialize 1<sup>st</sup> moment vector)
   v_0 \leftarrow 0 (Initialize 2<sup>nd</sup> moment vector)
   t \leftarrow 0 (Initialize timestep)
   while \theta_t not converged do
       t \leftarrow t + 1
       g_t \leftarrow \nabla_{\theta} f_t(\theta_{t-1}) (Get gradients w.r.t. stochastic objective at timestep t)
      m_t \leftarrow \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t (Update biased first moment estimate) v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2 (Update biased second raw moment estimate) \widehat{m}_t \leftarrow m_t/(1 - \beta_1^t) (Compute bias-corrected first moment estimate)
       \hat{v}_t \leftarrow v_t/(1-\beta_2^t) (Compute bias-corrected second raw moment estimate)
       \frac{v_t \leftarrow v_{t/1}}{\theta_t \leftarrow \theta_{t-1} - \alpha \cdot \widehat{m}_t/(\sqrt{\widehat{v}_t} + \epsilon) \text{ (Update parameters)}}{\text{Also add a bias correction term}}
   end while
                                                                                             for better conditioning in earlier iterations
   return \theta_t (Resulting parameters)
```

Kingma, D. P., & Ba, J. (2014). Adam: A method for stochastic optimization. arXiv preprint arXiv:1412.6980.



Many more variants

Adam with weight decay

https://arxiv.org/pdf/1711.05101.pdf

Layer-wise Adaptive Rate Scaling (LARS)

https://arxiv.org/pdf/1708.03888.pdf



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Using different optimizers in PyTorch

- Same as for vanilla SGD (which we've seen before)
- More details at:

https://pytorch.org/docs/stable/optim.html

```
optimizer = torch.optim.SGD(model.parameters(), lr=0.01, momentum=0.9)
optimizer = torch.optim.Adam(model.parameters(), lr=0.0001)
```

Remember to save the optimizer state if you are using momentum / ADAM



ADAM in PyTorch

$$m_t := \alpha \cdot m_{t-1} + (1 - \alpha) \cdot \frac{\partial \mathcal{L}}{\partial w_{i,j}}(t)$$

$$m_{t} := \alpha \cdot m_{t-1} + (1 - \alpha) \cdot \frac{\partial \mathcal{L}}{\partial w_{i,j}}(t)$$

$$r := \beta \cdot MeanSquare(w_{i,j}, t - 1) + (1 - \beta) \left(\frac{\partial \mathcal{L}}{\partial w_{i,j}(t)}\right)^{2}$$

CLASS torch.optim.Adam(params, 1r=0.001, betas=(0.9, 0.999), eps=1e-08, weight_decay=0, amsgrad=False)

[SOURCE]

Implements Adam algorithm.

It has been proposed in Adam: A Method for Stochastic Optimization.

- params (iterable) iterable of parameters to optimize or dicts defining parameter groups
- Ir (float, optional) learning rate (default: 1e-3)
- betas (Tuple[float, float], optional) coefficients used for computing running averages of gradient and its square (default: (0.9, 0.999))
- eps (float, optional) term added to the denominator to improve numerical stability (default: 1e-8)

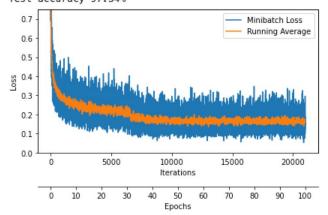
Source: https://pytorch.org/docs/stable/optim.html

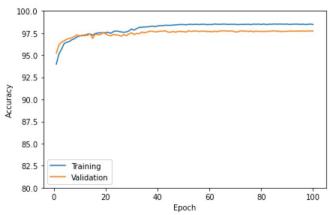


ADAM in PyTorch

sgd-scheduler-momentum.ipynb

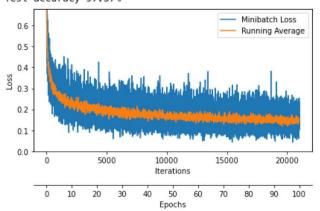
Epoch: 100/100 | Train: 98.47% | Validation: 97.73% Time elapsed: 4.60 min Total Training Time: 4.60 min Test accuracy 97.34%

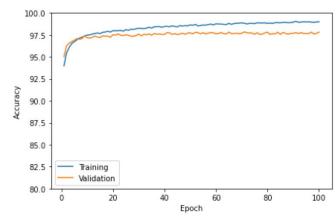




adam.ipynb

Epoch: 100/100 | Train: 99.00% | Validation: 97.82% Time elapsed: 4.71 min Total Training Time: 4.71 min Test accuracy 97.37%







Optimization is not a settled science

Improving Generalization Performance by Switching from Adam to SGD

Nitish Shirish Keskar, Richard Socher

(Submitted on 20 Dec 2017)

Despite superior training outcomes, adaptive optimization methods such as Adam, Adagrad or RMSprop have been found to generalize poorly compared to Stochastic gradient descent (SGD). These methods tend to perform well in the initial portion of training but are outperformed by SGD at later stages of training. We investigate a hybrid strategy that begins training with an adaptive method and switches to SGD when appropriate. Concretely, we propose SWATS, a simple strategy which switches from Adam to SGD when a triggering



Optimization is not a settled science

NEURAL NETWORKS (MAYBE) EVOLVED TO MAKE ADAM THE BEST OPTIMIZER

by bremen79

https://parameterfree.com/2020/12/06 /neural-network-maybe-evolved-tomake-adam-the-best-optimizer/

DEC 06

Disclaimer: This post will be a little different than my usual ones. In fact, I won't prove anything and I will just briefly explain some of my conjectures around optimization in deep neural networks. Differently from my usual posts, it is totally possible that what I wrote is completely wrong



"it is known that Adam will not always give you the best performance, yet most of the time people know that they can use it with its default parameters and get, if not the best performance, at least the second best performance on their particular deep learning problem."

"Usually people try new architectures keeping the optimization algorithm fixed, and most of the time the algorithm of choice is Adam. This happens because, as explained above, Adam is the default optimizer."

Questions?

