Efficient adjustment for complex covariates:
Gaining efficiency with DOPE

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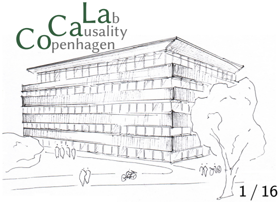


Outline

1. Covariate adjustment and motivation
2. Efficiency theory
3. Debiased Outcome-adapted Propensity Estimation

Adjusted treatment-specific mean of outcome

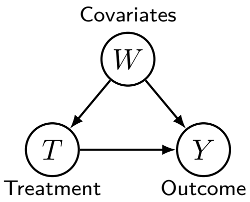
Template observation $T \in \{0,1\}$, $W \in \mathbb{R}^p$, $Y \in \mathbb{R}$.

- Adjusted mean for treatment $t \in \{0,1\}$ is

$$\chi_t \coloneqq \mathbb{E}\big[\mathbb{E}[\,Y \mid T = t, W]\big].$$

- Under the assumption of *no unmeasured confounding*:

$$\chi_t = \mathbb{E}\big[Y \mid \mathrm{do}(T = t)\big] \quad \text{and} \quad \chi_1 - \chi_0 = \mathrm{ATE}.$$



Estimation of adjusted mean

- Given i.i.d. observations $(T_i, W_i, Y_i)_{i\in[n]}$, we could produce an estimate $\widehat{g}(t, \cdot\,)$ of

  $$g(t, \cdot\,) := \mathbb{E}[\,Y \,|\, T = t, W = \cdot\,]$$

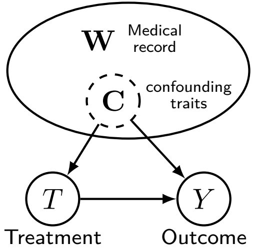
  and then use the estimator $\widehat{\chi}_t^{\mathrm{reg}} := \frac{1}{n}\sum_{i=1}^n \widehat{g}(t, W_i)$.
- $\widehat{\chi}_t^{\mathrm{reg}}$ is not $\sqrt{n}$-consistent – unless restrictive assumptions on g.
- The *AIPW estimator* is given by

  $$\widehat{\chi}_t^{\mathrm{aipw}} := \frac{1}{n}\sum_{i=1}^n \left(\widehat{g}(t, W_i) + \frac{\mathbb{1}(T_i = t)(Y_i - \widehat{g}(t, W_i))}{\widehat{m}_t(W_i)} \right),$$

  where $\widehat{m}_t(\cdot) = \widehat{\mathbb{P}}(T = t \,|\, W = \cdot)$ is an estimate of $m_t(\cdot) = \mathbb{P}(T = t \,|\, W = \cdot)$.

Challenges with complex and unstructured covariates

To meet the assumption of no unmeasured confounding, suppose we record a large/complex covariate $\mathbf{W}$.

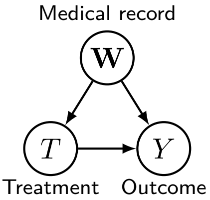


Problem 1

Medical record is difficult to model.

If $\mathbf{W}$ is a text variable we could:

- Apply a pretrained text embedding $\rightarrow \mathbf{Z} = \varphi(\mathbf{W}) \in \mathbb{R}^d$.
- Do standard adjustment based on $\mathbf{Z}$.



Embeddings need fine-tuning[1].

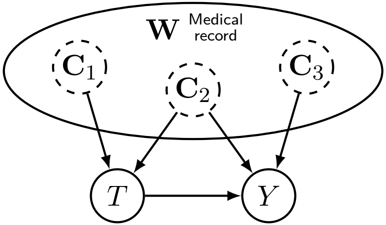
Is there an optimal way to fine-tune the embedding?

[1] Veitch, Wang, and Blei (2019) and Veitch, Sridhar, and Blei (2020)

Problem 2

Medical record might be highly predictive of treatment assignment

- Problematic for inverse propensity weights.
- Suppose that confounding traits can categorized:



- Can we formally and practically distinguish the information in $\mathbf{W}$?

Adjustment for covariate transformations

It can be sensible to adjust for $\mathbf{Z} = \varphi(\mathbf{W})$ rather than $\mathbf{W}$ itself.
Suppose $(T, \mathbf{W}, Y) \sim P$ for some $P \in \mathcal{P}$.

- The adjusted mean for $\mathbf{Z}$ is denoted by:

$$\chi_t(\mathbf{Z}; P) := \mathbb{E}_P[\mathbb{E}_P[Y \mid T = t, \mathbf{Z}]]$$

- We want $\mathbf{Z}$ such that

$$\mathbf{Z} \textit{ valid transformation} \quad \overset{\text{def}}{\iff} \quad \chi_t(\mathbf{Z}; P) = \chi_t(\mathbf{W}; P) \text{ for all } P \in \mathcal{P}.$$

Connections to adjustment sets

Example 1

Given a DAG $\mathcal{D}$ on the nodes $(T, \mathbf{W}, Y) \in \mathbb{R} \times \mathbb{R}^p \times \mathbb{R}$, let $\mathcal{P}$ contain all distributions consistent w.r.t. $\mathcal{D}$. Assume $\mathbf{W}$ is a *valid adjustment set*, equivalent with

$$\forall P \in \mathcal{P}: \qquad \chi_t(\mathbf{W}, P) = \mathbb{E}_P[\, Y \,|\, \mathrm{do}(T = t)\,].$$

Then

- A subset $\mathbf{Z} \subseteq \mathbf{W}$ is a valid adjustment set if and only if it is valid as a transformation.
- The *optimal adjustment set* is obtained by a pruning procedure[2]:
  iteratively remove covariates that are not predictive of outcome conditionally on treatment and remaining covariates.

[2] Henckel, Perković, and Maathuis (2022) and Rotnitzky and Smucler (2020)

Non-graphical example

Example 2

Suppose again that $\mathbf{W} \in \mathbb{R}^p$ and

$$Y = g\big(T, \|\mathbf{W}\|\big) + \varepsilon_Y, \qquad \mathbb{E}[\varepsilon_Y | T, \mathbf{W}] = 0,$$

for an unknown function g. Then

- $\mathbf{W}$ is *a priori* the only valid adjustment set.
- $\mathbf{Z} = \|\mathbf{W}\|$ is a valid transformation of $\mathbf{W}$. Follows from:

$$\mathbb{E}[Y \mid T, \mathbf{W}] = g(T, \mathbf{Z}) = \mathbb{E}[Y \mid T, \mathbf{Z}].$$

- For AIPW, should we estimate the propensity score based on $\mathbf{W}$ or $\mathbf{Z}$?

Efficiency bound

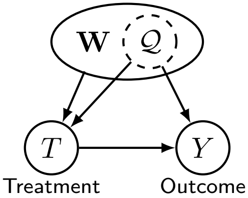
Hahn (1998) semiparametric efficiency bound:
If $\mathcal{P}$ is sufficiently dense, all RAL estimators of $\chi_t(\mathbf{W}; P)$ will have asymptotic variance of at least

$$\mathbb{V}_t(\mathbf{W}; P) := \mathrm{Var}_P\Big(\text{influence function of } \chi_t(\mathbf{W}; P)\Big).$$

- The bound can usually be improved if $\mathcal{P}$ is consistent w.r.t. a DAG (Example 1).
- Under rate conditions, AIPW/TMLE achieve this efficiency bound.
- If $\mathbf{Z}$ is a valid transformation of $\mathbf{W}$, then the efficiency bound is at most $\mathbb{V}_t(\mathbf{Z}; P)$.
- Note that $\chi_t(\mathbf{Z}; P)$ and $\mathbb{V}_t(\mathbf{Z}; P)$ depend only on the information in $\mathbf{Z} = \varphi(\mathbf{W})$.

Conditional outcome information

1. The information in $\sigma(\mathbf{W})$ that is "minimally sufficient" for prediction of $Y \mid T = t$ should be more efficient than $\mathbf{W}$ for adjustment:



2. We construct this information $\mathcal{Q}$ and prove that it is optimal to adjust for among all transformations that preserve the conditional outcome.
3. In particular, for all $P \in \mathcal{P}$ it holds that $\mathbb{V}_t(\mathcal{Q}; P) \leq \mathbb{V}_t(\mathbf{W}; P)$.

Debiased Outcome-adapted Propensity Estimation

General estimation framework for any outcome regression method with predictions of the form

$$\widehat{\mathbb{E}}[Y \mid T = t, \mathbf{W} = \mathbf{w}] = \widehat{h}(t, \widehat{\varphi}(\mathbf{w})).$$

Abridged version:

1. regress $(Y_i)_{i\in[n]}$ onto $(T_i, \mathbf{W}_i)_{i\in[n]}$ to obtain estimates $\widehat{\varphi}(\cdot)$ and $\widehat{h}(\cdot,\cdot)$.
2. regress $(T_i)_{i\in[n]}$ onto $(\widehat{\varphi}(\mathbf{W}_i))_{i\in[n]}$ to obtain an estimate of the form $\widehat{m}_t(\cdot) = \widetilde{m}_t(\widehat{\varphi}(\cdot))$.
3. Compute AIPW estimate based on nuisance estimates $\widehat{h}(t, \widehat{\varphi}(\cdot))$ and $\widehat{m}_t(\cdot)$.

Related to existing works and estimation methods[3].

[3] Shortreed and Ertefaie (2017), van de Geer (2019), Ju, Benkeser, and van der Laan (2020), and Benkeser, Cai, and van der Laan (2020).

Example in single-index model

For $\mathbf{W} \in \mathbb{R}^p$, the single-index model assumes that

$$Y = h(T, \mathbf{W}^\top \theta) + \varepsilon, \qquad \mathbb{E}[\varepsilon \mid T, \mathbf{W}] = 0,$$

where $h(\cdot, \cdot)$ and $\theta \in \mathbb{R}^p$ are unknown.

- Can apply DOPE with $\widehat{\varphi}(\mathbf{w}) = \mathbf{w}^\top \widehat{\theta}$, and with h and θ estimated using any single-index regression method.
- Paper contains a simulation study of this procedure as well as application to real data.

Conclusion

- Some ideas for efficient adjustment in graphical models generalize to a non-graphical setting.
- It can pay off to adjust for an outcome-adapted transformation in certain situations.
- See arXiv preprint[4] for further details, in particular: general information-based adjustment framework, efficiency results, asymptotic analysis of DOPE and an application to real data.

Thank you for listening 😎

[4] Christgau and Hansen (2024)

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