Powerlaw Explained

A Guided Tour Through the Mysteries of Force-based Motion Planning

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Overview

1. Basics of Motion Planning
2. Great, now what does that actually mean?
3. Explaining my Visualization Choices
4. Conclusions and Future Work

Overview

Motion Planning

- Motion planning is one of the most fundamental skills a human being possesses
- Being able to avoid collisions allows us to interact with other humans
- Teaching a robot how to approach this problem is very hard

Problem Definition

- Given a set of agents A, each agent has:
  - p^t position at time t
  - v^t velocity at time t
  - g a goal position
  - r a radius
- Each agent is non-holonomic, DOF of the control space equals DOF of the state space
- We want:
  - $|\mathrm{p}_a^{-1} - \mathrm{g}_a| < \epsilon$
  - $|\mathrm{p}_a^t - \mathrm{p}_b^t| < r_a + r_b$

Goal Driving Force

- Defined as the vector pointing directly at the goal
- Getting to the goal is a robots prime directive

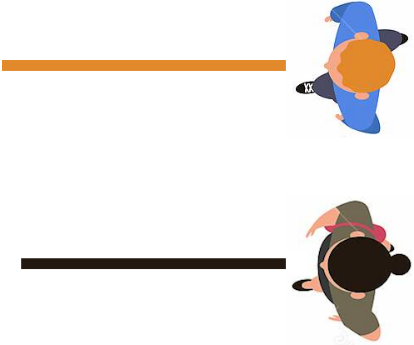
$$\mathsf{F}_{goal} = \mathsf{g} - \mathsf{p}$$

Current Ecosystem

- Geometric approaches
  - RVO & ORCA
  - **PowerLaw**
  - Helbing
- Learning approaches
  - KDMA
  - CrowdNav/CADRL
  - NavDreams

Time to Collision

- Humans use *Time-to-Collision (ttc)*, or τ, as a metric for avoiding collisions



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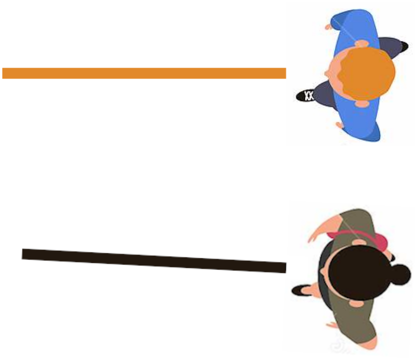

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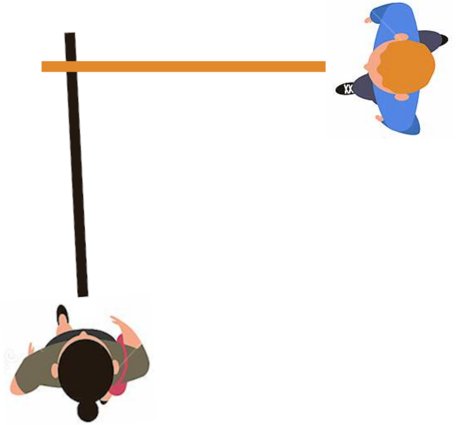
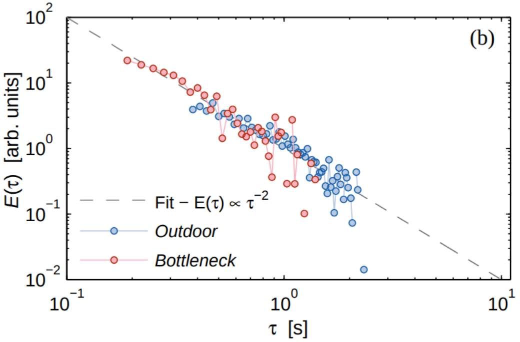
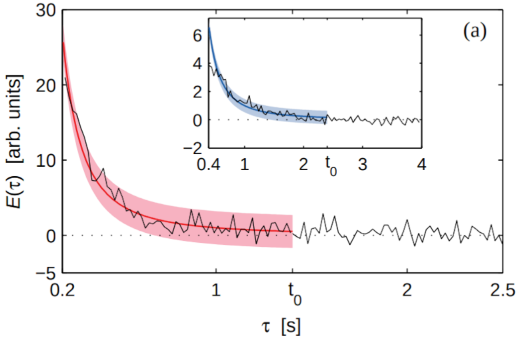


Powerlaw

- Humans use *Time-to-Collision (ttc)*, or τ, as a metric for avoiding collisions
- When plotting the ttc against a pairwise density function a clear trend emerges





Powerlaw

- From that we can generate a model for approximating the energy of a state based on the ttc:

$$E(\tau) = \frac{k}{\tau^2} e^{-\tau/\tau_0}$$

where τ is the ttc,

k is a constant that sets the units,

and τ_0 is the time horizon

Powerlaw

- This directly implies that the gradient of the energy is the repulsive force experienced by pedestrians

$$\mathsf{F}(\tau) = -\nabla_{\mathsf{r}} \left(\frac{k}{\tau^2} e^{-\tau/\tau_0} \right)$$

where ∇_{r} is the spacial gradient

- Integrating this force results in a collision free velocity
- Combining this with the goal-directed force satisfies both goal-directed behavior and collision-free behavior

High Level

- There is a force driving each agent to the goal
- Each agent enacts some sort of force on every other agent
- This inter-agent force is sufficient to avoid all collisions between agents
- This combination results in a SOTA decentralized motion planning algorithm

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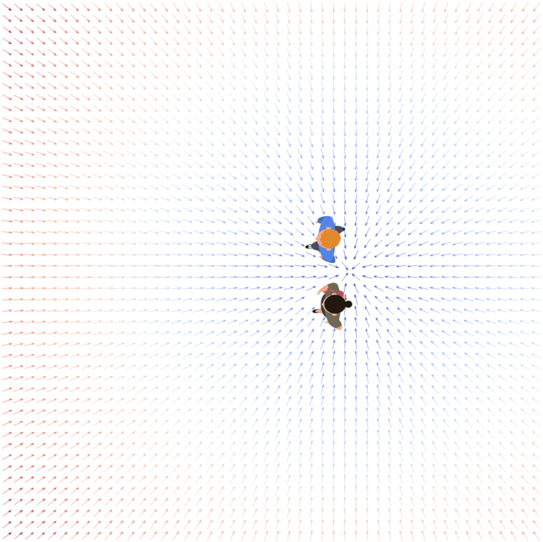
Context

- Thus far everything has been abstract, so lets contextualize

Goal Driving Force



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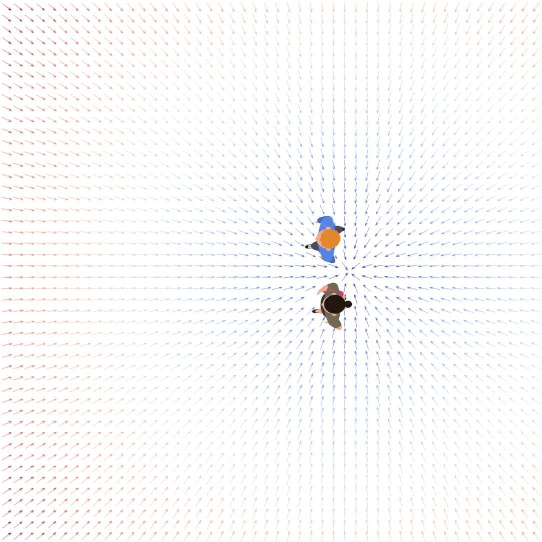
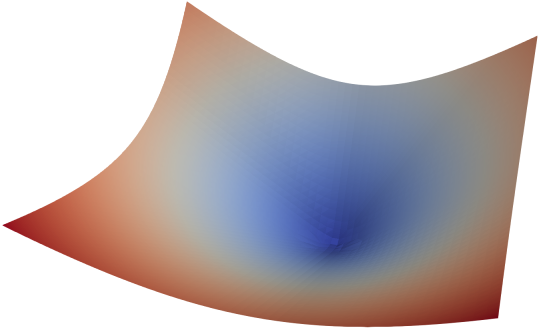
$$F_{goal} = g - p$$

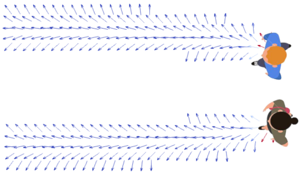
Goal Driving Force



Goal Driving Surface



Combined Induced Forces



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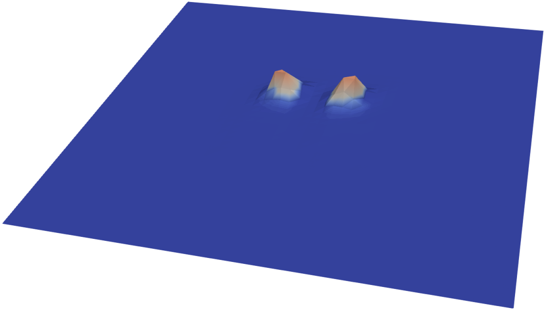
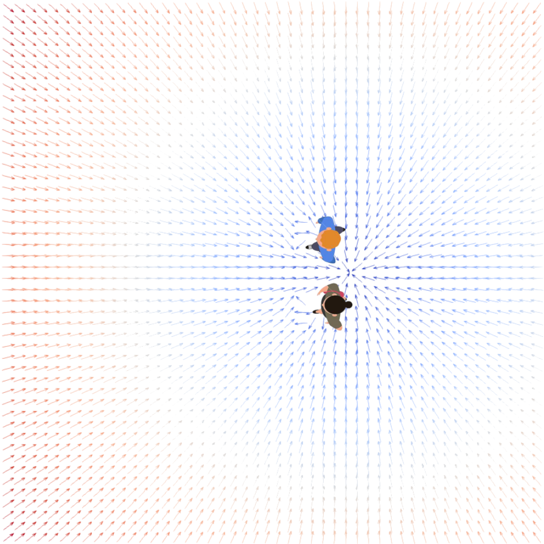
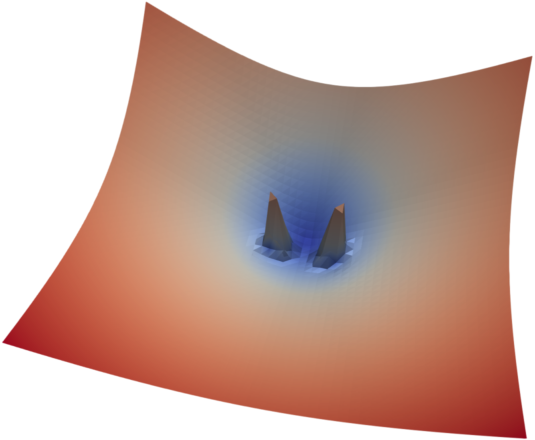


Powerlaw Forces



Powerlaw Surface



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Simplicity of the Output

- Motion planning algorithms work mostly with velocities and forces
- While there are a decent number of steps in the pipeline, they all deal with something that is relatively basic to visualize
- I wanted to build intuition in a similar way to the way a spiral wishing well works

Spiral Wishing Well

Vector Fields

- I couldn't just jump straight to a surface
- Vector fields are a good middle ground that show both directionality and magnitude of velocities
- Still have the problem of not really being able to convey how "strong" the repulsive force is

Delaunay Triangulation

- With the Powerlaw algorithm being based on something similar to gradient descent I knew I would have to develop some sort of surface to convey this
- Delanunay was a simple way to create that surface
- I could vary which of the magnitudes to plot as the z axis, the spacial derivative is something I left to the reader

Overview

Conclusions

- Bringing nice visualizations to fruition is both incredibly frustrating and rewarding
- A lot of my intuition as to how Powerlaw would look was wrong
- As robots become more involved in daily life it will be important to understand how they work

Future Work

- Visualize in velocity space rather than positional space
- Include a “true” geometric planner like ORCA
- Try to visualize something non-numerical, like CrowdNav