# Powerlaw Explained

## A Guided Tour Through the Mysteries of Force-based Motion Planning

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# Overview

1. Basics of Motion Planning
2. Great, now what does that actually mean?
3. Conclusions, Contributions, and Future Work

# Overview

# Problem Definition

- Given a set of agents $A$, each agent has:
  - $\mathrm{p}^t$ position at time $t$
  - $\mathrm{v}^t$ velocity at time $t$
  - $\mathrm{g}$ a goal position
  - $r$ a radius
- Each agent is non-holonomic, DOF of the control space equals DOF of the state space
- We want:
  - $|\mathrm{p}_a^{-1} - \mathrm{g}_a| < \epsilon$
  - $|\mathrm{p}_a^t - \mathrm{p}_b^t| < r_a + r_b$

# Motion Planning

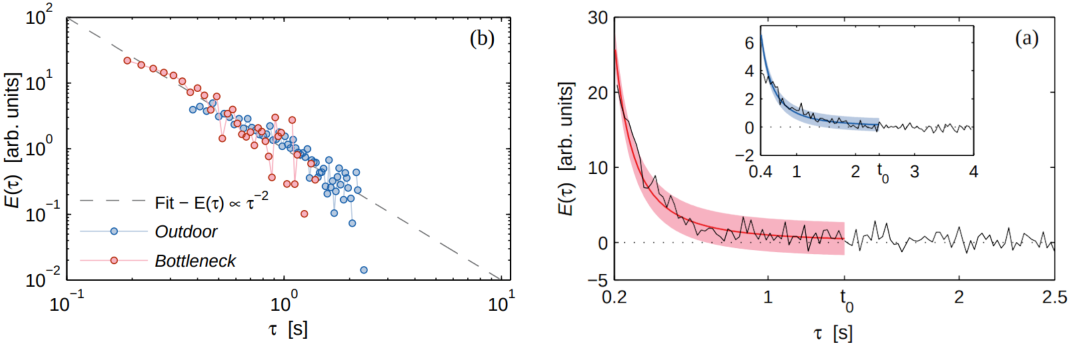
- Motion planning is one of the most fundamental skills a human being possesses
- Being able to avoid collisions allows us to interact with other humans
- Teaching a robot how to approach this problem is very hard

# Current Ecosystem

- Geometric approaches
  - RVO & ORCA
  - **PowerLaw**
  - Helbing
- Learning approaches
  - KDMA
  - CrowdNav/CADRL
  - NavDreams

# Powerlaw

- Humans use *Time-to-Collision (ttc)*, or $\tau$, as a metric for avoiding collisions
- When plotting the ttc against a pairwise density function a clear trend emerges



# Powerlaw

- From that we can generate a model for approximating the energy of a state based on the ttc:

$$E(\tau) = \frac{k}{\tau^2} e^{-\tau/\tau_0}$$

where $\tau$ is the ttc,

$k$ is a constant that sets the units,

and $\tau_0$ is the time horizon

# Powerlaw

- This directly implies that the gradient of the energy is the repulsive force experienced by pedestrians

$$\mathsf{F}(\tau) = -\nabla_{\mathsf{r}} \left( \frac{k}{\tau^2} e^{-\tau/\tau_0} \right)$$

where $\nabla_{\mathsf{r}}$ is the spacial gradient

- This force is calculated for each agent and combined with a goal driving force
- Integrating this force results in a collision free and goal directed velocity

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# High Level

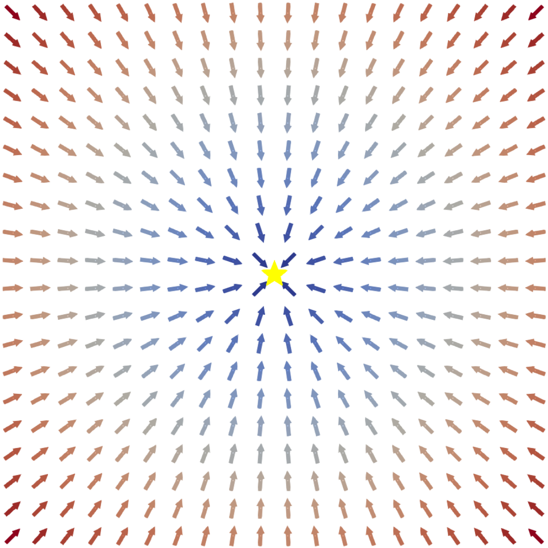
- There is a force driving each agent to the goal
- Each agent enacts some sort of force on every other agent
- This inter-agent force is sufficient to avoid all collisions between agents
- This combination results in a SOTA decentralized motion planning algorithm

# Goal Driving Force

- Defined as the vector pointing directly at the goal
- Can be scaled to promote collision-free decisions over goal-directed ones

$$\mathsf{F}_{goal} = \mathsf{g} - \mathsf{p}$$

# Goal Driving Force



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