

# Midterm

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Analysis of Linear Systems

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1a.

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

calculated in matlab

$$x_1 + 4x_3 = \bigodot_{\mathbb{R}^5} \implies -4x_3 = x_1$$

$$x_2 + 6x_3 = \bigodot_{\mathbb{R}^5} \implies -6x_3 = x_2$$

$$\mathcal{N}(A) = \text{span} \left( \left\{ \begin{bmatrix} -4 \\ -6 \\ 1 \end{bmatrix} \right\} \right)$$

$$\mathcal{R}(A^T) = \text{span} \left( \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\} \right)$$

1b.

$$U = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ -6 \\ 1 \end{bmatrix} \right\}$$

$$P_1 = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$$