

Homework 8

Alex Day

Analysis of Linear Systems

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3a.

$$\begin{bmatrix} \langle y_1, y_1 \rangle & \langle y_1, y_2 \rangle & \langle y_1, y_3 \rangle \\ \langle y_2, y_1 \rangle & \langle y_2, y_2 \rangle & \langle y_2, y_3 \rangle \\ \langle y_3, y_1 \rangle & \langle y_3, y_2 \rangle & \langle y_3, y_3 \rangle \end{bmatrix} \alpha \begin{bmatrix} \langle y_1, x \rangle \\ \langle y_2, x \rangle \\ \langle y_3, x \rangle \end{bmatrix}$$

$$\alpha = \begin{bmatrix} -0.0415 & 0.3518 & -0.9677 \end{bmatrix}^T$$

$$m_0 = \alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3$$

$$m_0 = \begin{bmatrix} 0.0461 & 1.2826 & 0.8233 & 0.2227 \end{bmatrix}$$

3b.

4a.

$$\begin{bmatrix} \langle y_1, y_1 \rangle & \langle y_1, y_2 \rangle \\ \langle y_2, y_1 \rangle & \langle y_2, y_2 \rangle \end{bmatrix} \alpha \begin{bmatrix} \langle y_1, x \rangle \\ \langle y_2, x \rangle \end{bmatrix}$$

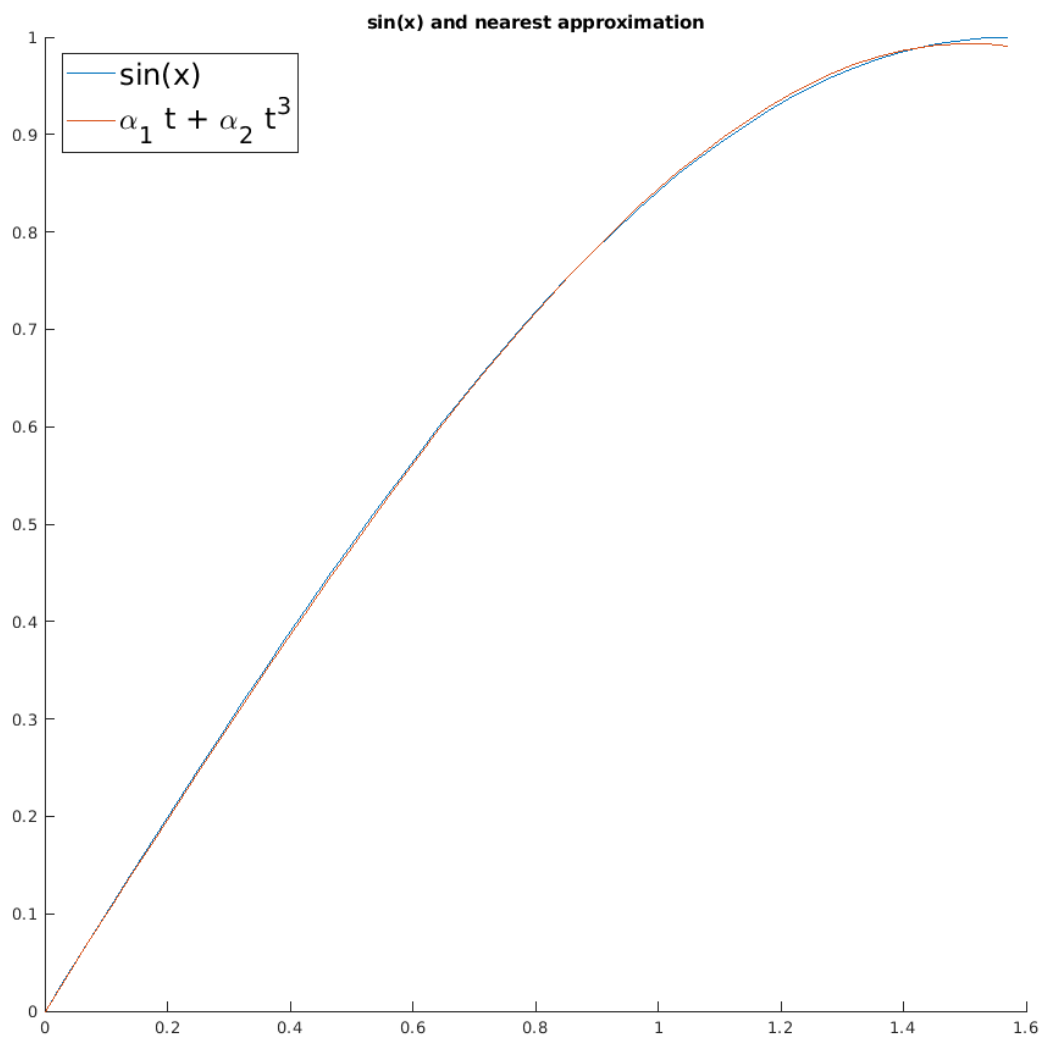
$$\alpha = \begin{bmatrix} 0.9888 & -0.1451 \end{bmatrix}^T$$

$$m_0 = \alpha_1 y_1 + \alpha_2 y_2$$

$$m_0 = \frac{(3360 * (\pi^2 - 10) * t^3)}{\pi^7} - \frac{(240 * (2 * \pi^2 - 21) * t)}{\pi^5}$$

4b.

$$\|x(\dot{}) - x_a(\dot{})\|_2 \approx 0.00347617397$$



4c.

$$\begin{aligned}\hat{v}_1 &= \frac{t}{\|t\|} \\ &= \frac{t}{\sqrt{\pi^{\frac{3}{24}}}}\end{aligned}$$

$$\begin{aligned}\hat{v}_2 &= \langle \hat{v}_1, y_2 \rangle v_1 \\ &= \left(\int_0^{\frac{\pi}{2}} \left(\frac{t}{\sqrt{\pi^{\frac{3}{24}}}} t^3 \right)^2 dt \right) \frac{t}{\sqrt{\pi^{\frac{3}{24}}}}\end{aligned}$$