Homework 5

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Analysis of Linear Systems

September 14, 2021

1c. A matrix M is symmetric if and only if $M = M^T$. This implies that the matrix takes the form $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ where $a_{12} = a_{21}$. The basis for matricies of this form is:

$$\boxed{U = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}}$$

1d.

$$\mathcal{L}_M(Q) = \begin{bmatrix} 2 & -2 \\ 6 & 5 \end{bmatrix}^T Q + Q \begin{bmatrix} 2 & -2 \\ 6 & 5 \end{bmatrix}$$

$$\begin{aligned} \mathcal{L}_M\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) &= \begin{bmatrix} 2 & -2 \\ 6 & 5 \end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 6 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 4 & -2 \\ -2 & 0 \end{bmatrix} = 4u_1 + 0u_2 + (-2)u_3 \\ &= \begin{bmatrix} 4 & 0 & -2 \end{bmatrix}_U \end{aligned}$$

$$\begin{aligned} \mathcal{L}_M\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) &= \begin{bmatrix} 2 & -2 \\ 6 & 5 \end{bmatrix}^T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 6 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 6 \\ 6 & 10 \end{bmatrix} = 0u_1 + 10u_2 + 6u_3 \\ &= \begin{bmatrix} 0 & 10 & 6 \end{bmatrix}_U \end{aligned}$$

$$\begin{aligned} \mathcal{L}_M\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) &= \begin{bmatrix} 2 & -2 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 6 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 12 & 7 \\ 7 & -4 \end{bmatrix} = 12u_1 + (-4)u_2 + 7u_3 \\ &= \begin{bmatrix} 12 & -4 & 7 \end{bmatrix}_U \end{aligned}$$

$$L_M = \begin{bmatrix} 4 & 0 & -2 \\ 0 & 10 & 6 \\ 12 & -4 & 7 \end{bmatrix}$$

1f.

$$\begin{aligned}
\mathcal{L}_M(Q) &= \begin{bmatrix} -10 & 0 \\ 0 & -20 \end{bmatrix} \\
&= L_m Q \\
&= \begin{bmatrix} 4 & 0 & -2 \\ 0 & 10 & 6 \\ 12 & -4 & 7 \end{bmatrix} Q
\end{aligned}$$