Homework 2

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Analysis of Linear Systems

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6a. $\forall x \in X, 0 \cdot x = \odot$

Proof. Let $x \in X$. So,

$$\begin{aligned}
0x &= 0x \\
&= (0+0)x && \text{(addative identity)} \\
&= 0x + 0x && \text{(distribution)} \\
0x + (-0x) &= 0x + 0x + (-0x) \\
\odot &= 0x + \odot && \text{(addative inverse)} \\
\odot &= 0x && \text{(addative identity)}
\end{aligned}$$

$\square$

6a. $\forall x \in X, (-x) = (-1) \cdot x$

Proof. Let $x \in X$. So,

$$\begin{aligned}
x + (-1) \cdot x &= x + (-1) \cdot x \\
&= 1 \cdot x + (-1) \cdot x && \text{(addative identity)} \\
&= (1-1) \cdot x && \text{(distribution)} \\
&= 0 \cdot x && \text{(addative inverse)} \\
&= 0 && \text{(see 6a.)} \\
x + (-1) \cdot x &= 0 \\
(-1) \cdot x &= -x
\end{aligned}$$

$\square$

7a. $W_1 \cap W_2$ is a subspace of $(V, \mathscr{F})$

1. $W_1 \cap W_2 \subset V$ This is true by the definition of intersection
2. $W_2 \cap W_2$ is closed under vector addition

$$\begin{aligned}
&\text{let } y_1, y_2 \in W_1 \cap W_2 \\
&y_1, y_2 \in W_1 \wedge y_1, y_2 \in W_2 && \text{(defition of intersection)} \\
&y_1 + y_2 \in W_1 \wedge y_1 + y_2 \in W_2 && \text{(defition of vector space)} \\
&\text{therefore } y_1 + y_2 \in W_1 \cap W_2
\end{aligned}$$

3. $W_2 \cap W_2$ is closed under scalar multiplication

$$\begin{aligned}
&\text{let } y_1 \in W_1 \cap W_2, \alpha \in \mathscr{F} \\
&y_1 \in W_1 \wedge y_1 \in W_2 && \text{(defition of intersection)} \\
&\alpha y_1 \in W_1 \wedge \alpha y_1 \in W_2 && \text{(defition of vector space)} \\
&\text{therefore } \alpha y_1 \in W_1 \cap W_2
\end{aligned}$$

7b.

$$\text{Let} \qquad \begin{aligned} W_1 &= \{x \in \mathbb{R} | c^T x = \odot, c^T = \begin{bmatrix} 1 & 1 \end{bmatrix}\} \\ W_2 &= \{x \in \mathbb{R} | c^T x = \odot, c^T = \begin{bmatrix} 1 & -1 \end{bmatrix}\} \end{aligned}$$

The set $W_1 \cup W_2$ is not closed under vector addition:

$$
\begin{aligned}
w_1 &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \in W_1 \\
w_2 &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \in W_2
\end{aligned}
$$

$$
w_3 = w_1 + w_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}
$$

$$
\begin{aligned}
&w_3 \notin W_1 \land w_3 \notin W_2 \\
&w_3 \notin W_1 \cap W_2
\end{aligned}
$$