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HW5 : Counting Problems
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1) Unusual has 5 unique letters
{U, N, S, A, L} is the only unique subset
and 5! different strings can be formed, 5! = 120.

2) $\binom{13}{2} \cdot \binom{4}{2} \cdot \binom{4}{2} \binom{11}{1} \cdot \binom{4}{1}$ ← suit for 5th card.
← 5th card
↑ 13 types of cards chosing 2 pairs
↑ suits for pair 1
↑ suits for pair 2

$= 78 \cdot 6 \cdot 6 \cdot 44 =$ 123552 different ways.

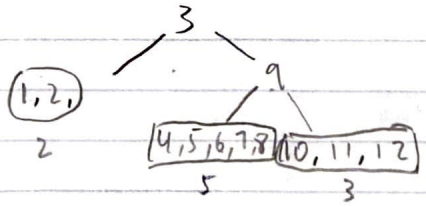
3) 16 songs, 7 couples, 1 couple has at most 1 song

$n_{Cr} = \frac{n!}{r!\,(n-r)!}$, so $16C_1 \cdot 15_{C6}$ ← 6 couples
↑ 1 couple fighting

$= \frac{16}{1!(16-1)!} \cdot \frac{15!}{6! \cdot (15-6)!}$

$= 16 \cdot 5005 =$ 80,080 different ways

4) Nodes: 1, 2, 4, 5, 6, 7, 7, 10, 11, 12 root = 3, n.c = 9



for nodes n: $\frac{2n\,cn}{n+1}$

2: $\frac{4c_2}{3} = 2$

5: $\frac{10c_5}{6} = 42$

3: $\frac{6c_3}{4} = 5$

total bsts: $2 \cdot 42 \cdot 5 =$ 420

5) $10 \cdot \binom{10}{4} + 10\binom{10}{3} = 2100 + 1200 = \boxed{3300}$