Satellite Interceptor Control

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February 2024

1 Overview

Ideally, a satellite interceptor would be launched as late as possible to ensure minimum time-to-kill. Therefore, the point of interception to be targeted lies on the plane that includes the launch site and is normal to the orbit of the target. For a zero inclination, zero eccentricity target with an interceptor launched from the equator, the point of interception is directly overhead the launch site. For cases where the target trajectory passes to either side of the launch site, the point is off to that side.

For our two-stage interceptor, both stages operate under a pure open loop control system. Guidance calculates the correct pitch rate to guarantee (approximately) the trajectory passes through the chosen intercept point, and then the vehicle flies with that parameter. When the payload separates from the second stage, it uses a reaction control system to closed loop refine the closest approach to be at the intercept point and occurring at the same time. We call this last step the terminal homing phase.

2 Open Loop Guidance

The steps of this algorithm are as follows:

1. Calculate the point of interception as lying both on the target trajectory and on the plane normal to the target trajectory passing through the launch point x_0
2. Calculate the pitch rate $\dot{\theta}$ required to fly through the intercept point x_i
3. Calculate the launch time t_0 such that the interceptor passes through the intercept point at the same time as the target t_i
4. Fly the vehicle using the calculated pitch rate

Linear tangent steering assumes that the interceptor pitches over at a constant rate. In this case, where the pitching occurs only in the direction of the intercept point, we call this the pitch rate. Thus, the problem is finding the pitch rate that intercepts the target trajectory at the intercept point. We will simplify by assuming the pitch rate is linearly dependent on the angle off up that the intercept point lies at. The gain on this dependence is found experimentally.

Our first big problem is therefore defining the intercept point x_i. This point is the closest point in the target spacecraft's trajectory to the launch site. We use the golden section method to iteratively search for the intercept point, using the distance from the launch site as the optimizing function. The minimum bound is the current time and the maximum bound is one hour from the current time (roughly one orbit for a target spacecraft at 500km up, about half the maximum altitude of the interceptor).

Using the intercept point, the next problem is determining the launch time. For this we need time-of-flight to the intercept point. Let us first define a method for finding the position at a given time since launch, represented in the ship-raw reference frame native to kOS. This will give us the time-of-flight and thus define how early we need to launch the interceptor. We are going to make a lot of assumptions here under the assumption that all the error we introduce by making those assumptions will be corrected in the terminal homing phase. Thus the trajectory is defined by:

$$\dot{x}(t) = \int \ddot{x}(t)dt$$

$$x(t) = x_0 + \int \dot{x}(t)dt$$

The forces acting on the interceptor are thrust, drag, and gravity. Let's assume the interceptor has two stages, acting under different accelerations each time. For stage n:

$$\ddot{x}_n(t) = \frac{1}{m(t)}(\bar{T_n} + \bar{F_d}) + \bar{g}$$

Appreciable drag is only present for about thirty seconds, so let's go ahead and assume it to be zero. Let's also assume the thrust in each stage is constant. Therefore:

$$\ddot{x}_n(t) = \frac{\bar{T_n}}{m_0 - \dot{m}t} + \bar{g}$$

The thrust vector T_n changes as a function of time and pitch rate. We will use the launch-centered, raw-orientation frame defined by kOS to define our thrust vector in terms of pitch rate and time. We take the normalized up vector and rotate it by the pitch angle (defined as the pitch rate times time) to produce the thrust vector.

Notationally, subscripts represent the state after the numbered stage is cutoff (t_1 is time after first stage cutoff, m_1 is mass after stage 1 cutoff, etc). We will numerically integrate to find the position as a function of time. Using Euler's method of numerical integration, the acceleration, velocity, and position at each point during first stage burn is (subscript n denotes the nth step in the integration process):

$$m_n = m_{n-1} - \dot{m}\delta t$$

$$\ddot{x}_n = \frac{\bar{T_n}}{m_n} + \bar{g}$$

$$\dot{x}_n = \dot{x}_{n-1} + \ddot{x}_n \delta t$$

$$x_n = x_{n-1} + \dot{x}_n \delta t$$

This is initialized at the launch point. The second stage uses the same equations, but initialized at first stage burnout. After all stages are spent, the final iteration continues initialized at MECO as:

$$\ddot{x}_n = \bar{g}$$

$$\dot{x}_n = \dot{x}_{n-1} + \ddot{x}_n \delta t$$

$$x_n = x_{n-1} + \dot{x}_n \delta t$$

At each iteration, the distance to the intercept point is calculated as $||x_n - x_i||$. Iteration stops when the distance to the intercept point stops decreasing. The time of flight is then calculated as $t_f = n\delta t$. Interceptor launch is commanded when the current time is $t_i - t_f$.

3 Closed Loop Guidance

This algorithm continuously calculates the minimum separation Δx to the intercept pointx_i and the time separation Δt to the intercept time t_i and uses the reaction control system to minimize them.