An Open-Source Framework for the Determination of Asteroid Orbits and Impact Probabilities

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Recent developments in planetary defense have led to a large increase in the amount of data related to the tracking of asteroids. Determining the orbits and trajectories of these asteroids is critical to assessing their probability of impacting Earth. Traditionally, this orbit determination process has been carried out by specialized models and algorithms at institutions dedicated to astrodynamics. However, the wealth of data now being delivered by sensors may require a more diverse system of orbit determination. This paper presents an open-source framework, written in MATLAB, that empowers users to individually determine the trajectories and impact probabilities of asteroids given optical observation data. A solar system dynamics model, observations model, and a batch least-square estimator were developed and shown to be accurate in comparison to the methods of institutions like the Jet Propulsion Laboratory (JPL).

Nomenclature

α	=	Right ascension
δ	=	Declination
r	=	Asteroid position
R	=	Earth position
$\hat{r}_{ECI}$	=	Normalized asteroid position in Earth-centered inertial frame
τ	=	Light time, asteroid-to-Earth
c	=	Speed of light
P	=	Covariance matrix
Φ	=	State transition matrix
P_c	=	Probability of collision

I. Background

On On 27 December 2024, the Earth-crossing asteroid 2024 YR_4 was discovered by the Asteroid Terrestrial-impact Last Alert System (ATLAS) [1]. Initial orbit determination showed that this asteroid was an Earth-crossing asteroid (ECA), meaning it made close approaches to the orbit of Earth. Estimates of the impact probability soon showed a dangerously high probability of collision P_c. In February 2025, 2024 YR_4's estimated impact probability peaked at over $P_c > 3\%$, a new record for planetary defense. Soon after, however, new data revealed that the asteroid's orbit was not endangering the Earth, and its risk level was downgraded significantly.

In the few months the asteroid was considered a threat, many different observatories and sensors were trained on it, gathering data that could be used to refine the orbit. The orbit determination algorithms used to process these observations into a best-fit orbit are generally complex and slow, used only by large institutions. However, with modern computers, individuals could be capable of running their own orbit determination processes against any datasets they chose. This could enable faster and more widespread processing of asteroid orbits than previously achieved. Additionally, wider verification and replication of results could be achieved. We considered it possible to replicate higher accuracy published results locally. An open-source package of the orbit determination and probability of collision estimation was therefore desired.

II. Methodology

Estimating an asteroid's probability of impacting Earth first requires a precise knowledge of the asteroid state (position and velocity, or some other form of orbital elements). This itself requires a few items. First, a dynamics model is necessary to estimate how the asteroid's state changes over time. An observation model is also needed to convert an asteroid's state into theoretical observations, with a form that matches whatever data is available on the asteroid. These can be used by some form of orbit determination algorithm (in our case batch least-squares) along with any *a priori* information available to form an estimated state. Once the state is determined, the estimated state and covariance are used to form an estimate of the collision probability. Each of these steps is analyzed in this section.

A. Dynamics Model

A dynamics model is a mathematical function F that describes how a system's state X changes with time.

$$\frac{dX}{dt} = F(X, t) \tag{1}$$

For the case of predicting a body's position, this function usually estimates the forces on a body and uses them to estimate how the position and velocity change as a result. For a position r, velocity $\dot{r}$, and acceleration $\ddot{r}$, the dynamics model sets the rate of change of the position to be the velocity $\dot{r}$ and the rate of change of velocity to be the acceleration, which is calculated.

The acceleration on a body in space can be modeled in many different ways. The most significant accelerations by far are the gravitational ones. For an asteroid, these are dominated by the gravity of the Sun and any nearby planets. Our dynamics model simply sums the gravitational effects of the bodies expected to be the largest contributors to the asteroid, i.e. the Sun, Venus, Earth, the Moon, Mars, and Jupiter. These were chosen to predict the orbit of 2024 YR_4 and other similar Earth-crossers to high accuracy. Asteroids and comets that venture beyond Jupiter, especially those that closely approach Saturn, Uranus, or Neptune, should not be expected to be well-modeled by this approach.

The acceleration due to each body, modeled by Eq. (2), was summed across all bodies. For Jupiter and Mars, the system gravitational parameter were used instead of those for the planets themselves to avoid modeling the moons of these planets [2]. For Earth's moon, the Moon was modeled individually, necessitated by the close approaches of Earth-crossers to the Earth-Moon system. Further refinements of gravity were neglected, including all nonspherical gravity terms and the gravity of any other bodies.

$$\ddot{r} = -\sum \frac{\mu_{body}}{r^2}\hat{r} \tag{2}$$

Also modeled were relativistic correction forces as described in Montenbruck & Gill page 111 [3] and shown below.

$$\ddot{r} = +\frac{\mu}{r^2}\left(\left(4\frac{\mu}{c^2 r} - \frac{v^2}{c^2}\right)\mathbf{e}_r + 4\frac{v^2}{c^2}\left(\mathbf{e}_r \cdot \mathbf{e}_v\right)\mathbf{e}_v\right) \tag{3}$$

These were disabled after comparisons of the model with these corrections on versus off revealed that the deviations were tiny and yet incurred a considerable performance burden.

Other forces not modeled by us that typically affect asteroids are solar radiation pressure and the Yarkovsky effect, which we considered too difficult to model precisely in the absence of physical data to implement.

The observations were assumed to be accurate to $0.01s$ in right ascension and 0.1" in declination as this is the accuracy they are reported to [4].

B. Observations

The premier organization for handling the observational data of minor bodies is the Minor Planet Center (MPC). They coordinate the dissemination of observational data related to all known minor bodies. This code base is designed to ingest observations in the MPC's format for optical astrometric observations of bodies [4]. Importantly, the data for 2024 YR_4 was published in this format and downloaded [5].

This format consists of UTC timestamped right ascension and declination values from a variety of different observatories. For ingestion into the orbit determination algorithm, these were converted from their native hours/minutes/seconds and degrees/arcminutes/arcseconds into decimal degrees. The observation model calculated theoretical observation values from the time and asteroid state. The asteroid state was first calculated factoring in light time delay by using the following fixed-point iteration scheme in the solar system barycentric frame to determine the light time [3].

$$\tau_{i+1} = \frac{1}{c}(r(t-\tau_i) - R(t) \tag{4}$$

Next, right ascension α and declination δ were calculated form the state by placing the asteroid position into the Earth-centered inertial frame and applying Eq. (6) and (7).

$$r_{ECI} = r - R \tag{5}$$

$$\alpha = \text{atan2}(\hat{r}_{ECI,y}, \hat{r}_{ECI,x}) \tag{6}$$

$$\delta = \arcsin(\hat{r}_{ECI,z}) \tag{7}$$

The observation model was extensively tested by propagating initial conditions provided by JPL [6] and comparing the calculated observations to the actual data from the MPC.

C. Batch Least-Squares Estimation

The batch least-squares estimator was set up as described by Tapley, Schutz, and Born [7]. The state and state transition matrix were numerically integrated together using MATLAB's built-in `ode45` to form a nominal trajectory that was iteratively improved.

To check convergence, the batch estimator was given an initial condition equal to JPL's orbit determination solution for that epoch, sourced from Horizons [6], plus a perturbation equal to a distance of roughly one Earth-Moon distance and 100 meters per second. The batch estimator should be able to converge from this perturbed initial state to the true initial state, with the performance easily apparent from how close it converged to the original JPL initial conditions.

D. Impact Probability

The impact probability calculation makes use of the MATLAB version of NASA's Conjunction Assessment Risk Analysis (CARA) tools [8]. These use Foster's method to calculate a probability of collision P_c. Our code base simply provides the Earth-centered asteroid position, velocity, and covariance matrices.

The position and velocity at closest approach to Earth are simple to find using traditional numerical integration, with some interpolation to search for the actual time at closest approach. Finding the covariance matrix at the time of closest approach requires taking the covariance matrix from the batch estimator (at the initial epoch) and propagating it using the state transition matrix (propagated along with the state).

$$P(t) = \Phi(t, t_0) P(t_0) \Phi(t, t_0)^T \tag{8}$$

These were plugged into the CARA tools' 2D Foster solver *Pc2D_Foster.m* using a hard-body radius equal to the Earth's radius to find the collision probability. This should be zero for 2024 YR_4.

III. Results

A. Dynamics Model Verification

The force model was verified by propagating the initial conditions found by JPL [6] from October 2024 to December 2032 and checking whether the 2032 close approach matched JPL's predicted closest approach in time and distance.

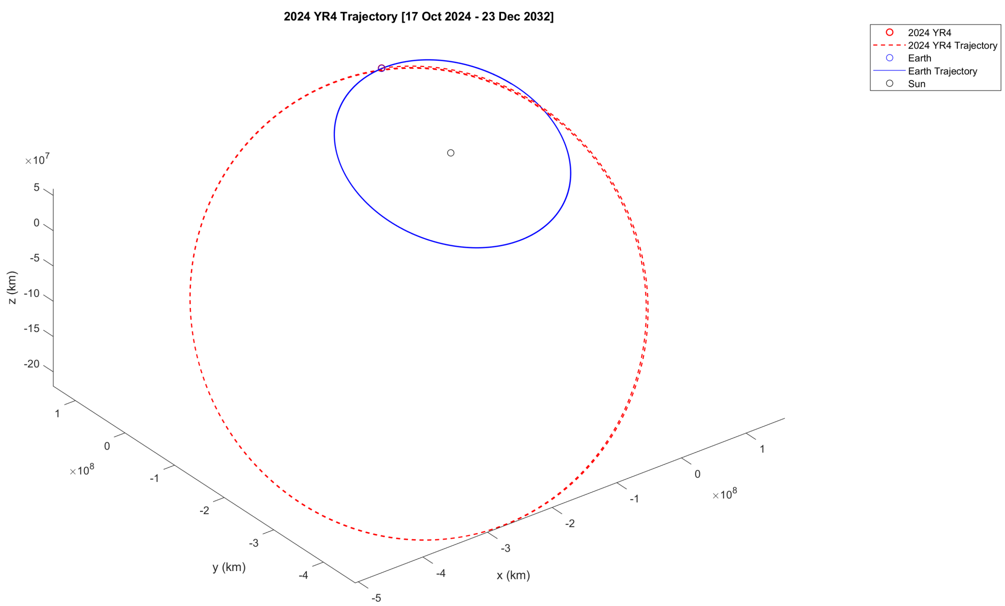

Fig. 1 Trajectory of 2024 YR_4 across three close approaches

In Figure 1, the trajectories of Earth and 2024 YR_4 are clearly visible. The circles indicate the position of the two bodies during the 2032 close approach. The fact that the two circles are on top of each other indicates that our model is accurate. Indeed, our model predicts a minimum distance during this approach of $d = 239,382$ kilometers, while JPL predicts a minimum distance of $d = 259,653$. A difference this small (20,000 km) after eight years of extremely chaotic orbital propagation is a testament to the accuracy of our dynamics.

B. Observation Model Verification

Our observation model was tested similarly by integrating from JPL's initial conditions and testing whether the observation values predicted by our model matched the data gathered by the MPC. A plot comparing our predicted right ascension and declination values to the truth is shown in Figure 2.

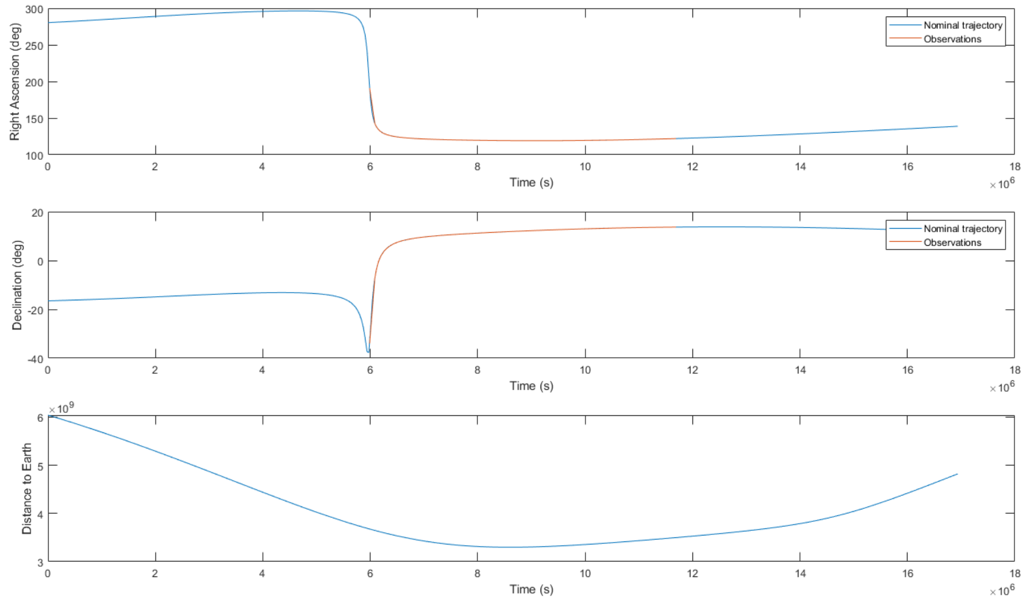

Fig. 2 Observation from the MPC compared against predicted observations from an integrated nominal trajectory

It is clearly evident how closely these data align. Calculating the residuals between them leaves a value within floating point precision of zero. This confirms our observation model.

C. Batch Convergence

The convergence of the batch least-square estimator was checked by checking that feeding in a perturbed set of initial conditions from JPL would return to the true initial conditions. As you can see in Figure 3, the batch estimator successfully reduced the error against the observations over a series of iterations.

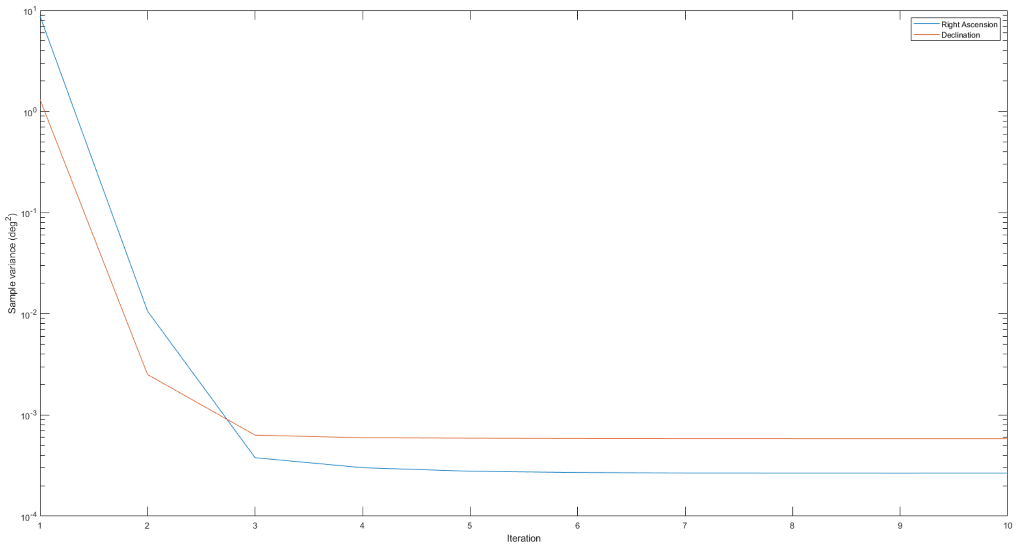

Fig. 3 Residuals in each observation type over the batch estimator's iterations

With a perturbation applied to the initial conditions on the order of 384, 000 km and 100 m/s, the batch estimator converged to a solution within 11, 000 km and 121.6 m/s of the solution given by JPL [6].

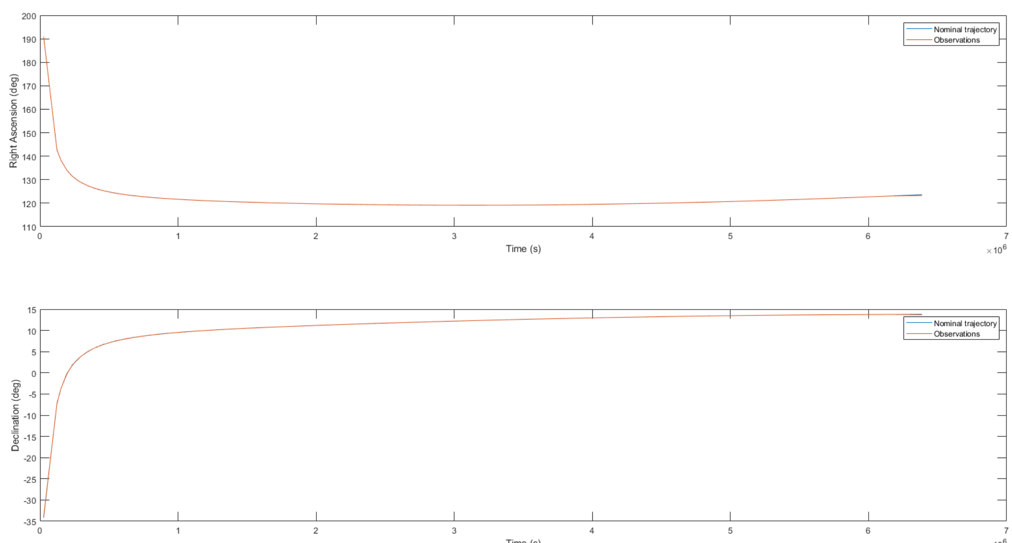

Fig. 4 Residuals in each observation type over the length of the nominal trajectory

The observations predicted by the nominal trajectory and the true observations can be compared. In Figure 4, the nominal trajectory recreates the true observations very closely. This, at least, shows that our model is converging to something.

D. Impact Probability

Using the NASA CARA tools to calculate impact probability [8], we find a collision probability of $P_c = 0$. This is expected and matches the impact probability predicted elsewhere.

One original goal of this project was to recreate the approximately 3% chance of collision predicted using only the first few months of observations of 2024 YR_4. Unfortunately, observations over such a small time proved difficult to converge too, and the highly chaotic system meant that small deviations in the converged-to solution produced a final orbit that was not very close to Earth during the 2032 flyby. In the absence of real data, the only verification of the collision probability estimator was by manually adjusting the flyby to be closer and observing nonzero values of P_c.

IV. Conclusion

This project was successful in providing an open-source toolkit for determining the orbits of asteroids using the most common astrometric observations. This would theoretically allow any user to produce their own set of determined orbits without reference to any institution, allowing for quick replication of results. The dynamics model was shown to be highly accurate to what institutions like JPL use and the observation model was practically exact.

However, the solar system is a chaotic place. The solutions converged to by the batch estimator showed some residual against the solutions published by JPL. This residual was not a problem when considering all the data available on 2024 YR_4, but became more of an issue when trying to use a reduced dataset, such as when trying to recreate the 3% odds of collision reported in February 2025. Orbit determination solutions that nearly exactly matched the data were sometimes inaccurate due to the short observation arc. In conclusion, we showed that it was possible to reproduce asteroid orbit determination results using simpler models while accepting some mild error.

An important question is what differentiates the models of institutions like JPL from ours. Our model converged to a slightly different value compared to JPL, which may seem insignificant but leads to vastly different states after multiple Earth flyby cases due to the chaos of the system. JPL is evidently using a slightly different dynamics model. Their model likely includes all of the bodies in the system for which gravitational parameters have been estimated. Additionally, their model probably includes relativistic corrections. These effects only manifest over very large time scales. They are practically negligible in the short term, and adding them to our model would have increased the > 10 minute runtime of the batch estimator considerably. This is something they can afford to do with their vast computational resources.

Future work should include identifying the largest sources of error in the dynamics model. Additionally, testing against other bodies with historically nonzero probabilities of collision like 99942 Apophis could yield interesting results, especially when greater observation arcs are available. Also, cleaning up the code into a manageable set of functions would be useful for end users.

Appendix - Setup Instructions

The code for this project is accessible both as a .zip file attached here and as a GitHub repository available at this link: `https://github.com/AlexanderEvitt/AsteroidImpactCalculator`

Clone or unzip the code to get started. Once everything is loaded, `test_model.m` can be used to test the dynamics model using initial conditions from JPL Horizons [6]. `test_obs_model.m` can be used to test that the observation model is correct by comparing real observations to calculated ones based on the JPL initial conditions. The file `estimate_trajectory.m` can be used to run the batch estimator and the file `collision_probability.m` can be used to estimate the odds of collision of an asteroid given a trajectory. To avoid having to run the batch estimator (which takes minutes), you can load `converged_data.mat` to see the results of the estimation process. The other files represents functions used in the completion of these tasks.

References

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