Transformations in Three Dimensions

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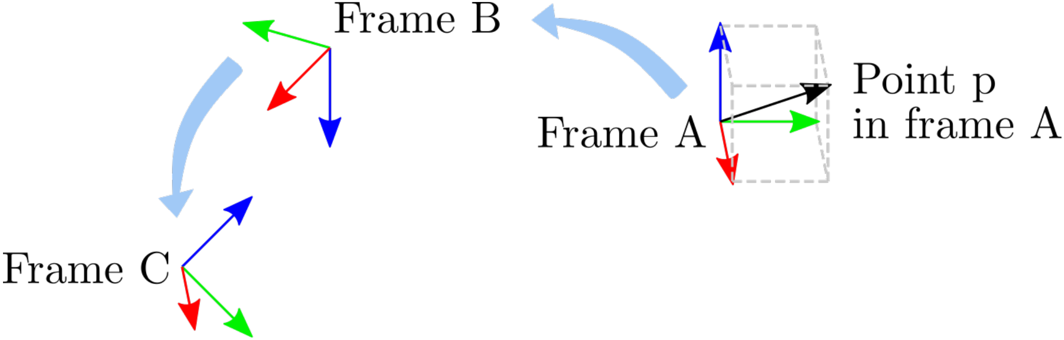
DFKI GmbH, Robotics Innovation Center



pytransform3d

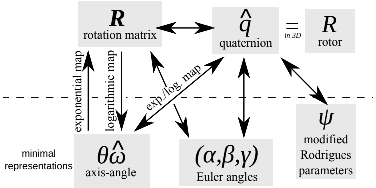
git

`https://github.com/dfki-ric/pytransform3d`

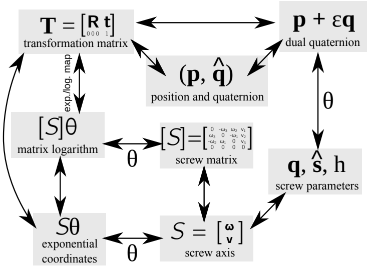
Why?

Why?

pytransform3d.rotations

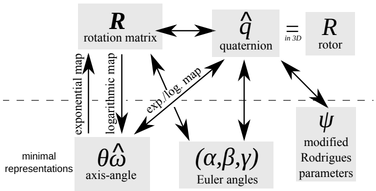


pytransform3d.transformations

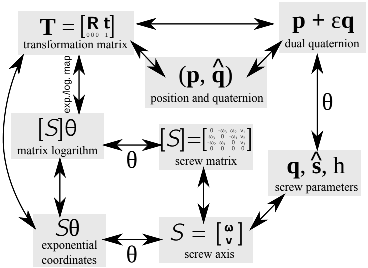


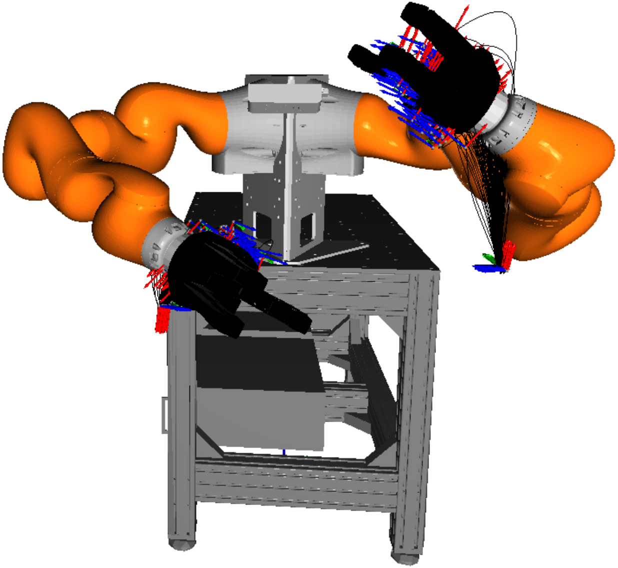
Why?

pytransform3d.rotations

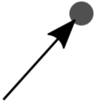
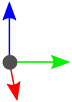


pytransform3d.transformations

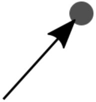
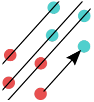
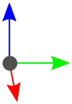
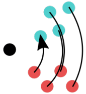




Introduction to 3D Rigid Transformations

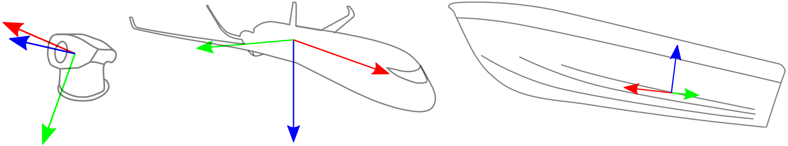
Representation	Term	
$\mathbb{R}^3$	position	
$SO(3)$	orientation	
$SE(3)$	pose	

Introduction to 3D Rigid Transformations

Representation	Term	Displacement
$\mathbb{R}^3$	position 	translation 
$SO(3)$	orientation 	rotation 
$SE(3)$	pose	rigid transformation

Frames

- A *coordinate reference* **frame** is defined by an origin (position) and orientation.
- It is attached to a rigid body.

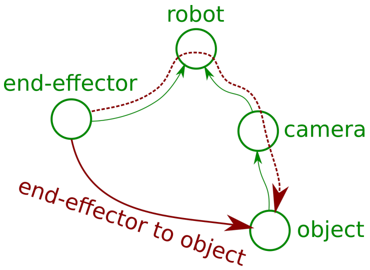


We will use RGB arrows to display frames.

Graphs of Transformations

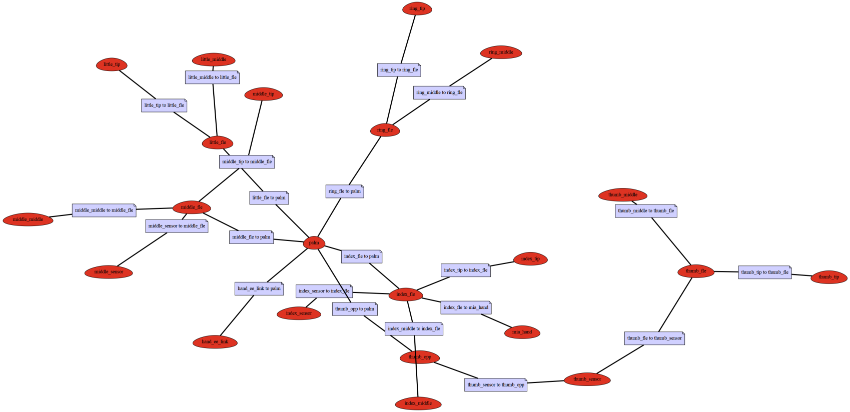
Examples: kinematic structures of robots (and other articulated bodies), estimated states, trajectories

```
from pytransform3d.transform_manager import
    TransformManager


tm = TransformManager()
tm.add_transform("end-effector", "robot", ee2robot)
tm.add_transform("object", "camera", object2cam)
tm.add_transform("camera", "robot", cam2robot)

ee2object = tm.get_transform(
    "end-effector", "object")
```



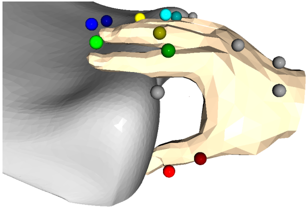
Robot Kinematics (Example: Robotic Hand)

```python
from pytransform3d.urdf import UrdfTransformManager
tm = UrdfTransformManager()
with open("robot.urdf", "r") as f:
    robot_urdf = f.read()
tm.load_urdf(robot_urdf)
tm.write_png(graph_filename)
```



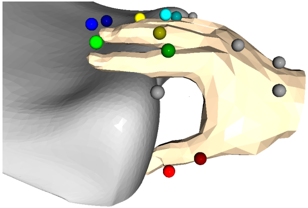
Application: Motion Transfer to Robotic Hands

Transfer of Motions to Robotic Hand



`https://github.com/dfki-ric/hand_embodiment` (Fabisch et al. 2022)

Transfer of Motions to Robotic Hand



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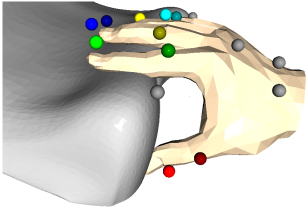

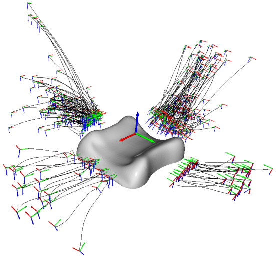

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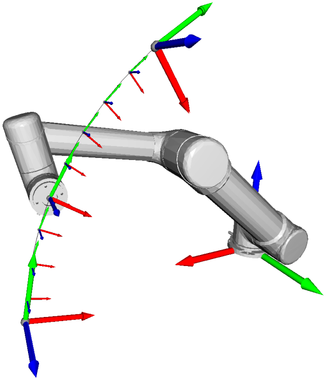
Application: Imitation Learning

Imitation Learning



Given: one or more solutions to a problem (trajectories)

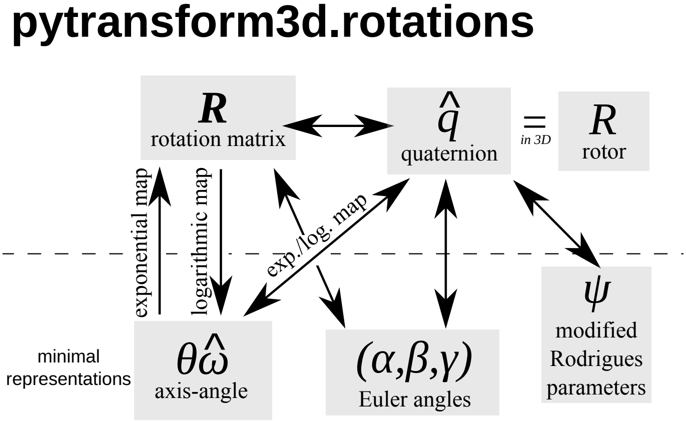
How can we represent orientation?

Library: `https://github.com/dfki-ric/movement_primitives`

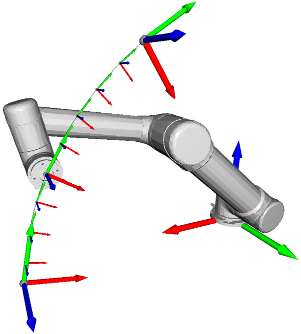
SO(3) (SO: special orthogonal group)

— group of all rotations in 3D

— represented by 3D rotation matrices



Imitation Learning



Problems

- Rotation matrices have 6 constraints that are not easy to enforce
- Minimal representations $\in \mathbb{R}^3$ have singularities
- Quaternions $q \in \mathbb{S}^3$ have an ambiguity: $q \equiv -q$

Quaternions - Pitfalls

We want to use quaternions (Ude et al. 2014)

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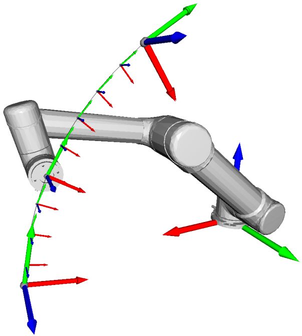
Be careful:

- Quaternions $q \in \mathbb{S}^3$ have an ambiguity: $q \equiv -q$

```
from pytransform3d.rotations import pick_closest_quaternion
q1 = pick_closest_quaternion(q1, q2)
```

- 2 conventions: **Hamilton** vs. JPL (Sommer et al. 2018)
- 4 numbers, e.g., $(0, 1, 0, 0)$ – **scalar first** (w, x, y, z) or last (x, y, z, w)

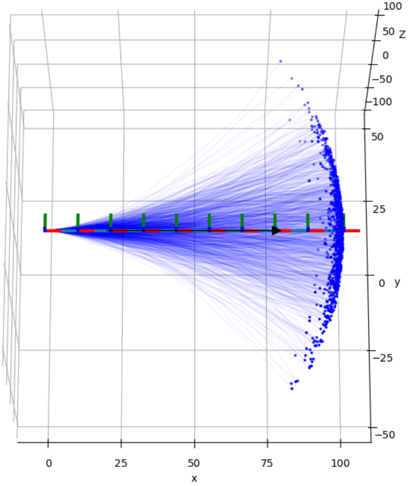
Visualizer – matplotlib-like interface to Open3D



```
import pytransform3d.visualizer as pv


fig = pv.figure()
fig.plot_transform(s=0.3)

fig.plot_graph(
    urdf_transform_manager,
    "ur5_base_link",
    show_collision_objects=True,
    show_frames=True)

fig.plot_transform(ee2base_start)
fig.plot_transform(ee2base_end)

pv.Trajectory(trajectory).add_artist(fig)

fig.view_init()
fig.show()
```

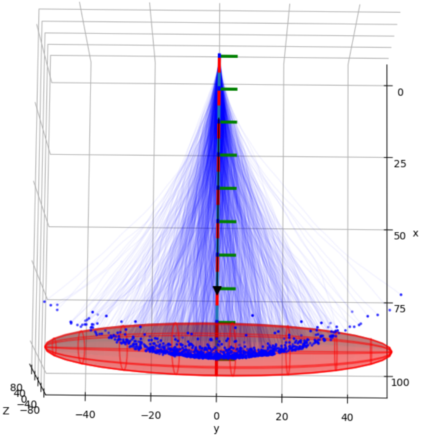
Application: Uncertain Transformations

Concatenation of Uncertain Transformations



- Uncertainty of angular velocity about z-axis
- Blue: Monte Carlo (MC) sampling of trajectories

Concatenation of Uncertain Transformations



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- Red: Gaussian of MC-sampled final positions

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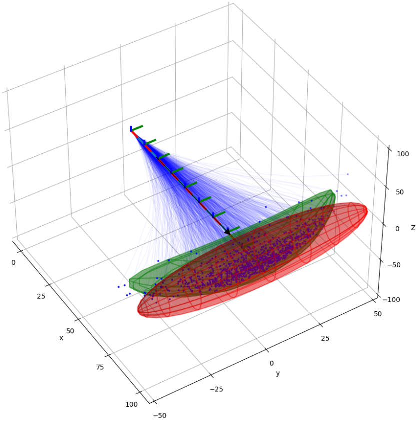

- Uncertainty of angular velocity about z-axis
- Blue: Monte Carlo (MC) sampling of trajectories
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- Green: Propagated uncertainty in *exponential coordinates* (**banana distribution**)

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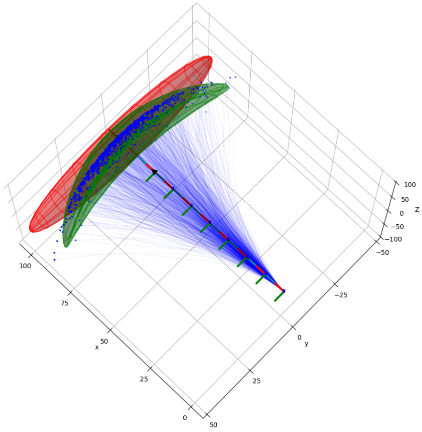

- Uncertainty of angular velocity about z-axis
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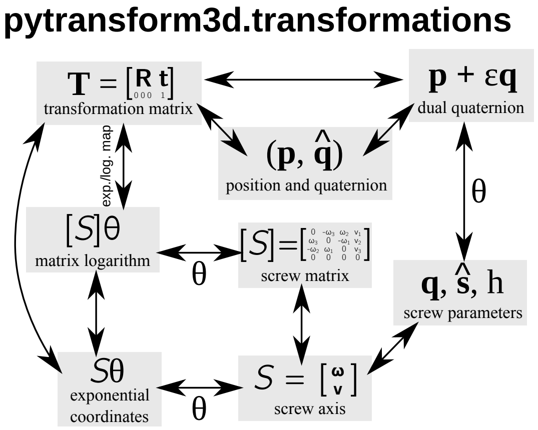
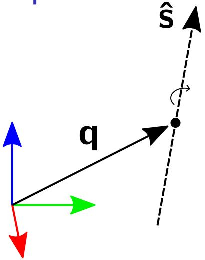
SE(3) (SE: special Euclidean group)

— group of all proper rigid transformations in 3D
— represented by transformation matrices

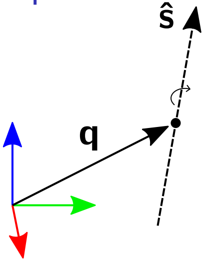


Exponential Coordinates (Screw Theory)



$$h = \frac{\text{translation}}{\text{rotation}}$$

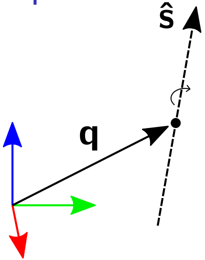
Exponential Coordinates (Screw Theory)



$$h = \frac{\text{translation}}{\text{rotation}}$$

$$\begin{aligned} \text{Angle:}\quad & \theta \in \mathbb{R} \\ \text{Screw axis:}\quad & \begin{bmatrix} \hat{\boldsymbol{s}} \\ \boldsymbol{q} \times \hat{\boldsymbol{s}} + h\hat{\boldsymbol{s}} \end{bmatrix} = \mathcal{S} \in \mathbb{R}^6 \\ \text{Exp. coord.:}\quad & \mathcal{S}\theta \in \mathbb{R}^6 \end{aligned}$$

Exponential Coordinates (Screw Theory)



$$h = \frac{\text{translation}}{\text{rotation}}$$

Exponential map:
$Exp(\mathcal{S}\theta) = \boldsymbol{T} \in SE(3)$

Angle: $\theta \in \mathbb{R}$

Screw axis: $\begin{bmatrix} \hat{\mathbf{s}} \\ \boldsymbol{q} \times \hat{\mathbf{s}} + h\hat{\mathbf{s}} \end{bmatrix} = \mathcal{S} \in \mathbb{R}^6$

Exp. coord.: $\mathcal{S}\theta \in \mathbb{R}^6$

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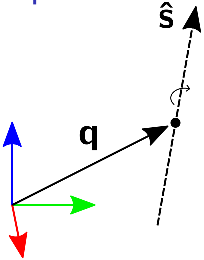


$$h = \frac{\text{translation}}{\text{rotation}}$$

Exponential map:
$Exp(\mathcal{S}\theta) = \boldsymbol{T} \in SE(3)$

```
import pytransform3d.transformations as pt
pt.transform_from_exponential_coordinates(
    Stheta)
```

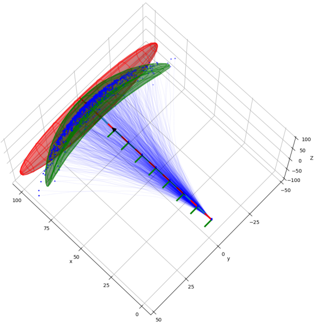
Angle: $\theta \in \mathbb{R}$

Screw axis: $\begin{bmatrix} \hat{\mathbf{s}} \\ \boldsymbol{q} \times \hat{\mathbf{s}} + h\hat{\mathbf{s}} \end{bmatrix} = \mathcal{S} \in \mathbb{R}^6$

(Lynch and Park 2017)

Exp. coord.: $\mathcal{S}\theta \in \mathbb{R}^6$

Concatenation of Uncertain Transformations



The **banana distribution** is Gaussian in exponential coordinates:

$$\boldsymbol{T} = Exp(\mathcal{S}\theta)\,\overline{\boldsymbol{T}}, \quad \text{with} \quad \mathcal{S}\theta \sim \mathcal{N}\left(0, \boldsymbol{\Sigma}_{6\times 6}\right)$$

(Long et al. 2012; Barfoot and Furgale 2014)

Concatenation of Uncertain Transformations

```python
import pytransform3d.uncertainty as pu
# estimated mean pose (transformation matrix, 4x4)
T_est = ...
# covariance in exponential coordinates (6x6)
cov_est = ...
# mean offset (transformation matrix, 4x4)
T_vel = ...
# covariance in exponential coordinates (6x6)
cov_vel = ...

T_est, cov_est = pu.concat_globally_uncertain_transforms(
        T_est, cov_est,
        T_vel, cov_vel)
```

Probabilistic Robot Kinematics

Examples > 3D Visualizations > Probabilistic Product of Exponentials

Funding

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pytransform3d

- a library for rigid transformations in 3D
- organizes complex graphs of transformations
- coupling with matplotlib for quick visualization
- a matplotlib-like interface to Open3D's visualizer
- operations for representations of rotation and translation
- conversions between representations
- clear documentation of conventions

Backup

Application: Collision Detection

Collision Detection

- Broad phase collision detection with AABBs
- GJK algorithm (Gilbert et al. 1988)

Collision Detection

Library: `https://github.com/AlexanderFabisch/distance3d`

```
from pytransform3d.urdf import UrdfTransformManager
from distance3d import broad_phase


# Load graph of transformations
robot_tree = UrdfTransformManager()
with open(filename, "r") as f:
    robot_urdf = f.read()
robot_tree.load_urdf(robot_urdf)

# Define configuration of robot
for joint_name in ["joint%d" % i for i in range(1, 7)]:
    robot_tree.set_joint(joint_name, 0.7)

# Construct bounding volume hierarchy for broad phase
robot_bvh = broad_phase.BoundingVolumeHierarchy(
    robot_tree, "robot_arm")
robot_bvh.fill_tree_with_colliders(robot_tree)
```

Transformation Matrices

$$SE(3) = \left\{ \boldsymbol{T} = \begin{pmatrix} \boldsymbol{R} & \boldsymbol{t} \\ 0 & 1 \end{pmatrix} \in \mathbb{R}^{4\times4} | \boldsymbol{R} \in SO(3), \boldsymbol{t} \in \mathbb{R}^3 \right\}$$

$$\begin{pmatrix} \boldsymbol{R} & \boldsymbol{t} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

How should we define a probability distribution in $SE(3)$?

Modeling Transformations

Mathematical notation: $\boldsymbol{T}_{BA}$ for transformation *from frame* `A` *to frame* `B`. In concatenation, read from right to left:

$$\boldsymbol{T}_{CB}\boldsymbol{T}_{BA} = \boldsymbol{T}_{C\cancel{B}}\boldsymbol{T}_{\cancel{B}A} = \boldsymbol{T}_{CA}.$$

In code we should prefer the notation `A2B` for a transformation *from frame* `A` *to frame* `B`:

```python
from pytransform3d.transformations import concat
A2B = ...  # transformation from frame A to frame B
B2C = ...  # transformation from frame B to frame C
A2C = concat(A2B, B2C)
```

Imitation Learning

Literature I

Barfoot, Timothy D. and Paul T. Furgale (2014). "Associating Uncertainty With Three-Dimensional Poses for Use in Estimation Problems". In: *IEEE Transactions on Robotics* 30.3, pp. 679–693. DOI: 10.1109/TRO.2014.2298059.

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