University of Southern Denmark

DM852

Heuristics & Approximations Algorithms

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1 Introduction

This project shows metaheuristic algorithms, resolving the Capacitated Vehicle Routing Problem (CVRP). The difference between metaheuristics and heuristics is in the solution part. For heuristics, you are trying your best to find a solution, even though it is not optimal. The algorithm is adapted to the problem in such a greedy approach, it can get stuck in a local optimum. This is fine since it is the idea of heuristics: "just solve the problem".
Metaheuristics are less greedy and tends to be more problem independent. They accept temporary solutions and allow "bad" steps as an attempt to get a better solution. You have a local- and global optimum to keep track of the best solution found. You can say metaheuristics are exploting heuristics to avoid getting trapped.
In this metaheuristics implementation project, we have chosen to implement two algorithms for CVRP: Simulated Annealing (SA) and Ant Colony Optimization (ACO). The SA algorithm is the one we have uploaded for electronic submisison at http://valkyria.imada.sdu.dk/D0App/. The ACO algorithm is implemented but does not perform well. We will compare the algorithms with our previous heuristic / local search project and lastly, show our results of computation for our two algorithms.

— Alexander & Narongrit

2 Simulated Annealing

The Simulated Annealing (SA) algorithm is inspired from the annealing process in metal, where you are altering the physical state of the metal by heating and cooling it. The inspiration can be used in computer science as well. We can use the algorithm on CVRP by starting off by generating a solution to the problem and not "care" about the initial solutions. The better steps we are taking, and better solutions we are finding, the more careful we are going to be in finding a solution. In the beginning we allow random and bad steps to be performed, while later, we make better calculations.
We have developed an algorithm to solve CVRP using Simulated Annealing technique by relating CVRP with existed problems i.e. Bin packing problem and Knapsack problem. Using this approach, we can guarantee maximum amount of vehicles necessary required with approximation analysis on bin packing hueristic algorithm. In this demonstration, we use well studied algorithm First-Fit-Decreasing which is guaranteed to use no more than $\frac{11}{9}\mathrm{OPT} + 1$ number of required vehicles[1]. With combination of simple Traveling Salesman Problem's local search algorithm, 2-opt, we are able create the initial feasible solution as described in 2. Next step of Simulated Annealing involved creating new feasible solution by altering current solution. Which in CVRP case, is to perform points exchange action between routes. In order to allow as much possibility of exchanges as it could without breaking capacity constrain, we choose simple algorithm to find exchanges possibility based on Knapsack problem 3.

Algorithm 1 Simulated annealing with First-Fit-Decreasing and Knapsack combination

```
Require: customers: List of customer coordinate with it's capacity
Require: route_cap(route): returns capacity of route
Require: route_cost(route): returns capacity of route
Require: tsp_local_search(route): performs travelling salesman local search and returns improved route
Require: is_accept(best_cost, cost, temp): an acceptance criterion function returns boolean based on Metopolis condition
Require: max_cap: Maximum capacity for each route
Require: depot: Depot point
function SIMULATED_ANNEALING_FFD(customers, temp, alpha, no_improve_limits)
    routes ← CVRP_FIRST_FIT_DECREASING(customers, depot)
    best_cost ← total cost of routes
    best_routes ← routes
    current_cost ← best_cost
    current_routes ← routes
    curr_no_improve_trying ← 0
    while True do
        if curr_no_improve_trying == no_improve_limits then break
        origin_route_id, target_route_id ← randomly pick 2 distinct index of routes
        origin_route_point ← randomly pick element from route[origin_route_id]
        upper_bound, lower_bound ← maximum and minimum capacity such that solution still feasible after exchange action
        target_neighbors_set ← KNAPSACK_COMBINATION(
            route[target_route_id], lower_bound, upper_bound)
        if len(target_neighbors_set) == 0 then continue
        target_neighbors ← randomly pick an element from target_neighbors_set
        new_route_i, new_route_i ← exchange neighbors between two route and assign to new variables
        new_route_i ← tsp_local_search(new_route_i)
        new_route_j ← tsp_local_search(new_route_j)
        old_tours_cost ← route_cost(route[origin_route_id]) + route_cost(route[target_route_id])
        new_tours_cost ← route_cost(new_route_i) + route_cost(new_route_j)
        new_cost ← current_cost − old_tours_cost + new_tours_cost
        if is_accept(best_cost, new_cost, temp) then
            update temp with alpha
            update current_routes with new routes
            curr_no_improve_trying ← 0
            if new_cost < best_cost then
                update best_cost and best_routes with new routes
        else
            curr_no_improve_trying ← curr_no_improve_trying + 1
    return best_routes
```

Algorithm 2 First-Fit-Decreasing for CVRP

Require: *customers*: List of customer coordinate with it's capacity
Require: *tsp_local_search*(*route*): performs travelling salesman local search and returns improved route
Require: *max_cap*: Maximum capacity for each route
Require: *depot*: Depot point
Require: *route_cost*(*route*): returns capacity of route
Ensure: *routes*: solution route

```
function CVRP_FIRST_FIT_DECREASING(customers)
    sorted_capacity_points ← sort customer points by it's capacity
    routes ← []
    add a route with depot to routes
    for all point in sorted_capacity_points do
        is_fit ← False
        for all route in routes do
            if route_cost(route) + point.capacity ≤ max_cap then
                add point to route
                is_fit ← True
                break
        if !is_fit then
            add new route with depot and point to routes
    for all route in routes do
        route ← tsp_local_search(route)
    return routes
```

Algorithm 3 Knapsack Combination

Require: *customers*: List of customer coordinate with it's capacity
Require: max_cap: Maximum capacity for each route
Require: *depot*: Depot point
Require: max_points_amt: Number of maximum combination points
Require: $route\_cap(route)$: returns capacity of route
Ensure: $combination\_set$: possible combination of points in route s.t. their capacity not exceeds max_cap

function KNAPSACK_COMBINATION($route, lower\_bound, upper\_bound$)
    $points \leftarrow$ randomly pick max_points_amt points from $route$ but $depot$
    $combination\_set \leftarrow []$
    function KNAPSACK_COMBINATION_RECUR($current\_point\_id, current\_capacity, knapsack$)
        if $current\_point\_id == len(points)$ **then**
            if $route\_cap(knapsack) > lower\_bound$ **then**
                add knapsack to $combination\_set$
        $knapsack\_combination\_recur($
            $current\_point\_id + 1, current\_capacity, knapsack)$
        if $current\_cap + points[current\_point\_id] \leq max\_cap$ **then**
            add $points[current\_point\_id]$ to $knapsack$
            $current\_cap \leftarrow points[current\_point\_id].capacity$
            $knapsack\_combination\_recur($
            $current\_point\_id + 1, current\_capacity, knapsack)$
    $knapsack\_combination(0, 0, [])$
    return $combination\_set$

---

instance	**CHH**		**SA**		**instance**	**CHH**		**SA**	
	k	**cost**	**k**	cost		**k**	**cost**	**k**	cost
A-n32-k05	5	867	5	980	CMT01	6	604	5	553
A-n33-k05	5	743	5	661	CMT02	11	920	10	894
A-n33-k06	6	766	6	816	CMT03	8	985	8	987
A-n34-k05	6	889	5	778	CMT04	12	1196	12	1568
A-n36-k05	5	862	5	807	CMT05	17	1496	17	3406
A-n37-k05	5	741	5	669	CMT11	8	1111	7	1517
A-n37-k06	7	1112	6	949	CMT12	10	909	10	1180
A-n38-k05	6	822	5	730	Golden_01	9	6055	9	8796
A-n39-k05	5	883	5	827	Golden_02	10	9121	10	14227
A-n39-k06	6	887	6	999	Golden_03	9	12144	9	17920
A-n44-k06	6	1029	6	939	Golden_04	10	15644	10	24338
A-n45-k06	7	1014	6	978	Golden_05	5	7330	5	10954
A-n45-k07	7	1250	7	1161	Golden_06	7	9698	7	14509
A-n46-k07	7	997	7	1039	Golden_07	9	11655	9	17513
A-n48-k07	7	1232	7	1128	Golden_08	10	12835	10	19443
A-n53-k07	8	1142	7	1054	Golden_09	15	590	14	653
A-n54-k07	8	1258	7	1217	Golden_10	16	763	16	843
A-n55-k09	9	1158	9	1300	Golden_11	18	963	18	1090
A-n60-k09	9	1412	9	1394	Golden_12	20	1192	19	1199
A-n61-k09	11	1300	9	1111	Golden_13	28	972	26	1175
A-n62-k08	8	1379	8	1363	Golden_14	31	1194	30	1471
A-n63-k09	10	1764	9	1676	Golden_15	35	1501	33	1743
A-n63-k10	11	1504	10	1442	Golden_16	39	1880	37	2092
A-n64-k09	9	1530	9	1501	Golden_17	22	774	23	1289
A-n65-k09	10	1274	9	1260	Golden_18	28	1108	29	2350
A-n69-k09	10	1297	9	1266	Golden_19	34	1578	35	3279
A-n80-k10	10	1903	10	1920	Golden_20	39	2084	40	4309

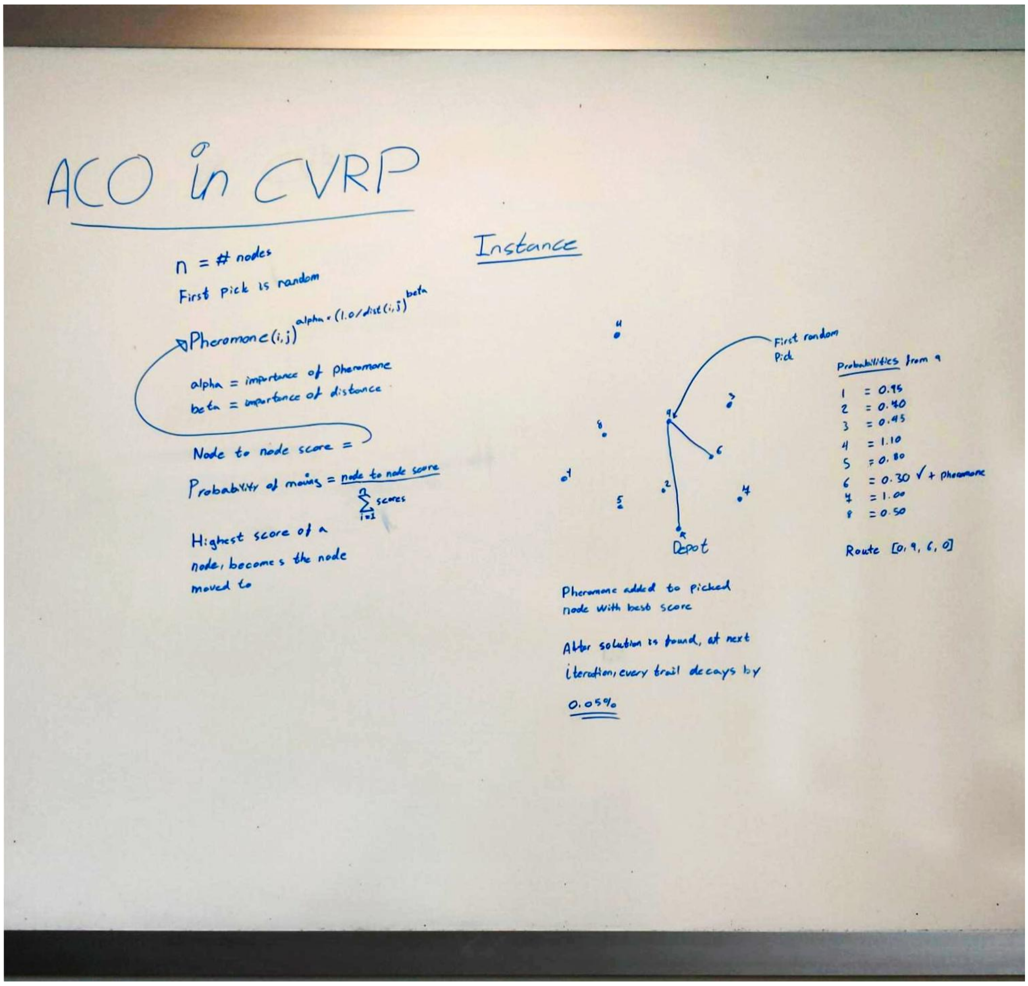
Table 1: Result comparing between Heuristic algorithm and Metahuristic running with time limit 240 seconds on an Intel(R) Core(TM) i7-2600 CPU @ 3.40GHz with 4 GB allocated RAM running Ubuntu 16.04.

3 Ant Colony Optimization

The Ant Colony Optimization (ACO) algorithm comes from the evolutionary algorithms, where computer science meets nature. In this algorithm, we have led us to be inspired by how ants find food in the nature. They are starting by spreading out in some area, randomly. When they seem to find trails, which could indicate to be good trails to find food, they leave pheromone behind them. The pheromone is used by other ants to determine their probability of taking a trail. If they are standing in a cross-section and they have to choose, they are taking the path with the highest pheromone. At some point in their exploration to find the best trail to obtain food, all the ants are using the same trails to get food and get back safe home.
We can apply the logic of the ants in the CVRP as well. By letting the instance the space of which the requests are "plotted" be the area to find a solution, we can start by sending one "ant" out to a point. From this point, we are going to calculate every probability of moving to the next point the one with the highest "score". We can calulate the probabilty of moving by the following:
First we establish the initial pheromone levels from all the points to every counter other point:

$$M = \begin{bmatrix} 0 & 0.50 & 0.20 \\ 0.125 & 0 & 0.60 \\ 1.20 & 0.355 & 0 \end{bmatrix}$$

Figure 1: Illustration of the ACO algorithm, showing how it can be used, and how it process next moves



4 BOXPLOT FIGURES

References

[1] Minyi Yue. A simple proof of the inequality ffd(l) $\leq$ 11/9 opt(l) + 1, $\forall$l for the ffd bin-packing algorithm. *Acta Mathematicae Applicatae Sinica*, 7(4):321–331, 1991.

Appendices