Machine Learning course,
advanced track

Lecture 15: Generative models

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MIPT
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References

This lecture structure was inspired a lot by the series of Habr posts on Autoencoders and GANs in Keras

Outline

- Generative models overview
- Simple Autoencoders (not generative models!)
- Variational Autoencoders (VAE)
  - Conditional VAE
- Generative Adversarial Networks
  - Conditional GAN

And a lot of gif images.

Generative models taxonomy

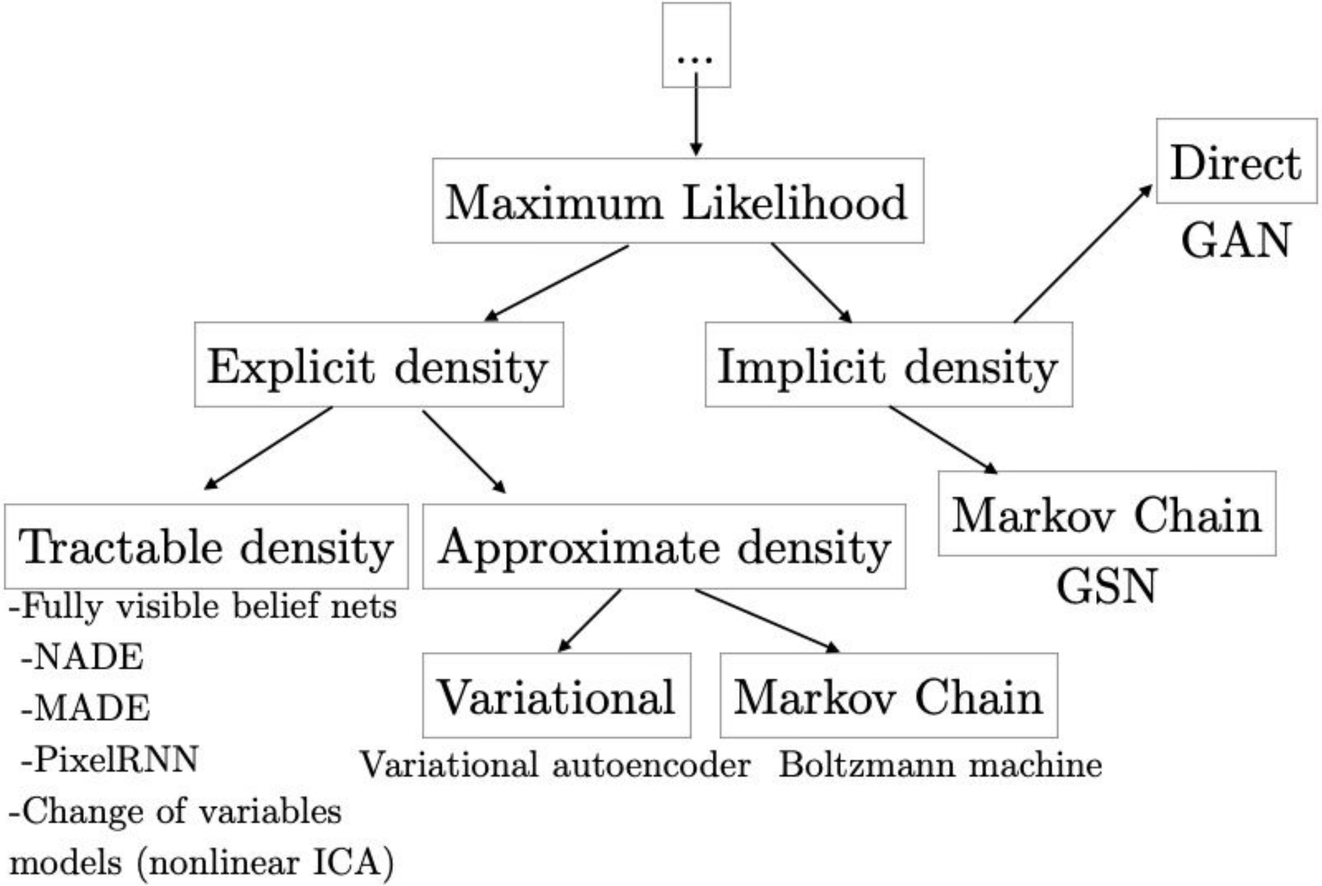


Image source: [Tutorial on Generative Adversarial Networks, Ian Goodfellow, NIPS, 2016]

Generative models taxonomy

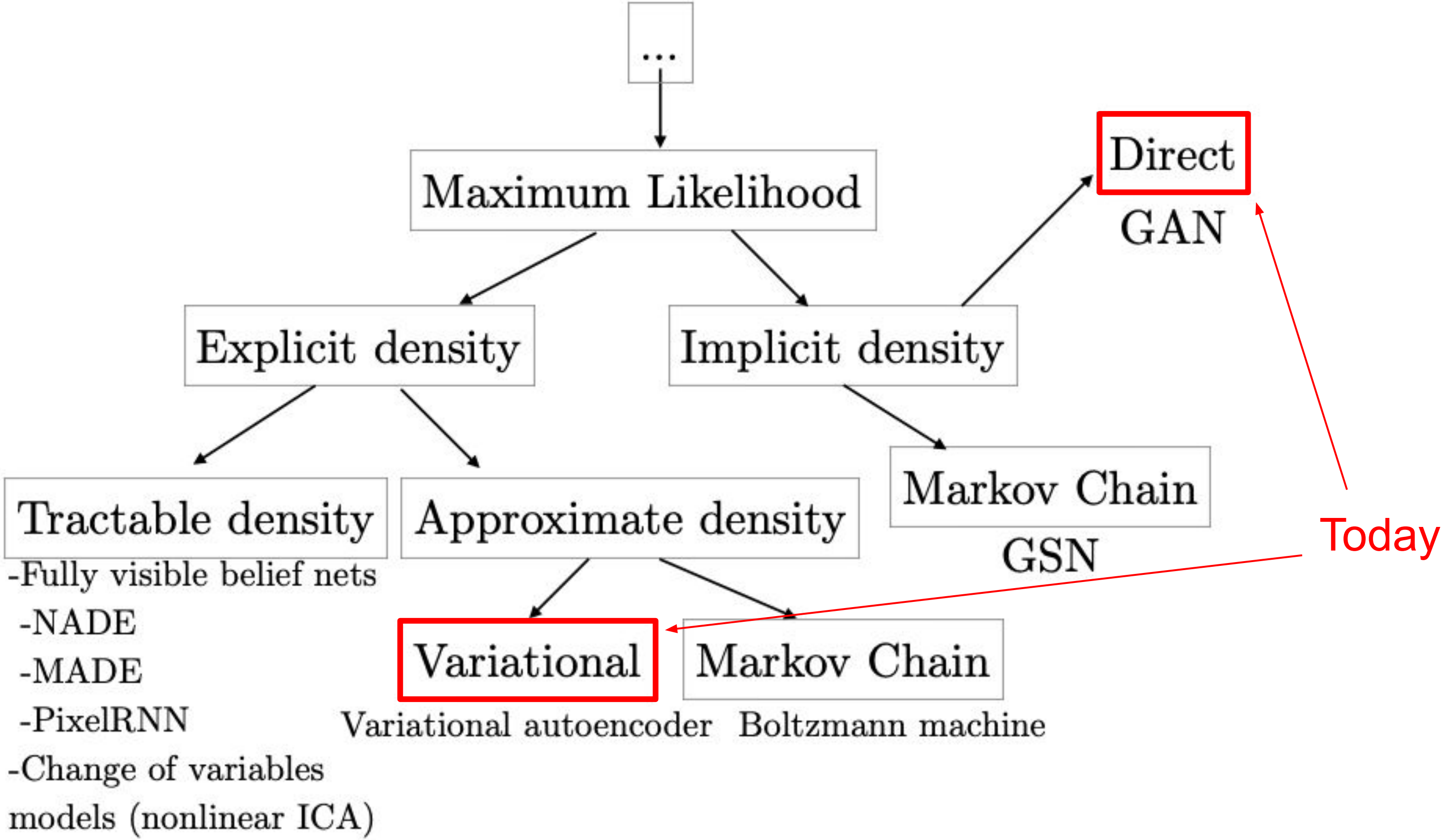


Image source: [Tutorial on Generative Adversarial Networks, Ian Goodfellow, NIPS, 2016]

Autoencoders

Denote **z** as encoded with encoder E input **x**

$$\mathbf{z} = E(\mathbf{x}, \boldsymbol{\theta}_E)$$

Decoder D recovers **x** from latent representation

$$\hat{\mathbf{x}} = D(\mathbf{z}, \boldsymbol{\theta}_D)$$

Optimal parameters learned w.r.t. loss function L

$$[\boldsymbol{\theta}_E, \boldsymbol{\theta}_D] = \arg\min L(\hat{\mathbf{x}}, \mathbf{x})$$

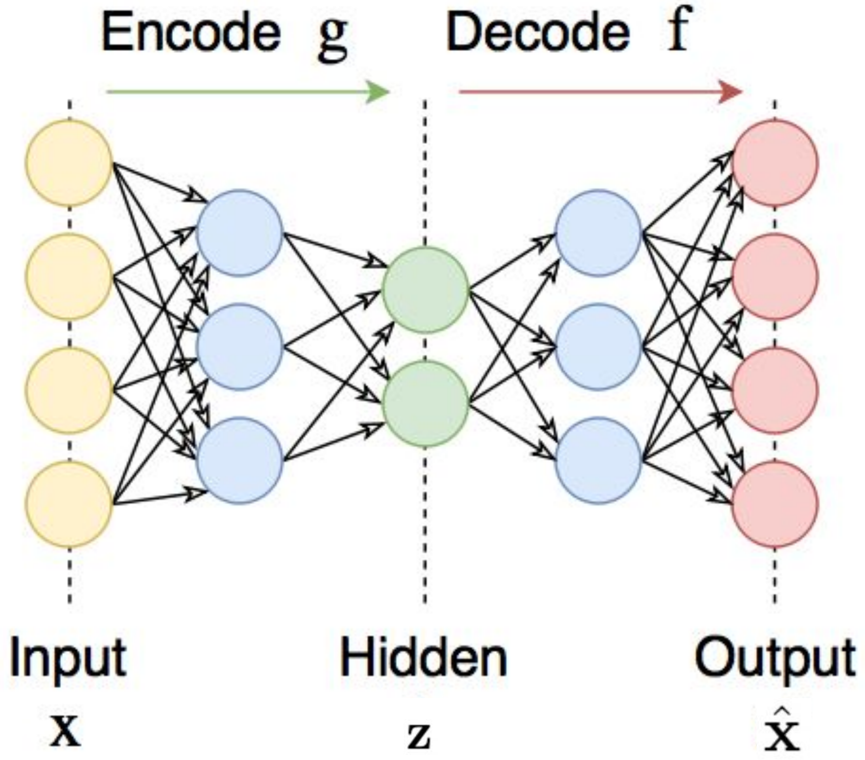


Image source: [Habr post on autoencoders and GANs]

Autoencoders

Denote **z** as encoded with encoder E input **x**

$$\mathbf{z} = E(\mathbf{x}, \boldsymbol{\theta}_E)$$

Decoder D recovers **x** from latent representation

$$\hat{\mathbf{x}} = D(\mathbf{z}, \boldsymbol{\theta}_D)$$

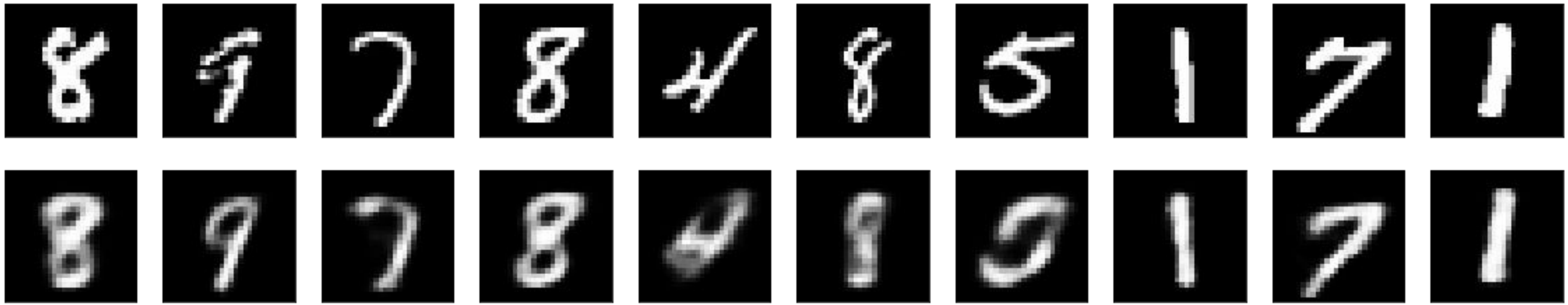
Simple example: PCA

Optimal parameters learned w.r.t. loss function L

$$[\boldsymbol{\theta}_E, \boldsymbol{\theta}_D] = \arg\min L(\hat{\mathbf{x}}, \mathbf{x})$$

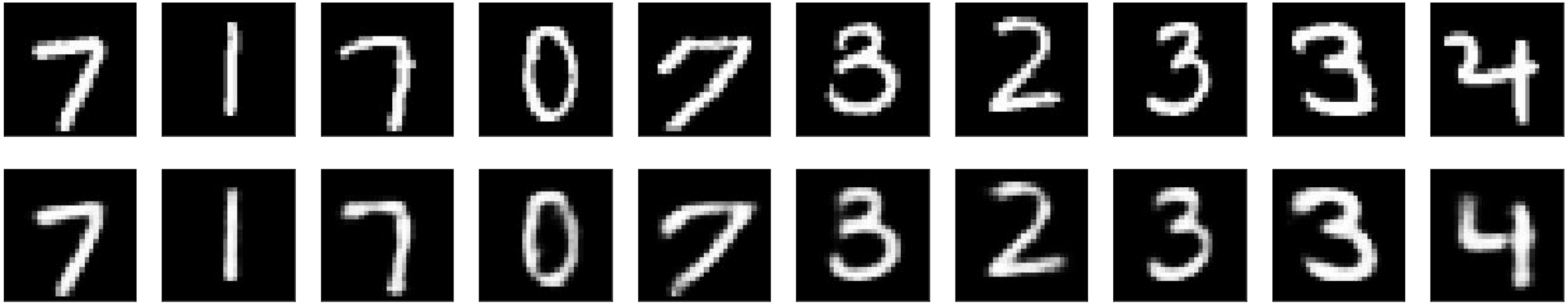
Image source: [Habr post on autoencoders and GANs]()

PCA performance on MNIST



16 components

Convolutional performance on MNIST

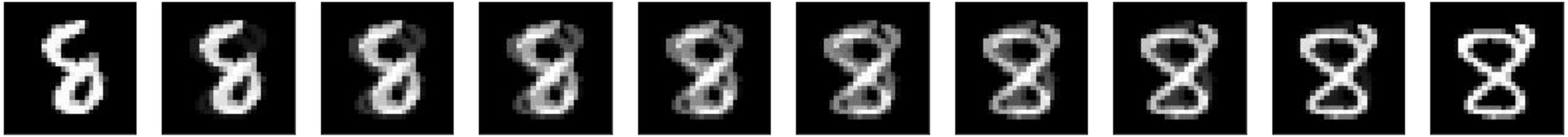


7 x 7 latent space

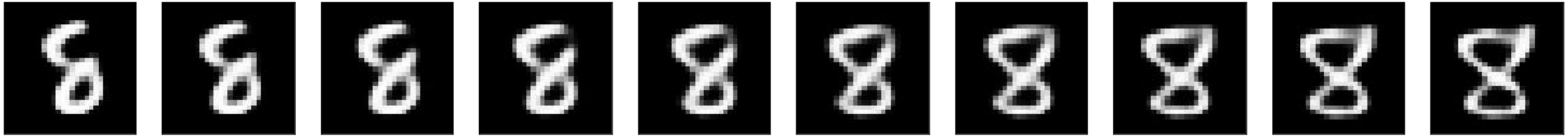
Homotopy between samples

10 steps between samples

- In original feature space (28 x 28):



- In latent space (7 x 7):



Latent space structure

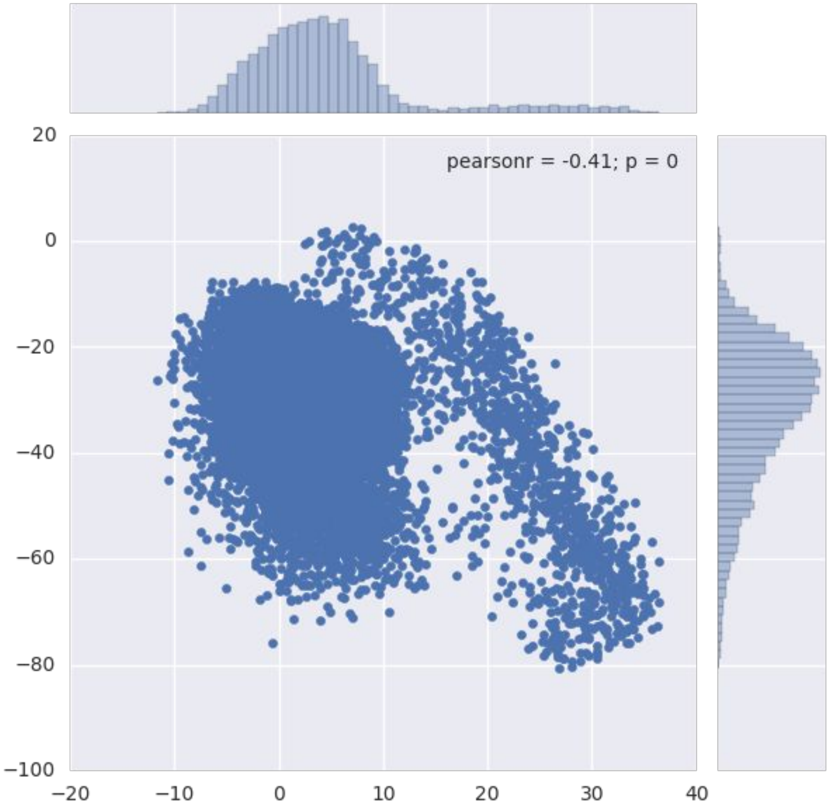


Image source: [Habr post on autoencoders and GANs]

VAE intuition

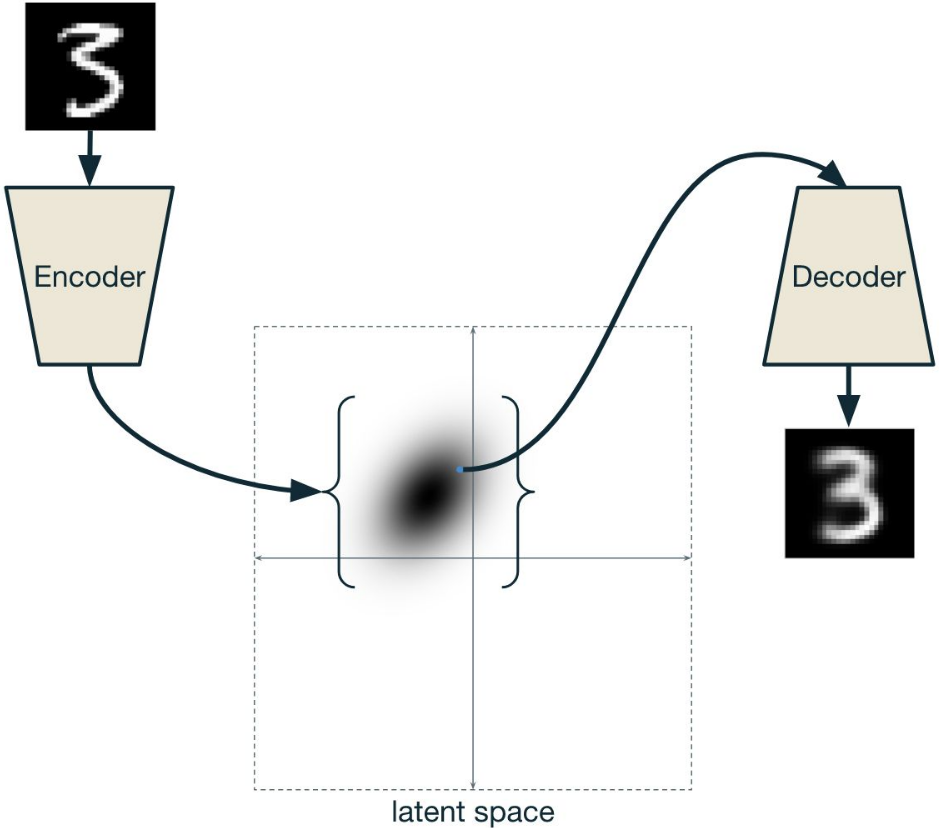


Image source: [Conditional Variational Autoencoders by Isaac Dykeman, 2016]

KL divergence

Denote distributions $Q(z)$ and $P(z|X)$.

Kullback–Leibler divergence is defined as

$$\mathcal{D}\left[Q(z) \| P(z|X)\right] = E_{z \sim Q}\left[\log Q(z) - \log P(z|X)\right]$$

KL divergence

$$\mathcal{D}\left[Q(z)\|P(z|X)\right] = E_{z\sim Q}\left[\log Q(z) - \log P(z|X)\right]$$

Applying the Bayes rule:

$$\mathcal{D}\left[Q(z)\|P(z|X)\right] = E_{z\sim Q}\left[\log Q(z) - \log P(X|z) - \log P(z)\right] + \log P(X)$$

KL divergence

$$\mathcal{D}\left[Q(z)\|P(z|X)\right] = E_{z\sim Q}\left[\log Q(z) - \log P(z|X)\right]$$

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$$\mathcal{D}\left[Q(z)\|P(z|X)\right] = E_{z\sim Q}\left[\log Q(z) - \log P(X|z) - \log P(z)\right] + \log P(X)$$

$$\log P(X) - \mathcal{D}\left[Q(z)\|P(z|X)\right] = E_{z\sim Q}\left[\log P(X|z)\right] - \mathcal{D}\left[Q(z)\|P(z)\right]$$

KL divergence

$$\mathcal{D}\left[Q(z)\|P(z|X)\right] = E_{z\sim Q}\left[\log Q(z) - \log P(z|X)\right]$$

Applying the Bayes rule:

$$\mathcal{D}\left[Q(z)\|P(z|X)\right] = E_{z\sim Q}\left[\log Q(z) - \log P(X|z) - \log P(z)\right] + \log P(X)$$

$$\log P(X) - \mathcal{D}\left[Q(z)\|P(z|X)\right] = E_{z\sim Q}\left[\log P(X|z)\right] - \mathcal{D}\left[Q(z)\|P(z)\right]$$

$$\boxed{\log P(X) - \mathcal{D}\left[Q(z|X)\|P(z|X)\right] = E_{z\sim Q}\left[\log P(X|z)\right] - \mathcal{D}\left[Q(z|X)\|P(z)\right]}$$

This equation is the core of Variational Autoencoders

Structure of the latent space

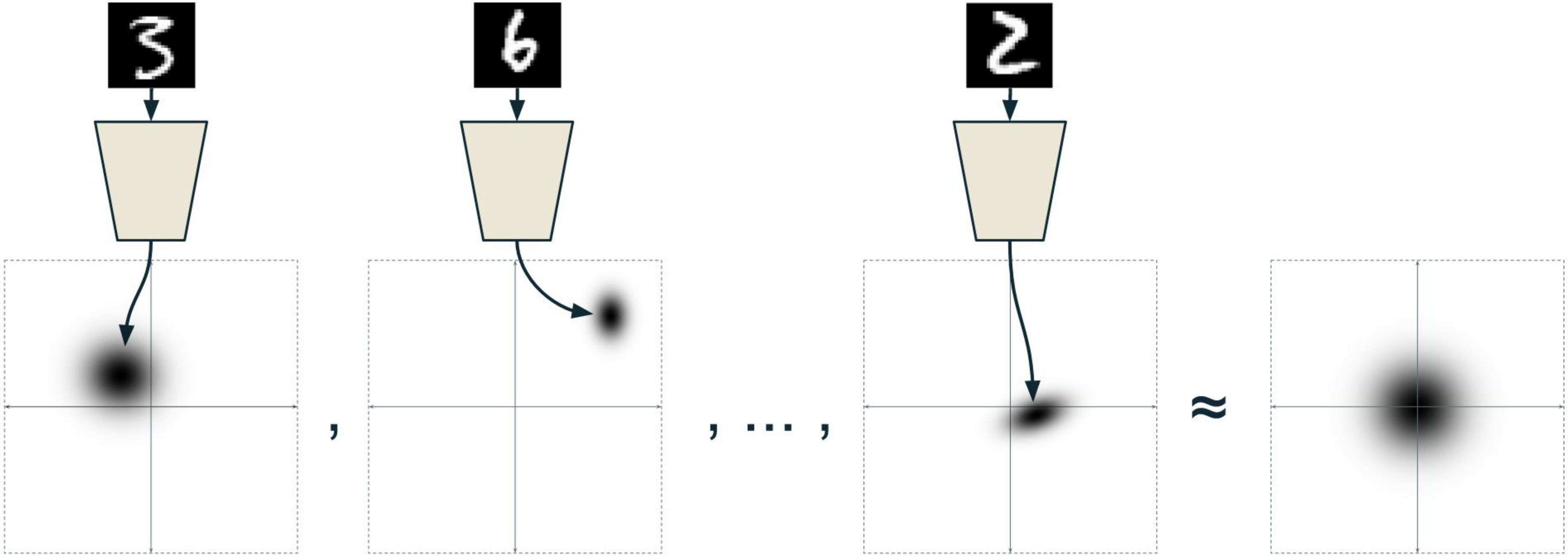


Image source: [Conditional Variational Autoencoders by Isaac Dykeman, 2016]

VAE so far

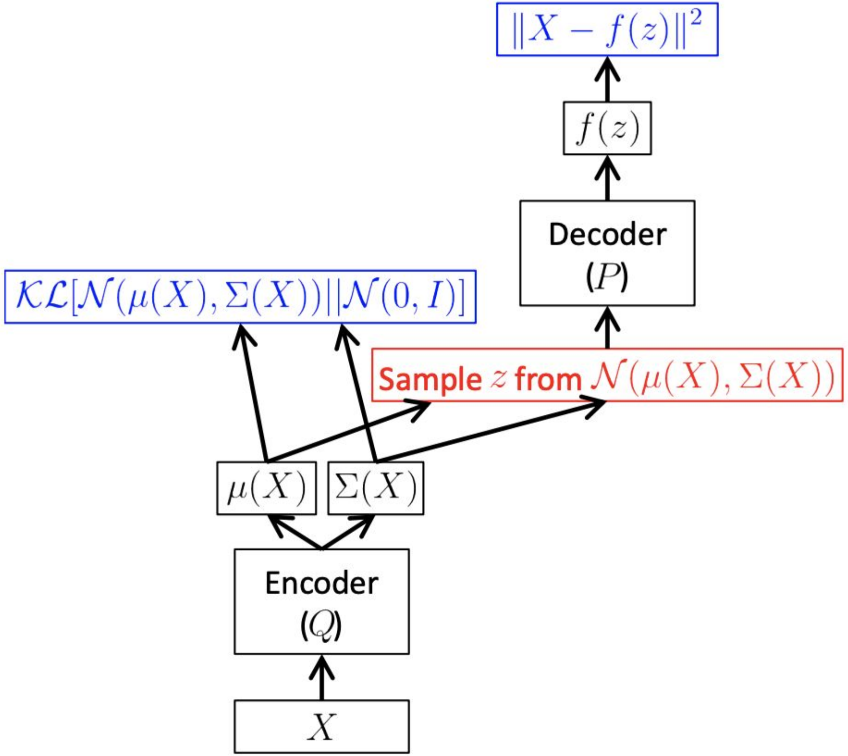


Image source: [Tutorial on Variational Autoencoders by Carl Doersch, 2016]

VAE so far

$$\mathcal{D}[\mathcal{N}(\mu(X),\Sigma(X))\|\mathcal{N}(0,I)] =$$
$$\frac{1}{2}\left(\mathrm{tr}\left(\Sigma(X)\right) + (\mu(X))^\top(\mu(X)) - k - \log\det\left(\Sigma(X)\right)\right)$$

Try to derive it by yourself

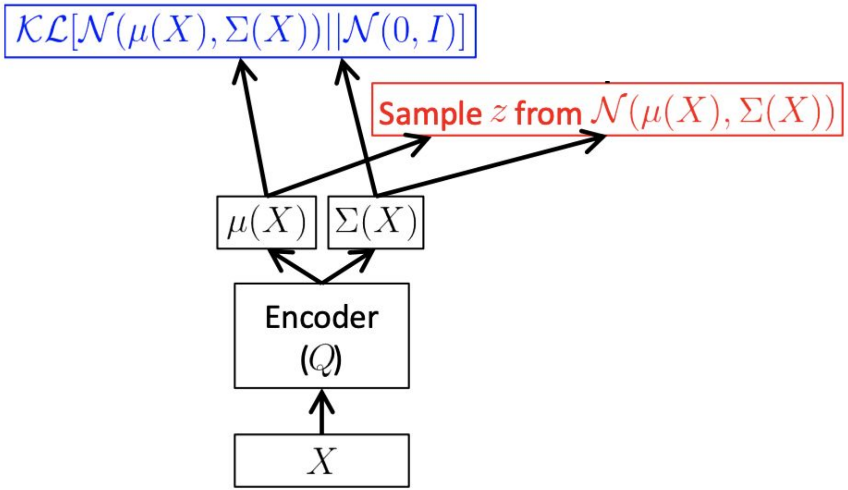


Image source: [Tutorial on Variational Autoencoders by Carl Doersch, 2016]

VAE so far

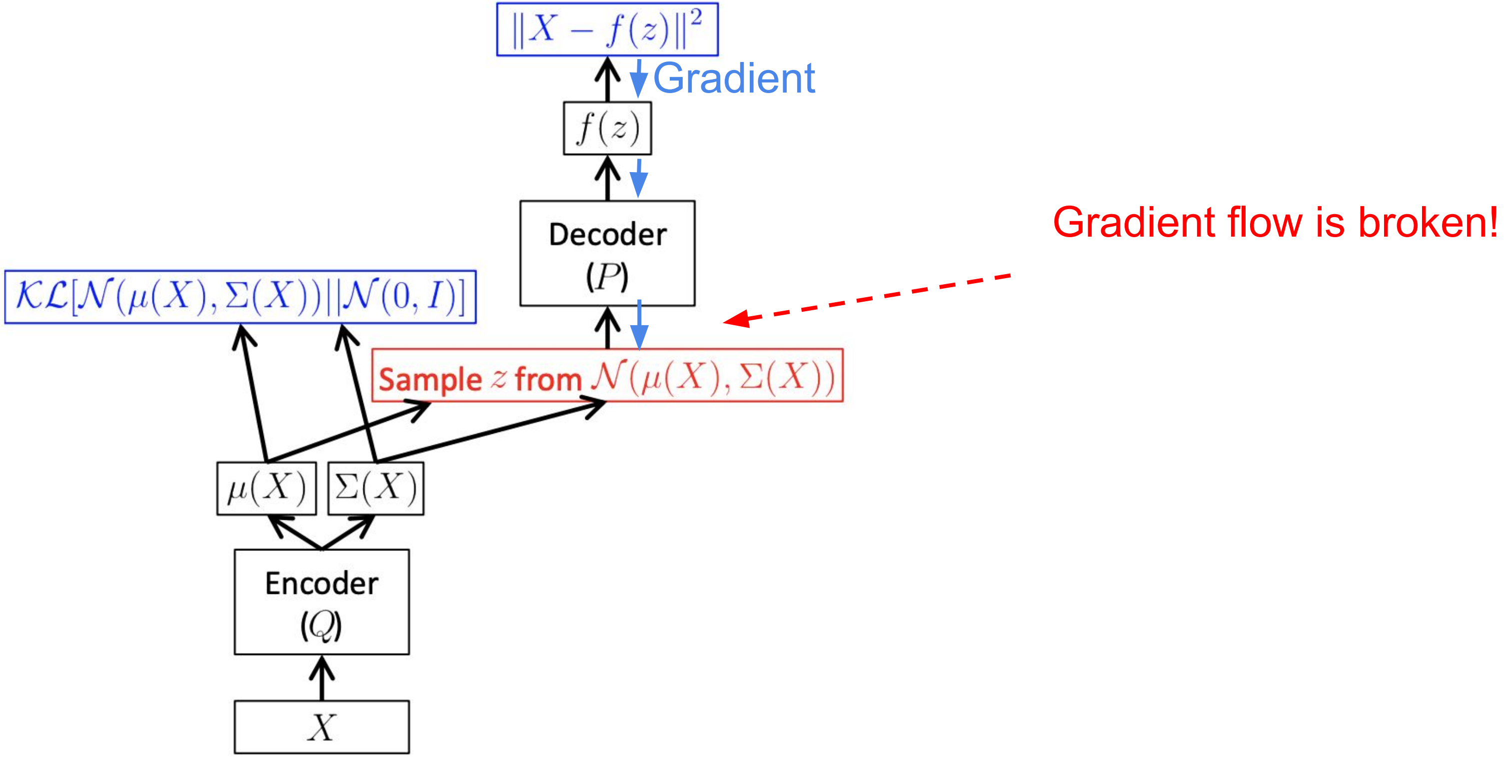
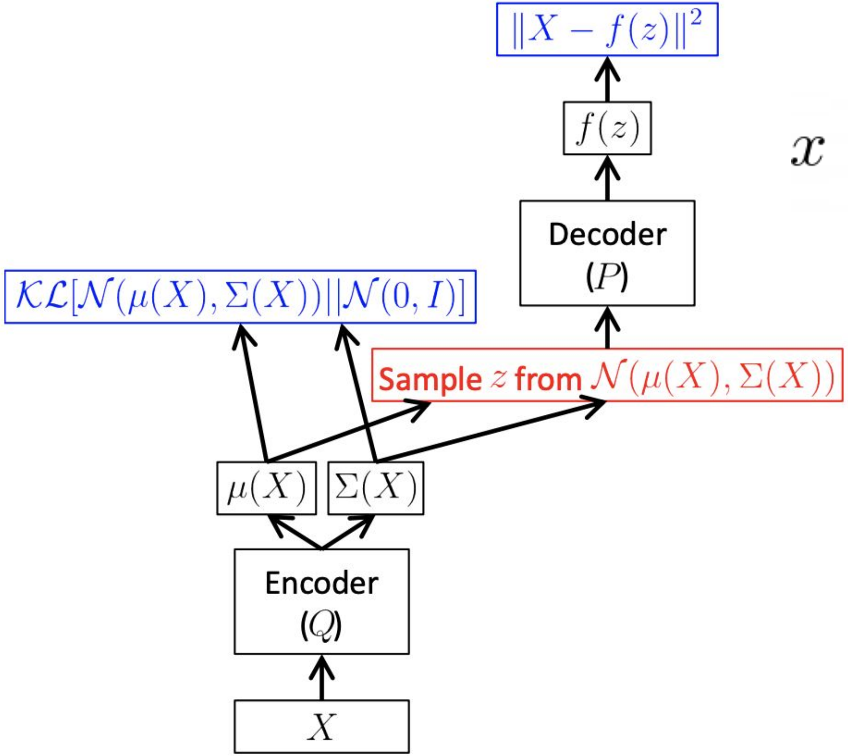


Image source: Tutorial on Variational Autoencoders by Carl Doersch, 2016

Reparametrization trick



$$x \sim \mathcal{N}(\mu, \sigma^2), \quad z = \frac{x - \mu}{\sigma} \sim \mathcal{N}(0, 1)$$

$$\Rightarrow x = \sigma(z + \mu) \sim \mathcal{N}(\mu, \sigma^2)$$

Image source: [Tutorial on Variational Autoencoders by Carl Doersch, 2016]

Reparametrization trick

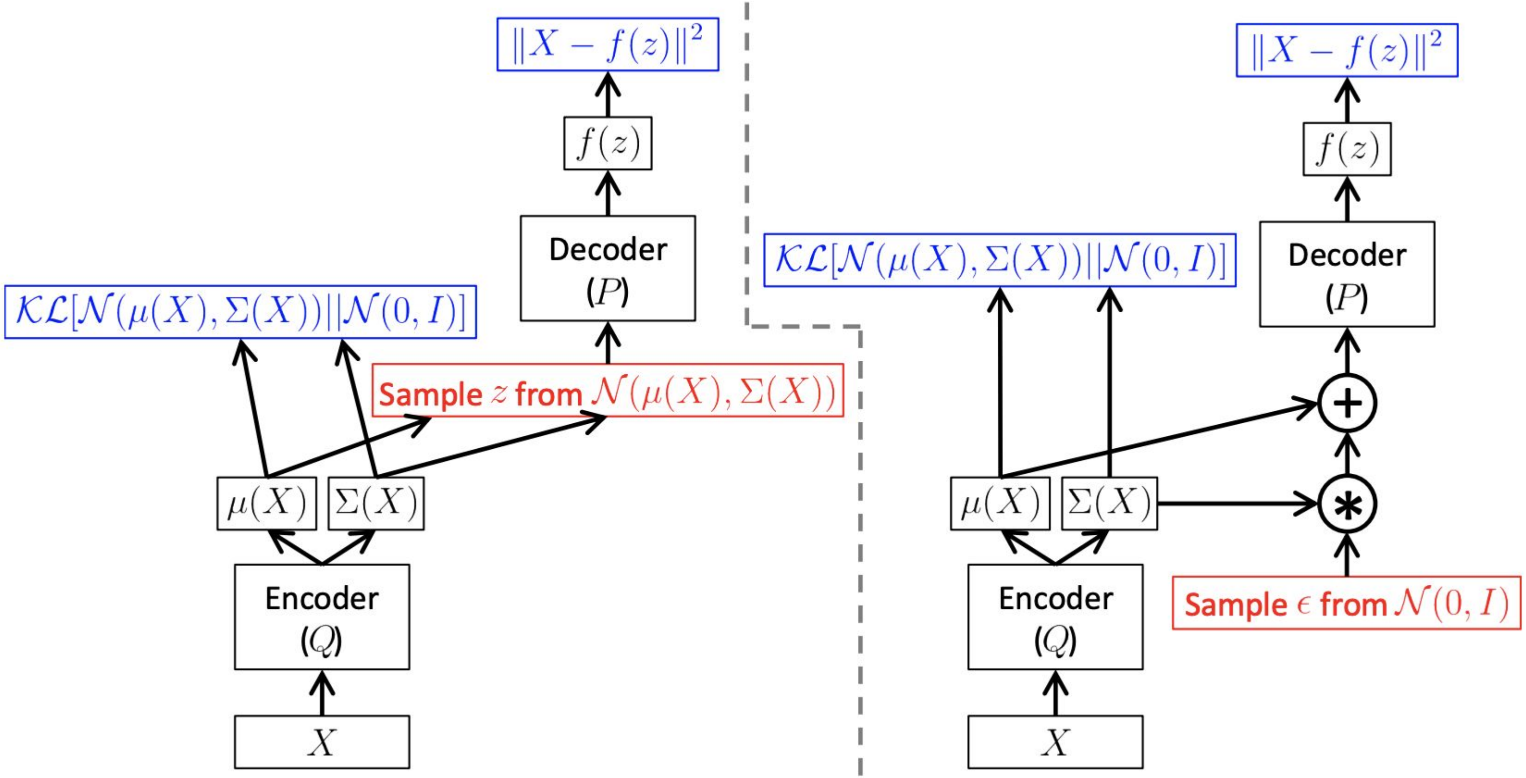
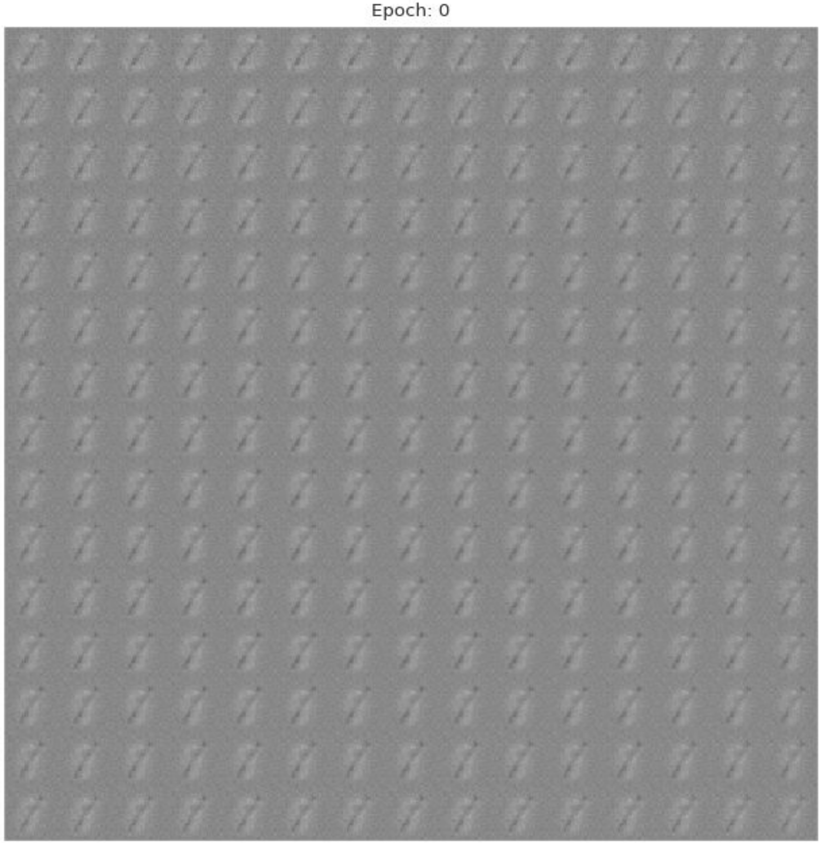


Image source: Tutorial on Variational Autoencoders by Carl Doersch, 2016

VAE manifold



Structure of the latent space

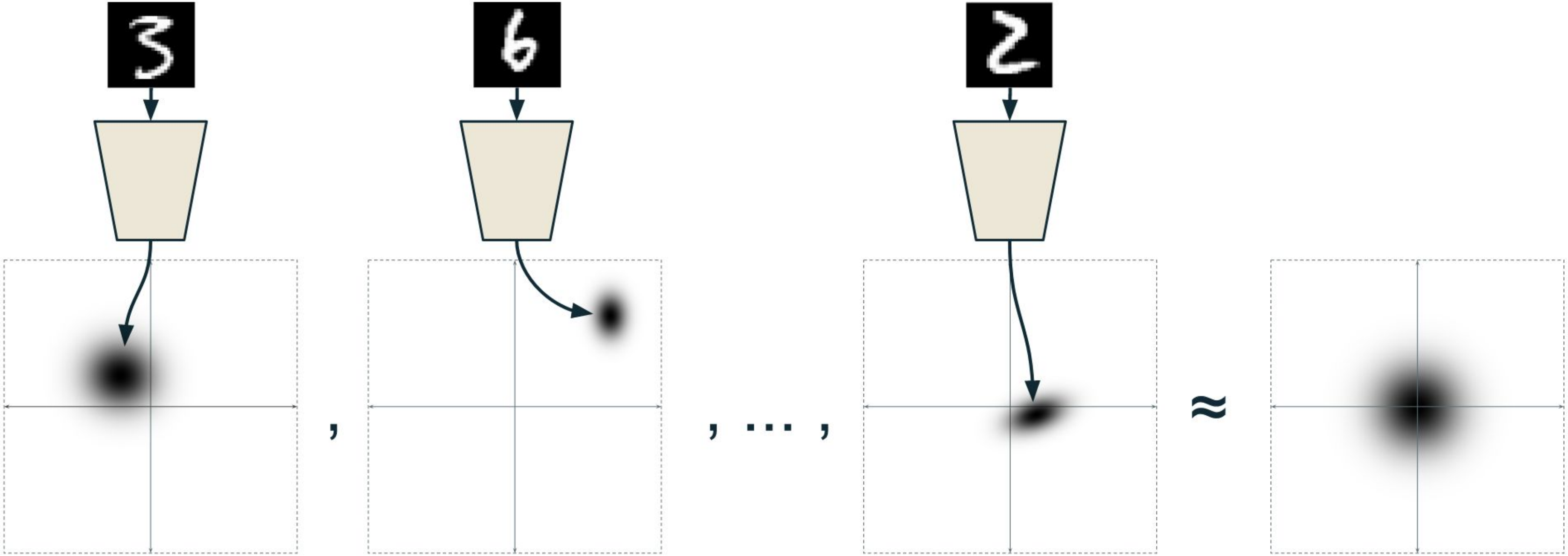
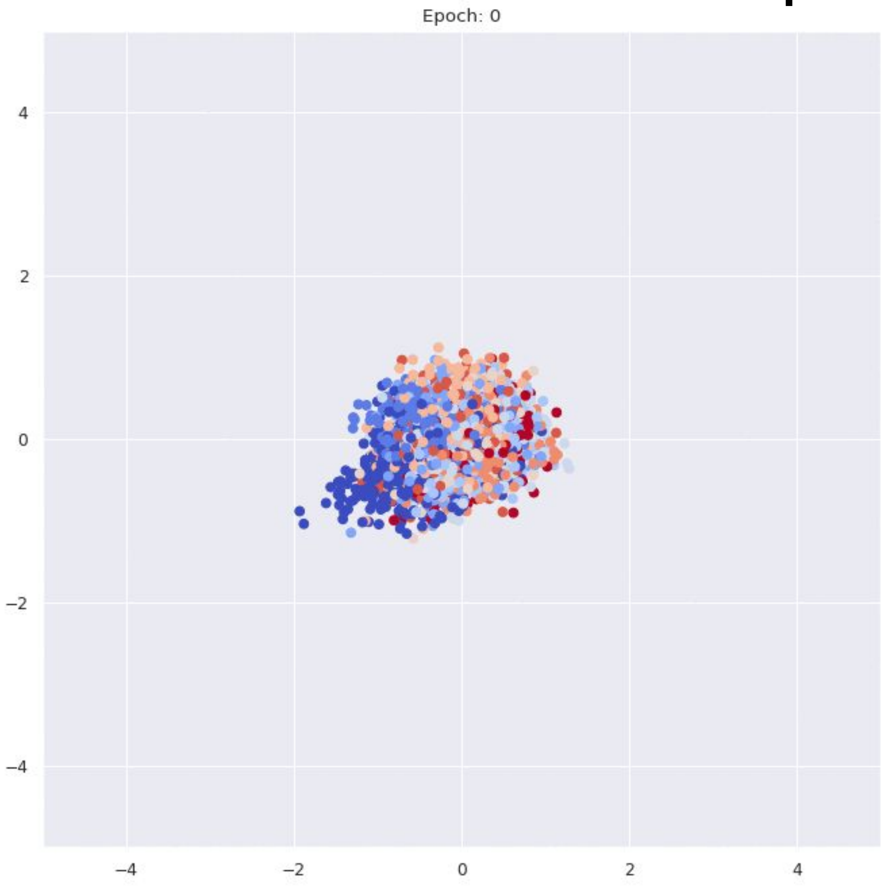


Image source: [Conditional Variational Autoencoders by Isaac Dykeman, 2016]

VAE latent space distribution



Conditional VAE intuition

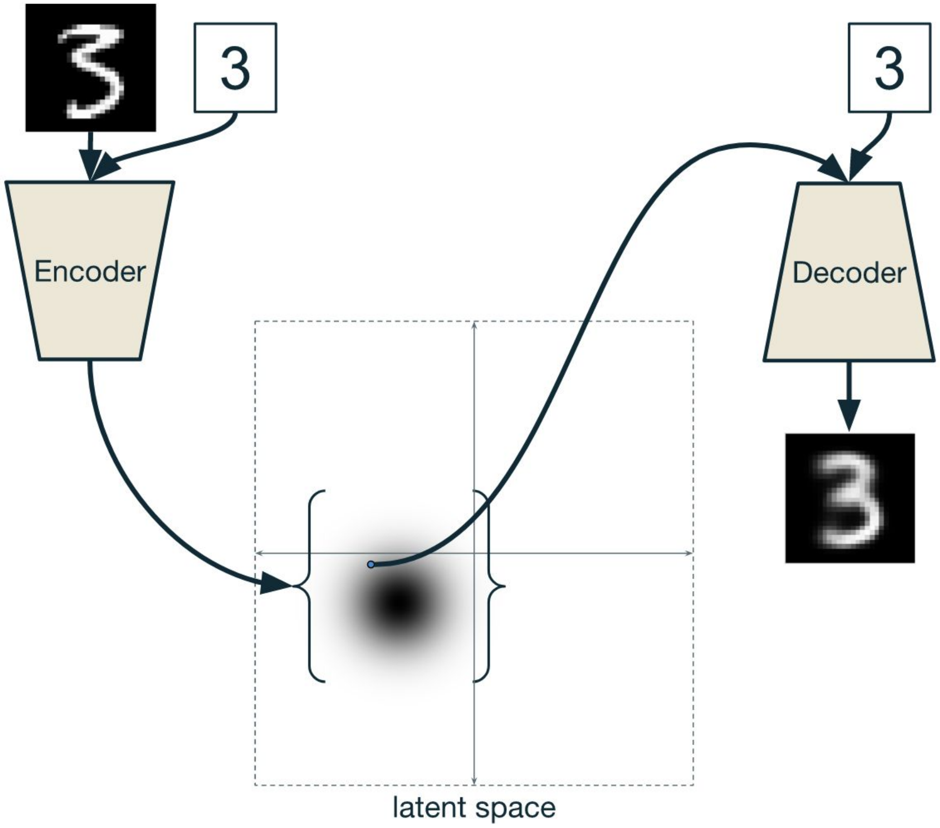


Image source: [Conditional Variational Autoencoders by Isaac Dykeman, 2016]

Conditional VAE

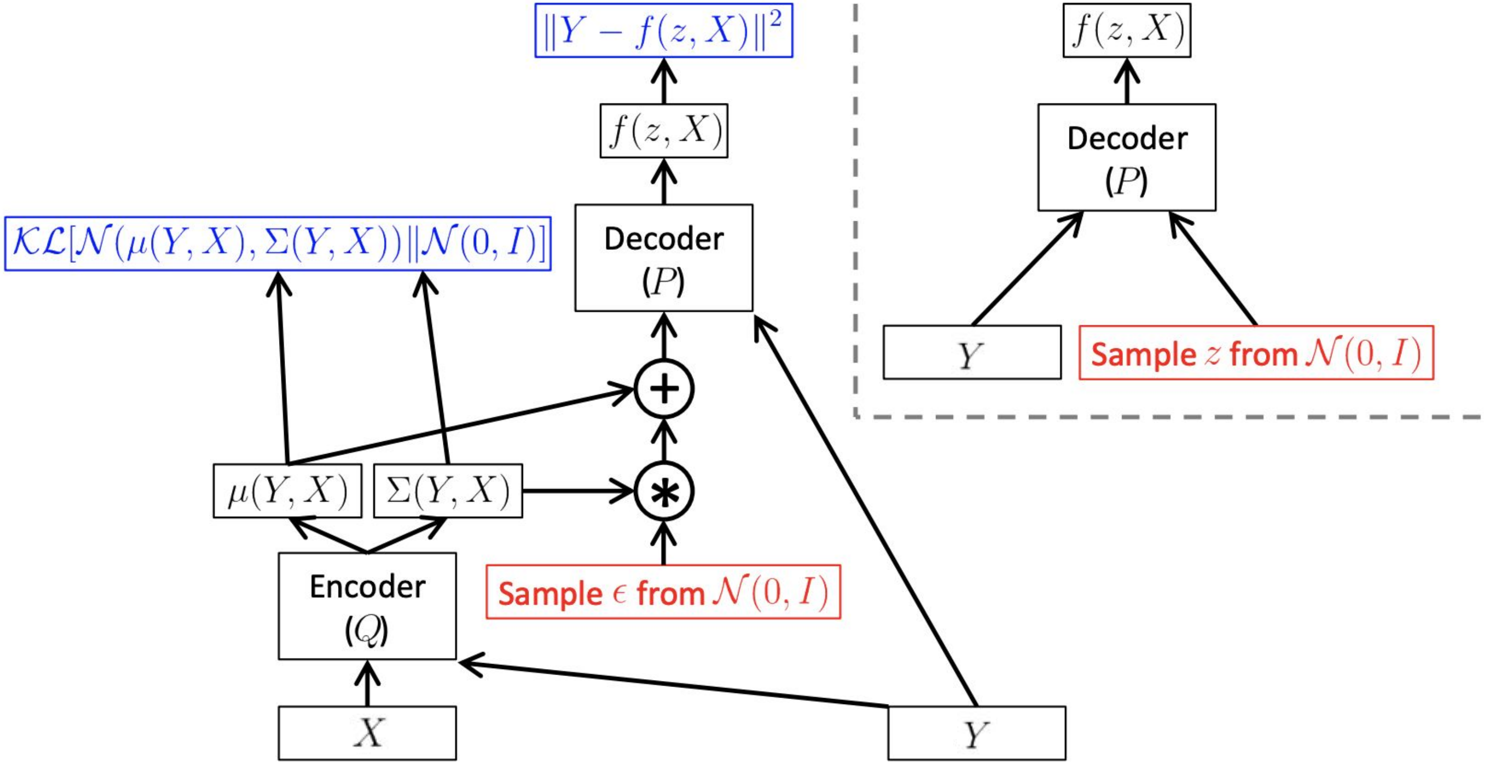
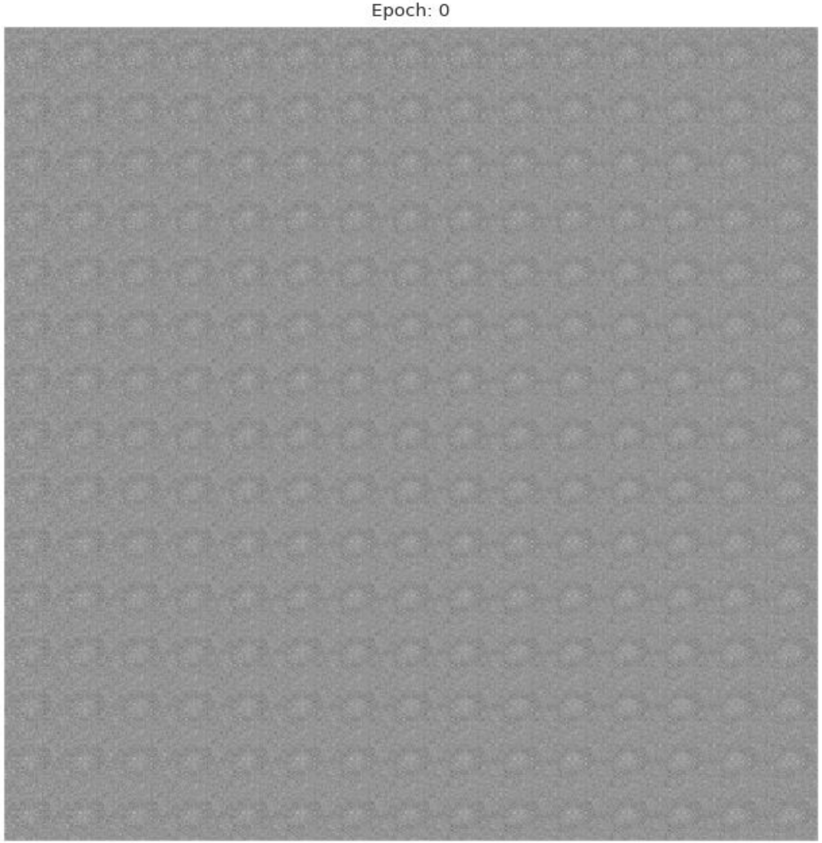
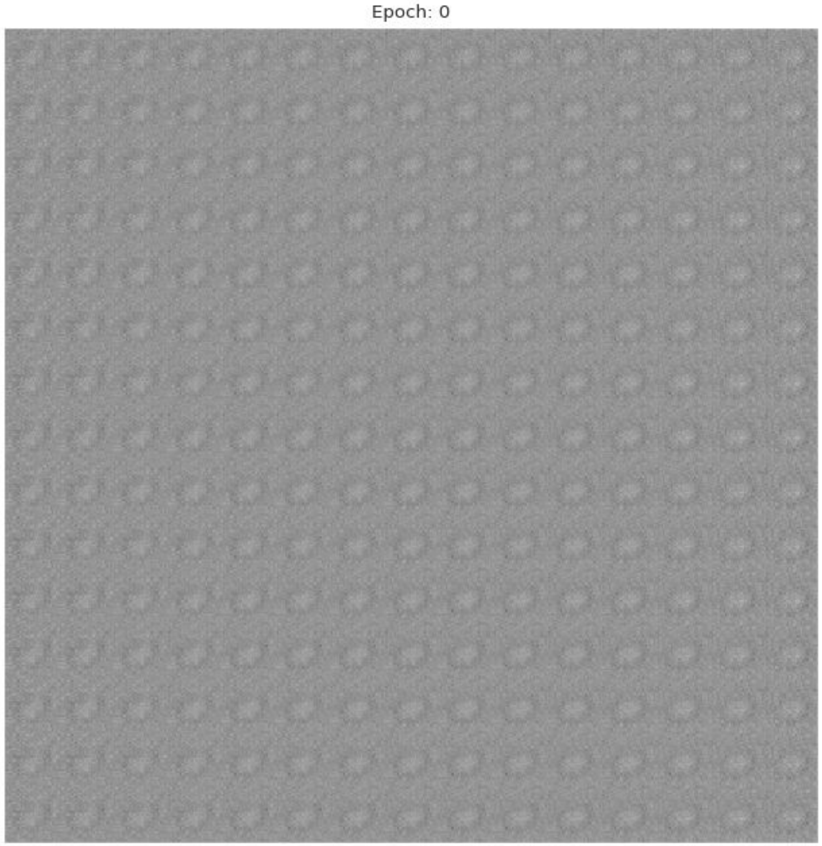


Image source: [Tutorial on Variational Autoencoders by Carl Doersch, 2016]

cVAE manifold



cVAE manifold



Transferring style with cVAE

Seed label is 1

Once again

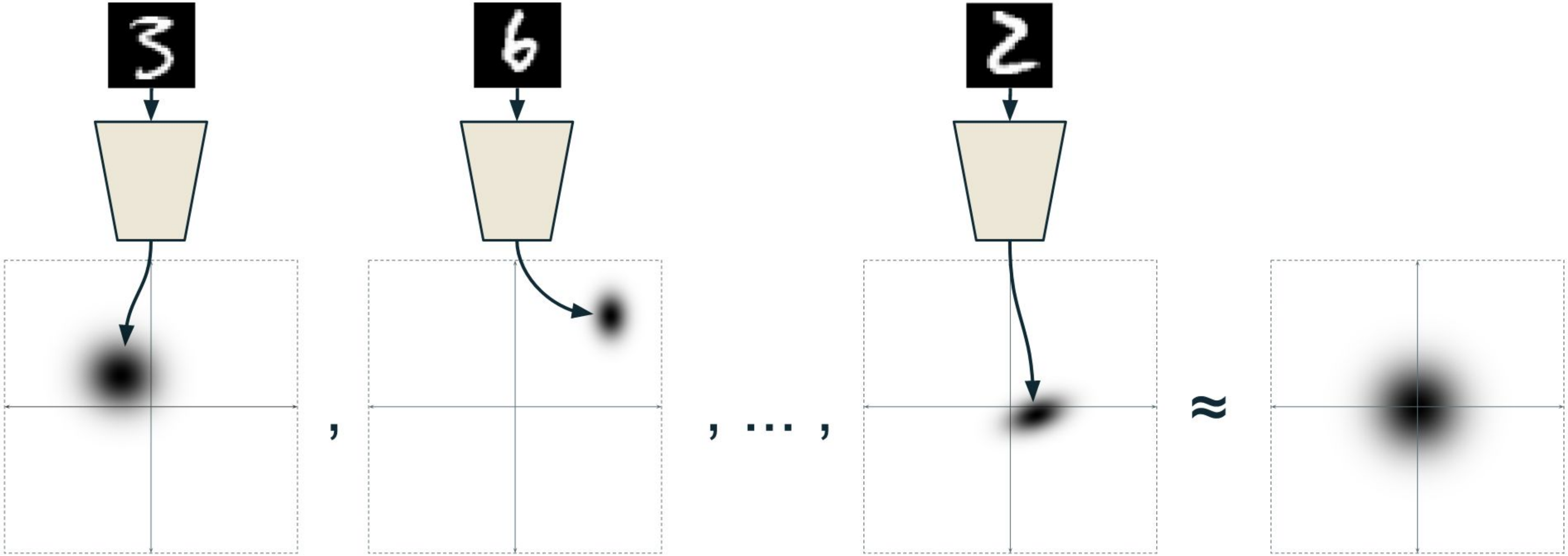
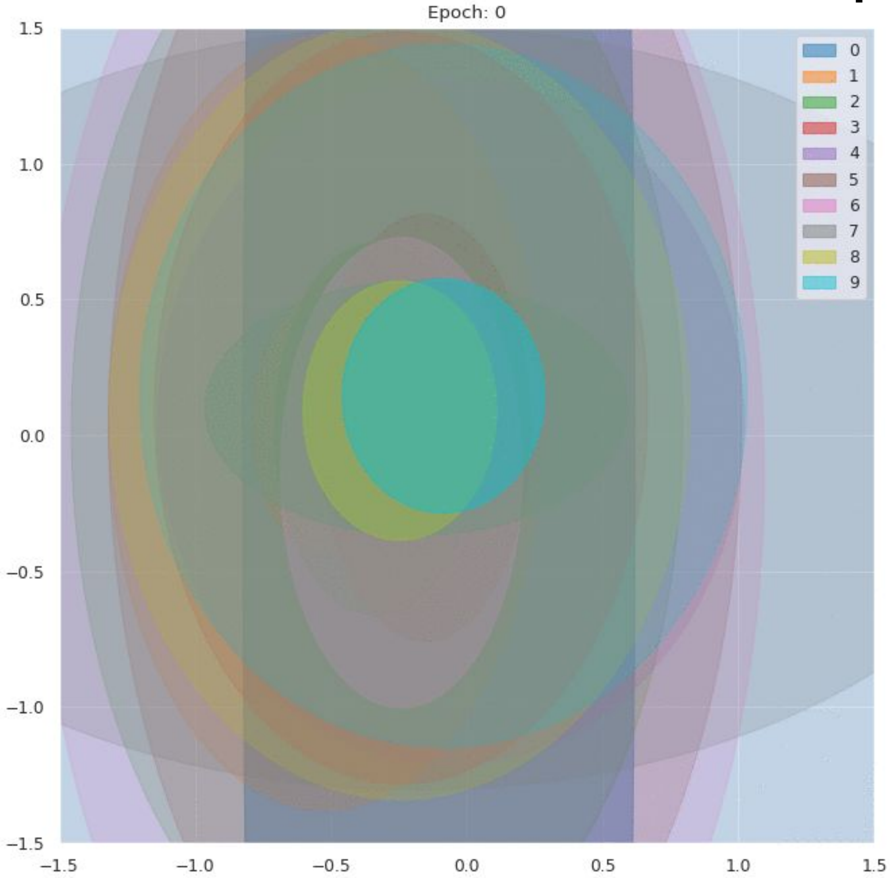


Image source: [Conditional Variational Autoencoders by Isaac Dykeman, 2016]

cVAE latent space distribution



GAN

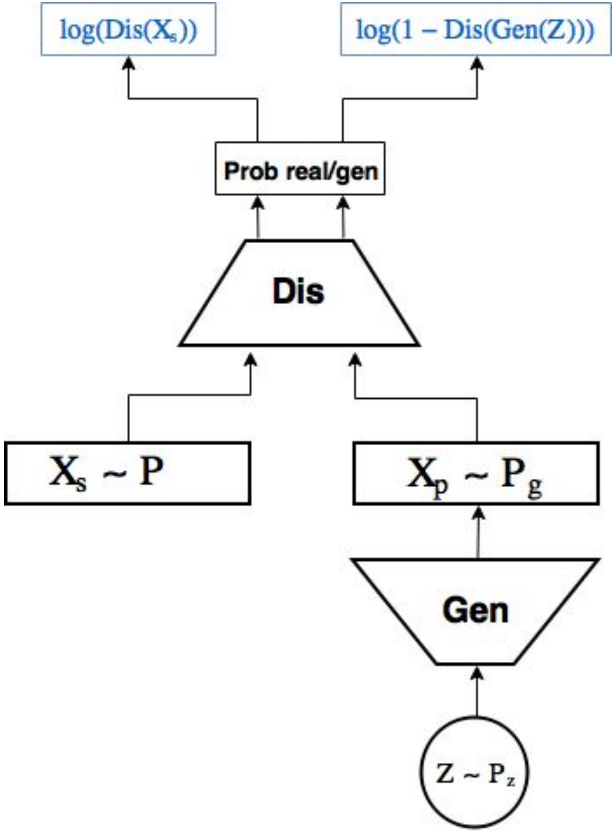
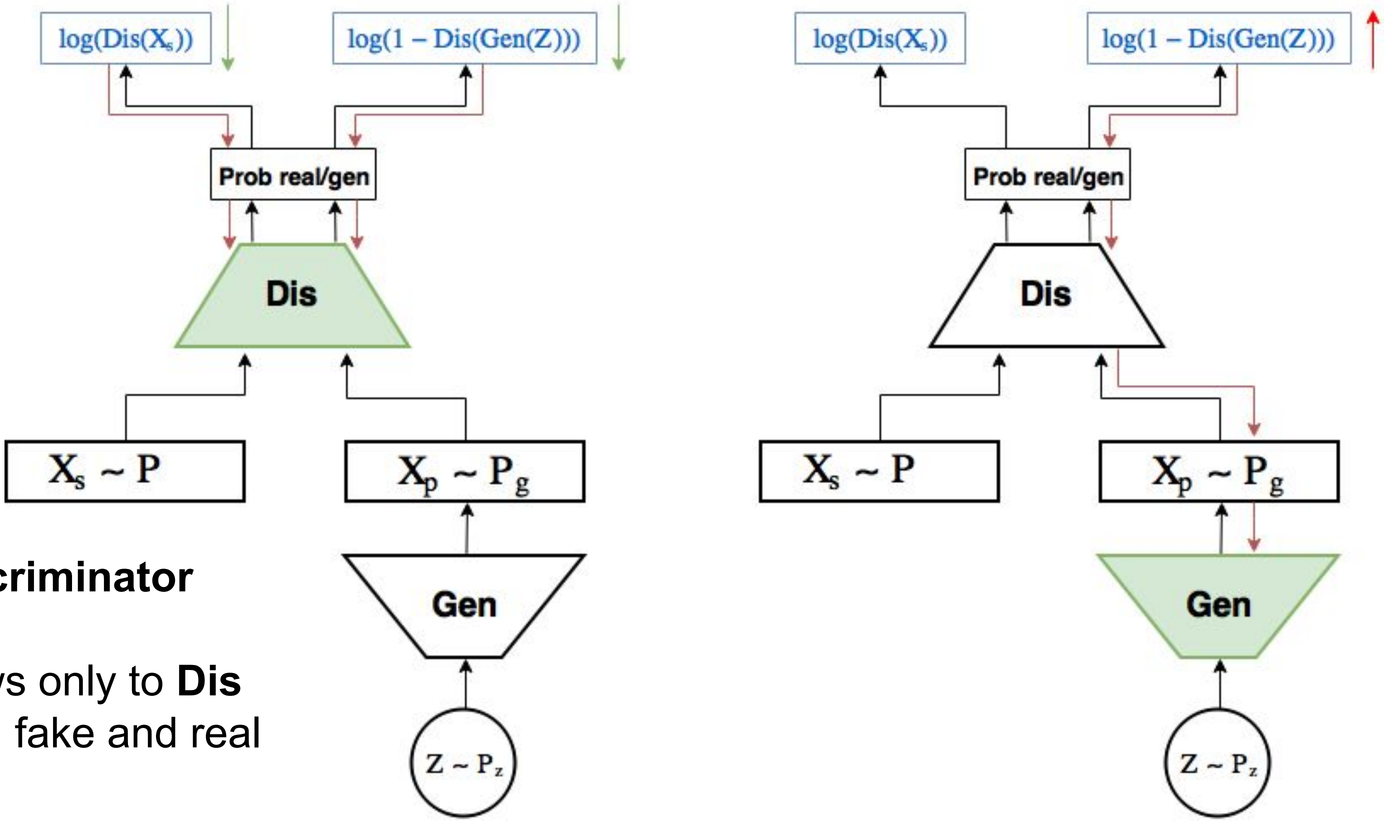


Image source: Habr post on autoencoders and GANs

Training GAN

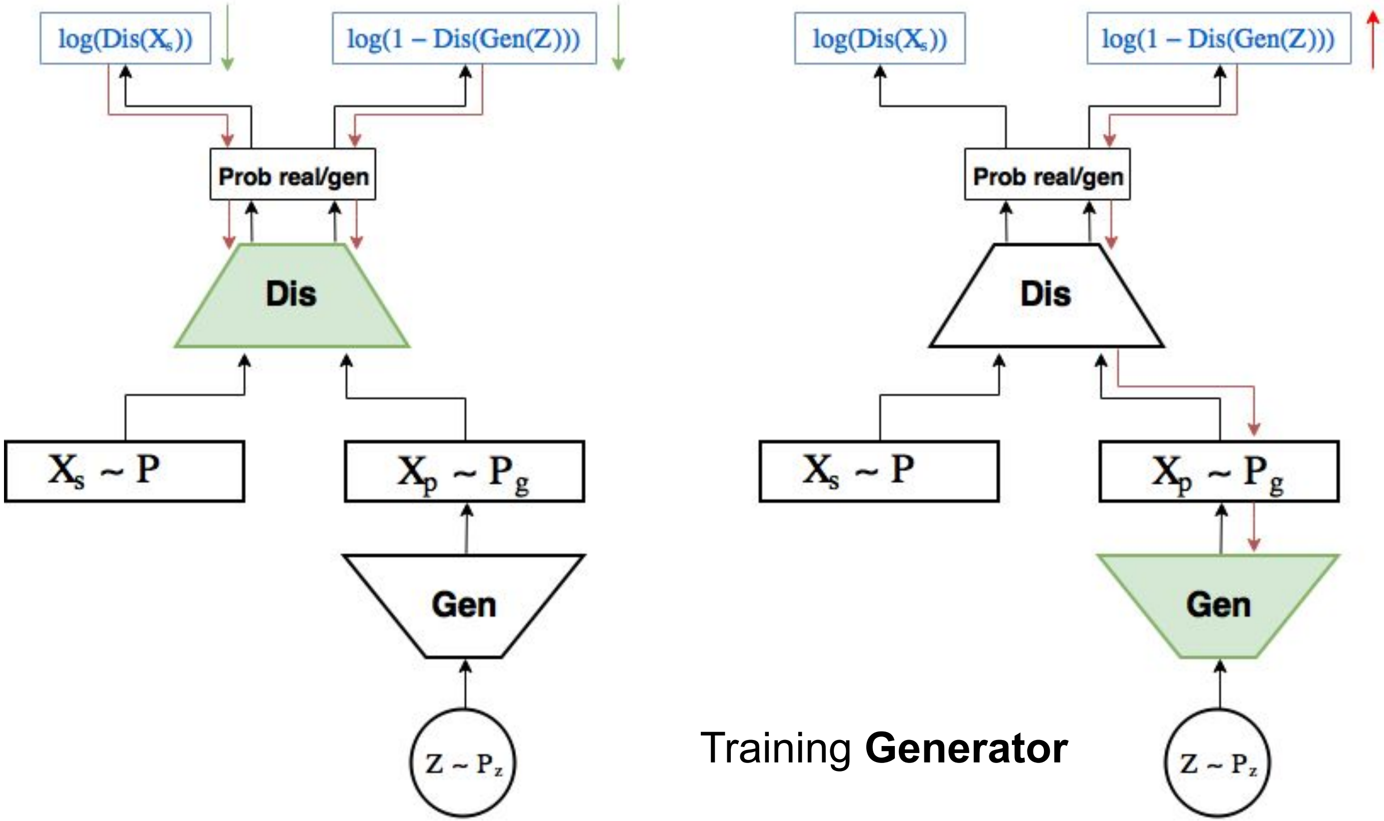


Training **Discriminator**

Gradient flows only to **Dis** to distinguish fake and real examples

Image source: [Habr post on autoencoders and GANs]

Training GAN



Training **Generator**

Gradient flows to **Gen** with **Dis** weights freezed to fool the Discriminator

Image source: [Habr post on autoencoders and GANs]

Optimization process in GAN

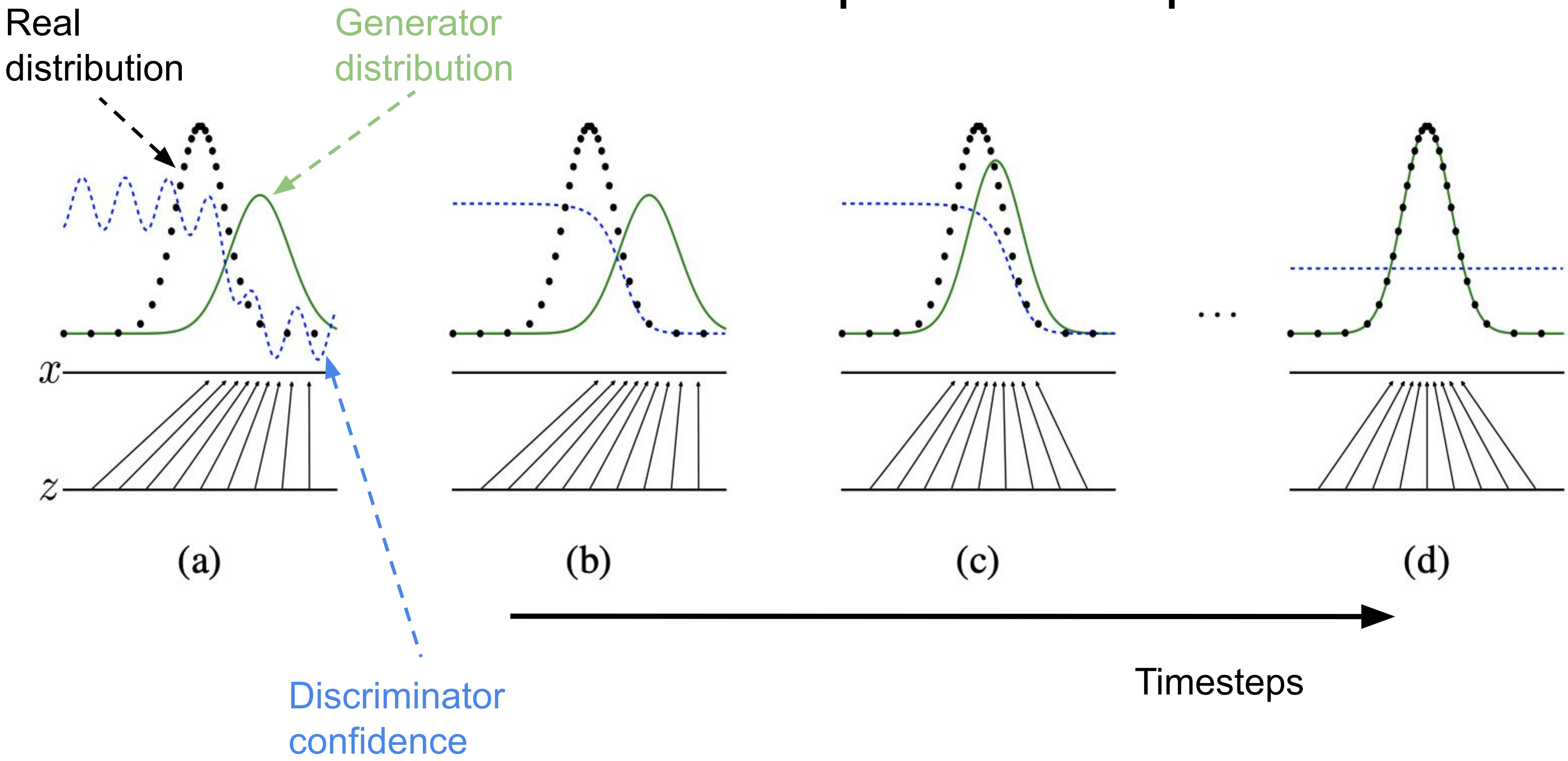


Image source: [Generative Adversarial Nets, Ian J. Goodfellow et al, 2014]

GAN manifold

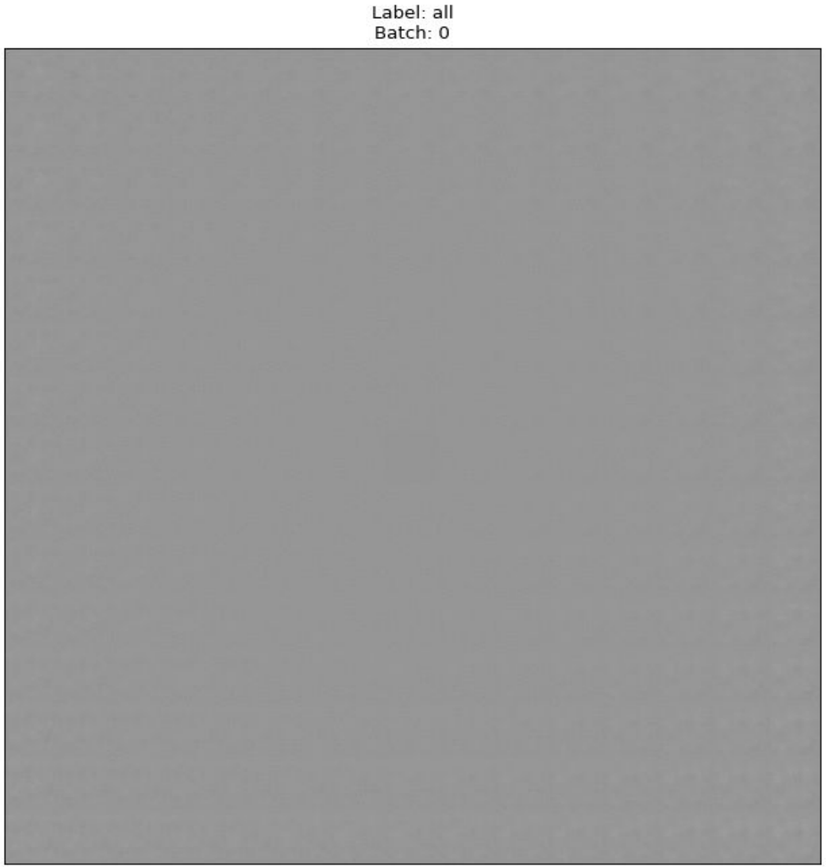


Image source: [Habr post on autoencoders and GANs]

Conditional GAN

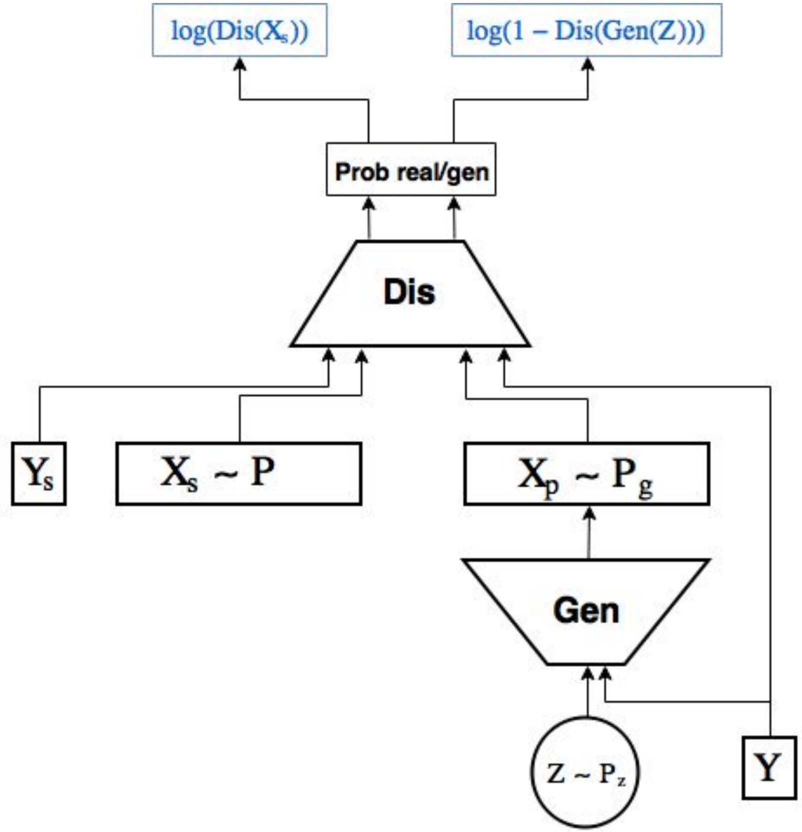
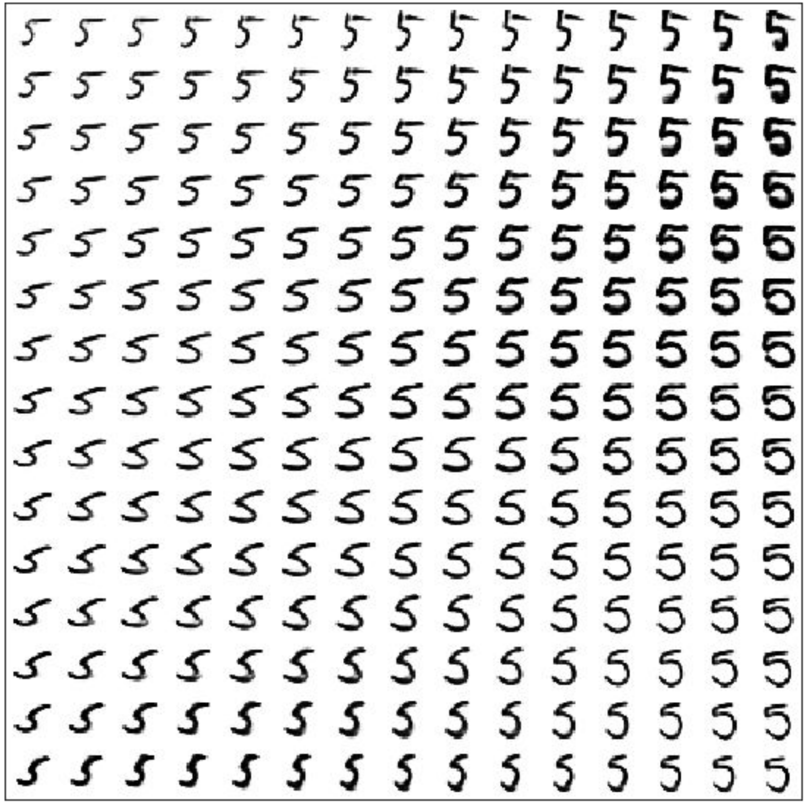


Image source: [Habr post on autoencoders and GANs]

cGAN manifolds



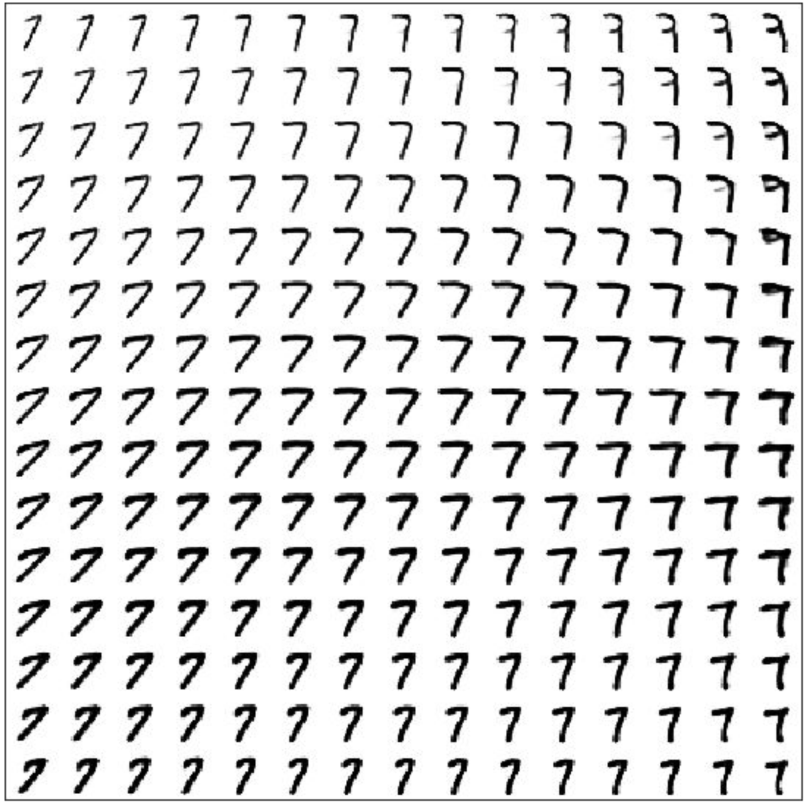


Image source: [Habr post on autoencoders and GANs]

Some more combinations

Image source: [Guiness World Records blog](), original video [PIKOTARO - PPAP (Pen Pineapple Apple Pen)]()

Simple GAN

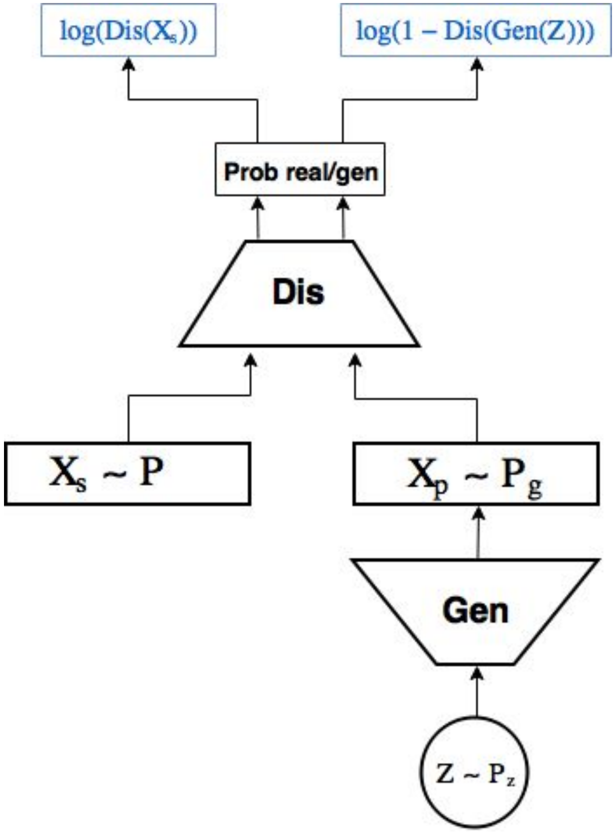
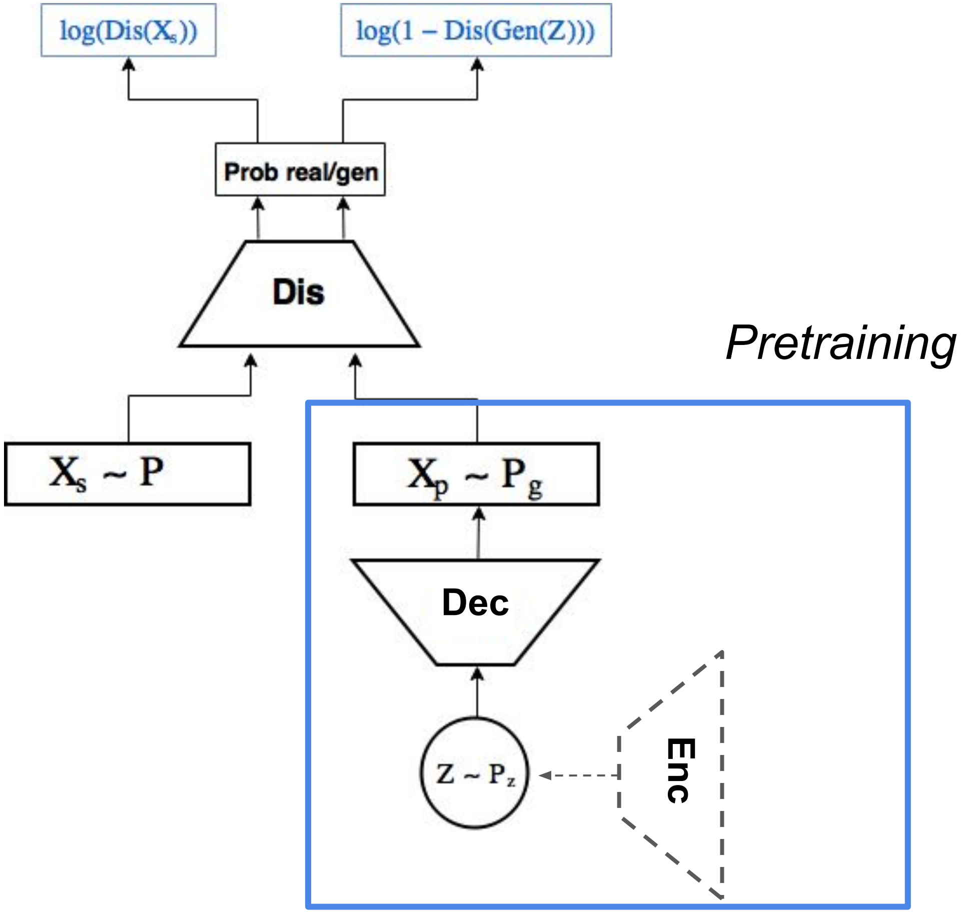


Image source: [Habr post on autoencoders and GANs]()

VAE/GAN



Partial image source: [Habr post on autoencoders and GANs]

VAE/GAN original illustration

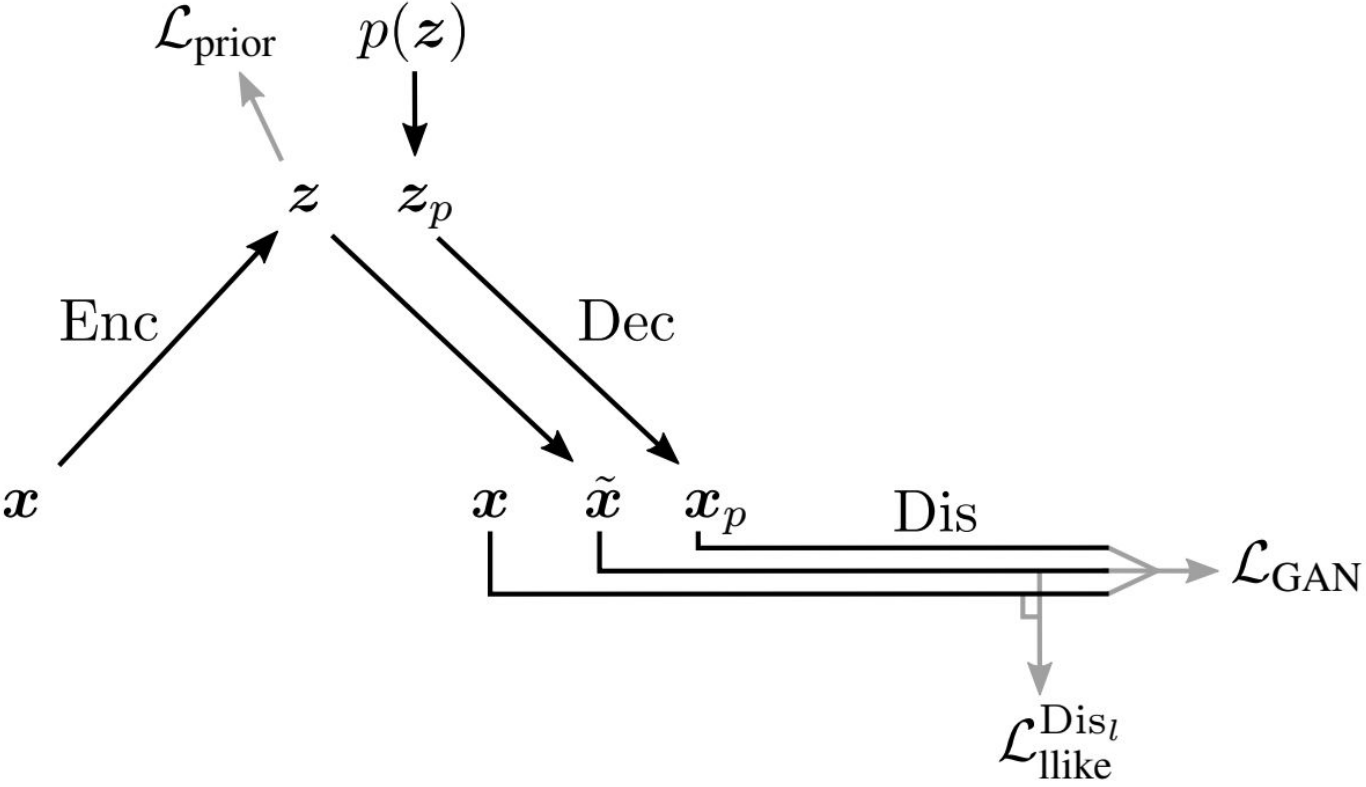


Image source: [Autoencoding beyond pixels using a learned similarity metric, Anders Boesen Lindbo Larsen et al, 2016]()

Q & A and farewell

(We still will write some code!)

This was the last lecture. Thank you for your attention.

Our course took 28 weeks and almost a year.

Machine Learning and Deep Learning worlds are very big, and we have only peeked at them. But it is still a lot.

It was not always smoothly, but we hope the journey was interesting.

Good luck

ml-mipt team