Exercise 1:

Remind yourself that

- $\not{p}\not{p} = p^2 \mathbb{1}$
- $\dfrac{1}{\not{p}-m} = \dfrac{\not{p}+m}{p^2-m^2}$
- $\gamma^\mu \gamma_\mu = 4 \cdot \mathbb{1}$
- $\gamma^\mu \gamma^\rho \gamma_\mu = -2\gamma^\rho$
- $\mathrm{Tr}(\gamma^\mu \gamma^\nu) = 4 g^{\mu\nu}$

Exercise 2:

Show that translational invariance of $G^{\Phi_1 \dots \Phi_n}(x_1, \dots, x_n)$ implies momentum conservation:

$$\tilde{G}^{\Phi_1 \dots \Phi_n}(p_1, \dots, p_n) = (2\pi)^4 \, \delta^{(4)}(p_1 + \dots + p_n) \, f(p_1, \dots, p_{n-1})$$

$$\tilde{f}(p) = \int d^4x \, e^{-ipx} f(x) \, , \quad f(x) = \int \frac{d^4p}{(2\pi)^4} e^{+ipx} \tilde{f}(p)$$

Exercise 3

- consider the Lagrangian of the free photon field (+ gauge fixing term)

$$\mathcal{L}_{A,0} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 \qquad (F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu)$$

$\Rightarrow$ e.o.m. $\quad \left[g^{\mu\nu} \Box - \left(1 - \frac{1}{\xi}\right) \partial^\mu \partial^\nu \right] A_\nu(x) = 0$

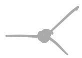
The Green's function is defined via

$$\left[g_{\mu\nu} \Box - \left(1 - \frac{1}{\xi}\right) \partial_\mu \partial_\nu \right] \Delta_F^{\nu\rho}(x) \overset{!}{=} g_\mu^{\ \rho}\, \delta(x)$$

Derive $\Delta_F^{\mu\nu}$ in momentum space using

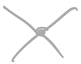
$$\Delta_F^{\mu\nu} = \int \frac{d^4p}{(2\pi)^4} e^{ipx} \Delta_F^{\mu\nu}(p)$$

Exercise 4

(a) draw all graphs to the 2-pt function $G^{\phi\phi}$
up to $\mathcal{O}(g^2)$ in $\mathcal{L}_{int} = \frac{g}{3!}\phi^3$. ()
determine the symmetry factors.

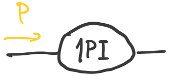
- connected graphs $G_{con}^{\Phi_1 \dots \Phi_n}(x_1, \dots x_n) := G^{\Phi_1 \dots \Phi_n}(x_1, \dots x_n)\Big|$ only connected graphs

$$\hookrightarrow \quad G_{con}^{\Phi_1 \dots \Phi_n} = \frac{\delta}{i\delta J_1} \cdots \frac{\delta}{i\delta J_n} Z_{con}[\{J\}]\Big|_{J=0} \quad (Z_{con} = \ln(Z))$$

(b) draw all connected graphs to the 2-pt function $G^{\phi \dots \phi}$
up to $\mathcal{O}(\lambda^2)$ in $\mathcal{L}_{int} = -\frac{\lambda}{4!}\phi^4$. ()
determine the symmetry factors.

Exercise 5

- consider a scalar theory with an arbitrary interaction ($\mathcal{L}_{int}$)

Let $i\Sigma_1^{\phi\phi}(p^2) \equiv$ 

be the sum of all 1PI graphs

"self energy"

↪ not allowed 

- Give a graphical representation for the 2-pt function $G^{\phi\phi}(p,-p)$ to all orders using $i\Sigma^{\phi\phi}(p^2)$

- resum the diagrammatic series ("Dyson series")

- give an expression for the 2-pt vertex function $\Gamma^{\phi\phi}(p,-p)$

Exercise 6

- the generating functional $\Gamma[\{\Phi\}]$ for vertex functions is the Legendre-transformation of $Z_{con}[\{J\}]$

$$\Gamma^{\Phi_1 \cdots \Phi_n}(x_1, \dots, x_n) = \frac{\delta}{\delta \Phi_1(x_1)} \cdots \frac{\delta}{\delta \Phi_n(x_n)} \Gamma[\{\Phi\}]$$

$\Leftrightarrow$ one can show that at tree-graph level $\rightsquigarrow$ Appendix

$$\Gamma^{(0)}[\{\Phi\}] = i \int d^4x \, \mathcal{L}(\{\Phi(x)\})$$

$\Rightarrow$ very easy derivation of Feynman rules $\left(\Gamma^{(0)\,\Phi_1 \cdots \Phi_n}\right)$

1. collect terms in $i\mathcal{L}$ that contain the fields $\Phi_1 \cdots \Phi_n$
2. replace all derivatives by "$-i$" the incoming momentum $(\partial_\mu \to -i p_\mu)$
3. sum over all permutations of indices & momenta of identical external fields
4. drop all fields.

derive the Feynman rules for

(a) $\mathcal{L}_g = g A^\mu (B \partial_\mu C - C \partial_\mu B)$

(b) $\mathcal{L}_g = g A^\mu A_\mu C D$

(c)  from $\bar{\psi}(i\not{\partial} - m)\psi$