$$\begin{cases} y' = f(t,y) \\ y(t_0) = y_0 \end{cases}$$

A NUMERICAL SOLUTION IS n^{th} ORDER ACCURATE IF THE ERROR IS PROPORTIONAL TO THE STEP SIZE Δt TO THE nth POWER

$$y_{n+1} = y_{n-1} + 2\Delta t\, f_n$$

$$f_n = \frac{y_{n+1} - y_{n-1}}{2\Delta t}$$

TAYLOR SERIES EXPANSION FOR y_{n+1}: $y_{n+1} = y_n + \Delta t\, f_n + \frac{\Delta t^2}{2} f_n' + \frac{\Delta t^3}{6} f_n'' + \frac{\Delta t^4}{24} f_n''' + O(\Delta t^5)$

TAYLOR SERIES EXPANSION FOR y_{n-1}: $y_{n-1} = y_n - \Delta t\, f_n + \frac{\Delta t^2}{2} f_n' - \frac{\Delta t^3}{6} f_n'' + \frac{\Delta t^4}{24} f_n''' + O(\Delta t^5)$

$$y_{n+1} - y_{n-1} = \frac{2\Delta t}{1} f_n + \frac{2\Delta t^3}{6} f_n'' + O(\Delta t^5)$$

$$\frac{y_{n+1} - y_{n-1}}{2\Delta t} = f_n + \frac{\Delta t^2}{6} f_n'' + O(\Delta t^4)$$

$$f_n = f_n + \underbrace{\frac{\Delta t^2}{6} f_n'' + O(\Delta t^4)}_{\text{ERROR}}$$

THE ORDER OF ACCURACY OF THE SCHEME IS 2