

For  $i$  in 1 to  $n$  we have  $y_i(s)$  where  $s \in S$  - functional response

For  $j$  in 1 to  $p$  we have functional covariates  $x_{ij}(t)$  where  $t \in T_j$

Functional response	Functional covariate 1	... $j$ ...	Functional covariate $p$
$y_1(s)$	$x_{11}(t)$		$x_{p1}(t)$
$y_2(s)$	$x_{12}(t)$		$x_{p2}(t)$
... $i$ ...			
$y_n(s)$	$x_{1n}(t)$		$x_{pn}(t)$

## Concurrent functional linear model

Assumption:

1.  $T_j = S$

Model:

$$y_i(s) = \alpha(s) + \sum_{j=1}^p x_{ij}(s) \psi_j(s) + e_i(s)$$

$\alpha(s)$  - functional intercept

$\psi_j(s)$  - functional regressor coefficients

$e_i(s)$  - functional zero-mean error

## Non concurrent (More general approach)

Assumption:

1.  $T_j \neq S$

Model:

$$y_i(s) = \alpha(s) + \sum_{j=1}^p \int x_{ij}(t) \psi_j(t, s) + e_i(s)$$

*? We are not really interested in it cause for us  $T_j = S$  (1)*

## Historical functional linear model

Assumption:

1.  $T_j = S$

Model:

$$T(s) = \{t \in T : t < s\}$$

$$y_i(s) = \alpha(s) + \sum_{j=1}^p \int_{T(s)} x_{ij}(t) \psi_j(t, s) + e_i(s)$$

*? Note (2)*

*? Интегральное уравнение по ядро  $\psi_j(t, s)$  наоборот неизвестно (Неоднородное уравнение Фредгольма второго рода)?*

## Introduced model

? Motivation (3)

? Каждый эксперимент независимый или мы просто добавляем дозу?

## Our model

$$y_i(d) = \sum_{j=1}^p k_{ji}(d)\beta_j(d) + e(d) \text{ or can be written as } y_i(d) = \sum_{j=1}^p k_{ij}(d)\beta_j(d) + e_i(d)$$

\* So domains of  $y_i(\cdot)$  and  $k_{ij}(\cdot)$  with  $\beta_j(\cdot)$  are the same. We can use **Concurrent functional linear model**

\* Do we need other model (non concurrent or historical)?

## Sparsity

?Note (4)