NOTES: Possible Source of Error When Using the Kater Pendulum

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Possible source of error when using the Kater pendulum

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The Kater pendulum has long been known as an instrument used for an accurate determination of the acceleration due to gravity, g, and is frequently encountered by the student in the undergraduate physics laboratory. It consists of a physical pendulum usually in the form of a metal bar having two fixed knife-edged pivots located on opposite sides of the center of mass. The period about each pivot can be changed by adjusting the positions of movable masses clamped to the bar. By careful adjustment, the periods of oscillation about each of the pivots can be made equal. What may not be generally known is that if the center of mass is halfway between the pivots, g cannot be determined from measurements of the equal periods without a separate measurement of the radius of gyration.

The period of oscillation about pivot 1 is easily derived[1] and is given by

$$T_1 = 2\pi \left(\frac{k^2 + \ell_1{}^2}{g\ell_1} \right)^{1/2}, \tag{1}$$

where k is the radius of gyration of the pendulum about the center of mass and ℓ_1 is the distance from the pivot to the center of mass. A similar expression exists for T_2, the period about pivot 2, where ℓ_2 is the distance from pivot 2 to the center of mass. The fact that the pivots are fixed results in an equation of constraint,

$$\ell_1 + \ell_2 = \ell, \tag{2}$$

where ℓ is the distance between the pivots and can accurately be determined experimentally by using a cathetometer.

The difference in the squares of the periods is

$$\begin{aligned} T_1{}^2 - T_2{}^2 &= \frac{4\pi^2}{g} \left(\frac{k^2 + \ell_1{}^2}{\ell_1} - \frac{k^2 + \ell_2{}^2}{\ell_2} \right) \\ &= \frac{4\pi^2}{g} (\ell_1 - \ell_2)\left(1 - \frac{k^2}{\ell_1 \ell_2}\right), \end{aligned} \tag{3}$$

and if $T_1 = T_2$, either

$$\ell_1 = \ell_2 \tag{4}$$

or

$$k^2 = \ell_1 \ell_2 . \tag{5}$$

In the case of Eq. 4, the center of mass is halfway between the pivots but k is unspecified and can take on an infinite number of values, depending on the locations of the adjustable masses. Since the equal periods are functions of k, according to Eq. 1, they also have an infinite number of values.

If, on the other hand, ℓ_1 does not equal ℓ_2, then k is determined by Eq. 5, and substituting this into Eq. 1 we get

$$\begin{aligned} T_1 &= 2\pi \left(\frac{\ell_1 \ell_2 + \ell_1{}^2}{g\ell_1} \right)^{1/2} = 2\pi \left(\frac{\ell_1 + \ell_2}{g} \right)^{1/2} \\ &= 2\pi \left(\frac{\ell}{g} \right)^{1/2}, \end{aligned} \tag{6}$$

a unique value for the equal periods. The solution of Eq. 6 for g is obvious and can thus be determined with great precision.

References

1. W. Arthur and S. K. Fenster, *Mechanics* (Holt, Rinehart and Winston, Inc., New York, 1969), Chap. 12, p. 454.

The measurement of "g" in an elevator

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The Behr free-fall apparatus was used to obtain records of a freely falling body in the cage of an hydraulic passenger elevator. Tapes were made for the acceleration and deceleration during the upward and downward motions. The bob was released as soon as the elevator was started. Control data were taken with the elevator at rest. The average velocity-time graphs for the accelerated motion are shown in Fig. 1. It is evident that the elevator accelerated uniformly during the recorded time interval. The values of "g" for the stationary, ascending, and descending conditions were calculated from the slope to be 9.84, 10.36, and 9.10 m/sec^2, respectively. The uncertainty from reading the graph was $-$.02 m/sec^2. Thus, the acceleration relative to ground of the ascending elevator was 0.52 m/sec^2; of the descending, - 0.74 m/sec^2.

With the elevator in what was judged to be uniform motion, the "stop" button was pressed and a record was made immediately afterward. The average velocity-time graphs for this motion are displayed in Fig. 2. The deceleration of the elevator was also uniform. The values of "g" during this phase were 9.76, 8.70, and 11.20 ± .02 m/sec^2, respectively, for the stationary,