
PHYSICS

The Photoelectric Effect in the Semiclassical Theory

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This work is devoted to an analysis of effects accompanying light–matter interaction and resulting in secondary emission and electron emission similar to spontaneous-emission effects caused by the electron-subsystem heating due to absorption of electromagnetic-wave energy.

Born and Wolf [1] wrote that the description of the electromagnetic field as a certain substance not reducible to anything else does not lead to any difficulties. In contrast to this commonly accepted view, the field loses its material substance essence in quantum electrodynamics. Annihilation of the electromagnetic field and its transformation to a probability wave describing the location of an electromagnetic-energy quantum at a certain point results from an ambiguous field-quantization procedure encountering mathematical obstacles, as well. One such obstacle is the *ad hoc* procedure of instantaneous collapse of the electromagnetic-field wave function and the creation of a photon at an electron (or atom or another particle) interacting with it. Another, no less paradoxical result of wave transformation into a particle is the frequently encountered persistent intention to ignore the fact that light propagation is related to the transverse (with respect to the propagation direction) oscillations of the electric field \mathbf{E} and magnetic field \mathbf{H} and reduction of scattering of the electromagnetic mass of the field of an electrically charged particle to the purely mechanical problem of encountering solid particles. This set of nontrivial procedures was proposed by A. Einstein in 1905 in his attempt to explain the photoelectric effect. He assumed that if an electromagnetic-field quantum with the energy $h\nu$ encounters an electron in metal, then the electron leaves the metal with the energy E_{kin} equal to the difference of $h\nu$ and the bonding energy E_n of the electron in the metal:

$$h\nu = E_n + E_{\text{kin}}. \quad (1)$$

This equation derived by Einstein is cited frequently in numerous publications devoted to quantum electrodynamics. For example, already in 2005 in his Nobel lecture, R.J. Glauber called Einstein's explanation of the photoelectric effect naive, but nevertheless said that Einstein was right and, indeed, light comprises quantized energy packets and both processes, i.e., the photoelectric and Compton effects, proceed “in accordance with the same rules which govern a billiard-ball encounter” [2]. Equation (1) and the statement by R.J. Glauber are evidently wrong. Even in the case of a free electron, $E_n \ll h\nu$, which is the most favorable for maximizing the electron kinetic energy, the electron velocity is determined by the scattered-wave momentum, but not by its energy. The energy is spent not only and not just for a billiardlike encounter, but for re-emission of the wave incident at the electron and boosting not only the translational electron motion in the wave-propagation direction determined by the Poynting vector \mathbf{S} , but also its oscillatory motion in the transverse direction under the action of the force $e\mathbf{E}$, where $\mathbf{E} = E_0 \cos \omega t$.

Let us construct another model for the photoelectric effect based not on criticizing and revising the field-quantization procedure, but on its complete abandonment. The quantum electrodynamics seemingly offering simple answers to certain questions posed by experimental observations does actually obscure the previously clear concept of the electromagnetic field, and it brings new strong postulates in the physical sense without solving the problems of understanding experimental data. To avoid difficulties in representing light in the form of the Maxwell field, one should “maximally extend the list of semiclassical calculations” [3] (recall the history of appearance and eventual discarding of the Newtonian hypothesis of light-corpuscule emanation).

The semiclassical approximation means an approximation in which the electromagnetic field remains a field, while the structure of matter interacting with it is described by the quantum theory of its structure. The same approach was once used by M. Laue who discovered X-ray diffraction by crystals whose atomic-structure sizes turned out to be comparable to the radiation wavelength.

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Efforts aimed at finding semiclassical solutions for the photoelectric-effect phenomenon were attempted earlier. In this respect, a case study is the work [4] in which irradiation of a simple system comprising the fundamental energy level n and continuous spectrum of excited states k by a monochromatic wave $\mathbf{E} = \mathbf{E}_0 \cos \omega t$ is considered. Using methods of quantum perturbation theory, we can write the formula for the probability of the transition from the state n to the state k during the time t under the action of periodic perturbation by the field \mathbf{E} :

$$P(t) = 4 \left| g_k \frac{e E_0}{2 \hbar} \right|^2 \frac{\sin^2 \left(\frac{\omega_k - \omega}{2} t \right)}{\left[\frac{t}{2} (\omega_k - \omega) \right]^2} t^2, \quad (2)$$

Here, ω_k is the resonance frequency of the electron transition from n to k , $\omega_k = E_k / \hbar$, g_k is the matrix element of the transition from n to k , and $|\mathbf{E}_0|^2$ is the electromagnetic-wave energy.

Next, since, as is shown in [4], averaging over k yields a formula correctly describing the field-energy absorption, many researchers were convinced by the semiclassical description of both the field-energy absorption and the photoelectric effect [5]. Unfortunately, this is not exactly true. Emission of an electron by an atomic system of absorbed energy implies that the oscillation energy of a certain electron absorbing the energy is always E_k and the oscillation with smaller amplitude does not free the electron or causes its transition to another level. Equation (2) does not imply that the energy reaches E_k in some samples.

Nevertheless, Eq. (2) providing for the semiclassical description of radiation-energy absorption rather than electron emission returns us back to the classical understanding of light absorption and the complex refractive index which do not depend on the energy of radiation incident on the matter, but resonantly depend on the ratio of the radiation energy and the oscillation frequencies of the bound electrons. According to Eq. (2), the electromagnetic-radiation energy is always absorbed at any radiation frequency. In contrast to Eq. (1), the light is always rescattered by electrons in the matter.

Hereafter, we consider only the absorbed energy and the way through which it leads to the appearance of photoelectrons. Let us turn back to the above-described simplified model of an atom and consider only one level k assuming that the factor g_k is close to unity. The motion of an electron at the level k is determined by the transition frequency ω_k and written as follows:

$$\ddot{r} + \omega_k r + \gamma \dot{r} = \frac{e}{m} E_0 \exp(-i\omega t), \quad (3)$$

where $\gamma = (2e^2/3mc^3)\omega_k^2$ determines the radiative damping. The electron-oscillation amplitude r and the scattering cross section σ_e are given by the formulas

$$r = \frac{e}{m} E_0 \frac{\exp(-i\omega t)}{\omega_k^2 - \omega^2 - i\omega\gamma}, \quad (4)$$

$$\sigma_0 = \frac{8\pi}{3} \left(\frac{e^2}{mc^2} \right)^2 \frac{\omega^4}{(\omega_k^2 - \omega^2)^2 + \omega^2 \gamma^2}.$$

Equation (4) shows that the oscillation amplitude and, therefore, the scattering cross section increase as the excitation frequency ω approaches ω_k . If $\gamma = 0$, then r and σ_0 are infinitely large for $\omega = \omega_k$. This leads to the result forced by quantum electrodynamics: an electron “engulfs” the electromagnetic field in the entire space. Consider the real case where $\gamma \neq 0$. In the case of resonance, the entire energy scattered by electrons is absorbed by them rather than re-emitted; i.e., it is spent to increase the electron-gas temperature. Relaxation of the excited electrons either generates photoelectrons or results in energy losses via other channels. Let the electron transitions from the level n to the level k be the predominant channel of relaxation losses since the excitation is resonant. Let us estimate the number of such electrons gaining the necessary energy $E_k = \hbar \nu_k$.

So far, no special constraints have been imposed on the radiation source. However, in view of our purposes, it is assumed that the radiation source is a single atom radiating a wave with the frequency ω and the photon energy $\hbar \nu$. The emission of wave by the atom is assumed spherically symmetric or dipole (a spherical photon is emitted), so that the metal sample at the distance R from the source is exposed to the wave with the energy density $\hbar \nu / 4\pi R^2$. It is known from the experiments on the photoelectric effect that the entire incident-wave energy is absorbed by a thin near-surface layer of metal if $\omega = \omega_k$. Let us find this part of the photoelectric effect from Eq. (4). The relaxation time τ for conductance electrons in metals is about 10^{-14} s, their mean free path is $l \approx 10^{-6}$ cm, and the surface density is about 10^{16} , so that the number of electrons in the near-surface region of up to 100 atomic layers is about $N = 10^{18} \text{ cm}^{-2}$ and $N_\sigma = 10^{18} \sigma_0$ within the area σ_0 of the scattering cross section. Thus, Eq. (4) implies that $\sigma_0 = (3/2\pi)\lambda^2$ for $\omega = \omega_k$ and $\sigma_0 \approx 2 \times 10^{-9} \text{ cm}^2$ and $N_\sigma \approx 2 \times 10^9$ for $\lambda = 0.6 \times 10^{-4}$.

All N_σ electrons are instantaneously thermalized, within about the time $\tau \approx 10^{-14}$ s after the beginning of irradiation. The energy of a fraction of the single spherical photon absorbed by such electrons in the near-surface layer is $U_0 = \hbar \nu_k / 4\pi R^2$, $N_\sigma \sigma_0 \frac{\hbar \nu_k}{4\pi R^2} \geq \frac{\hbar \nu_k}{4\pi R^2}$. If the

electromagnetic-gas energy is equilibrium during the time necessary for switching on the energy-loss chan-

nels, then the energy distribution of the excited-electron gas can be approximated by the Boltzmann distribution

$$F(E_k) = \exp\left(-\frac{h\nu_k \cdot 4\pi R^2}{h\nu_k}\right) = \exp(-4\pi R^2); \quad (5)$$

where R is in cm. If N_a are emitted in a source during a

short time, then $F(E_k) = \exp\left(-\frac{4\pi R^2}{N_a}\right)$. It is assumed

that $N = 10^{18}$ electrons capable of becoming free from the barrier are located near the surface. Since the distribution $F(E)$ yields the fraction of electrons which absorbed the necessary energy, the number of photo-

electrons is $N_{ph} = SN_a \exp\left(-\frac{4\pi R^2}{N_a}\right)$, where S is the detector area. If $S = 1 \text{ cm}^2$, then $R = 1 \text{ cm}$, $N_a = 1$, and $N_{ph} \geq 1$.

The above estimates show that a detector can indeed completely absorb the incident light energy. The probability of the appearance of the photoelectric-effect is also not small even if the illuminance is very low. Thus, the phenomenon of the photoelectric effect and all its features implied by Eq. (2) can be explained without employing the procedure of wave function collapse.

In conclusion, we note that the above statements are qualitative, but the above estimates meet the goal of demonstrating the capability of classical electrodynamics described by the Maxwell equations to treat such secondary-emission effects as the photoelectric effect or fluorescence. Of course, the semiclassical description spoils the simplicity and beauty of the photoelec-

tric effect and other quantum theories of absorption, but eliminates troubles caused by field quantization. The theory of absorption and emission remains quantum from the viewpoint of matter structure and classical for the electromagnetic field. It is treated in the context of the known concept of light absorption, refraction, and spontaneous emission.

The above analysis also highlights the differences between the quantum and semiclassical predictions of the photoelectric-effect behavior. For example, the temperature of an electron gas heated by a field decreases exponentially with an increase in distance, while the probability of a detector encounter by a photon is probably $S/4\pi R^2$. Hence, if a radiating atom is surrounded by a spherical detector, then a quantum counter detects photon radiation anyway, whereas the probability of triggering a semiclassical detector decreases exponentially as its radius increases.

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