Rule-based Graph Repair using Minimally Restricted Consistency-Improving Transformations

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Abstract

Contents

1 Preliminaries

Definition 1.1 (**subgraph**). *Let G_1 and G_2 be graphs. The graph G_2 is called a* subgraph *of G_1 if an injective morphism $f : G_2 \to G_1$ exists. We use the notation $G_2 \subseteq G_1$ if G_2 is a subgraph of G_1 and $G_2 \subset G_1$ if f is not bijective.*

Definition 1.2 (**minimal uppergraph**). *Let G_1 and G_2 be graphs with $G_1 \subseteq G_2$. A graph C is called an* minimal uppergraph *of G_1 w.r.t G_2, if $G_1 \subset C \subseteq G_2$ and no graph $C' \subset C$ with $G_1 \subset C \subseteq G_2$ exists. The set of minimal uppergraphs of G_1 w.r.t. G_2 is denoted by $\mathcal{U}(G_1, G_2)$. If $G_1 = G_2$, we set $\mathcal{U}(G_1, G_2) = \{G_1\}$.*

Definition 1.3 (**overlap**). *Let G and G' be graphs. A graph H is called an* overlap of G and G' *if morphisms $p : G \hookrightarrow H$ and $p' : G' \hookrightarrow H$ such that p and p' are jointly surjective. The set of all overlaps of G and G' is denoted by* $\mathrm{ol}(G, G')$.

Definition 1.4 (**overlap at morphism**). *Let C,G and C' with $C \subset C'$ be graphs and $p : C \hookrightarrow G$ a morphism. A graph H is called an* overlap of G and C' at p *if a morphism $p' : C' \hookrightarrow H$ with $p'|_C = p$ exists. The set of all overlaps of G and C' at p is denoted by* $\mathrm{ol}_p(G, C')$.

Definition 1.5 (**partial morphism**). *Let $f : G_1 \to G_2$ and $g : G_3 \to G_4$ be morphisms. The morphism g is called a* partial morphism *of f if $G_3 \subseteq G_1$, $G_4 \subseteq G_2$ and $f|_{G_3} = g$.*

Definition 1.6 (**nested graph condition**). *A* graph condition *over a graph C_0 is inductively defined as follows:*

- *true is a graph condition over every graph.*
- *$\exists(a : C_0 \hookrightarrow C_1, d)$ is a graph condition over C_0 if a is a injective graph morphism and d is a graph condition over C_1.*
- *$\neg d$ is a graph condition over C_0 if d is a graph condition over C_0.*
- *$d_1 \wedge d_2$ and $d_1 \vee d_2$ are graph conditions over C_0 if d_1 and d_2 are graph conditions over C_0.*

Conditions over the empty graph $\emptyset$ are called constraints. *Every injective morphism $p : C_0 \hookrightarrow G$ satisfies true. An injective morphism p satisfies $\exists(a : C_0 \hookrightarrow C_1, d)$ if there exists an injective morphism $q : C_1 \hookrightarrow G$ such that $q \circ a = p$ and q satisfies c. An injective morphism satisfies $\neg d$ if it does not satisfy d, it satisfies $d_1 \wedge d_2$ if it satisfies d_1 and d_2 and it satisfies $d_1 \wedge d_2$ if it satisfies d_1 or d_2. A graph G satisfies a constraint c, $G \models c$, if $p : \emptyset \hookrightarrow G$ satisfies c. We use the abbreviation $\forall(a : C_0 \hookrightarrow C_1, d) := \neg\exists(a : C_0 \hookrightarrow C_1, \neg d)$.*

The nesting level nl *of a condition is defined as* $\mathrm{nl}(\mathsf{true} = 0$ *and* $\mathrm{nl}(\exists(a : P \to Q, d)) := \mathrm{nl}(d) + 1$.

Definition 1.7 (**alternating quantifier normal form (ANF)**[1])**.** *A graph condition c is in* alternating normal form *(ANF) if it is of the form*

$$c = Q(a_1 : C_0 \hookrightarrow C_1, \overline{Q}(a_2 : C_1 \hookrightarrow C_2, Q(a_3 : C_2 \hookrightarrow C_3, \overline{Q}(a_4 : C_3 \hookrightarrow C_4, \ldots))))$$

with $Q \in \{\exists, \forall\}$ and $\overline{Q} = \exists$ if $Q = \forall$, $\overline{Q} = \forall$ if $Q = \exists$.

2 partial consistency improving

2.1 extended alternating quantifier normal form

Definition 2.1 (**extended alternating quantifier normal form**)**.** *A condition c is in* extended alternating quantifier normal form *(EANF) if it is in ANF, universally bound and ends with a condition of the form $\exists(a_k : C_k \hookrightarrow C_{k+1}, e)$ with $e \in \{\mathsf{true}, \mathsf{false}\}$.*

Lemma 2.2. *Any constraint in ANF can be transformed into an equivalent constraint in EANF.*

Proof. Let c be a constraint in ANF. If c is universally bound and ends with a condition of the form $\exists(a_k : C_k \hookrightarrow C_{k+1}, e)$ with $e \in \{\mathsf{true}, \mathsf{false}\}$, c is already in EANF. We construct the equivalent constraint in EANF by two steps and show that, after each step, the derived constraint is equivalent to c.

1. If $c = \exists(a_1 : \emptyset \hookrightarrow C_0, e)$ is existentially bound, we show that c is equivalent to $d := \forall(a_0 : \emptyset \hookrightarrow \emptyset, c)$. Let G be a graph.

   "$\Longrightarrow$": Let $p : \emptyset \hookrightarrow G$ be a morphism with $p \models c$, therefore a morphism $q : C_0 \to G$ with $q \models e$ and $p = q \circ a_0$ exists. Then, $p \models d$, since p is the only morphism from $\emptyset$ to G and $p = p \circ a_1$ and $p \models c$.

   "$\Longleftarrow$": Let $p : \emptyset \hookrightarrow G$ be a morphism with $p \models d$, therefore all morphisms $q : \emptyset \hookrightarrow G$ with $p = q \circ a_0$ satisfy c. With $p = p \circ a_0$, $p \models c$ follows immediately.

2. If c ends with a condition of the form $d = \forall(a_k : C_k \hookrightarrow C_{k+1}, e)$ with $e \in \{\mathsf{true}, \mathsf{false}\}$. We show that d is equivalent to $d' = \forall(a_k : C_k \hookrightarrow C_{k+1}, \exists(a_{k+1} : C_{k+1} \hookrightarrow C_{k+1}, e))$. Let G be a graph.

   (a) If $e = \mathsf{true}$. Let $p : C_k \hookrightarrow G$ be a morphism, we show that $p \models d$ and $p \models d'$. Since every morphism satisfies true, a morphism $q : C_k \hookrightarrow G$ with $p = a_k \circ q$ and $p \not\models e$ cannot exist, therefore $p \models d$. The morphism a_{k+1} has to be the identity on C_{k+1} and therefore for every $q : C_k \hookrightarrow G$ with $p = a_k \circ q$ is holds that $q = a_{k+1} \circ q$ and $q \models e$. It follows that $p \models d'$.

   (b) If $e = \mathsf{false}$:

   "$\Longrightarrow$": Let $p : C_k \hookrightarrow G$ be a morphism with $p \models d$. Since $e = \mathsf{false}$, no morphism $q : C_{k+1} \hookrightarrow G$ with $p = a_k \circ q$ exists. Therefore, no morphism $q : C_{k+1} \hookrightarrow G$ with $p = a_k \circ q$ and $q \not\models \exists(a_{k+1} : C_{k+1} \hookrightarrow C_{k+1}, e)$ exists. It follows, that $p \models d'$.

"$\Longleftarrow$": Let $p : C_k \hookrightarrow G$ be a morphism with $p \models d'$. Since no morphism satisfies false, no morphism $q : C_{k+1} \hookrightarrow G$ satisfies $\exists(a_{k+1} : C_{k+1} \hookrightarrow C_{k+1}, e)$. Hence, there does not exist a morphism $q : C_{k+1} \hookrightarrow G$ with $p = q \circ a_k$ and $p \models d$ follows.

□

2.2 conditions up to layer

Definition 2.3 (**Layer of a subcondition**). *Let c be a condition and d a subcondition of c. The layer of d is defined as* $\mathrm{lay}(d) := \mathrm{nl}(c) - \mathrm{nl}(d) - 1$.

Definition 2.4 (**substitution at layer**). *Let $c = Q(a : C_0 \hookrightarrow C_1, d)$ be a condition in ANF, such that the subcondition of c with layer $0 \leq k \leq \mathrm{nl}(c)$ is a condition over C_k. Let e be a condition over C_k. The* substitution in c at layer k with e, $\mathrm{sub}(k, c, e)$, *is recursively defined as:*

1. *If $k = 0$:*
$$\mathrm{sub}(0, c, e) := e$$
2. *If $k > 0$:*
$$\mathrm{sub}(k, c, e) := Q\big(a : C_0 \hookrightarrow C_1, \mathrm{sub}(k-1, d, e)\big)$$

Definition 2.5 (**Condition up to layer**). *Let c be a condition in ANF and d be the subcondition of c at layer $0 \leq k \leq \mathrm{nl}(c)$. The* condition up to layer k *of c,* $\mathrm{cond}(k, c)$, *is defined as*

$$\mathrm{cond}(k, c) := \begin{cases} \mathrm{sub}(k, c, \mathsf{true}) & \text{, if } k = 0 \vee d \text{ is existentially bound} \\ \mathrm{sub}(k, c, \mathsf{false}) & \text{, if } d \text{ is universally bound.} \end{cases}$$

Definition 2.6 (**Satisfaction up to layer**). *Let G be a graph and c be a condition over C_0. A morphism $p : C_0 \hookrightarrow G$* satisfies c up to layer k, *$p \models_k c$, if*

$$p \models \mathrm{cond}(k, c).$$

A graph G satisfies a constraint c up to layer k, $G \models_k c$, if $q : \emptyset \hookrightarrow G$ satisfies $\mathrm{cond}(k, c)$. *The biggest k with $G \models_k c$ such that no $j > k$ with $G \models_j c$ exists is denoted by $c_{\max}$.*

Lemma 2.7. *Let G be a graph $p : C_0 \hookrightarrow G$ a morphism and c a condition over C_0 in ANF with $p \models_k c$. If the subcondition $d = Q(a_k : C_{k-1} \hookrightarrow C_k, e)$ of c at layer k is universally bound, then for any condition f over C_k it holds that*

$$p \models \mathrm{sub}(k, c, f).$$

Proof. Let k be the smallest number such that $p \models_k c$ and the subcondition of c with layer k is universally bound, let $d = \forall(a_k : C_{k-1} \hookrightarrow C_k, e)$ be this subcondition. Let $q : G_{k-1} \to G$ be a morphism such that $q \models \forall(a_k : C_{k-1} \hookrightarrow C_k, \mathsf{false})$. This must exist, since $p \models_k c$ and k is the smallest number such that $p \models_k v$ and the subcondition of c with layer k is universally bound.

Therefore, there does not exist a morphism $q' : C_k \to G$ with $q = q' \circ a_k$. Hence, for every condition f over C_k a morphism $q' : C_k \to G$ with $q \not\models f$ and $q = q' \circ a_k$ cannot exist. It follows immediately that $q \models \forall(a_k : C_{k-1} \hookrightarrow C_k, f)$. ☐

Lemma 2.8. *Let G be a graph, $p : C_0 \to G$ a morphism and c a condition over C_0 in ANF with $p \models_k c$. If the subcondition d of c with* $\mathrm{lay}(d) = k$ *is universally bound,*

$$p \models_k c \implies p \models c.$$

Proof. Follows immediately by using lemma 2.7 and setting f to the subcondition of c with layer $k+1$. ☐

Lemma 2.9. *Let c be a condition in ANF over C_0 and $p : C_0 \hookrightarrow G$ a morphism with $p \models_k c$. Let $d = Q(a_{k+2} : C_{k+1} \hookrightarrow C_{k+2}, e)$ be the subcondition of c with layer $k+2$. There does exist a graph $C_{k+1} \subseteq C' \subseteq C_{k+2}$ such that*

$$p \models \mathrm{sub}\big(k+1, c, Q(a_{k+2} : C_{k+1} \hookrightarrow C', f)\big)$$

with f being a $\overline{Q}$ bound condition over C'.

Proof. If $p \models c$, we can choose $C' = C_{k+2}$ and $f = e$.

If $p \not\models c$, there does not exists a j with $p \models_j c$ and the subcondition of c with layer j is universally bound and $Q = \exists$ follows immediately. We choose $C' = C_{k+1}$ and $f = \mathsf{true}$. Let $q : C_k \to G$ with $p = q \circ a_k \circ \ldots \circ a_1$ and $q \circ \ldots \circ a_\ell$ satisfying the condition up to $\ell - k$ of the subcondition of c at layer ℓ for all $0 \leq \ell \leq k$. This morphism must exists since $p \models_k c$ and $p \not\models c$. Let $q' : C_{k+1} \to G$ be a morphism with $q = q' \circ a_{k+1}$. Since $C' = C_{k+1}$, the morphism a'_{k+2} has to be the identity and therefore $q' = q' \circ a'_{k+2}$. It follows that $q' \models \exists(a'_{k+2} : C_{k+1} \hookrightarrow C', \mathsf{true})$ and therefore $p \models \mathrm{sub}\big(k+1, c, Q(a_{k+2} : C_{k+1} \hookrightarrow C', f)\big)$. ☐

Definition 2.10 (partial condition). *Let c be a condition in ANF over C_0. Let d be the subcondition of c at layer $k+1$. The* partial condition of c at layer k with C', $\mathrm{part}(k, c, C')$ *is defined as:*

1. *If d is universally bound, let $e = \exists(a : C_{k+1} \hookrightarrow C_{k+2}, f)$ be the subcondition of c at layer $k+2$ with $C_{k+1} \subseteq C' \subseteq C_{k+2}$:*

$$\mathrm{part}(k, c, C') := \mathrm{sub}\big(k+2, c, \exists(a : C_{k+1} \hookrightarrow C', \mathit{true})\big)$$

2. *If $d = \exists(a : C_k \hookrightarrow C_{k+1}, f)$ is existentially bound with $C_k \subseteq C' \subseteq C_{k+1}$:*

$$\mathrm{part}(k, c, C') := \mathrm{sub}\big(k+1, c, \exists(a : C_{k+1} \hookrightarrow C', \mathit{true})\big)$$

Definition 2.11 (**biggest partially satisfying condition**)**.** *Let G be a graph, c a condition over C_0 and $p : C_0 \hookrightarrow G$ a morphism with $p \models_k c$.*

A partial condition $c = \mathrm{part}(c_{\max}, c, C')$ with $p \models c$ is a biggest partially satisfying condition *if there does not exist a graph $C' \subset C''$ with $p \models \mathrm{part}(c_{max}, c, C'')$. The graph C' is called a* biggest partially satisfying graph.

The set of biggest partially satisfying conditions of c is denoted by $\mathcal{P}_c^G$.

The set of all biggest partially satisfying graphs is denoted by $\mathcal{G}_c^G$.

2.3 minimal consistency improving

Definition 2.12 (**number of violations**)**.** *Let G be a graph and c a constraint in EANF. The* number of violations $\mathrm{nvc}(j, G)$ at layer j in G *is defined as:*

1. *If $j < c_{\max}$:*
$$\mathrm{nvc}(j, G) := 0$$
2. *If $j = c_{\max}$, let $d = \forall(a_k : C_j \hookrightarrow C_{j+1}, e)$ be the subcondition of c at layer $j + 1$.*
$$\mathrm{nvc}(j, G) := \sum_{C \in \mathcal{G}_c} \sum_{C' \in \mathcal{U}(C, C_{j+1})} |\{q \mid q : C_{j+1} \hookrightarrow G \wedge q \not\models \mathrm{part}(1, e, C')\}|$$
3. *If $j > c_{\max}$:*
$$\mathrm{nvc}(j, G) := \infty$$

Definition 2.13 (**minimal consistency improving**)**.** *Let a graph G, a rule r and a constraint c in ANF be given.*

A transformation $G \Longrightarrow_{r,m} H$ is called minimal consistency improving*, if*

$$\mathrm{nvc}(k, H) < \mathrm{nvc}(k, G)$$

for any $0 \leq k \leq \mathrm{nl}(c)$. A rule r is called minimal consistency improving*, if all of its applications to graphs G with $G \not\models c$ are.*

Lemma 2.14. *Let a graph G, a morphism $p : C_0 \to G$ and a constraint c in ANF over C_0 with $p \models_k c$ be given. Then, $p \models_j c$ for all $j < k$ such that the subcondition of c at layer j is existentially bound.*

Proof. 1. The subcondition of c at layer k is existentially bound: If an $j < k$ with $p \models_j c$ exists such that the subcondition of c at layer j is universally bound, let j_1 be the smallest of these. With lemma 2.7 follows that $p \models_{j_2} c$ for all $j_1 < j_2$. Let $\ell < j_1$, such that the subcondition of c at layer ℓ is existentially bound and let $d = \exists(a_\ell : C_\ell \hookrightarrow C_{\ell+1}, e)$ be the condition up to layer $j_1 - \ell$ of the subcondition of c at layer ℓ. Since $\ell < j_1$, a morphism $q : C_\ell \to G$ with $q \models d$ must exists and therefore a morphism $q' : C_{\ell+1} \to G$ with $q = q' \circ a_k$ must exists. If follows that $q \models \exists(a_\ell : C_\ell \hookrightarrow C_{\ell+1}, \mathsf{true})$ and with that $p \models_\ell c$.

2. The subcondition of c at layer k is universally bound: With lemma 2.7 follows that $p \models_{k+1} c$. Since c is in ANF, 1. can be applied to $k+1$.

□

Theorem 2.1. *Let a graph G, a rule r and a constraint c in ANF be given. Let $k < \mathrm{nl}(c)$ be the biggest number, such that $G \models_k c$. A transformation $G \Longrightarrow_{r,m} H$ is minimal consistency improving if $G \models_j c$ and $k < j$.*

Proof. No $\ell > k$ with $G \models_\ell c$ exists and $G \models_k c$. Hence, $\mathrm{nvc}(k, G) > 0$ and $\mathrm{nvc}(k, G) \neq \infty$. Since $j > k$, $\mathrm{nvc}(k, H) = 0$ and it follows immediately that the transformation is minimal consistency improving. □

Definition 2.15 (direct minimal consistency improving). *Let G be a graph, r a plain rule and c a constraint in EANF. Let $d = \forall(a_k : C_{k-1} \hookrightarrow C_k, e)$ be the condition at layer $k = \mathrm{c_{max}} + 1 \leq \mathrm{nl}(c)$ of c and*

$$\mathbf{G} := \bigcup_{C' \in \mathcal{G}_c^G} \mathcal{U}(C', C_{k+1})$$

be the set of all minimal upper-graphs of all biggest partially satisfying graphs. A transformation $t : G \Longrightarrow_{r,m} H$ is called direct minimal consistency improving *if $G \models_{k-1} c$ and equations (2.1), (2.2) and (2.3) hold.*

Every occurrence of C_k in G that satisfies $\mathrm{part}(1, e, C')$ *for any $C' \in \mathbf{G}$ still satisfies* $\mathrm{part}(1, e, C')$ *in H.*

$$\forall p : C_k \hookrightarrow G\Big(\bigwedge_{C' \in \mathbf{G}} (p \models \mathrm{part}(1, e, C') \wedge \mathrm{tr}_t \circ p \textit{ is total}) \\ \Longrightarrow \mathrm{tr}_t \circ p \models \mathrm{part}(1, e, C')\Big) \tag{2.1}$$

Every new inserted occurrence of C_k by t satisfies $\mathrm{part}(1, e, C')$ *for all $C' \in \mathbf{G}$.*

$$\forall p' : C_k \hookrightarrow H\Big(\neg \exists p : C_k \hookrightarrow G(p' = \mathrm{tr}_t \circ p) \implies \big(\bigwedge_{C' \in \mathbf{G}} p' \models \mathrm{part}(1, e, C')\big)\Big) \tag{2.2}$$

At least one occurrence of C_k in G that does not satisfy $\mathrm{part}(1, e, C')$*, for any $C' \in \mathbf{G}$, either has been destroyed by t or satisfies* $\mathrm{part}(1, e, C')$ *in H.*

$$\exists p : C_k \hookrightarrow G\Big(\bigvee_{C' \in \mathbf{G}} (p \not\models \mathrm{part}(1, e, C') \wedge \mathrm{tr}_t \circ p \textit{ is not total} \\ \vee (\mathrm{tr}_t \circ p \textit{is total } \wedge \mathrm{tr}_t \circ p \models \mathrm{part}(1, e, C')))\Big) \tag{2.3}$$

Lemma 2.16. *Let a graph G, a constraint c and a direct minimal improving transformation $t : G \Rightarrow_{r,m} H$ w.r.t. c be given. Then, t is also a minimal improving transformation.*

Proof. Let G be a graph with $k = \mathrm{c}_{\max}$ and $G \models \mathrm{part}(k, c, C)$ with $\mathrm{part}(k, c, C) \in \mathcal{P}_c^G$ Let d be the subcondition of c at layer $k + 1$.

1. We show that equations (2.1) and (2.2) imply that $\mathrm{nvc}(k, H) \leq \mathrm{nvc}(k, G)$. Assume that $\mathrm{nvc}(k, H) > \mathrm{nvc}(k, G)$. Therefore, a morphism $p : C_k \hookrightarrow G$ with $p \not\models \mathrm{part}(1, d, C')$ for any $C' \in \mathcal{U}(C, C_{k+1})$ exists, such that either 1a or 1b is satisfied.

    (a) There does exist a morphism $q' : C_k \hookrightarrow G$ with $q' \models \mathrm{part}(1, d, C')$ and $p = \mathrm{tr}_t \circ q'$.

    (b) There does not exist a morphism $q : C_k \hookrightarrow G$, such that $p = \mathrm{tr}_t \circ q$.

    This is a contradiction, if 1a is satisfied, q' does not satisfy equation (2.1) and if 1b is satisfied q does not satisfy equation (2.2).

2. Since (2.3) is satisfied, a morphism $p : C_k \hookrightarrow G$ with $p \not\models \mathrm{part}(1, d, C')$, such that either $\mathrm{tr} \circ p$ is total and $p \models \mathrm{part}(1, d, C')$ or $\mathrm{tr} \circ p$ is not total exists, for any $C' \in \mathbf{G}$. In both cases the following holds

    $$\begin{aligned} p &\in \{q \mid q : C_k \hookrightarrow G \wedge q \not\models \mathrm{part}(1, e, C')\} \wedge \\ \mathrm{tr} \circ p &\notin \{q \mid q : C_{k+2} \hookrightarrow H \wedge q \not\models \mathrm{part}(1, e, C')\}. \end{aligned}$$

    With that and 1 it follows that

    $$|\{q \mid q : C_k \hookrightarrow G \wedge q \not\models \mathrm{part}(1, e, C')\}| < |\{q \mid q : C_{k+2} \hookrightarrow H \wedge q \not\models \mathrm{part}(1, e, C')\}|.$$

With 1 and 2 follows that $\mathrm{nvc}(k, G) < \mathrm{nvc}(k, G)$ and therefore t is a minimal improving transformation. □

3 application condition

Definition 3.1 (extended overlap). *Let G and $C_0 \subseteq C_1$ be graphs. Let C be an overlap of C_0 and G with the overlap morphism: $q : C_0 \hookrightarrow C$.*

Let $p = L \hookleftarrow K \hookrightarrow R$ be a plain rule with $K = C_0$, $L = K$ and $R = C_1$. The graph H, derived by the transformation

$$C \Longrightarrow_{p,q} H$$

is called the extended overlap of C with C_1. *The extended overlap of an overlap C with an graph C_1 is denoted by $\mathrm{eol}(C, C_1)$.*

Lemma 3.2. *Let graphs G, $C_0 \subset C_1$ and an overlap C of G and C_0 be given. Then, $\mathrm{eol}(C, C_1)$ is an overlap of G and C_1 and $C \subset \mathrm{eol}(C, C_1)$.*

Proof. The graph $\mathrm{eol}(C, C_1)$ is constructed by the transformation $C \Longrightarrow_{p,q} \mathrm{eol}(C, C_1)$ with $p = C_0 \hookleftarrow C_0 \hookrightarrow C_1$. Since the comatch $n : C_1 \hookrightarrow \mathrm{eol}(C, C_1)$ exists, $\mathrm{eol}(C, C_1)$ is an overlap of G and C_1. Because $L = K$, the transformation does not delete any elements and $C \subset \mathrm{eol}(C, C_1)$ follows. □

Definition 3.3 (**overlap shift**). *Let $r = L \hookleftarrow K \hookrightarrow R$ be a plain rule, C a graph and C' an overlap of C and L with morphisms $p : L \hookrightarrow C'$, $k : K \hookrightarrow C'$, $c : C \hookrightarrow C'$ and the partial morphism $q : R \hookrightarrow C'$. We define*

$$
\begin{aligned}
D := \{e \in C' \mid (&\exists e' \in L : p(e') = e \\
&\vee \exists e' \in R : q(e') = e) \\
&\wedge \exists e' \in C : c(e') = e\}
\end{aligned}
\tag{3.1}
$$

Let $r = L \hookleftarrow K' \hookrightarrow R$ be the rule with

$$K' := K \cup D$$

The graph H derived by the transformation $G \Rightarrow_{r,p} H$ is called the overlap shifted graph *of C'. The overlap shifted graph of an graph C is denoted by* $\mathrm{ols}(C)$.

Definition 3.4. *Let $r = L \hookleftarrow K \hookrightarrow R$ be a plain rule and c a constraint in EANF. Let $d = \forall(a_k : C_{k-1} \hookrightarrow C_k, \exists(a_{k+1} : C_k \hookrightarrow C_{k+1}, e))$ be the subcondition of c at layer k with $k = 2i - 1$ for an $i \in \mathbb{N}$. The application condition* ap_k *of the condition at layer k of c with $C' \in \mathcal{U}(C_k, C_{k+1})$ is defined as:*

$$
\begin{aligned}
\mathrm{ap}(k, C') :=& \Big(\bigvee_{P \in \mathrm{ol}(L, C_k)} \mathrm{nex}(P, C') \wedge (\mathrm{rep}(P, C') \vee \mathrm{del}(P, C'))\Big) \wedge \\
& \Big(\bigwedge_{P \in \mathrm{ol}(L, C_k)} \mathrm{ex}(P, C')\Big) \wedge \\
& \Big(\bigwedge_{P \in \mathrm{ol}(R, C_{k+1})} \mathrm{rem}(P, C')\Big)
\end{aligned}
\tag{3.2}
$$

with

1.
$$\mathrm{nex}(P, C') := \exists(a : L \hookrightarrow P, \neg\exists(b : P \hookrightarrow \mathrm{eol}(P, C'), \textsf{true}))$$

2.
$$\mathrm{rep}(P, C') := \mathrm{Left}(\forall(a : R \hookrightarrow \mathrm{ols}(P), \exists(b : \mathrm{ols}(P) \hookrightarrow \mathrm{ols}(\mathrm{eol}(P, C')), \textsf{true}), r)$$

3. *Let $i_1 : L \hookrightarrow P$ and $i_2 : C_k \hookrightarrow P$ be the overlap morphisms of P:*
$$\mathrm{del}(P, C') := \begin{cases} \exists(L \hookrightarrow P, \textsf{true}) & \text{, if } i_1(L \setminus K) \cap i_2(C_k \setminus C_{k-1}) \neq \emptyset \\ \textsf{false} & \text{, otherwise} \end{cases}$$

4. *Let $i' : L \hookrightarrow P$ and $i_j : C_j \hookrightarrow P$ be the inclusion morphisms for all $j \leq k$, let E be the set of all existentially bound graphs C_j with $j \leq k$:*
$$\mathrm{ex}(P, C') := \begin{cases} \neg\exists(L \hookrightarrow P, \textsf{true}) & \text{, if } \bigcup_{C_j \in E} \big(i_j(C_j \setminus C_{j-1}) \cap i'(L \setminus K)\big) \neq \emptyset \\ \textsf{true} & \text{, otherwise} \end{cases}$$

5. *Let $i' : R \hookrightarrow P$ and $i_j : C_j \hookrightarrow P$ be the inclusion morphisms for all $j \leq k$, let U be the set of all universally bound graphs C_j with $j \leq k$:*

$$\mathrm{rem}(P, C') := \begin{cases} \mathrm{Left}(\neg\exists(R \hookrightarrow P, \textsf{true}), r) & \text{, if } \bigcup_{C_j \in U}\big(i_j(C_j \setminus C_{j-1}) \cap i'(R \setminus K)\big) \neq \emptyset \\ \textsf{true} & \text{, otherwise} \end{cases}$$

Lemma 3.5. *Let G be a graph, c a constraint in EANF, with $G \not\models c$, and r a plain rule. Let $d = \forall(a_k : C_{k-1} \hookrightarrow C_k, \exists(b : C_k \hookrightarrow C_{k+1}, e))$ be the subcondition of c at layer $\mathrm{c_{max}}$. Then, every application of $(r, \mathrm{ap}(\mathrm{c_{max}}, C'))$ with*

$$C' \in \bigcap_{C \in \mathcal{G}_c^G} \mathcal{U}(C, C_{k+1})$$

is direct minimal consistency improving.

Proof. □

Lemma 3.6. *Let G be a graph, c a constraint in EANF, with $\mathrm{c_{max}} < \mathrm{nl}(c)$, and $r = L \hookleftarrow K \hookrightarrow R$ a plain rule. Let $d = \forall(a_k : C_{k-1} \hookrightarrow C_k, \exists(b : C_k \hookrightarrow C_{k+1}, e))$ be the subcondition of c at layer $\mathrm{c_{max}} + 1$ and $\mathrm{ap}(\mathrm{c_{max}}, C')$ the application condition constructed by definition 3.4 with $C' \in \mathcal{U}(C, C_{k+1})$ for any $C \in \mathcal{G}_c^G$. The following simplifications can be applied:*

1. *Let $P \in \mathrm{ol}(L, C_k)$. If an injective morphism $p : P \hookrightarrow G$ does not exist, $\mathrm{nex}(P, C')$ can be replaced by* false.
2. *If $(L \setminus K) \cap C_k = \emptyset$, every $\mathrm{del}(P, C')$ can be replaced by* false *and $\mathrm{ex}(P, C')$ can be replaced by* true.
3. *If $(R \setminus K) \cap C' = \emptyset$, every $\mathrm{rep}(P, C')$ can be replaced by* false.
4. *If 2. and 3. apply, $\mathrm{ap}(k, C')$ can be replaced by* false.
5. *If $(R \setminus K) \cap C_{k+1} = \emptyset$, every $\mathrm{rem}(P, C')$ can be replaced by* true.

Proof. Let $m : L \hookrightarrow G$ be the match of the application of r. We show the simplifications by showing that the replaced parts of $\mathrm{ap}(\mathrm{c_{max}}, C')$ will be evaluated to the values they have been replaced with, if the required conditions are met. Note that $C_j \subseteq C_{k'}$ for all $0 \leq j \leq k'$ and $k' \in \{0, \ldots, \mathrm{nl}(c)\}$.

- Proof of 1.: Let $P \in \mathrm{ol}(L, C_k)$ with $a : L \hookrightarrow P$, such that no morphism $p : P \hookrightarrow G$ exists. Therefore, no morphism $q : P \hookrightarrow G$ with $m = p \circ a$ exists and $\mathrm{nex}(P, C')$ will be evaluated to false.
- Proof of 2.: Let $(L \setminus K) \cap C_k = \emptyset$ and $P \in \mathrm{ol}(L, C_k)$ with $i' : L \hookrightarrow P$ and $i_j : C_j \hookrightarrow P$ for all $j \leq k$. If follows that $i'(L \setminus K) \cap i_k(C_k \setminus C_{k-1}) = \emptyset$ and with that $\mathrm{del}(P, C') = \textsf{false}$ for every $P \in \mathrm{ol}(L, C_k)$. Similar it follows that

  $$\bigcup_{j \leq k} i_j(C_j \setminus C_{j-1}) \cap i'(L \setminus K) = \emptyset$$

  and with that $\mathrm{ex}(P, C') = \textsf{true}$ for every $P \in \mathrm{ol}(L, C_k)$.

- Proof of 3.: work in progress.
- Proof of 4.: Let the simplifications 2. and 3. apply. Every $\mathrm{del}(P, C')$ and every $\mathrm{rep}(P, C')$ have been replaced by false. Therefore, the expression

$$\bigvee_{P \in \mathrm{ol}(L, C_k)} \mathrm{nex}(P, C') \wedge (\mathrm{rep}(P, C') \vee \mathrm{del}(P, C'))$$

will be evaluated to false and with that $\mathrm{ap}(k, C')$ will also be evaluated to false.

- Proof of 5.: Let $(R \setminus K) \cap C_{k+1} = \emptyset$ and $P \in \mathrm{ol}(R, C_{k+1})$ with the morphisms $i' : R \hookrightarrow P$ and $i_j : C_j \hookrightarrow P$ for all $j \leq k$. If follows that

$$\bigcup_{j \leq k} i'(R \setminus K) \cap i_j(C_j \setminus C_{j-1}) = \emptyset.$$

Therefore $\mathrm{rem}(P, C') = \mathsf{true}$ for all $P \in \mathrm{ol}(R, C_{k+1})$.

□

3.1 potentially minimal improving rules

Definition 3.7 (potentially minimal improving rule). *Let a constraint c and a plain rule $r = L \hookleftarrow K \hookrightarrow R$ be given. The rule r is called* potentially minimal improving *w.r.t c at layer k with $C_k \subset P \subseteq C_{k+1}$ and $k \in \{1, 3, \ldots, \mathrm{nl}(c)\}$, if*

$$(L \setminus K) \cap (C_{k-1} \cup (C_{k+1} \setminus C_k)) = \emptyset \tag{3.3}$$

and

$$(R \setminus K) \cap C_k = \emptyset \tag{3.4}$$

and either 1. or 2. applies.

1. *The rule r deletes elements of $C_k \setminus C_{k-1}$:*

$$L \subseteq C_k \wedge L \setminus K \neq \emptyset \tag{3.5}$$

Then, r is called a deleting potentially minimal improving rule.

2. *The rule r creates an instance of P:*

$$L = C_k \wedge P \subseteq R \tag{3.6}$$

Then, r is is called an inserting potentially minimal improving rule.

Definition 3.8 (appl. conditions for potentially minimal improving rules). *Let a constraint c in EANF and a potentially minimal improving rule $r = L \hookleftarrow K \hookrightarrow R$ w.r.t c at layer k with $C_k \subseteq P \subseteq C_{k+1}$ be given. We define the application condition for r as:*

1. *If r is a deleting potentially minimal improving rule:*

$$\mathrm{ap}_{\mathrm{pi}}(j,P) := \begin{cases} \exists\big(a_0 : L \hookrightarrow C_k, \neg\exists(a_1 : C_k \hookrightarrow C_{k+1}, \mathsf{true})\big) & , \text{ if } j = k \\ \mathsf{false} & , \text{ if } j \neq k. \end{cases}$$

2. *If r is an inserting potentially minimal improving rule:*

$$\mathrm{ap}_{\mathrm{pi}}(j,P) := \begin{cases} \neg\exists(b : L \hookrightarrow P, \mathsf{true}) & , \text{ if } j = k \\ \mathsf{false} & , \text{ if } j \neq k \end{cases}$$

Theorem 3.1. *Let a graph G, a constraint c in EANF, with $G \models \mathrm{part}(\mathrm{c_{max}}, c, C)$ and $\mathrm{part}(\mathrm{c_{max}}, c, C) \in \mathcal{P}_c^G$, and a potentially minimal improving rule $r = L \hookleftarrow K \hookrightarrow R$ at layer $\mathrm{c_{max}}$ with $C_{\mathrm{c_{max}}+1} \subseteq P \subseteq C_{\mathrm{c_{max}}+2}$ be given. Then, $r' = (r, \mathrm{ap}_{\mathrm{pi}}(\mathrm{c_{max}}+1, P))$ is a direct minimal consistency improving rule.*

Proof. Let $t : G \Longrightarrow_{r',m} H$ be a transformation, $k = \mathrm{c_{max}}+1$ and e be the subcondition of c at layer $k+1$. We show that t is a direct minimal consistency improving transformation. Firstly, we show that equation (2.1) is satisfied. Let $p : C_k \hookrightarrow G$ be a morphism. If r is a deleting potentially minimal improving rule, either 1. or 2. applies, if r is an inserting and not a deleting potentially minimal improving rule, only 2. applies, because r cannot destroy any occurrences of C_k in G.

1. If $p(C_k) \cap m(L \setminus K) \neq \emptyset$, $\mathrm{tr}_t \circ p$ is not total, since at least one element of $p(C_k)$ has been deleted by t and p does satisfy $\bigwedge_{C' \in \mathbf{G}}(p \models \mathrm{part}(1, e, C') \wedge \mathrm{tr}_t \circ p \text{ is total}) \implies \mathrm{tr}_t \circ p \models \mathrm{part}(1, e, C'))$.
2. If $p(C_k) \cap m(L \setminus K) = \emptyset$, $\mathrm{tr}_t \circ p$ is total, since no element of $p(C_k)$ has been deleted by t. Because (3.3) holds, t does not delete any elements of $C_{k+1} \setminus C_k$ and therefore $p \models \mathrm{part}(1, e, C') \implies \mathrm{tr}_t \circ p \models \mathrm{part}(1, e, C')$ for all $C_k \subseteq C' \subseteq C_{k+1}$.

With 1. and 2. follows that (2.1) is satisfied.

Secondly, we show that equation (2.2) is satisfied. Let $p' : C_k \hookrightarrow H$ be a morphism. Because (3.4) is satisfied, t does not create any elements of C_k and therefore, there must exist an morphism $p : C_k \hookrightarrow G$ with $\mathrm{tr}_t \circ p = p'$. If follows that (2.2) is satisfied.

Lasty, we show that (2.3) is satisfied. We consider the cases that r is a deleting minimal potentially improving rule and that r is an inserting minimal potentially improving rule.

1. If r is a deleting potentially minimal improving rule, the condition $\mathrm{ap}_{\mathrm{pi}}(k, P) = \exists\big(a_0 : L \hookrightarrow C_k, \neg\exists(a_1 : C_k \hookrightarrow C_{k+1}, \mathsf{true})\big)$ is satisfied by m. Therefore a morphism $p : C_k \hookrightarrow G$ with $p \not\models \neg\exists(a_1 : C_k \hookrightarrow C_{k+1}, \mathsf{true}) = \mathrm{part}(1, e, C_{k+1})$ and $m = p \circ a_0$ must exist. Since r is a deleting rule, at least one element of $p(C_k)$ has been deleted by t and therefore $\mathrm{tr}_t \circ p$ is not total. It follows that (2.3) is satisfied.

2. If r is a inserting potentially minimal improving rule, $\text{ap}_{\text{pi}}(k, P) = \neg\exists(b : L \hookrightarrow P, \mathsf{true})$ is satisfied by m. Because $L = C_k$, $m \models \neg\exists(b : C_k \hookrightarrow P, \mathsf{true}) = \text{part}(1, e, P)$. Since (3.6) is satisfied, $\text{tr} \circ p$ is total and $\text{tr}_t \circ p \models \text{part}(1, e, P)$. Therefore, (2.3) is satisfied. In total follows that t is a direct minimal consistency improving transformation and therefore r' is direct minimal consistency improving rule.

□

Definition 3.9 (**repairing rule set**). *Let a constraint c in EANF and a set of rules $\mathcal{R}$ be given. Then, $\mathcal{R}$ is called a* repairing rule set for c at layer k *if for all graphs G with $k = \text{c}_{\max}$ a sequence*

$$G = G_0 \Rightarrow_{r_0} \ldots \Rightarrow_{r_{n-1}} G_n = H$$

exists, such that $r_j \in \mathcal{R}$ for all $j \in \{0, \ldots, n-1\}$ and $H \models_{k+2} c$.

Corollary 3.10. *Let a constraint c in EANF and a set of rules $\mathcal{R}$ be given. If $\mathcal{R}$ is a repairing rule set for c at layer k, $\mathcal{R}$ is a repairing rule set w.r.t c at layer j for all $k < j \leq \text{nl}(c)$.*

Corollary 3.11. *Let a constraint c in EANF and a repairing rule set $\mathcal{R}$ for c at layer k, for all $k \in \{1, 3, \ldots, \text{nl}(c)\}$, be given. Then, for all graphs G, a sequence*

$$G = G_0 \Rightarrow_{r_0} \ldots \Rightarrow_{r_{n-1}} G_n = H$$

exists, such that $r_j \in \mathcal{R}$ for all $j \in \{0, \ldots, n-1\}$ and $H \models c$.

Definition 3.12 (**decomposition of a graph**). *Let graphs $G_0 \subset G_1$ be given.*

A decomposition of G_1 with G_0 is a set

$$\mathbf{P} := \{P_v \mid v \in V_{C_1 \setminus C_0}\}$$

of subgraphs of C_1, such that every P_v is constructed in the following way: $G_0 \subset P_v$, $v \in P$ and for all nodes $v' \in P \setminus C_k$ is holds that P contains all edges $e \in E_{G_1 \setminus G_0}$ and all nodes $u \in V_{G_1}$ such that either $\text{tar}(e) = v' \wedge \text{src}(e) = u$ or $\text{src}(e) = u \wedge \text{tar}(e) = v'$ holds.

Lemma 3.13. *Let graphs $G_0 \subset G_1$ and a decomposition $\mathbf{P}$ of G_1 with G_0 be given. Then, for each pair $P, P' \in \mathbf{P}$ with $P \neq P'$ the following holds:*

$$(P \setminus C_k) \cap (P' \setminus C_k) = \emptyset$$

Proof. Assume that $(P \setminus C_k) \cap (P' \setminus C_k) \neq \emptyset$, therefore a node $v \in G_1 \setminus G_0$ with $v \in P \cap P'$. By the construction of P and P' it follows that $P = P_v$ and $P' = P_v$ and therefore $P = P'$. □

Lemma 3.14. *Let graphs $G_0 \subset G_1$ and a decomposition $\mathbf{P}$ of G_1 with G_0 be given. Then,*

$$G_1 = \bigcup_{P \in \mathbf{P}} P.$$

Proof. Let $\mathbf{P} = \{P_v \mid v \in V_{C_1 \setminus C_0}\}$ and $H = \bigcup_{P \in \mathbf{P}} P$. Firstly, we show that $H \subseteq G_1$. Since every $P_v \in \mathbf{P}$ is a subgraph of G_1 it follows that $V_H \subseteq V_{G_1}$ and $E_H \subseteq E_{G_1}$

Secondly, we show that $G_1 \subseteq H_1$. Let $u \in V_{G_1}$ be a node, if $u \in V_{G_0}$, then u is contained in every P_v and therefore $u \in V_H$. Otherwise, if $u \notin V_{G_0}$, then u is contained in P_u and therefore $V_H \subseteq V_{G_1}$. Let $e \in E_{G_1}$ be an edge. If $e \in E_{G_0}$, then e is contained in every P_v and therefore $e \in E_H$. Otherwise, if $e \notin E_{G_0}$, either $\mathrm{src}(e) \notin V_{G_0}$, and therefore $e \in P_{\mathrm{src}(e)}$ or $\mathrm{tar}(e) \notin V_{G_0}$ and therefore $e \in P_{\mathrm{tar}(e)}$. It follows that $e \in E_H$ and with that $E_{G_1} \subseteq E_H$. $\square$

Theorem 3.2. *Let a constraint c in EANF and a set of rules $\mathcal{R}$ be given. Then, $\mathcal{R}$ is a repairing set of c at layer $k \leq \mathrm{nl}(c)$ if either 1 or 2 applies.*

1. *For any universally bound graph C_j at layer $j \leq k$ of c, $(r, \mathrm{app_{pi}}(j, C_{j+1})) \in \mathcal{R}$ and $r = L \hookleftarrow K \hookrightarrow R$ is a deleting potentially minimal improving rule at layer j with C_{j+1}, such that r only deletes edges of C_j:*

2. *A decomposition $\mathbf{P}$ of C_k with C_{k-1} exists, such that for each $P \in \mathbf{P}$ a rule $(r, \mathrm{app_{pi}}(k, P)) \in \mathcal{R}$ exists, such that r is an inserting basic improving rule at layer k with $P \cup C_{k-1}$.*

Proof. Let a constraint c in EANF, a rule set $\mathcal{R}$ and a graph G with $k = \mathrm{c_{max}}$ and $\mathrm{c_{max}} < \mathrm{nl}(c)$ be given. We show that a sequence $G = C'_0 \Rightarrow \ldots \Rightarrow C'_n = H$ with rules of $\mathcal{R}$ exists, such that $H \models_{k+2} c$ if 1. or 2. of theorem **??** is satisfied.

1. Assume that 1. of theorem 3.2 holds. Let $(r, \mathrm{app_{pi}}(j, C_{j+1})) \in \mathcal{R}$, such that $r = L \hookleftarrow K \hookrightarrow R$ is a deleting potentially minimal improving rule at layer $j \leq k$ with C_{j+1} and C_j is a universally bound graph of c. Then, $\mathrm{app_{pi}}(j, C_{j+1}) = \exists\big(a : L \hookrightarrow C_j, \neg\exists(a_j : C_j \hookrightarrow C_{j+1}, \mathsf{true})\big)$. Let $q : C_j \hookrightarrow G$ be a morphism such that $q \not\models \exists(a_j : C_j \hookrightarrow C_{j+1}, \mathsf{true})$. Since $L \subseteq C_j$, we can construct a morphism $m_1 : L \hookrightarrow G$ with $m_1(L) = q(L)$. It holds that $m_1 = q \circ a_0$, therefore $m_1 \models \mathrm{app_{pi}}(j, C_{j+1})$. Since r only deletes edges, a transformation $t : G = G_0 \Rightarrow_{r,m_1} G_1$ exists and $\mathrm{tr}_t \circ p$ is not total. Because r does not insert any elements of C_j:

   $$|\{q : C_k \hookrightarrow G_0 \mid q \not\models d\}| < |\{q : C_k \hookrightarrow G_1 \mid q \not\models d\}|$$

   with $d = \exists(a_j : C_j \hookrightarrow C_{j+1}, \mathsf{true})$. By iteratively applying this construction, we generate a finite sequence of transformations

   $$G = G_0 \Rightarrow_{r,m_1} G_1 \Rightarrow_{r,m_2} \ldots \Rightarrow_{r,m_n} G_n = H$$

   such that $|\{q : C_k \hookrightarrow G_n \mid q \not\models d\}| = 0$ and therefore $H \models_j c$. With lemma 2.8, $H \models_{k+2} c$ and $H \models c$ follows.

2. Assume that 2. of theorem 3.2 holds. Let $r_0 = (r, \mathrm{app_{pi}}(k, P)) \in \mathcal{R}$ be an inserting basic improving rule of c at layer k with $P \in \mathbf{P}$. Then, $\mathrm{app_{pi}}(k, P) = \neg\exists(b : L \hookrightarrow P, \mathsf{true})$. Let $q_0 : C_k \hookrightarrow G$ be a morphism, such that $q_0 \not\models \exists(a'_k :$

$C_k \hookrightarrow P, \mathsf{true})$. Since $L = C_k$, we set $m_0 : C_k \hookrightarrow G$ with $m_0 = q_0$. It follows that $m_0 \models \neg\exists(a'_k : C_k \hookrightarrow P, \mathsf{true}) = \mathrm{ap}_{\mathrm{pi}}(k, P)$. Because r does not delete any elements, a transformation $t_0 : G \Longrightarrow_{r_0, m_0} G_1$ exists and $\mathrm{tr}_t \circ q \models \exists(a'_k : C_k \hookrightarrow P, \mathsf{true})$. We set $q_1 = \mathrm{tr}_{t_0} \circ q_0$ and apply the same method to q_1.

By iteratively applying this, we can construct a finite sequence of transformations

$$G \Longrightarrow_{r_0, m_0} G_0 \Longrightarrow_{r_1, m_1} \ldots \Longrightarrow_{r_n, m_n} G_n$$

such that $m_i = \mathrm{tr}_{t_{i-1}} \circ \ldots \circ \mathrm{tr}_{t_0} \circ m_0$ and $q \models \exists(b_i : C_k \hookrightarrow P_i \cup C_k, \mathsf{true})$ for all $P_i \in \mathbf{P}$ with $q = \mathrm{tr}_{t_n} \circ q_0$. Let $p_i : P_i \cup C_k \hookrightarrow G_n$ be the morphism, such that $m_n = p_i \circ b_i$.

Now, we can construct a morphism $p : C_{k+1} \hookrightarrow G$ with

$$p(e) := \begin{cases} p_1(e) & \text{,if } e \in P_1 \\ & \vdots \\ p_j(e) & \text{,if } e \in P_j \end{cases}$$

.

Since $q(e) = p_i \circ b_i(e)$ and $q(e) = p_\ell \circ b_\ell(e)$ and b_i and b_ℓ are both partial morphisms of a_k, it follows that $p_i(e) = p_\ell(e)$ Because $P_i \cap P_\ell)C_k = \emptyset$ for all $i \neq \ell$, p is a morphism and by the definition of p it follows that $q = p \circ a_k$ and therefore $q \models \exists(a_k : C_k \hookrightarrow C_{k+1}, \mathsf{true})$.

By iteratively applying this to all occurrences of C_k that do not satisfy $\exists(a_k : C_k \hookrightarrow C_{k+1}, \mathsf{true})$ the derived graph graph H does not contain any occurrences of C_k not satisfying $\exists(a_k : C_k \hookrightarrow C_{k+1}, \mathsf{true})$ and therefore $H \models_{k+2} c$.

□

Corollary 3.15. *If a set of rule $\mathcal{R}$ is a repairing set of c at layer $k \leq \mathrm{nl}(c)$ and 1. of theorem 3.2 applies, then $\mathcal{R}$ is a repairing set of c at layer j for all $k \leq j \leq \mathrm{nl}(c)$.*

References

[1] C. Sandmann and A. Habel. Rule-based graph repair. *arXiv preprint arXiv:1912.09610*, 2019.