Complexity and Representations of Controllers in Reactive Synthesis

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Dagstuhl Seminar – Stochastic Games

Talk overview

Strategies are at the center of game-theoretic approaches to reactive synthesis.

Goal of this talk

Motivate and explain a **multifaceted vision** of strategy complexity.

In this talk, we will discuss:

- the classical **Mealy machine** model of strategies;
- **alternative** models of strategies.

The goal is not to be **exhaustive**, but explain some questions that interest our research group.

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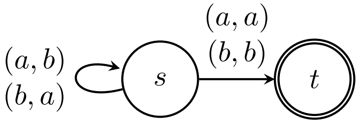
Games

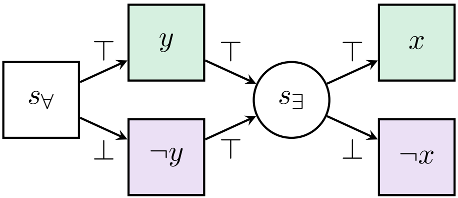


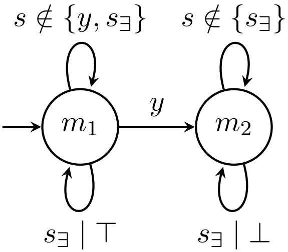
Two-player concurrent stochastic game $\mathcal{G}$

- Finite set of **states** S;
- Finite sets of **actions** $A^{(1)}$ and $A^{(2)}$;
- Probabilistic **transition** function $\delta\colon S \times \bar{A} \to \mathcal{D}(S)$,

- **Play**: $\pi = s_0\bar{a}_0s_1\bar{a}_1\ldots$ s.t. $\delta(s_\ell, \bar{a}_\ell)(s_{\ell+1}) > 0$ for all $\ell \in \mathbb{N}$,
- **History**: prefix h of a play ending in a **state**.
- **Strategy**: function $\sigma_i\colon \mathsf{Hist}(\mathcal{G}) \to \mathcal{D}(A^{(i)})$.
- Strategies σ_1, σ_2 + initial state $s \rightsquigarrow$ **distribution** $\mathbb{P}_s^{\sigma_1,\sigma_2}$ over plays.

Strategies and memory

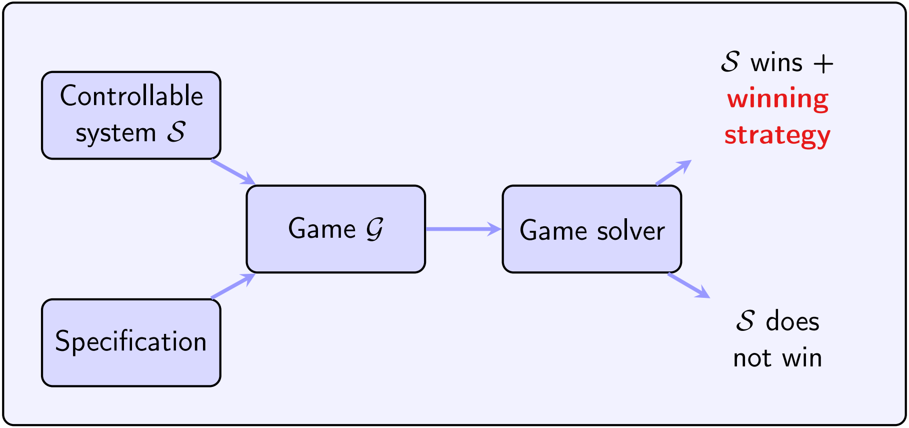




Representation of strategies via Mealy machines with randomisation

- Set of **memory states** M;
- initial **memory distribution** μ_{init};
- **next-move** function $\alpha_{\text{nxt}} \colon M \times S \to \mathcal{D}(A^{(i)})$;
- memory **update** function $\alpha_{\text{up}} \colon M \times S \times \bar{A} \to \mathcal{D}(M)$.

The synthesis pipeline



Strategy $\approx$ blueprint for a controller of the system.

$\rightsquigarrow$ Simpler strategies are preferable.

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Strategy complexity via memory

- The complexity of strategies is traditionally measured by the size of their **memory**.
- Memory requirements for optimal strategies in games have been thoroughly studied.

A glimpse into known results on memory

- Characterisations and one-to-two player lifts (e.g., [GZ05; Bou+22]).
- Characterisations for memory requirements via universal graphs (e.g., [Ohl23; CO23]).
- Refining memory bounds/computing optimal bounds (e.g., [Bou+23; Mai24]).
- Trading memory for **randomisation** (e.g., [CAH04; CRR14]).

Strategy complexity in general

- Memory size does **not fully describe** the complexity of strategies.
- Other aspects also play a role in the complexity of strategies.
- **Major question**: what makes a strategy complex?

Our vision

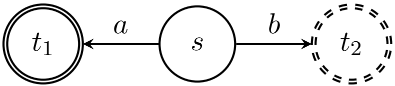
Strategy complexity is **multifaceted**: various factors contribute to the complexity of a strategy.

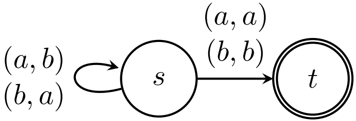
We are interested in identifying relevant **factors that contribute to complexity**, not limited to model-based measures of complexity.

- **Next step**: a brief look into **randomisation**.

Not all randomisation is the same

Some specifications require **randomisation** to be satisfied.





$\bigwedge_{j \in \{1,2\}} \mathbb{P}_s^{\sigma_1}(\mathsf{Reach}(\{t_j\})) \geq \frac{1}{2}$

One-off random choice between two pure memoryless strategies.

$\forall \sigma_2,\ \mathbb{P}_s^{\sigma_1,\sigma_2}(\mathsf{Reach}(\{t\})) = 1$

σ_1 must randomise its choice **infinitely often** to win.

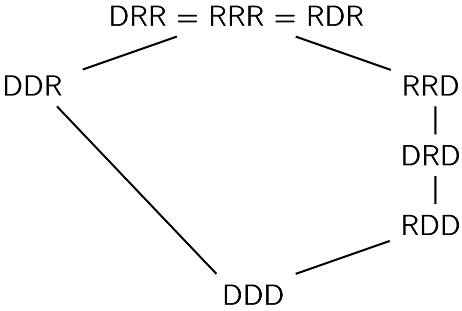
Theorem (Consequence of [Ete+08])

*A one-off choice between d **pure finite-memory strategies** is sufficient to satisfy multiple reachability queries with d **objectives**.*

Randomisation and finite memory [MR22]

A class of Mealy machines is denoted by **XYZ** where X, Y, Z $\in$ {D, R} where D stands for deterministic and R for random, and

- X characterises initialisation,
- Y characterises the next-move function,
- Z characterises updates.



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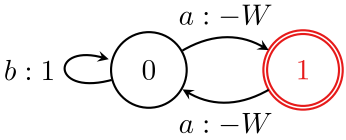
Memory does not tell the whole story (1/2)

Counter-based strategies

Memory and **randomisation** do **not fully reflect** the complexity of a strategy.

- We consider a game with an **energy-Büchi** objective [CD12], where $W \in \mathbb{N}$.



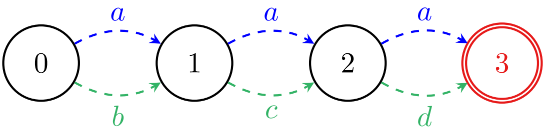
- Need memory **exponential** in the binary encoding of W to satisfy the energy-Büchi objective.
- **Polynomial** representation with a **counter**-based approach.

Memory does not tell the whole story (2/2)

Action choices influence simplicity

Memory and **randomisation** do **not fully reflect** the complexity of a strategy.

- The goal is to move to 3.
- We compare two strategies σ_1 and σ_2.



$\rightarrow$ Strategy σ_1 is **simpler to represent** than σ_2

- The **action choices** can impact how concise the strategy can be made.

Alternative representations

- **Different representations** of a given strategy provide different information on its complexity.
- There has been a recent surge in works using **alternative strategy models**. For instance:
  - Decision trees for memoryless strategies [Brá+15; JKW23]
  - Turing machine-based models [Gel14]
  - Programmatic representations of strategies [SFM24]

Some questions

- What are the **links** between different strategy models?
- How **relevant** are some models for practical endeavours?

Conclusion

In a nutshell

We are interested in developing deeper insight on **strategy complexity** and studying **alternative strategy models**.

Advertisement

There is an opening for a **postdoc position** on Formal Methods/AI for Controller Synthesis in **Mickaël Randour**'s team in UMONS, in **Mons, Belgium**.

Information on the project can be found online.[1]

Thank you for your attention.

[1] https://math.umons.ac.be/staff/Randour.Mickael/controllers.html

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