Cross-dating of intra-annual wood density series

Calculations

Absolute to Normalized Coordinates

given

$\mathrm{Y} = \langle 5, 3, 2 \rangle$

calculation

$$\mathrm{mean(Y)} = \frac{10}{3}$$

$$\mathrm{sd(y)} = \sqrt{\frac{\sum_{i=1}^{3}\left(y_i - mean(Y)\right)^2}{n-1}} = \sqrt{\frac{\left(5-\frac{10}{3}\right)^2 + \left(3-\frac{10}{3}\right)^2 + \left(2-\frac{10}{3}\right)^2}{2}} \approx 1.527525$$

$$\frac{5-\frac{10}{3}}{1.527525} \approx 1.09109$$

$$\frac{3-\frac{10}{3}}{1.527525} \approx -0.2182179$$

$$\frac{2-\frac{10}{3}}{1.527525} \approx -0.8728717$$

Hint: R-Console was used for calculation.

Double Weighting

given

$weighting = 3$

$bestYears \ = \langle 1992, 1502, 1493, 1801, 1723 \rangle$ // by rank

calculation

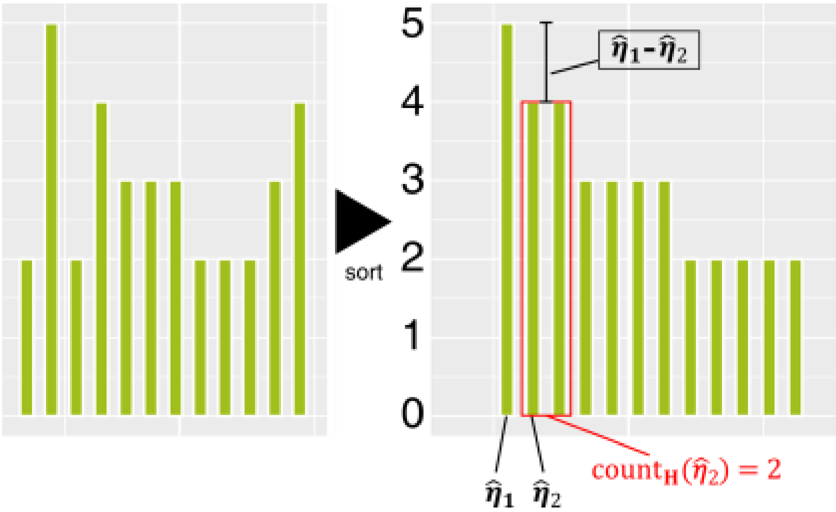
$output = \langle 1992, 1992, 1992, 1502, 1502, 1493, 1801, 1723 \rangle$

1992 has rank 1, so it is contained 3 times in the list.
1502 has rank 2 (less important), so it is contained 2 times in the list.
1493 has rank 3, so it is contained once.

The same holds for 1801, 1723, since there are no ranks lower 1.

(so, it is weighted by rank and by length of sample which is here 3)

Δ Peak



$$\Delta\ \text{Peak} = \frac{5-4}{2} = \frac{1}{2}$$

Empirical Distribution

given

$\mathbf{S}_{\mathrm{S}}^{\mathrm{C}} = \{\varsigma_1 = 2, \varsigma_2 = 15, \varsigma_3 = 5, \varsigma_4 = 52, \varsigma_5 = 3\}$

calculation

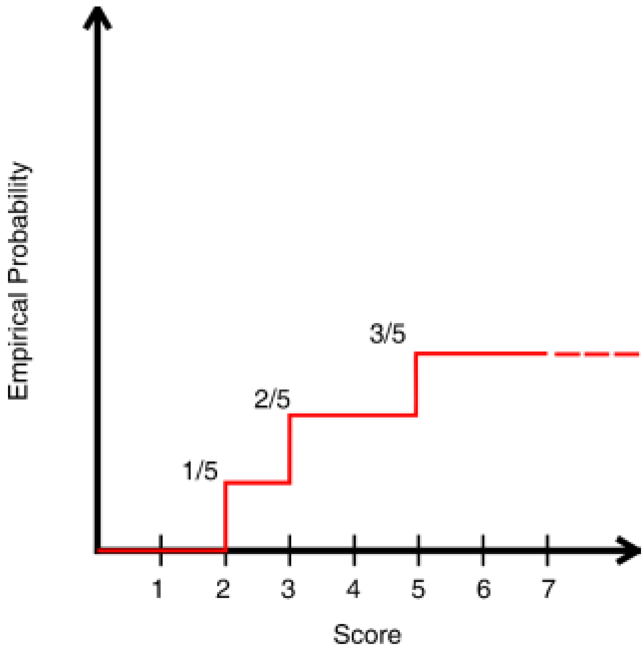
$$\hat{F}(\varsigma = 2) = \frac{1}{5} \cdot (1) = \frac{1}{5}$$

$$\hat{F}(\varsigma = 3) = \frac{1}{5} \cdot (1+0+0+0+1) = \frac{2}{5}$$

$$\hat{F}(\varsigma = 5) = \frac{1}{5} \cdot (1+0+1+0+1) = \frac{3}{5}$$

$$\hat{F}(\varsigma = 15) = \frac{1}{5} \cdot (1+1+1+0+1) = \frac{4}{5}$$

$$\hat{F}(\varsigma = 52) = \frac{1}{5} \cdot (1+0+1+0+1) = \frac{3}{5}$$



Gaps in Samples

given

Sample: ABCD[-gap-]XYZ

first sample:

1	2	3	4	5	6	7	8	9	10
A	B	C	D						

Shift	Profil A	Profil B	Profil C	Profil D
1	0.5	0.6	0.7	0.8
2	0.9	1.0	1.1	1.2
3	1.3	1.4	1.5	1.6

for gap-size 1:

1	2	3	4	5	6	7	8	9	10
					X	Y	Z		

Shift	Profil X	Profil Y	Profil Z
1	1.6	1.7	1.8
2	1.9	2.0	2.1
3	2.2	2.3	2.4

for **gap-size 2**:

1	2	3	4	5	6	7	8	9	10
						X	Y	Z	

Shift	Profil X	Profil Y	Profil Z
1	1.9	2.0	2.1
2	2.2	2.3	2.4

orange parts are equal

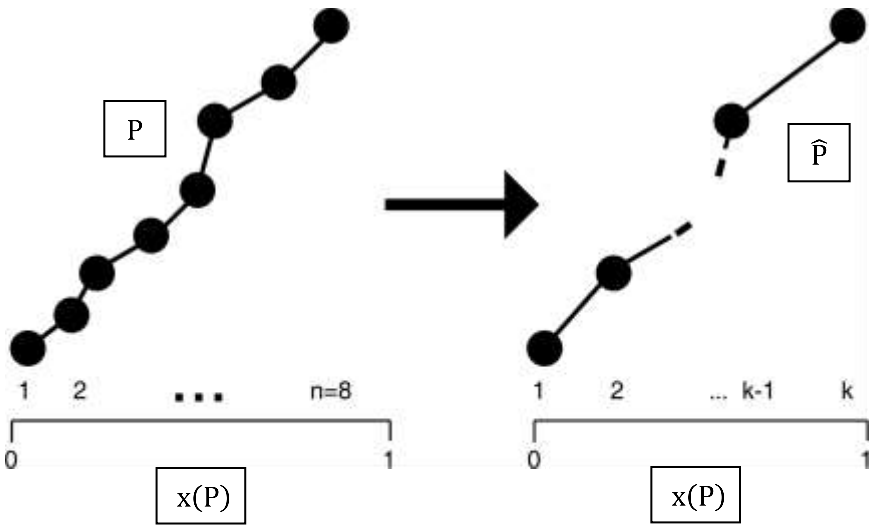
⇒ you have not to test for different gap sizes

example: value under gap-size = 2 & shift 1

$0.5 + 0.6 + 0.7 + 0.8 + 0 + 0 + 1.9 + 2.0 + 2.1 = 8.6$

Interpolation

$\alpha_k(\mathrm{P}) = \widehat{\mathrm{P}}$ (interpolation in interval of $x(\mathrm{P})$)



Kendall

given data sorted with respect to the value of the first **sequence of values**

i	$r(v_i^a)$	$r(v_i^b)$	q_i
5	1	2	1
6	2	3	1
7	3.5	5	2
4	3.5	1	0
3	5	4	0
2	6	7.5	2
8	7	7.5	1
1	8	6	0

$$\tau = \frac{\#concordant - \#discordant}{S} = \frac{C - D}{\sqrt{(C + D + T_a) \cdot (C + D + T_b)}}$$

$concordant$:	$v_i^a < v_j^a \wedge v_i^b < v_j^b$	|	$v_i^a > v_j^a \wedge v_i^b > v_j^b$	(sort order agreement)
$discordant$:	$v_i^a < v_j^a \wedge v_i^b > v_j^b$	|	$v_i^a > v_j^a \wedge v_i^b < v_j^b$	(sort order disagreement)
T_a:	$v_i^a = v_j^a \wedge v_i^b \neq v_j^b$			(bindings in a)
T_b:	$v_i^a \neq v_j^a \wedge v_i^b = v_j^b$			(bindings in b)

calculation

$r(v_i^a)$	1	2	3.5	3.5	5	6	7	8
$r(v_i^b)$	2	3	5	1	4	7.5	7.5	6

it holds:

if $\boldsymbol{r(v_i^b) - r(v_j^b) < 0}$

then **concordant** because $r(v_i^a) - r(v_j^a) < 0$ holds,
since ranks $r(v_i^a)$ of first sequence are sorted in ascending order

else if $\boldsymbol{r(v_i^b) - r(v_j^b) > 0}$

then **discordant**

else…

Hint: count boxes $\frac{K \cdot (K-1)}{2} = \frac{8 \cdot 7}{2} = 28$ (that is the number of comparisons)

1-2	1-3.5	1-3.5	1-5	1-6	1-7	1-8
2-3	**2-5**	**2-1**	**2-4**	**2-7.5**	**2-7.5**	**2-6**
-	-	+	-	-	-	-

2-3.5	2-3.5	2-5	2-6	2-7	2-8
3-5	**3-1**	**3-4**	**3-7.5**	**3-7.5**	**3-6**
-	+	-	-	-	-

3.5-3.5	3.5-5	3.5-6	3.5-7	3.5-8
5-1	**5-4**	**5-7.5**	**5-7.5**	**5-6**
a	+	-	-	-

3.5-5	3.5-6	3.5-7	3.5-8
1-4	**1-7.5**	**1-7.5**	**1-6**
-	-	-	-

5-6	5-7	5-8
4-7.5	**4-7.5**	**4-6**
-	-	-

6-7	6-8
7.5-7.5	**7.5-6**
b	+

7-8
7.5-6
+

C: 21 D: 5 T_a: 1 T_b: 1

(but also, we got ties and this had also to be considered and this is done with the formula below)

$$\tau = \frac{C-D}{\sqrt{(C+D+T_a)\cdot(C+D+T_b)}} = \frac{21-5}{\sqrt{27\cdot 27}} \approx 0.5925926$$ (correct)

Mean Distance

given

$\mathrm{Y^a} = \langle 2, 3, 5\rangle$

$\mathrm{Y^b} = \langle 3, 1, 2\rangle$

calculation

$$\mathrm{dist}_{avg}^{y}\left(\mathrm{P^a}, \mathrm{P^b}\right) = \frac{1}{n} \cdot \sum_{i=1}^{n} \left|y_i^a - y_i^b\right| = \frac{|2-3| + |3-1| + |5-2|}{3} = \frac{1+2+3}{3} = 2$$

Median

$\mathbf{R} = \{r_1 = 1,\ r_2 = 6,\ r_3 = 7,\ r_4 = 23,\ r_5 = 65,\ r_6 = 76\}$ with $r_1 \leq r_2 \leq \cdots \leq r_u$

$\Rightarrow u = 6$

$$\mathrm{median}(\mathbf{R}) = \begin{cases} r_{\frac{u+1}{2}} & , \quad u\ odd \\ \frac{1}{2}\left(r_{\frac{u}{2}} + r_{\frac{u}{2}+1}\right) & , \quad u\ even \end{cases}$$

$$= \frac{1}{2}\left(r_{\frac{6}{2}} + r_{\frac{6}{2}+1}\right)$$

$$= \frac{1}{2}(r_3 + r_4)$$

$$= \frac{1}{2}(7 + 23) = 15$$

Normalization of Coordinates

given

$\mathrm{Y} = \langle 2 \quad 3 \quad 6\rangle$

calculation

$$y_1^{std} = \frac{2 - \mathrm{mean(Y)}}{\sigma(\mathrm{Y})}$$

$$= \frac{2 - \frac{11}{3}}{\sqrt{\frac{1}{2}} \cdot \sqrt{\left(2 - \frac{11}{3}\right)^2 + \left(3 - \frac{11}{3}\right)^2 + \left(6 - \frac{11}{3}\right)^2}}$$

$$= \frac{-\frac{5}{3}}{\sqrt{\frac{1}{2}} \cdot \sqrt{\left(-\frac{5}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{7}{3}\right)^2}}$$

$$\approx -0.801$$

Powerset Table

score table:

year	score1	score2	score3
1992	0.1	0.2	0.3
1993	0.4	0.5	0.6
1994	0.7	0.8	0.9

power set table:

year	score1	score2	score3	score1 + score2	score1 + score3	score2 + score3	score1 + score2 + score3
1992	0.1	0.2	0.3	0.3	0.4	0.5	0.6
1993	0.4	0.5	0.6	0.9	1.0	1.1	1.5
1994	0.7	0.8	0.9	1.5	1.6	1.7	2.4

Spearman

given

$\mathrm{V}^{\mathrm{a}} = \langle 4, 2, 3 \rangle$

$\mathrm{V}^{\mathrm{b}} = \langle 2, 5, 6 \rangle$

calculation

$\varrho\left(\mathrm{V}^{\mathrm{a}}, \mathrm{V}^{\mathrm{b}}\right)$

$= \rho\left(r(\mathrm{V}^{\mathrm{a}}), r\left(\mathrm{V}^{\mathrm{b}}\right)\right)$

$= \rho\left((3,1,2), (1,2,3)\right)$

$$= \frac{(1,-1,0)\cdot(-1,0,1)}{2} = \frac{-1+0+0}{2} = -0.5$$

Tukey's Biweight Robust Mean

given

$\mathbf{V}_{\leq} = \{2, 3, 5\}_{\leq}$

$c = 9$

Calculation (write m instead of Zeta)

$\mathrm{median} = 3$

$\mathrm{MAD}_v = \mathrm{median}\{|2-3|, |3-3|, |5-3|\} = \mathrm{median}\{0, 1, 2\} = 1$

$$\zeta_1 = \frac{v - \tilde{v}}{c_{tun} \cdot \mathrm{MAD}_v + \varepsilon} = \frac{2-3}{9 \cdot 1 + 0.0001} = \frac{-1}{9.0001} \approx -0.111$$

$$\zeta_2 = \frac{3-3}{9 \cdot 1 + 0.0001} = 0$$

$$\zeta_3 = \frac{5-3}{9 \cdot 1 + 0.0001} = \frac{2}{9.0001} \approx 0.222$$

$$w(\zeta_1) = \begin{cases} (1-\zeta_1^2)^2 & , |\zeta_1| \leq 1 \\ 0 & , |\zeta_1| > 1 \end{cases} = \left(1 - \left(\frac{-1}{9.0001}\right)^2\right)^2 \approx 0.975$$

$$w(\zeta_2) = (1 - 0^2)^2 = 1$$

$$w(\zeta_3) = \left(1 - \left(\frac{2}{9.0001}\right)^2\right)^2 \approx 0.904$$

$$\bar{y} = \frac{w(\zeta_1) \cdot 2 + w(\zeta_2) \cdot 3 + w(\zeta_3) \cdot 5}{w(\zeta_1) + w(\zeta_2) + w(\zeta_3)} = 3.288936942$$

```
> dplR::tbrm(c(2,3,5));
[1] 3.288936

> Math.tukeysBiweightRobustMean(a)
[1] 3.288937
```