Perinterial St. (Detta): Derivative w. v.t. the value of the unabliffe osset. [Black-Scholes:  $\frac{\partial V}{\partial S} = \overline{\Phi}(d_1) =: \Delta$ ]

Detintion St. (Gamma): Second abordative w. v.t. the underlying. [Black-Scholes:  $\frac{\partial^2 V}{\partial S^2} = \frac{\Phi'(d_1)}{S_0 \in V_1} =: T$ ]

Definition St. (Vega): The portion derivative to an option prior [Black-Scholes:  $\frac{\partial V}{\partial S} = \frac{\partial V}{\partial S} =$ 

The design in Discrete Time: Delta and Delta-Gamma

Recall the situation in continuous time Headging.

$$dV(+) = \frac{\partial V(+)}{\partial t} dt + \sum_{j=0}^{\infty} \frac{\partial V}{\partial S_{j}} dS_{j} + \frac{1}{2} \sum_{j,j=0}^{\infty} \frac{\partial^{2} V(+)}{\partial S_{j}} dS_{j} dS_{$$

Compare this to the replication portfolio:

$$\Delta \pi(t) = \sum_{i=0}^{\infty} \phi_i(t) \Delta S_i(t)$$

To ensure that the replication portfolio remains

self-financing we have to reformulate the cond

in discrete time!

$$\sum_{i=0}^{\infty} \left( \phi_i(t_k) - \phi_i(t_{k-1}) \right) S_i(t_k) = \emptyset$$

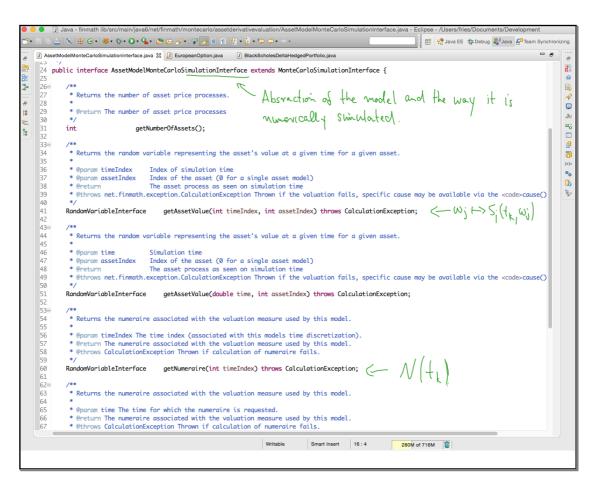
For example, we may ensure they have via

$$(x) \quad \phi_{o}(t_{k}) = \phi_{o}(t_{k-1}) - \frac{1}{S_{o}(t_{lc})} \stackrel{\sim}{\underset{i=1}{\sum}} (\phi_{i}(t_{lc}) - \phi_{i}(t_{k-1})) S_{i}(t_{k})$$

7.4.1 Delta Hedsing:

We choose  $\phi_{1,...,}\phi_{n}$  according to the delta theody, i.e.  $\phi_{i} = \frac{\partial V}{\partial S_{i}}$  for i=1,...,n

and then choose of according to (\*).



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🔝 AssetModelMonteCarloSimulationInterface.java 🔝 EuropeanOption.java 🕱 🗓 BlackScholesDeltaHedgedPortfolio.java
                                                                                                                                                                                                                                                           public class EuropeanOption extends AbstractAssetMonteCarloProduct {
              private final double maturity;
private final double strike;
private final Integer underlyingIndex;
private final String nameOfUnderliyng;
                                                                                                                                                                                                                                                           A<sup>9</sup>
□
                 * Construct a product representing an European option on an asset S (where S the asset with index 0 from the model - single asset case).

* ®param maturity T in the option payoff max(S(T)-K,0)

* ®param strike The strike K in the option payoff max(S(T)-K,0).

* ®param underlyingIndex The index of the underlying to be fetched from the model.
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                                                                                                                                                                                                                                                           °о
[]ь
               public EuropeanOption(double maturity, double strike, int underlyingIndex) {
                     super();
this.maturity
this.strike
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                     this.maturity = maturity;
this.strike = strike;
this.underlyingIndex
this.nameOfUnderlying = null; // Use
                                                                                // Use underlyingIndex
              }
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                **Construct a product representing an European option on an asset S (where S the asset with index 0 from the model - single asset case).

* @param maturity The maturity T in the option payoff max(S(T)-K,0).

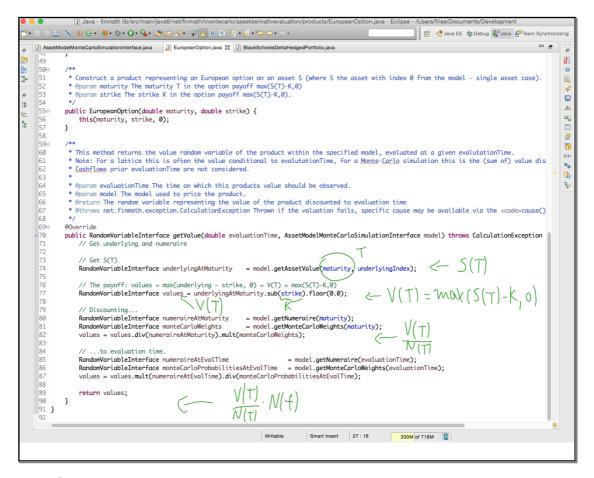
* @param strike The strike K in the option payoff max(S(T)-K,0).
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              public EuropeanOption(double maturity, double strike) {
                     this(maturity, strike, 0);
              }
                 * This method returns the value random variable of the product within the specified model, evaluated at a given evalutationTime.

* Note: For a lattice this is often the value conditional to evalutationTime, for a Monte-Carlo simulation this is the (sum of) value dis
                 * Cashflows prior evaluationTime are not considered
                * @param evaluationTime The time on which this products value should be observed.
* @param model The model used to price the product.
* @return The random variable representing the value of the product discounted to evaluation time
* @throws net.fimmath.exception.CalculationException Thrown if the valuation fails, specific cause may be available via the <code>cause()
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               public RandomVariableInterface getValue(double evaluationTime, AssetModelMonteCarloSimulationInterface model) throws CalculationException
                     // Get underlying and num
                                                                                                           Writable Smart Insert 27:16 309M of 716M
```



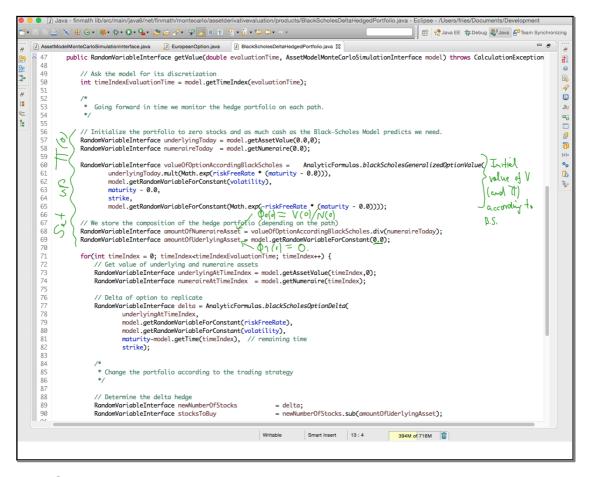
```
☑ AssetModelMonteCarloSimulationInterface.java

☑ EuropeanOption.java

☑ BlackScholesDeltaHedgedPortfolio.java
                                                                                                                                                                                                                          public class BlackScholesDeltaHedgedPortfolio extends AbstractAssetMonteCarloProduct {
                 Properties of the European option we wish to replicate
                                                                                                                                                                                                                          A<sup>0</sup>
□
             private final double maturity; < T } the spec. Of the product we like to healge.
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             (i.c. $ (~$$)).
               * ®param maturity
* ®param strike
* ®param riskFreeRate
* ®param volatility

Maturity of the option we wish to replicate.
Strike of the option we wish to replicate.
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                                                                                                                              Does not need to correspond to the
             public BlackScholesDeltaHedgedPortfolio(double maturity, double strike, double riskFreeRate, double volatility) { Sailly generate
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                   lic Blackschotesuertuneugen of coss
super();
this.maturity = maturity;
this.strike = strike;
this.riskFreeRate = riskFreeRate;
this.volatility = volatility;
                                                                                                                                                                                 N,S
             public RandomVariableInterface getValue(double evaluationTime, AssetModelMonteCarloSimulationInterface model) throws CalculationException
                   // Ask the model for its discretization
int timeIndexEvaluationTime = model.getTimeIndex(evaluationTime);
                    /*

* Going forward in time we monitor the hedge portfolio on each path.
                    // Initialize the portfolio to zero stocks and as much cash as the Black-Scholes Model predicts we need.
                   Random Variable Interface \ underlying Today = model.get Asset Value (0.0,0); \\ Random Variable Interface \ numeraire Today = model.get Numeraire (0.0); \\
                   RandomVariableInterface valueOfOptionAccordingBlackScholes =
                                                                                                               AnalyticFormulas.blackScholesGeneralizedOptionValue(
                              underlyingToday.mult(Math.exp(riskFreeRate * (maturity - 0.0))),
model.getRandomVariableForConstant(volatility),
```



```
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                                 ationInterface.java 📗 EuropeanOption.java 🗓 BlackScholesDeltaHedgedPortfolio.java 🛭
                   RandomVariableInterface valueOfOptionAccordingBlackScholes = AnalyticFormulas.blackScholesGeneralizedOptionValue(
underlyingToday.mult(Math.exp(riskFreeRate * (maturity - 0.0))),
                                                                                                                                                                                                                                  model.getRandomVariableForConstant(volatility),
maturity - 0.0,
strike,
                               model.getRandomVariableForConstant(Math.exp(-riskFreeRate * (maturity - 0.0))));
                   // We store the composition of the hedge portfolio (depending on the path)
RandomVariableInterface amountOfNumeraireAsset = valueOfOptionAccordingBlackScholes.div(numeraireToday);
RandomVariableInterface amountOfUderlyingAsset = model.getRandomVariableForConstant(0.0);
                   for(int timeIndex = 0; timeIndex<timeIndexEvaluationTime; timeIndex++) {</pre>
                         (Int cimelines = 0, timelines = 0, (%) (Get value of underlying and numeraire assets |
RandomVariableInterface underlyingAtTimeIndex = model.getAssetValue(timeIndex,0);
RandomVariableInterface numeraireAtTimeIndex = model.getNumeraire(timeIndex);
         + K-1
                         // Delta of option to replicate
                                   WariableInterface delta = AnalyticFormulas. blackScholesOptionDelta(underlyingAtTimeIndex, <math>5(\frac{1}{4} k^{-1}) model.getRandomVariableForConstant(riskFreeRate),
         tk
                                    model.getRandomVariableForConstant(volatility),
maturity-model.getTime(timeIndex), // remaining time
strike);
                                                                               T-+x-1
                                          5 K
                         /*
    * Change the portfolio according to the trading strategy
*/
                         // Determine the delta hedge
                         RandomVariableInterface newNumberOfStocks
                                                                                                       = delta;
                         RandomVariableInterface stocksToBuy
                                                                                                       = newNumberOfStocks.sub(amountOfUderlyingAsset);
                                                                                                       = stocksToBuy.mult(underlyingAtTimeIndex).div(numeraireAtTimeIndex);
                        RandomVariableInterface numeraireAssetsToSell
RandomVariableInterface newNumberOfNumeraireAsset
                                                                                                      = amountOfNumeraireAsset.sub(numeraireAssetsToSell);
                         // Update portfolio
amountOfNumeraireAsset = newNumberOfNumeraireAsset;
amountOfUderlyingAsset = newNumberOfStocks;
                   ^{\prime\ast} * At maturity, calculate the value of the replication portfolio
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```

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                     // Delta of option to replicate
RandomVariableInterface delta = AnalyticFormulas.blackScholesOptionDelta(
                               underlyingAtTimeIndex
                               model.getRandomVariableForConstant(riskFreeRate),
model.getRandomVariableForConstant(volatility),
moturity-model.getTime(timeIndex), // remaining time
strike);
                     /*
 * Change the portfolio according to the trading strategy
 */
                                                                                                                                                                                            °0
                     // Determine the delta hedge
                     RandomVariableInterface newNumberOfStocks
RandomVariableInterface stocksToBuy
                                                                                      = delta;
= newNumberOfStocks.sub(amountOfUderlyingAsset);
                     // Ensure self financing
                     RandomVariableInterface numeraireAssetsToSell = stocksToBuy.mult(underlyingAtTimeIndex).div(numeraireAtTimeIndex);
RandomVariableInterface newNumberOfNumeraireAsset
= amountOfNumeraireAsset.sub(numeraireAssetsToSell);
                     // Update portfolio
                     amountOfNumeraireAsset = newNumberOfNumeraireAsset;
amountOfUderlyingAsset = newNumberOfStocks;
                                                                                                                                            T= evaluation Time
                  * At maturity, calculate the value of the replication portfolio
                RandomVariableInterface portfolioValue = amountOfNumeraireAsset.mult(numeraireAtEvaluationTime)
.add(amountOfUderlyingAsset.mult(underlyingAtEvaluationTime));
                                                                                                                                    TT(T)=00(T)N(T)+0, [T]SA
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114 }
                return portfolioValue:
                                                                                Writable Smart Insert 72:60 306M of 717M
```

