Exercise 01

A discretization of the SDE

$$dS(t) = \mu(t, S(t))dt + \sigma(t, S(t))dW_t, S(0) = S_0 > 0$$

is already given in the class `EulerScheme`. There the process is calculated for each time-step by using the $Euler - Maryuama - Discretization$

$$S(t_i) = S(t_{i-1}) + \mu(t_{i-1}, S(t_{i-1}))\Delta t_{i-1} + \sigma(t_{i-1}, S(t_{i-1}))\Delta W_{t_{i-1}},$$

where $\Delta t_i := t_{i+1} - t_i$ and $\Delta W_{t_i} := W_{t_{i+1}} - W_{t_i}$. In order to incapsulate the Jump-part $J(t)$ only small changes are necessary: First of all we have to introduce a new private variable of type `PointProcessInterface` in our new class `JumpProcessEulerScheme`, which will simulate the Jumps $J(t)$. Similiarly to the original class the parameters λ, a and b of the compound poisson process are specified by constructing an object of type `CompoundPoissonProcess` and calling the constructor `JumpProcessEulerScheme`, which has inputparameters `BrownianMotionInterface brownianMotion` and `PointProcessInterface compoundPoissonProcess`. Then - instead of adding

$$\mu(t_{i-1}, S(t_{i-1}))\Delta t_{i-1} + \sigma(t_{i-1}, S(t_{i-1}))\Delta W_{t_{i-1}}$$

in each timestep - we additionally add $S(t_{i-1})\Delta J(t_{i-1})$, where again $\Delta J(t_i) := J(t_{i+1}) - J(t_i)$. Note that for this procedure it is inevitable to set the $statespacetransform$ to id, since we are adding $S(t-)J(t)$ and not $J^X(t)$.

Exercise 02

```
Printing results for:  riskFreeRate = 0.05, volatility = 0.3, nu = 0.15,
strike = 100.0, maturity = 2.0
---------------------------------
Merton-Jump-Diffusion-Model:
intensity | Option price
0.0         24,19920
0.4         24,89619
1.0         26,36171
1.4         26,62189
2.0         28,16875
---------------------------------
Option Price for the Black-Scholes-Model:  24,20361
```

When $\lambda = 0$ the values of the Merton-Jump-Diffusion-Model are nearly the same as in the Black-Scholes-Model. This is due to the fact that jump intensity is set to zero and thus no jumps occur. Consequently the modified Euler-Maryuama-Scheme described in exercise 1 simply performs a discretization of the black-scholes-model. The values should be exactly the same, but since we did not implement a log-euler-scheme, but the original black-scholes-model is simulated by one, small discrepancies are reasonable.

Exercise 03

1. If the market obeys the black-scholes-model exactly, the volatility does not change for different strikes. Thus it is sufficient to use one option to calibrate the model.

2. The implied volatility will be the same for every product, because the the prices of the underlying follow the black-scholes-model and thus - as above - the volatility does not depend on the strikes.

The plot of the implied volatilities retrieved by the marketdata are not flat as one could expect keeping 1. and 2. in mind. Actually the shape of a smile is visible. The Black Scholes model assumes that a stock price is driven by a continious diffusion model. But in reality stock prices are subject to more sources of randomness.

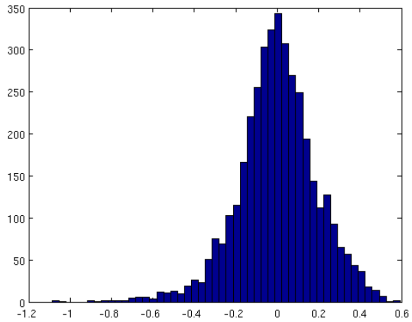
Exercise 04

```
Printing results for:  riskFreeRate = 0.05, volatility = 0.3,
intensity = 0.4, nu = 0.15, maturity = 2.0
---------------------------------
Merton-Jump-Diffusion-Model:
Strike | Option Price | implied volatility
30.0      73,2692       0,4931
40.0      64,3304       0,4073
50.0      55,6257       0,3609
60.0      47,4114       0,3410
70.0      39,9263       0,3336
80.0      33,2461       0,3293
90.0      27,4585       0,3273
100.0     22,5270       0,3261
110.0     18,3956       0,3255
120.0     14,9495       0,3248
130.0     12,1019       0,3240
140.0     9,7786        0,3233
---------------------------------
```

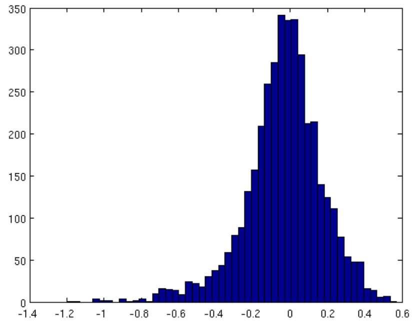
If an underlying is following the Merton Jump Diffusion model and we calculate the implied volatilities for options written on this underlying, we observe similarly to the real market data a volatility smile. The implied volatilities are slightly higher than the real one ($\sigma = 0.3$), since the additional jumps lead to higher fluctuation.

Exercise 06

Number of restructuring dates: 10



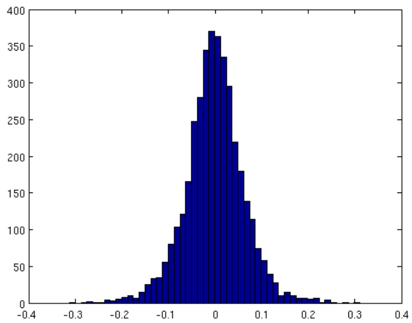
Black Scholes model



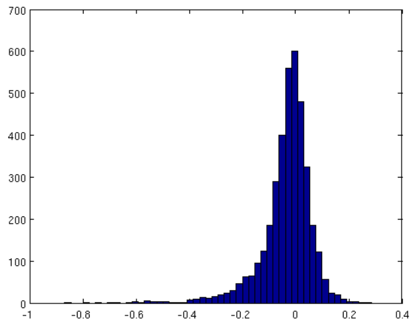
Merton Jump Diffusion model

	Black Scholes Model	Merton-Jump-Diffusion Model
Mean:	-0,000798947	-0,031588544
Variance:	0,033585186	0,039472002

Number of restructuring dates: 100



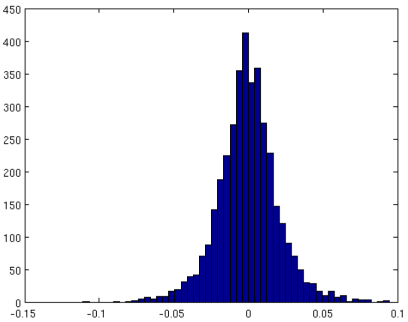
Black Scholes model



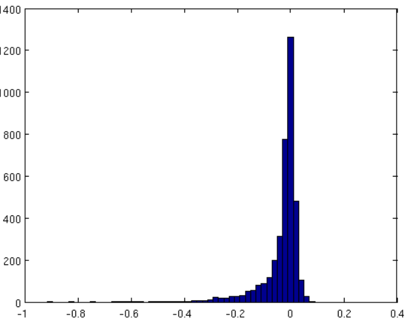
Merton Jump Diffusion model

	Black Scholes Model	Merton-Jump-Diffusion Model
Mean:	-0,000233700	-0,031529162
Variance:	0,003640516	0,008744096

Number of restructuring dates: 1000



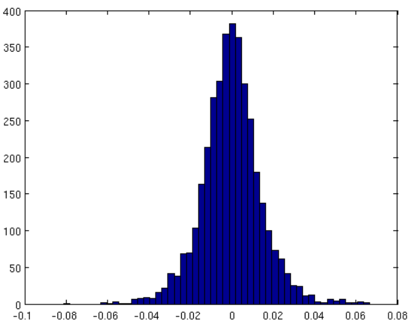
Black Scholes model



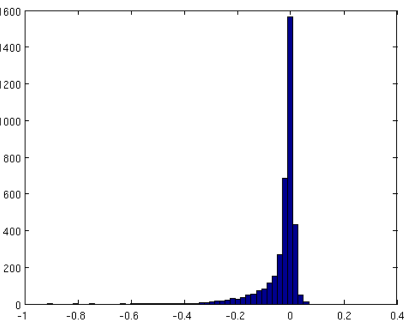
Merton Jump Diffusion model

```
          | Black Scholes Model | Merton-Jump-Diffusion Model
Mean:        -0,000046441         -0,030845866
Variance:     0,000363885          0,005082788
```

Number of restructuring dates: 2000



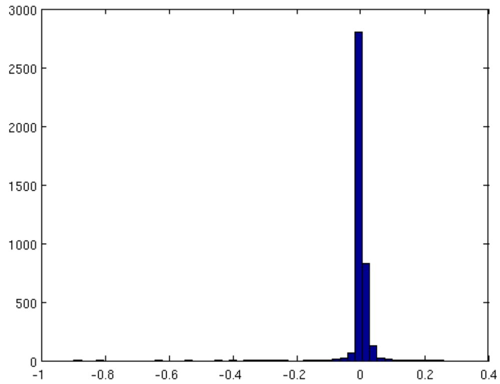
Black Scholes model



Merton Jump Diffusion model

```
          | Black Scholes Model | Merton-Jump-Diffusion Model
Mean:        -0,000247766         -0,030723511
Variance:     0,000184194          0,004757257
```

Delta-Gamma-Hedge for Merton Jump Diffusion model, hedging instrument: european call option with strike = 110, maturity = 5




```
          | Merton-Jump-Diffusion-Model via Delta-Gamma-Hedging
Mean:      0,004614378
Variance:  0,000895833
```