Exercise 01

A discretization of the SDE

$$dS(t) = \mu(t, S(t))dt + \sigma(t, S(t))dW_t, S(0) = S_0 > 0$$

is already given in the class `EulerScheme`. There the process is calculated for each time-step by using the $Euler - Maryuama - Discretization$

$$S(t_i) = S(t_{i-1}) + \mu(t_{i-1}, S(t_{i-1}))\Delta t_{i-1} + \sigma(t_{i-1}, S(t_{i-1}))\Delta W_{t_{i-1}},$$

where $\Delta t_i := t_{i+1} - t_i$ and $\Delta W_{t_i} := W_{t_{i+1}} - W_{t_i}$. In order to incorporate the Jump-part $J(t)$ only small changes are necessary: First of all we have to introduce a new private variable of type `PointProcessInterface` in our new class `JumpProcessEulerScheme`, which will simulate the Jumps $J(t)$. Similiar to the original class the parameters λ, a and b of the compound poisson process are specified by constructing an object of type `CompoundPoissonProcess` and calling the constructor `JumpProcessEulerScheme`, which has input parameters `BrownianMotionInterface brownianMotion` and `PointProcessInterface compoundPoissonProcess`. Then - instead of adding

$$\mu(t_{i-1}, S(t_{i-1}))\Delta t_{i-1} + \sigma(t_{i-1}, S(t_{i-1}))\Delta W_{t_{i-1}}$$

in each timestep - we additionally add $S(t_{i-1})\Delta J(t_{i-1})$, where again $\Delta J(t_i) := J(t_{i+1}) - J(t_i)$. Note that for this procedure it is inevitable to set the $statespacetransform$ to id, since we are adding $S(t-)J(t)$ and not $J^X(t)$.

Exercise 02

```
Printing results for:  riskFreeRate = 0.05, volatility = 0.3, nu = 0.15,
strike = 100.0, maturity = 2.0
--------------------------------
Merton-Jump-Diffusion-Model:
intensity | Option price
0.0         24,19920
0.4         24,89619
1.0         26,36171
1.4         26,62189
2.0         28,16875
--------------------------------
Option Price for the Black-Scholes-Model:  24,20361
```

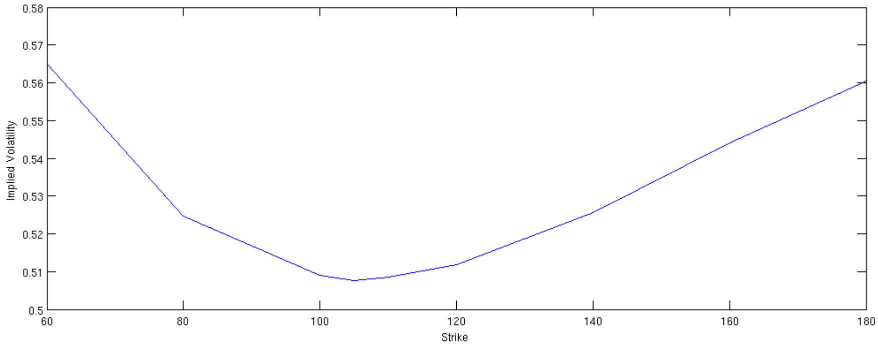
When $\lambda = 0$ the values of the Merton-Jump-Diffusion Model are nearly the same as in the Black-Scholes-Model. This is due to the fact that jump intensity is set to zero and thus no jumps occur. Consequently the modified Euler-Maryuama-Scheme described in exercise 1 simply performs a discretization of the Black-Scholes Model. The values should be exactly the same, but since we did not implement a log-Euler-Scheme, but the Black-Scholes Model in the `finmathlib` is simulated by one, small discrepancies are reasonable.

Exercise 03

1. If the market obeys the Black-Scholes Model exactly, the volatility does not change for different strikes. Thus it is sufficient to use one option to calibrate the model.

2. The implied volatility will be the same for every product, because the prices of the underlying follow the Black-Scholes Model and thus - as above - the volatility does not depend on the strikes.

Implied volatilities for the market data

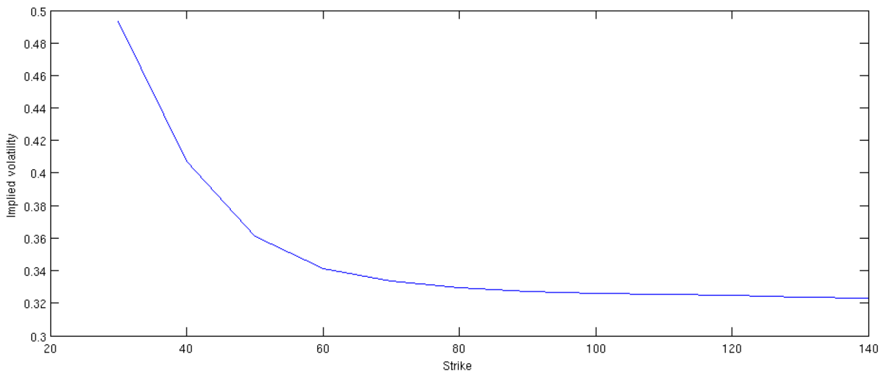


The plot of the implied volatilities retrieved by the market data are not flat as one could expect keeping 1. and 2. in mind. Actually, the shape of a smile is visible. The Black-Scholes Model assumes that a stock price is driven by a continious diffusion model. But in reality stock prices are subject to further sources of randomness.

Exercise 04

```
Printing results for:  initialValue = 100.0, riskFreeRate = 0.05,
volatility = 0.3, intensity = 0.4, nu = 0.15, maturity = 2.0
---------------------------------
Merton-Jump-Diffusion-Model:
Strike | Option Price | implied volatility
30.0      73,2692       0,4931
40.0      64,3304       0,4073
50.0      55,6257       0,3609
60.0      47,4114       0,3410
70.0      39,9263       0,3336
80.0      33,2461       0,3293
90.0      27,4585       0,3273
100.0     22,5270       0,3261
110.0     18,3956       0,3255
120.0     14,9495       0,3248
130.0     12,1019       0,3240
140.0     9,7786        0,3233
---------------------------------
```

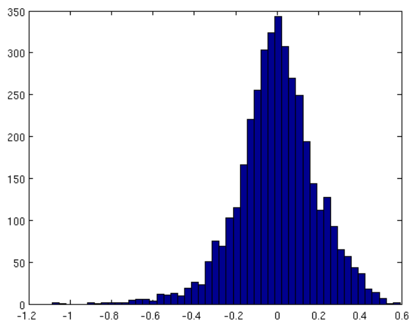
Implied volatilities for the Merton-Jump-Diffusion Model



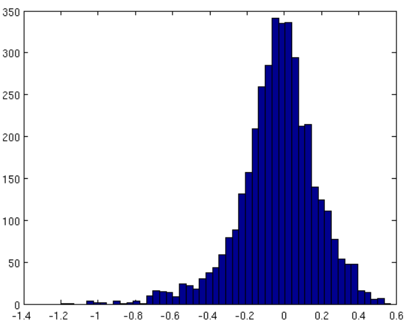
If an underlying is following the Merton-Jump-Diffusion Model and we calculate the implied volatilities for options written on this underlying, we observe similar to the real market data a volatility smile. The implied volatilities are slightly higher than the real one ($\sigma = 0.3$), since the additional jumps lead to higher fluctuation.

Exercise 06

Number of restructuring dates: 10



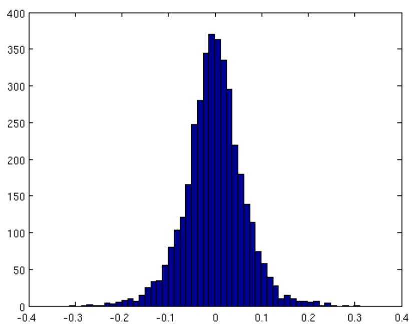
Black Scholes model



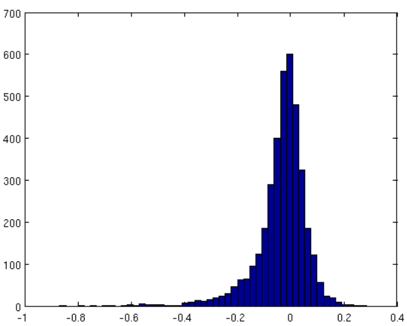
Merton Jump Diffusion model

```
          | Black Scholes Model | Merton-Jump-Diffusion Model
Mean:       -0,000798947          -0,031588544
Variance:    0,033585186           0,039472002
```

Number of restructuring dates: 100



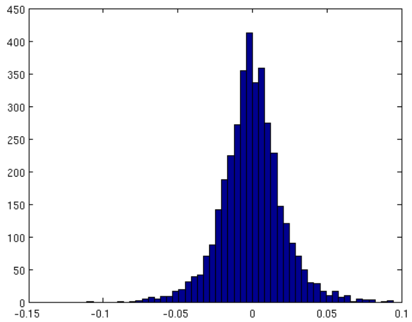
Black Scholes model



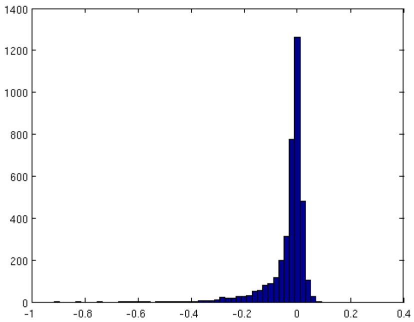
Merton Jump Diffusion model

```
          | Black Scholes Model | Merton-Jump-Diffusion Model
Mean:       -0,000233700          -0,031529162
Variance:    0,003640516           0,008744096
```

Number of restructuring dates: 1000



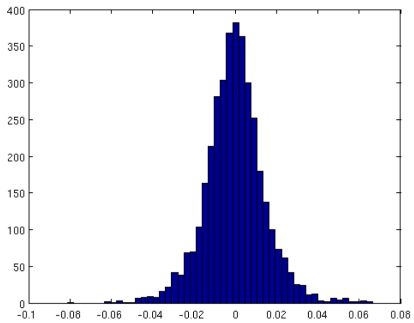
Black Scholes model



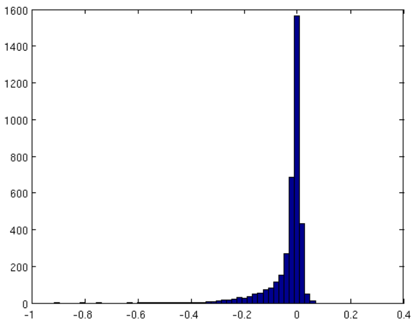
Merton Jump Diffusion model

	Black Scholes Model	Merton-Jump-Diffusion Model
Mean:	-0,000046441	-0,030845866
Variance:	0,000363885	0,005082788

Number of restructuring dates: 2000



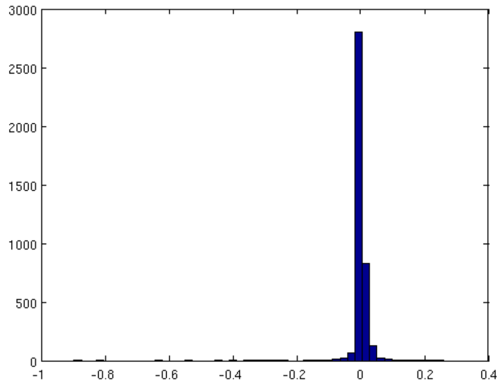
Black Scholes model



Merton Jump Diffusion model

	Black Scholes Model	Merton-Jump-Diffusion Model
Mean:	-0,000247766	-0,030723511
Variance:	0,000184194	0,004757257

Delta-Gamma-Hedge for Merton Jump Diffusion model, hedging instrument: european call option with strike = 110, maturity = 5




```
          | Merton-Jump-Diffusion-Model via Delta-Gamma-Hedging
Mean:      0,004614378
Variance:  0,000895833
```

By increasing the number of hedging times, the Relative P&L is tending to zero in both models. Since the hedging strategy is deduced from the Black-Scholes Model the mean and variance of the hedges for the options written on the Black-Scholes underlying are by a factor of 10 lower than the ones of the Merton-Jump-Diffusion Model. This seems to be natural due to the fact that higher fluctuations caused by random jumps are more difficult to control assuming to hedge an option written on a continious underlying. In order to handle this effect we decided to use the implied volatility for strike 100 calculated in **Exercise 04** in the hedge for the Merton-Jump-Diffusion Model. Actually this did not lead to any improvement, which is not surprising, because the hedging strategy still expects a Black-Scholes underlying.