

HOMEWORK 5

Due on 4/30, submit in HARDCOPY

1. (15%) Decipher the following ciphertext, which was encrypted with the Caesar cipher:
TEBKFKQEBZLROPBLCERJXKBSBKQP. Follow the example in Chapter 8.2.2 to find the key and the plaintext, and show the table in Figure 8-2 for this question. Make a program to compute the table (but no need to submit your program).

Plaintext	a	b	c	d	e	f	g	h	i	j	k	l	m
Ciphertext	D	E	F	G	H	I	J	K	L	M	N	O	P
Plaintext	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

TEBKFKQEBZLROPBLCERJXKBSBKQP
Wheninthecourseofhumanevents

Inserting spaces in all the appropriate places we have the final plaintext

When in the course of human events

2. (10%) Let k be the encryption key for a Caesar cipher. Then, the decryption key is $26-k$. One of the characteristics of a public key system is that the encryption and decryption keys are different. Why is the Caesar cipher not a public key system, even though its encryption and decryption keys are different?

Because in a Caesar cipher if you have the encryption or decryption key you can get the other and by only possessing one key. While in a public key system owning either key does not grant you access to the other, making it the more secure system.

3. (15%) We have a law: $(ab) \% n = ((a \% n) (b \% n)) \% n$. We want to compute $(35^{77}) \% 83$, i.e. 35 powers to 77 then modulo over 83.

(a) Show how to use the law to reduce the number of multiplications of this computation from 76 multiplications to 9 multiplications.

HOMEWORK 5

35^1
 $35^2 = 1225 \bmod 83 \Rightarrow 63 \bmod 83$
 $35^4 = (35^2)^2 = (63^2) \bmod 83 = 3969 \bmod 83 \Rightarrow 68 \bmod 83$
 $35^8 = (35^4)^2 = (68^2) \bmod 83 = 4624 \bmod 83 \Rightarrow 59 \bmod 83$
 $35^{16} = (35^8)^2 = (59^2) \bmod 83 = 3481 \bmod 83 \Rightarrow 78 \bmod 83$
 $35^{32} = (35^{16})^2 = (78^2) \bmod 83 = 6084 \bmod 83 \Rightarrow 25 \bmod 83$
 $35^{64} = (35^{32})^2 = (25^2) \bmod 83 = 625 \bmod 83 \Rightarrow 44 \bmod 83$

(b) Make a C or C++ or Java program to implement an integer exponentiation function `dexp(unsigned int x, unsigned int y, unsigned int n)` that returns $(x^y) \bmod n$ with reduced multiplications. Copy and paste your code in your homework submission.

```

template <typename T>
T modpow(T base, T exp, T modulus) {
    base %= modulus;
    T result = 1;
    while (exp > 0) {
        if (exp & 1) result = (result * base) % modulus;
        base = (base * base) % modulus;
        exp >>= 1;
    }
    return result;
}

```

(c) Use your program to get the result of $(35^{77}) \bmod 83$. Show the result.

Result is 27.

4. (10%) A byte-sum program exclusive or's all bytes in its input to produce a one-byte hash. Is this byte-sum program a secure hash function or not? Show one example of inputs to justify your answer.

HOMEWORK 5

Not a secure hash function, being that the output produced is always 1 byte that is of 8 bits hence it is always going to be easy for the attacker to find 2 inputs producing the same hash in roughly $O(2^4)$ operations.

For example by assuming that key is all 1's and first 8 bits will be taken from the resulting output as our hash Now say input is 101010101010. Key will be of length of input that is 12 and all 1 bits Therefore $101010101010 \text{ xor } 111111111111 = 010101010101$ Therefore $\text{hash}(\text{first 8 bits}) = 01010101$
Now consider the input - 101010101111 Key will be of length of input that is 12 and all 1 bits Therefore $101010101111 \text{ xor } 111111111111 = 010101010000$ Therefore $\text{hash}(\text{first 8 bits}) = 01010101$

Note that same hash is produced for 2 different inputs approving that hash function is not secured.

5. (10%) Multiply two large numbers p and q (you will need to find a tool or a library by yourself)

p =
0xc315d99cf91a018dafba850237935b2d981e82b02d994f94db0a1
ae40d1fc7ab9799286ac68d620f1102ef515b348807060e6caec532
0e3dceb25a0b98356399

q =
0xe90bbb3d4f51311f0b7669abd04e4cc48687ad0e168e7183a9de3
ff9fd2d2a3a50303a5109457bd45f0abe1c5750edfaff1ad87c13ee
d45e1b4bd2366b49d97f

p * q = b197 d3af e713 8165 82ee 988b 276f 6358 00f7 28f1 18f5
125d e1c7 c1e5 7f27 3835 1de8 ac64 3c11 8a54 80f8 67b6 d875 6021
9118 18e4 7095 2bd0 a526 2ed8 6b4f c4c2 b796 2cd1 97a8 bd8d 8ae3
f821 ad71 2a42 285d b67c 8598 3581 c4c3 9f80 dbb2 1bf7 00db d2ae
9709 f7e3 0776 9b5c 0e62 4b66 1441 c1dd b62e f1fe 7684 bbe6 1d8a
19e7

HOMEWORK 5

6. (10%) Factorize a short large number (you will need to download and use the tool yafu)

$N=359567260516027240236814314071842368703501656647819140843316303878351$

$59\ 567260\ 516027\ 240236\ 814314\ 071842\ 368703\ 501656\ 647819\ 140843\ 316303\ 878351$ (69 digits) = $17963\ 604736\ 595708\ 916714\ 953362\ 445519$ (35 digits) \times $20016\ 431322\ 579245\ 244930\ 631426\ 505729$ (35 digits)

Möbius: 1

$$n = a^2 + b^2 + c^2 + d^2$$

$a = 14289\ 016215\ 761299\ 320744\ 668512\ 106427$ (35 digits)

$b = 10797\ 671605\ 230392\ 476974\ 632177\ 292689$ (35 digits)

$c = 5553\ 744391\ 295291\ 603970\ 848491\ 758201$ (34 digits)

$d = 2820\ 901849\ 319697\ 198195\ 560861\ 354070$ (34 digits)

7. (10%) Run the CTF virtual box, read the partial solution of "Super Old Cipher". Show the flag.

Submit!

FLAG{CaEsaR_Is_EaSy}

Super Old Cipher - 30

Cryptography - Solved

Solve

Hint

8. (10%) Run the CTF virtual box, read the partial solution of "Messy AES". Show the flag.

Submit!

FLAG{looks_like_you_can_break_aes}

HOMEWORK 5

Messy Aes - 30

Cryptography - Solved

Solve

Hint