

National Research University Higher School of Economics

Youthful Passion Fruit

Alexander Nekrasov, Iurii Pustovalov, Alexey Vasilyev

Contest (1)

```
template.cpp
                                                                    42 lines
#ifdef LOCAL
#define _GLIBCXX_DEBUG
#endif
#include <bits/stdc++.h>
using namespace std:
using 11 = long long;
using ld = long double;
using ull = unsigned long long;
#define pbc push_back
#define mp make_pair
#define all(v) (v).begin(), (v).end()
#define vin(v) for (auto &el : a) cin >> el
mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
template <typename T1, typename T2> inline void chkmin(T1 &x, const T2 &y
    if (y < x) {
        x = y;
template <typename T1, typename T2> inline void chkmax(T1 &x, const T2 &y
    if (x < y) {
       x = y;
void solve() {
signed main() {
    cin.tie(0)->sync_with_stdio(0);
    cout.precision(20), cout.setf(ios::fixed);
    int t = 1;
    // cin >> t;
    while (t--) {
        solve():
genfolders.sh
                                                                     6 lines
chmod +x bld*
for f in {A..Z}
do
    mkdir $f
    cp main.cpp bld* $f
bld
                                                                     1 lines
g++ -std=c++20 -g -DLOCAL -fsanitize=address, bounds, undefined -o $1 $1.
     срр
bldf
                                                                     1 lines
g++ -std=c++20 -g -02 -o $1 $1.cpp
hacks.sh
                                                                     2 lines
UBSAN_OPTIONS=print_stacktrace=1 ./main
gdb rbreak regex
hash.sh
                                                                     3 lines
# Hashes a file, ignoring all whitespace and comments.
# Use for verifying that code was correctly typed.
```

cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum |cut -c-6

```
clion.cpp
```

set (CMAKE CXX STANDARD 20) set (CMAKE_CXX_FLAGS "-DLOCAL")

$_{_{2 \; \mathrm{lines}}} \, | \; \mathrm{C}{+}{+} \; (2)$

GpHashtable.cpp

Description: Hash map with mostly the same API as unordered_map, but $\sim 3x$ faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided). lines

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
const int RANDOM =
   chrono::high_resolution_clock::now().time_since_epoch().count();
   int operator()(int x) const { return x ^ RANDOM; }
gp_hash_table<int, int, hasher> table;
```

OrderedSet.cpp

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null_type.

Time: $\mathcal{O}(\log(n))$

```
<bits/extc++.h>, <bits/stdc++.h>
                                                               dff260, 37 lines
using namespace __gnu_pbds;
using namespace std;
template <typename T>
using ordered set =
    tree<T, null_type, less<>, rb_tree_tag,
         tree_order_statistics_node_update>;
int main() {
    ordered_set<int> X;
    X.insert(1);
    X.insert(2);
   X.insert(4):
    X.insert(8);
   X.insert(16);
    assert(*X.find_by_order(1) == 2);
    assert (*X.find by order(2) == 4);
    assert(*X.find_by_order(4) == 16);
    assert(X.find_by_order(6) == X.end());
    assert (X.order of kev(-5) == 0);
    assert (X.order_of_key(1) == 0);
    assert (X.order_of_key(3) == 2);
    assert (X.order_of_key(4) == 2);
    assert (X.order_of_key(400) == 5);
    // std::cout \ll *X.find_by\_order(1) \ll std::endl;
    // std::cout \ll *X.find_by_order(2) \ll std::endl;
    // std::cout << *X.find_by\_order(4) << std::endl;
    // std::cout \ll (end(X) = X.find_by\_order(6)) \ll std::endl; // true
    // std::cout \ll X. order\_of\_key(-5) \ll std::endl; // 0
    // std::cout \ll X. order\_of\_key(1) \ll std::endl; // 0
    // std::cout \ll X. order\_of\_key(3) \ll std::endl; // 2
    // std::cout \ll X. order\_of\_key(4) \ll std::endl;
    // std::cout \ll X. order_of_key(400) \ll std::endl; // 5
bitset.cpp
Description: bitset
                                                                521d1f, 2 lines
```

bs. Find first() bs._Find_next(idx) - returns right after

```
alloc.cpp
```

Description: fastalloc

8726b1, 11 lines

```
const int MAX_MEM = 1e8;
int mpos = 0;
char mem[MAX MEM];
inline void *operator new(size_t n) {
  assert((mpos += n) <= MAX_MEM);
  return (void *) (mem + mpos - n);
```

} // must have!

} // must have!

void operator delete(void *) noexcept {

void operator delete(void *, size_t) noexcept {

fastio Manacher AhoCorasick SuffixArray Lcp

```
fastio.cpp
Description: fastio
                                                             79fd14, 52 lines
inline int readChar();
template <class T = int>
inline T readInt();
template <class T>
inline void writeInt(T x, char end = 0);
inline void writeChar(int x);
inline void writeWord(const char *s);
static const int buf_size = 4096;
inline int getChar() {
 static char buf[buf_size];
  static int len = 0, pos = 0;
 if (pos == len) pos = 0, len = fread(buf, 1, buf_size, stdin);
 if (pos == len) return -1;
  return buf[pos++];
inline int readChar() {
 int c = getChar();
  while (c <= 32) c = getChar();
  return c:
template <class T>
inline T readInt() {
 int s = 1, c = readChar();
 if (c == '-') s = -1, c = getChar();
  while ('0' \le c \&\& c \le '9') x = x * 10 + c - '0', c = getChar();
  return s == 1 ? x : -x;
static int write_pos = 0;
static char write_buf[buf_size];
inline void writeChar(int x) {
 if (write pos == buf size)
   fwrite(write_buf, 1, buf_size, stdout), write_pos = 0;
  write buf[write pos++] = x;
template <class T>
inline void writeInt(T x, char end) {
 if (x < 0) writeChar('-'), x = -x;
  char s[24];
 int n = 0:
 while (x \mid | !n) s[n++] = '0' + x % 10, x /= 10;
  while (n--) writeChar(s[n]);
 if (end) writeChar(end);
inline void writeWord(const char *s) {
 while (*s) writeChar(*s++);
struct Flusher {
 ~Flusher() {
    if (write_pos) fwrite(write_buf, 1, write_pos, stdout), write_pos =
} flusher;
```

```
Strings (3)
Manacher.cpp
Description: Manacher algorithm
Time: \mathcal{O}(n)
                                                              a6ddfb, 27 lines
vector<int> manacherOdd(string s) {
   int n = s.size();
   vector<int> d1(n);
   int 1 = 0, r = -1;
   for (int i = 0; i < n; ++i) {</pre>
        int k = i > r ? 1 : min(d1[l + r - i], r - i + 1);
        while (i + k < n \&\& i - k >= 0 \&\& s[i + k] == s[i - k])
        d1[i] = k;
        if (i + k - 1 > r)
           1 = i - k + 1, r = i + k - 1;
vector<int> manacherEven(string s) {
   int n = s.size();
   vector<int> d2(n);
    1 = 0, r = -1;
    for (int i = 0; i < n; ++i) {</pre>
        int k = i > r ? 0 : min(d2[1 + r - i + 1], r - i + 1);
        while (i + k < n \&\& i - k - 1 >= 0 \&\& s[i + k] == s[i - k - 1])
            ++k:
        d2[i] = k;
       if (i + k - 1 > r)
           1 = i - k, r = i + k - 1;
AhoCorasick.cpp
Description: Build aho-corasick automaton.
Time: \mathcal{O}(n)
                                                               ae5fc2, 19 lines
int go(int v, char c);
int get link(int v) {
   if (t[v].link == -1)
        if (v == 0 || t[v].p == 0)
            t[v].link = 0;
            t[v].link = go(get_link(t[v].p), t[v].pch);
    return t[v].link;
int go(int v, char c) {
   if (t[v].go[c] == -1)
        if (t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
            t[v].go[c] = v == 0 ? 0 : go(get_link(v), c);
    return t[v].go[c];
SuffixArray.cpp
Description: Build suffix array
Time: \mathcal{O}(n\log(n))
                                                              5bd<u>011, 47 lines</u>
vector<int> buildSuffixArray(string &s) {
    // Remove, if you want to sort cyclic shifts
    s += (char)(1);
    int n = s.size();
    vector<int> a(n);
    iota(all(a), 0);
    stable_sort(all(a), [&](int i, int j) { return s[i] < s[j]; });</pre>
    vector<int> c(n);
    int cc = 0:
    for (int i = 0; i < n; i++) {
        if (i == 0 || s[a[i]] != s[a[i - 1]]) {
            c[a[i]] = cc++;
            c[a[i]] = c[a[i - 1]];
```

for (int L = 1; L < n; L *= 2) {

```
for (auto i : c) {
            cnt[i]++;
        vector<int> pref(n);
        for (int i = 1; i < n; i++) {</pre>
           pref[i] = pref[i - 1] + cnt[i - 1];
        vector<int> na(n);
        for (int i = 0; i < n; i++) {
            int pos = (a[i] - L + n) % n;
            na[pref[c[pos]]++] = pos;
        a = na;
        vector<int> nc(n);
        cc = 0:
        for (int i = 0; i < n; i++) {</pre>
            if (i == 0 || c[a[i]] != c[a[i - 1]] ||
                c[(a[i] + L) % n] != c[(a[i - 1] + L) % n]) {
                nc[a[i]] = cc++;
            } else {
                nc[a[i]] = nc[a[i - 1]];
    a.erase(a.begin());
    s.pop back();
    return a:
Lcp.cpp
Description: lcp array
Time: \mathcal{O}(n)
                                                               1cc27c, 43 lines
vector<int> perm;
vector<int> buildLCP(string &s. vector<int> &a) {
    int n = s size():
    vector<int> ra(n);
    for (int i = 0; i < n; i++) {
        ra[a[i]] = i;
    vector<int> lcp(n - 1);
    int cur = 0;
    for (int i = 0; i < n; i++) {
        cur--;
        chkmax(cur. 0):
        if (ra[i] == n - 1) {
            cur = 0;
            continue;
        int j = a[ra[i] + 1];
        while (s[i + cur] == s[j + cur]) cur++;
        lcp[ra[i]] = cur;
    perm.resize(a.size());
    for (int i = 0; i < a.size(); ++i) perm[a[i]] = i;</pre>
    return lcp:
int cntr[MAXN];
int spt[MAXN][lgg];
void build(vector<int> &a) {
    for (int i = 0; i < a.size(); ++i) {</pre>
        spt[i][0] = a[i];
    for (int i = 2; i < MAXN; ++i) cntr[i] = cntr[i / 2] + 1;</pre>
    for (int h = 1; (1 << (h - 1)) < a.size(); ++h) {</pre>
        for (int i = 0; i + (1 << (h - 1)) < a.size(); ++i) {
            spt[i][h] = min(spt[i][h-1], spt[i+(1 << (h-1))][h-1]
int getLCP(int 1, int r) {
    l = perm[l], r = perm[r];
    if (1 > r) swap(1, r);
    int xx = cntr[r - 1];
    return min(spt[1][xx], spt[r - (1 << xx)][xx]);</pre>
```

vector<int> cnt(n):

if (p == -1)

else {

st[cur].link = 0;

Eertree Suffix Automaton Prefix Z MinShift SA-IS

int q = st[p].next[c];

if (st[p].len + 1 == st[q].len)

```
Eertree.cpp
Description: Creates Eertree of string str
Time: \mathcal{O}(n)
                                                              7924c8, 40 lines
struct eertree {
   int len[MAXN], suffLink[MAXN];
    int to[MAXN][26];
   int numV, v;
    void addLetter(int n, string &str) {
        while (str[n - len[v] - 1] != str[n])
           v = suffLink[v];
        int u = suffLink[v];
        while (str[n - len[u] - 1] != str[n])
            u = suffLink[u];
        int u_ = to[u][str[n] - 'a'];
        int v_ = to[v][str[n] - 'a'];
        if (v == -1) {
            v_{-} = to[v][str[n] - 'a'] = numV;
            len[numV++] = len[v] + 2;
            suffLink[v_] = u_;
        v = v;
    void init() {
        len[0] = -1;
        len[1] = 0;
        suffLink[1] = 0;
        suffLink[0] = 0;
        numV = 2:
        for (int i = 0; i < 26; ++i) {
            to[0][i] = numV++;
            suffLink[numV - 1] = 1;
            len[numV - 1] = 1;
        v = 0;
    void init(int sz) {
        for (int i = 0; i < sz; ++i) {</pre>
            len[i] = suffLink[i] = 0;
            for (int j = 0; j < 26; ++j)
                to[i][j] = -1;
SuffixAutomaton.cpp
Description: Build suffix automaton.
Time: \mathcal{O}(n)
                                                              662a10, 45 lines
struct state {
   int len, link;
    map<char, int> next;
const int MAXLEN = 100000;
state st[MAXLEN * 2];
int sz. last:
void sa init() {
   sz = last = 0;
    st[0].len = 0;
    st[0].link = -1;
    ++82:
    // if you want to build an automaton for different strings:
    for (int i=0; i \le MAXLEN * 2; ++i)
            st[i].next.clear();
void sa_extend(char c) {
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    for (p = last; p != -1 \&\& !st[p].next.count(c); p = st[p].link)
       st[p].next[c] = cur;
```

```
st[cur].link = q;
            int clone = sz++:
            st[clone].len = st[p].len + 1;
            st[clone].next = st[q].next;
            st[clone].link = st[q].link;
            for (; p != -1 && st[p].next[c] == q; p = st[p].link)
                st[p].next[c] = clone;
            st[q].link = st[cur].link = clone;
    last = cur;
PrefixZ.cpp
Description: Calculates Prefix, Z-functions
Time: \mathcal{O}(n)
                                                               1c4e93, 25 lines
vector<int> pf(string s) {
   int k = 0;
    vector<int> p(s.size());
    for (int i = 1; i < s.size(); ++i) {</pre>
        while (k && s[i] != s[k])
           k = p[k - 1];
        k += (s[i] == s[k]);
        p[i] = k;
    return p;
vector<int> zf(string s) {
   int n = s.size();
    vector<int> z(n, 0);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r)
           z[i] = min(r - i + 1, z[i - 1]);
        while (i + z[i] < n \&\& s[z[i]] == s[i + z[i]])
            ++z[i];
        if (i + z[i] - 1 > r)
            1 = i, r = i + z[i] - 1;
   z[0] = n;
    return z;
MinShift.cpp
Description: Calculates min-cyclic-shift of s, Duval decomposition
Time: \mathcal{O}(n)
                                                               3f0fb9, 20 lines
string minshift(string s) {
   int i = 0, ans = 0;
    s += s;
   int n = s.size();
    while (i < n / 2) {
        ans = i:
        int j = i + 1, k = i;
        while (j < n && s[k] <= s[j]) {
            if (s[k] < s[i])
                k = i:
            else
                ++k;
            ++j;
        while (i <= k) {
            i += j - k;
    return s.substr(ans, n / 2);
SA-IS.cpp
Description: Build suffix array
Time: \mathcal{O}(n)
                                                                f90ffe, 87 lines
void induced sort(vector<int> &vec, int LIM, vector<int> &sa, vector<bool</pre>
                   vector<int> &fx) {
    vector<int> 1(LIM), r(LIM);
```

```
for (int c : vec) {
        if (c + 1 < LIM) {
            ++1[c + 1];
        ++r[c]:
   partial_sum(all(1), l.begin());
    partial_sum(all(r), r.begin());
    fill(all(sa), -1);
    for (int i = fx.size() - 1; i >= 0; --i) {
        sa[--r[vec[fx[i]]]] = fx[i];
    for (int i : sa) {
        if (i >= 1 && sl[i - 1]) {
           sa[l[vec[i-1]]++] = i-1;
    fill(all(r), 0);
    for (int c : vec) ++r[c];
    partial_sum(all(r), r.begin());
    for (int k = sa.size() - 1, i = sa[k]; k >= 1; --k, i = sa[k])
        if (i >= 1 && !sl[i - 1]) sa[--r[vec[i - 1]]] = i - 1;
vector<int> SA IS(vector<int> &vec, int LIM) {
    const int n = vec.size();
    vector<int> sa(n), fx;
   vector<bool> sl(n);
    sl[n - 1] = false;
    for (int i = n - 2; i >= 0; --i) {
        sl[i] = (vec[i] > vec[i + 1] | | (vec[i] == vec[i + 1] && sl[i +
             1]));
        if (sl[i] && !sl[i + 1]) {
            fx.pbc(i + 1);
   reverse(all(fx));
    induced_sort(vec, LIM, sa, sl, fx);
    vector<int> nfx(fx.size()), lmv(fx.size());
    for (int i = 0, k = 0; i < n; ++i) {
        if (!sl[sa[i]] && sa[i] >= 1 && sl[sa[i] - 1]) {
           nfx[k++] = sa[i];
    int cur = 0;
    sa[n - 1] = cur;
    for (int k = 1; k < nfx.size(); ++k) {</pre>
        int i = nfx[k - 1], j = nfx[k];
        if (vec[i] != vec[j]) {
            sa[j] = ++cur;
            continue:
       bool flag = false;
        for (int a = i + 1, b = j + 1;; ++a, ++b) {
            if (vec[a] != vec[b]) {
                flag = true;
                break:
            if ((!sl[a] && sl[a - 1]) || (!sl[b] && sl[b - 1])) {
                flag = !((!sl[a] \&\& sl[a - 1]) \&\& (!sl[b] \&\& sl[b - 1]));
                break:
        sa[i] = (flag ? ++cur : cur);
    for (int i = 0; i < fx.size(); ++i) {</pre>
        lmv[i] = sa[fx[i]];
    if (cur + 1 < (int)fx.size()) {</pre>
        auto lms = SA IS(lmv, cur + 1);
        for (int i = 0; i < fx.size(); ++i) {</pre>
           nfx[i] = fx[lms[i]];
    induced_sort(vec, LIM, sa, sl, nfx);
    return sa:
template <typename T>
vector<int> suffix_array(T &s, const int LIM = 128) {
    vector<int> vec(s.size() + 1);
```

copy(all(s), begin(vec));
vec.back() = (char)(1);

auto ret = SA_IS(vec, LIM);

ret.erase(ret.begin());

return ret;

Hungarian BlossomShrinking Lct

```
Graph (4)
Hungarian.cpp
Description: Hungarian algorithm
Time: \mathcal{O}\left(n^3\right)
                                                                5afee5, 41 lines
int n, m;
vector<vector<int>> a;
vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
for (int i = 1; i <= n; ++i) {</pre>
   p[0] = i;
    int j0 = 0;
    vector<int> minv(m + 1, INF);
    vector<char> used(m + 1, false);
        used[j0] = true;
        int i0 = p[j0], delta = INF, j1;
        for (int j = 1; j <= m; ++j)
            if (!used[j]) {
                int cur = a[i0][j] - u[i0] - v[j];
                if (cur < minv[j])</pre>
                   minv[j] = cur, way[j] = j0;
                 if (minv[j] < delta)</pre>
                    delta = minv[j], j1 = j;
        for (int j = 0; j <= m; ++j)</pre>
            if (used[j])
                u[p[j]] += delta, v[j] -= delta;
               minv[j] -= delta;
        j0 = j1;
    } while (p[j0] != 0);
        int j1 = way[j0];
        p[j0] = p[j1];
        j0 = j1;
    } while (j0);
// matching
vector<int> ans(n + 1);
for (int j = 1; j <= m; ++j) {
   ans[p[j]] = j;
// cost
int cost = -v[0];
BlossomShrinking.cpp
Description: Maximum matching in general graph
Time: \mathcal{O}\left(n^3\right)
                                                              23839d, 118 lines
struct Edge {
   int u, v;
const int N = 510;
int n, m;
vector<int> g[N];
vector<Edge> perfectMatching;
int match[N], par[N], base[N];
bool used[N], blossom[N], lcaUsed[N];
int lca(int u, int v) {
    fill(lcaUsed, lcaUsed + n, false);
    while (u ! = -1) {
        u = base[u];
        lcaUsed[u] = true;
        if (match[u] == -1)
            break;
        u = par[match[u]];
    while (v != -1) {
        v = base[v];
        if (lcaUsed[v])
            return v;
        v = par[match[v]];
```

assert (false);

return -1;

```
void markPath(int v, int myBase, int children) {
    while (base[v] != myBase) {
        blossom[v] = blossom[match[v]] = true;
        par[v] = children;
        children = match[v];
        v = par[match[v]];
int findPath(int root) {
    iota(base, base + n, 0);
    fill(par, par + n, -1);
    fill (used, used + n, false);
    queue<int> q;
    q.push(root);
    used[root] = true;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (auto to : q[v]) {
            if (match[v] == to)
                continue;
            if (base[v] == base[to])
                continue;
            if (to == root || (match[to] != -1 && par[match[to]] != -1))
                fill(blossom, blossom + n, false);
                int myBase = lca(to, v);
                markPath(v, myBase, to);
                markPath(to, mvBase, v);
                for (int u = 0; u < n; ++u)
                    if (!blossom[base[u]])
                        continue;
                    base[u] = myBase;
                    if (used[u])
                        continue;
                    used[u] = true;
                    q.push(u);
            } else if (par[to] == -1) {
                par[to] = v;
                if (match[to] == -1) {
                    return to;
                used[match[to]] = true;
                q.push(match[to]);
    return -1;
void blossomShrinking() {
    fill (match, match + n, -1);
    for (int v = 0; v < n; ++v) {
        if (match[v] != -1)
           continue:
        int nxt = findPath(v);
        while (nxt != -1) {
            int parV = par[nxt];
            int parParV = match[parV];
           match[nxt] = parV;
            match[parV] = nxt;
            nxt = parParV:
    for (int v = 0; v < n; ++v) {</pre>
        if (match[v] != -1 && v < match[v]) {</pre>
            perfectMatching.push_back({v, match[v]});
signed main() {
    cin >> n:
    int u, v;
    set<pair<int, int>> edges;
    while (cin >> u >> v) {
        --u;
        --∀;
        if (u > v)
```

NRU HSE MaxFlow 5

```
swap(u, v);
        if (edges.count({u, v}))
            continue;
        edges.insert({u, v});
        g[u].push_back(v);
        g[v].push_back(u);
    blossomShrinking();
    cout << perfectMatching.size() * 2 << '\n';</pre>
    for (auto i : perfectMatching) {
        cout << i.u + 1 << " " << i.v + 1 << "\n";
    return 0:
Lct.cpp
Description: link-cut tree
Time: O(n \log(n))
                                                              3d8a3f, 136 lines
#include <hits/stdc++ h>
using namespace std;
const int MAXN = 1e5 + 228;
struct node {
   node *ch[2];
    node *p;
   bool rev:
    int sz;
        ch[0] = ch[1] = p = NULL;
        rev = false;
        sz = 1;
};
int getsz(node *n) { return (n == NULL) ? 0 : n->sz; }
void pull(node *n) { n->sz = getsz(n->ch[0]) + getsz(n->ch[1]) + 1; }
void push (node *n) {
    if (n->rev) {
        if (n->ch[0]) {
            n->ch[0]->rev ^= 1;
        if (n->ch[1]) {
            n->ch[1]->rev ^= 1;
        swap(n->ch[0], n->ch[1]);
        n->rev = 0;
bool isRoot(node *n) {
    return n->p == NULL || (n->p->ch[0] != n && n->p->ch[1] != n);
int chnum(node *n) { return n->p->ch[1] == n; }
void attach(node *n, node *p, int num) {
    if (n != NULL)
        n->p = p;
    if (p != NULL)
        p->ch[num] = n;
void rotate(node *n) {
    int num = chnum(n);
    node *p = n->p;
    node *b = n \rightarrow ch[1 - num];
    n->p = p->p;
    if (!isRoot(p)) {
        p \rightarrow p \rightarrow ch[chnum(p)] = n;
    attach(p, n, 1 - num);
    attach(b, p, num);
    pull(p);
```

pull(n);

```
node *qq[MAXN];
void splay(node *n) {
   node *nn = n;
   int top = 0;
    qq[top++] = nn;
    while (!isRoot(nn)) {
        nn = nn->p;
        qq[top++] = nn;
    while (top) {
        push (gg[--top]);
    while (!isRoot(n)) {
        if (!isRoot(n->p)) {
            if (chnum(n) == chnum(n->p)) {
                rotate(n->p);
            } else {
                rotate(n);
        rotate(n):
void expose(node *n) {
    splay(n);
    n \rightarrow ch[1] = NULL;
    pull(n);
    while (n->p != NULL) {
        splay(n->p);
        attach(n, n->p, 1);
        pull(n->p);
        splay(n);
void makeRoot(node *n) {
   expose(n);
   n->rev ^= 1:
node *nodes[MAXN];
int main() {
   int n:
    for (int i = 0; i <= n; i++) {</pre>
        nodes[i] = new node();
    int q;
    cin >> q;
    while (a--) {
        string s;
        cin >> s;
        int u, v;
        cin >> u >> v;
        makeRoot (nodes[u]);
        makeRoot (nodes[v]);
        if (s == "get") {
            if (isRoot(nodes[u]) && u != v) {
                cout << "-1" << endl;
            } else {
                cout << getsz(nodes[v]) - 1 << endl;
        } else if (s == "link") {
            nodes[v]->p = nodes[u];
            push (nodes[v]);
            nodes[v] \rightarrow ch[1] = NULL;
            nodes[u]->p = NULL;
   }
```

```
MaxFlow.cpp Description: Dinic Time: O(n^2m)
```

1c1bc8, 72 lines

```
struct MaxFlow {
    const int inf = 1e9 + 20;
    struct edge {
       int a, b, cap;
    int n:
    vector<edge> e;
    vector<vector<int>> q;
    MaxFlow() {}
    int s, t;
    vector<int> d, ptr;
    void init(int n_, int s_, int t_) {
       s = s, t = t, n = n;
        q.resize(n);
        ptr.resize(n);
    void addedge(int a, int b, int cap) {
        g[a].pbc(e.size());
        e.pbc({a, b, cap});
        g[b].pbc(e.size());
        e.pbc({b, a, 0});
    bool bfs() {
        d.assign(n, inf);
        d[s] = 0;
        queue<int> q;
        q.push(s);
        while (q.size()) {
            int v = q.front();
            q.pop();
            for (int i : q[v]) {
                if (e[i].cap > 0) {
                    int b = e[i].b;
                    if (d[b] > d[v] + 1) {
                        d[b] = d[v] + 1;
                        q.push(b);
               }
        return d[t] != inf;
    int dfs(int v, int flow) {
        if (v == t) return flow;
        if (!flow) return 0;
        int sum = 0;
        for (; ptr[v] < g[v].size(); ++ptr[v]) {</pre>
            int b = e[q[v][ptr[v]]].b;
            int cap = e[g[v][ptr[v]]].cap;
            if (cap <= 0) continue;</pre>
            if (d[b] != d[v] + 1) continue;
            int x = dfs(b, min(flow, cap));
            int id = g[v][ptr[v]];
            e[id].cap -= x;
            e[id ^ 1].cap += x;
            flow -= x;
            sum += x;
        return sum;
    int dinic() {
        int ans = 0;
        while (1) {
            if (!bfs()) break;
            ptr.assign(n, 0);
            int x = dfs(s, inf);
            if (!x) break;
            ans += x;
        return ans:
};
```

MCMF.cpp

Description: Min cost

int a, b, cap, cost;

vector<edge> e; vector<vector<int>> q;

vector<ll> po;

int s, t;
int n;

MCMF MCMFfast GlobalMincut WeightedMatching

```
Time: \mathcal{O}(?)
                                                              32340a, 61 lines
struct MCMF {
   struct edge {
       int a, b, cap, cost;
   vector<edge> e;
   vector<vector<int>> q;
   int s, t;
   int n:
    void init(int N, int S, int T) {
       s = S, t = T, n = N;
       q.resize(N);
       e.clear();
   void addedge(int a, int b, int cap, int cost) {
       g[a].pbc(e.size());
        e.pbc({a, b, cap, cost});
       g[b].pbc(e.size());
        e.pbc({b, a, 0, -cost});
   int getcost(int k) {
        int flow = 0:
        int cost = 0;
        while (flow < k) {
           vector<int> d(n, INF);
            vector<int> pr(n);
            d[s] = 0;
            queue<int> q:
            q.push(s);
            while (q.size()) {
               int v = q.front();
                a.pop();
                for (int i : q[v]) {
                    int u = e[i].b;
                    if (e[i].cap && d[u] > d[v] + e[i].cost) {
                        d[u] = d[v] + e[i].cost;
                        pr[u] = i;
            if (d[t] == INF) return INF;
            int qf = k - flow;
            int v = t;
            while (v != s)
               int id = pr[v];
               chkmin(qf, e[id].cap);
               v = e[id].a;
            while (v != s) {
               int id = pr[v];
               e[id].cap -= qf;
               e[id ^ 1].cap += qf;
               cost += e[id].cost * af;
                v = e[id].a;
            flow += qf;
        return cost;
MCMFfast.cpp
Description: Min cost with potentials
Time: \mathcal{O}(?)
                                                              363228, 86 lines
struct MCMF {
 struct edge {
```

```
void init(int N, int S, int T) {
   s = S, t = T, n = N;
   g.resize(N);
   e.clear();
 void addedge(int a, int b, int cap, int cost) {
   g[a].pbc(e.size());
   e.pbc({a, b, cap, cost});
   a[b].pbc(e.size());
   e.pbc({b, a, 0, -cost});
 void calc_p() {
   po.assign(n, INF);
    vector<int> ing(n);
   queue<int> q;
   q.push(s);
    po[s] = 0;
    while (!q.emptv()) {
     int v = q.front();
     q.pop();
     inq[v] = 0;
     for (auto i : q[v]) {
       if (po[e[i].b] > po[v] + e[i].cost && e[i].cap) {
         po[e[i].b] = po[v] + e[i].cost;
         if (!inq[e[i].b]) q.push(e[i].b);
         ina[e[i].b] = 1;
 ll getcost(int k) {
   calc_p();
    int flow = 0:
   11 cost = 0;
    while (flow < k) {
     vector<ll> d(n, INF);
     vector<int> pr(n);
     d[s] = 0;
     set<pair<ll, int>> q;
     g.insert(mp(011, s));
     while (q.size()) {
       int v = q.begin()->second;
        q.erase(q.begin());
        for (int i : q[v]) {
         if (e[i].cap && d[u] > d[v] + e[i].cost + po[v] - po[e[i].b]) {
           g.erase(mp(d[u], u));
           d[u] = d[v] + e[i].cost + po[v] - po[e[i].b];
           q.insert(mp(d[u], u));
           pr[u] = i;
     if (d[t] == INF) return INF;
     for (int i = 0; i < n; ++i) {
       if (d[i] != INF) po[i] += d[i];
     int qf = k - flow;
     int v = t;
     while (v != s) {
       int id = pr[v];
        chkmin(qf, e[id].cap);
        v = e[id].a:
     while (v != s) {
       int id = pr[v];
       e[id].cap -= qf;
        e[id ^ 1].cap += qf;
        cost += 1ll * e[id].cost * gf;
        v = e[id].a;
     flow += af:
   return cost:
};
```

```
Time: \mathcal{O}\left(n^3\right)
                                                               7b8a6b, 35 lines
const int MAXN = 500;
int n, g[MAXN][MAXN];
int best_cost = 1000000000;
vector<int> best_cut;
void mincut() {
    vector<int> v[MAXN];
    for (int i = 0; i < n; ++i)</pre>
        v[i].assign(1, i);
    int w[MAXN];
    bool exist[MAXN], in_a[MAXN];
    memset (exist, true, sizeof exist);
    for (int ph = 0; ph < n - 1; ++ph) {
        memset (in a, false, sizeof in a);
        memset(w, 0, sizeof w);
        for (int it = 0, prev; it < n - ph; ++it) {</pre>
            int sel = -1:
            for (int i = 0; i < n; ++i)</pre>
                if (exist[i] && !in_a[i] && (sel == -1 || w[i] > w[sel]))
                     sel = i:
            if (it == n - ph - 1) {
                 if (w[sel] < best cost)</pre>
                     best_cost = w[sel], best_cut = v[sel];
                 v[prev].insert(v[prev].end(), v[sel].begin(), v[sel].end
                 for (int i = 0; i < n; ++i)
                     g[prev][i] = g[i][prev] += g[sel][i];
                 exist[sel] = false;
            } else {
                 in_a[sel] = true;
                 for (int i = 0; i < n; ++i)</pre>
                    w[i] += q[sel][i];
                prev = sel;
   }
WeightedMatching.cpp
Description: Max weighted matching
Time: \mathcal{O}(N^3) or so
                                                               c3f149, 193 lines
#define Dist(e) (lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2)
const int N = 1023, INF = 1e9;
struct Edge {
    int u. v. w:
} q[N][N];
int n, m, n_x, lab[N], match[N], slack[N], st[N], pa[N], flower_from[N][N
      ], S[N], vis[N];
vector<int> flower[N];
demiesint> a.
void update_slack(int u, int x) {
    if (!slack[x] || Dist(q[u][x]) < Dist(q[slack[x]][x])) slack[x] = u;
void set slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
        if (q[u][x].w > 0 && st[u] != x && S[st[u]] == 0) update slack(u,
void q_push(int x) {
    if (x <= n) return q.push back(x);</pre>
    for (int i = 0; i < flower[x].size(); ++i) q_push(flower[x][i]);</pre>
void set_st(int x, int b) {
    st[x] = b:
    if (x <= n) return:
    for (int i = 0; i < flower[x].size(); ++i) set st(flower[x][i], b);</pre>
int get pr(int b, int xr) {
    int pr = find(flower[b].begin(), flower[b].end(), xr) - flower[b].
         begin();
    if (pr % 2 == 1) {
        reverse(flower[b].begin() + 1, flower[b].end());
        return (int)flower[b].size() - pr;
```

GlobalMincut.cpp

Description: Global min cut

DominatorTree OrientedSpanningTree

```
} else return pr;
void set match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;</pre>
    Edge e = g[u][v];
    int xr = flower_from[u][e.u], pr = get_pr(u, xr);
    for (int i = 0; i < pr; ++i) set_match(flower[u][i], flower[u][i ^</pre>
         11);
    set_match(xr, v);
    rotate(flower[u].begin(), flower[u].begin() + pr, flower[u].end());
void augment(int u, int v) {
    int xnv = st[match[u]];
    set match(u, v);
    if (!xnv) return;
    set_match(xnv, st[pa[xnv]]);
    augment(st[pa[xnv]], xnv);
int get_lca(int u, int v) {
    static int t = 0;
    for (++t; u || v; swap(u, v)) {
       if (u == 0) continue;
        if (vis[u] == t) return u;
        vis[u] = t;
        u = st[match[u]];
        if (u) u = st[pa[u]];
    return 0:
void add blossom(int u, int lca, int v) {
    int b = n + 1:
    while (b <= n_x && st[b]) ++b;
    if (b > n_x) ++n_x;
    lab[b] = 0, S[b] = 0, match[b] = match[lca];
    flower[b].clear();
    flower[b].push_back(lca);
    for (int x = u, v; x != lca; x = st[pa[v]])
        flower[b].push_back(x), flower[b].push_back(y = st[match[x]]),
            a nush (v):
    reverse(flower[b].begin() + 1, flower[b].end());
    for (int x = v, y; x != lca; x = st[pa[y]])
        flower[b].push_back(x), flower[b].push_back(y = st[match[x]]),
    set_st(b, b);
    for (int x = 1; x \le n_x; ++x) g[b][x].w = g[x][b].w = 0;
    for (int x = 1; x \le n; ++x) flower from [b][x] = 0;
    for (int i = 0; i < flower[b].size(); ++i) {</pre>
        int xs = flower[b][i];
        for (int x = 1; x <= n_x; ++x) {</pre>
            if (q[b][x].w == 0 \mid | Dist(q[xs][x]) < Dist(q[b][x]))
                q[b][x] = q[xs][x], q[x][b] = q[x][xs];
        for (int x = 1; x <= n; ++x) if (flower_from[xs][x]) flower_from[</pre>
             bl[x] = xs;
    set slack(b);
void expand blossom(int b) {
    for (int i = 0; i < flower[b].size(); ++i) set st(flower[b][i],</pre>
         flower[b][i]);
    int xr = flower_from[b][g[b][pa[b]].u], pr = get_pr(b, xr);
    for (int i = 0; i < pr; i += 2) {
        int xs = flower[b][i], xns = flower[b][i + 1];
        pa[xs] = q[xns][xs].u;
        S[xs] = 1, S[xns] = 0;
        slack[xs] = 0, set slack(xns);
        q push (xns);
    S[xr] = 1, pa[xr] = pa[b];
    for (int i = pr + 1; i < flower[b].size(); ++i) {</pre>
        int xs = flower[b][i];
        S[xs] = -1, set_slack(xs);
   st[b] = 0:
bool on_found_Edge(const Edge &e) {
    int u = st[e.u], v = st[e.v];
   if (S[v] == -1) {
```

```
pa[v] = e.u, S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = 0;
        S[nu] = 0, q_push(nu);
    } else if (S[v] == 0) {
        int lca = get_lca(u, v);
        if (!lca) return augment(u, v), augment(v, u), 1;
        else add blossom(u, lca, v);
    return 0;
bool matching() {
    fill(S, S + n x + 1, -1), fill(slack, slack + n_x + 1, 0);
    for (int x = 1; x \le n_x; ++x) if (st[x] == x && !match[x]) pa[x] =
         0, S[x] = 0, q_{push}(x);
    if (q.empty()) return 0;
    while(1) {
        while (q.size()) {
            int u = q.front();
            q.pop_front();
            if (S[st[u]] == 1) continue;
            for (int v = 1; v <= n; ++v) {
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (Dist(g[u][v]) == 0) {
                        if (on_found_Edge(g[u][v])) return 1;
                        update slack(u, st[v]);
        int d = INF:
        for (int b = n + 1; b <= n_x; ++b) if (st[b] == b && S[b] == 1)
              chkmin(d, lab[b] / 2);
        for (int x = 1; x \le n x; ++x) {
            if (st[x] == x && slack[x]) {
                if (S[x] == -1)
                    d = min(d, Dist(g[slack[x]][x]));
                else if (S[x] == 0)
                    d = min(d, Dist(q[slack[x]][x]) / 2);
        for (int u = 1; u <= n; ++u) {
            if (S[st[u]] == 0) {
                if (lab[u] <= d) return 0;</pre>
                lab[u] -= d;
            } else if (S[st[u]] == 1)
                lab[u] += d;
        for (int b = n + 1; b <= n_x; ++b) {</pre>
            if (st[b] == b) {
                if (S[st[b]] == 0)
                    lab[b] += d * 2;
                else if (S[st[b]] == 1)
                    lab[b] -= d * 2;
        for (int x = 1; x \le n x; ++x) {
            if (st[x] == x && slack[x] && st[slack[x]] != x &&
                Dist(q[slack[x]][x]) == 0)
                if (on_found_Edge(g[slack[x]][x])) return 1;
        for (int b = n + 1; b <= n x; ++b)
            if (st[b] == b && S[b] == 1 && lab[b] == 0) expand_blossom(b)
    return 0;
pair<ll, int> weight_blossom()
    fill (match, match + n + 1, 0);
    n_x = n;
    int n matches = 0;
    ll tot weight = 0;
    for (int u = 0; u <= n; ++u) st[u] = u, flower[u].clear();</pre>
    int w_max = 0;
    for (int u = 1; u <= n; ++u) {</pre>
        for (int v = 1; v <= n; ++v) {</pre>
            flower from [u][v] = (u == v ? u : 0);
```

```
w_max = max(w_max, g[u][v].w);
    for (int u = 1; u <= n; ++u) lab[u] = w_max;</pre>
    while (matching()) ++n_matches;
    for(int u=1; u<=n; ++u)
        if (match[u]&&match[u]<u)
           tot_weight+=g[u][match[u]].w;
    return make_pair(tot_weight,n_matches);
DominatorTree.cpp
Description: Dominator tree
Time: ?
                                                              e82004, 52 lines
struct DominatorTree {
    vector<basic_string<int>> g, rg, bucket;
    basic_string<int> arr, par, rev, sdom, dom, dsu, label;
    int n. t:
    DominatorTree(int n): q(n), rq(n), bucket(n), arr(n, -1), par(n, -1)
        , rev(n, -1),
    sdom(n, -1), dom(n, -1), dsu(n, 0), label(n, 0), n(n), t(0) {}
    void add edge(int u, int v) {
        q[u] += v;
    void dfs(int u) {
        arr[u] = t;
        rev[t] = u:
        label[t] = sdom[t] = dsu[t] = t;
        for (int w : g[u]) {
            if (arr[w] == -1) {
                dfs(w);
                par[arr[w]] = arr[u];
            rg[arr[w]] += arr[u];
    int find(int u, int x=0) {
        if (u == dsu[u]) return x ? -1 : u;
        int v = find(dsu[u], x + 1);
        if (v < 0) return u;</pre>
        if (sdom[label[dsu[u]]] < sdom[label[u]])</pre>
           label[u] = label[dsu[u]];
        dsu[u] = v;
        return x ? v : label[u];
    vector<int> run(int root) {
        dfs(root);
        iota(dom.begin(), dom.end(), 0);
        for (int i = t - 1; i >= 0; --i) {
            for (int w : rq[i]) sdom[i] = min(sdom[i], sdom[find(w)]);
            if (i) bucket[sdom[i]] += i;
            for (int w : bucket[i]) {
               int v = find(w);
                if (sdom[v] == sdom[w]) dom[w] = sdom[w];
                else dom[w] = v;
            if (i > 1) dsu[i] = par[i];
        for (int i = 1; i < t; i++) if (dom[i] != sdom[i]) dom[i] = dom[</pre>
        vector<int> outside_dom(n, -1);
        for (int i = 1; i < t; i++) outside_dom[rev[i]] = rev[dom[i]];</pre>
        //-1 if vertex is not reachable
        return outside_dom;
};
OrientedSpanningTree.cpp
Description: Oriented Spanning Tree
Time: O(nlogn?)
                                                             3d7a73, 96 lines
struct RollbackUF {
    vector <int> p, sz;
    vector <int> changes;
```

RollbackUF(int n) {

```
p.resize(n):
        changes.reserve(n);
        sz.resize(n, 1);
        for (int i = 0; i < n; ++i) p[i] = i;
        return changes.size();
    int find(int v) {
        if (v == p[v]) return v;
        return find(p[v]);
    bool join(int a, int b) {
        a = find(a);
        b = find(b):
        if (a == b) return false;
        if (sz[a] > sz[b]) swap(a, b);
        changes.push back(a);
        sz[b] += sz[a];
        p[a] = b;
        return true;
    void rollback(int t) {
        while (changes.size() > t) {
            int v = changes.back();
            sz[p[v]] -= sz[v];
            p[v] = v;
            changes.pop_back();
struct Edge { int a, b; ll w; };
struct Node {
    Edge kev:
    Node *1, *r;
    ll delta:
    void prop() {
        kev.w += delta;
        if (1) 1->delta += delta;
        if (r) r->delta += delta;
        delta = 0;
    Edge top() { prop(); return key; }
Node *merge(Node *a, Node *b) {
    if (!a || !b) return a ?: b;
    a->prop(), b->prop();
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->1, (a->r = merge(b, a->r)));
    return a:
void pop(Node*\& a) { a->prop(); a = merge(a->1, a->r); }
pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
    RollbackUF uf(n);
    vector<Node*> heap(n);
    for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
    11 \text{ res} = 0;
    vi seen(n, -1), path(n), par(n);
    seen[r] = r;
    vector<Edge> Q(n), in(n, \{-1, -1\}), comp;
    deque<tuple<int, int, vector<Edge>>> cycs;
    for (int s = 0; s < n; ++s) {
        int u = s, \alpha i = 0, w;
        while (seen[u] < 0) {
            if (!heap[u]) return {-1, {}};
            Edge e = heap[u]->top();
            heap[u]->delta -= e.w, pop(heap[u]);
            O[qi] = e, path[qi++] = u, seen[u] = s;
            res += e.w, u = uf.find(e.a);
            if (seen[u] == s) {
                Node* cyc = 0;
                int end = qi, time = uf.time();
                    cyc = merge(cyc, heap[w = path[--qi]]);
                } while (uf.join(u, w));
                u = uf.find(u), heap[u] = cyc, seen[u] = -1;
                cycs.push_front({u, time, {&Q[qi], &Q[end]}});
```

```
for (int i = 0; i < qi; ++i) {
        in[uf.find(Q[i].b)] = Q[i];
}

for (auto& [u, t, comp] : cycs) { // restore so l ( optional )
        uf.rollback(t);
        Edge inEdge = in[u];
        for (auto& e : comp) in[uf.find(e.b)] = e;
        in[uf.find(inEdge.b)] = inEdge;
}

for (int i = 0; i < n; ++i) par[i] = in[i].a;
return {res, par};</pre>
```

Geometry (5)

```
Point.cpp
```

Description: struct Point

80dfd5, 80 lines

```
const ld EPS = 1e-7;
ld sa(ld x) {
    return x * x;
int sign(ld x) {
    if (x < -EPS) {
        return -1:
    if (x > EPS) {
        return 1:
    return 0;
#define vec point
struct point \{//\% - cross, * - dot
    ld x, v:
    auto operator<=>(const point&) const = default;
ld operator*(const point &a, const point &b) {
    return a.x * b.x + a.y * b.y;
ld operator% (const point &a, const point &b) {
    return a.x * b.y - a.y * b.x;
point operator-(const point &a, const point &b) {
    return {a.x - b.x, a.y - b.y};
point operator+(const point &a, const point &b) {
    return {a.x + b.x, a.v + b.v};
point operator*(const point &a, ld b) {
    return {a.x * b, a.y * b};
point operator/(const point &a, ld b) {
    return {a.x / b, a.y / b};
bool operator<(const point &a, const point &b) {
    if (sign(a.v - b.v) != 0) {
        return a.y < b.y;</pre>
    } else if (sign(a.x - b.x) != 0) {
        return a.x < b.x;</pre>
    return 0:
ld len2 (const point &a)
    return sq(a.x) + sq(a.y);
ld len(const point &a) {
    return sgrt(len2(a));
point norm(point a) {
   return a / len(a);
int half (point a) {
    return (sign(a.y) == -1 || (sign(a.y) == 0 && a.x < 0));
point ort (point a) {
    return {-a.v, a.x};
point turn(point a, ld ang) {
    return {a.x * cos(ang) - a.y * sin(ang), a.x * sin(ang) + a.y * cos(
ld getAngle(point &a, point &b)
    return atan2(a % b, a * b);
bool cmpHalf(const point &a, const point &b) {
   if (half(a) != half(b)) {
        return half(b);
    } else {
        int sqn = siqn(a % b);
```

if (!san) {

} else {

return len2(a) < len2(b);

return sgn == 1;

Line Intersections Tangents Hull IsInPolygon

```
Line.cpp
Description: struct Line
                                                             887306, 26 lines
struct line {
   ld a, b, c:
    void norm() {
        // for half planes
        ld d = len({a, b});
       assert(sign(d) > 0);
        a /= d;
       b /= d;
        c /= d;
    ld eval(point p) const { return a * p.x + b * p.y + c; }
    bool isIn(point p) const { return sign(eval(p)) >= 0; }
   bool operator==(const line &other) const {
        return sign(a * other.b - b * other.a) == 0 &&
               sign(a * other.c - c * other.a) == 0 &&
               sign(b * other.c - c * other.b) == 0;
line getln(point a, point b) {
   line res;
    res.a = a.y - b.y;
   res.b = b.x - a.x;
    res.c = -(res.a * a.x + res.b * a.y);
    res.norm();
    return res;
```

Intersections.cpp

```
Description: Geometry intersections
                                                             45d7d9, 75 lines
bool isCrossed(ld lx, ld rx, ld ly, ld ry) {
    if (lx > rx)
        swap(lx, rx);
    if (ly > ry)
    return sign(min(rx, ry) - max(lx, ly)) >= 0;
// if two segments [a, b] and [c, d] has AT LEAST one common point ->
bool intersects (const point &a, const point &b, const point &c, const
     point &d) {
    if (!isCrossed(a.x, b.x, c.x, d.x))
        return false:
    if (!isCrossed(a.y, b.y, c.y, d.y))
        return false:
    if (sign((b-a) % (c-a)) * sign((b-a) % (d-a)) == 1) return 0;
    if (sign((d - c) % (a - c)) * sign((d - c) % (b - c)) == 1) return 0;
    return 1;
//intersecting lines
bool intersect(line 1, line m, point &I) {
    1d d = 1.b * m.a - m.b * 1.a;
    if (sign(d) == 0) {
       return false;
    1d dx = m.b * 1.c - m.c * 1.b;
    1d dy = m.c * 1.a - 1.c * m.a;
    I = \{dx / d, dy / d\};
    return true;
//intersecting circles
int intersect (point o1, ld r1, point o2, ld r2, point &i1, point &i2) {
    if (r1 < r2) {
       swap(o1, o2);
        swap(r1, r2);
    if (sign(r1 - r2) == 0 \&\& len2(o2 - o1) < EPS) {
```

```
return 3;
   1d ln = len(o1 - o2);
    if (sign(ln - r1 - r2) == 1 || sign(r1 - ln - r2) == 1) {
    ld d = (sq(r1) - sq(r2) + sq(ln)) / 2 / ln;
    vec v = norm(o2 - o1);
    point a = o1 + v * d;
    if (sign(ln - r1 - r2) == 0 || sign(ln + r2 - r1) == 0) {
       i1 = a:
        return 1;
   v = ort(v) * sqrt(sq(r1) - sq(d));
    i1 = a + v;
   i2 = a - v;
    return 2:
//intersecting line and circle, line should be normed
int intersect (point o, ld r, line l, point &i1, point &i2) {
    ld len = abs(l.eval(o));
    int sqn = sign(len - r);
    if (sqn == 1) {
        return 0;
    vec v = norm(vec{1.a, 1.b}) * len;
    if (sign(l.eval(o + v)) != 0) {
        v = vec{0, 0} - v;
    point a = o + v:
    if (sqn == 0) {
       i1 = a;
        return 1;
    v = norm(\{-1.b, 1.a\}) * sqrt(sq(r) - sq(len));
   i1 = a + v;
    i2 = a - v;
    return 2:
Tangents.cpp
Description: Tangents to circles.
// tangents from point to circle
int tangents(point &o, ld r, point &p, point &i1, point &i2) {
    ld ln = len(o - p);
    int sqn = sign(ln - r);
    if (sgn == -1) {
        return 0:
    } else if (san == 0) {
        i1 = p:
        return 1;
    } else {
        ld x = sq(r) / ln;
        vec v = norm(p - o) * x;
        point a = o + v;
```

```
c73373, 43 lines
        v = ort(norm(p - o)) * sqrt(sq(r) - sq(x));
        i1 = a + v:
        i2 = a - v;
        return 2;
void _tangents(point c, ld r1, ld r2, vector<line> &ans) {
    1d r = r2 - r1;
    1d z = sq(c.x) + sq(c.v);
    ld d = z - sq(r);
    if (sign(d) == -1)
        return;
    d = sart(abs(d));
    line 1:
   1.a = (c.x * r + c.v * d) / z;
    1.b = (c.y * r - c.x * d) / z;
   1.c = r1;
    ans.push_back(1);
// tangents between two circles
vector<line> tangents(point o1, ld r1, point o2, ld r2) {
    vector<line> ans;
```

```
9
    for (int i = -1; i <= 1; i += 2)
        for (int j = -1; j <= 1; j += 2)
            _{\text{tangents}} (o2 - o1, r1 * i, r2 * j, ans);
    for (int i = 0; i < (int)ans.size(); ++i)</pre>
       ans[i].c \rightarrow ans[i].a \star o1.x + ans[i].b \star o1.y;
Hull.cpp
Description: Polygon functions
                                                              fc1928, 16 lines
vector<point> hull(vector<point> p, bool need_all=false) {
  sort(all(p));
   p.erase(unique(all(p)), end(p));
   int n = p.size(), k = 0;
   if (n <= 2) return p;</pre>
   vector<point> ch(2 * n);
   ld th = need_all ? -EPS : +EPS; // 0 : 1 if int
   for (int i = 0; i < n; ch[k++] = p[i++]) {
     while (k \ge 2 \&\& (ch[k-1] - ch[k-2]) % (p[i] - ch[k-1]) < th)
   for (int i = n - 2, t = k + 1; i >= 0; ch[k++] = p[i--]) {
     while (k \ge t \&\& (ch[k-1] - ch[k-2]) % (p[i] - ch[k-1]) < th)
  ch resize(k - 1):
  return ch:
IsInPolygon.cpp
Description: Is in polygon functions
bool isOnSegment (point &a, point &b, point &x) {
   if (sign(len2(a - b)) == 0) {
        return sign(len(a - x)) == 0;
    return sign((b - a) % (x - a)) == 0 && sign((b - x) * (a - x)) <= 0;
    // optional (slower, but works better if there are some precision
    // problems) return sign((b-a).len() - (x-a).len() - (x-b).len
    // == 0:
int isIn(vector<point> &p, point &a) {
    int n = p.size();
    // depends on limitations(2*MAXC + 228)
    point b = a + point\{2e9 + 228, 1\};
    int cnt = 0:
    for (int i = 0; i < n; ++i) {
       point x = p[i];
        point y = p[i + 1 < n ? i + 1 : 0];
        if (isOnSegment(x, y, a)) {
            // depends on the problem statement
            return 1:
        cnt += intersects(x, y, a, b);
    return 2 * (cnt % 2 == 1);
    /*optional (atan2 is VERY SLOW)!
```

 $ld \ ans = 0;$

 $int \ n = p.size();$

Point x = p/i;

return true;

 $return \ abs(ans) > 1:*/$

x = x - a:

y = y - a;

for (int i = 0; i < n; ++i) {

if (isOnSegment(x, y, a)) {

 $ans \neq = atan2(x \land y, x * y);$

Point y = p[i + 1 < n ? i + 1 : 0];

// depends on the problem statement

bool isInTriangle (point &a, point &b, point &c, point &x) {

return sign((b-a) % (x-a)) >= 0 && sign((c-b) % (x-b)) >= 0

```
NRU HSE
           sign((a - c) % (x - c)) >= 0;
// points should be in the counterclockwise order
bool isInConvex(vector<point> &p, point &a) {
   int n = p.size();
   assert (n >= 3);
   // assert(isConvex(p));
    // assert(isCounterclockwise(p));
   if (sign((p[1] - p[0]) % (a - p[0])) < 0)
       return 0;
    if (sign((p[n-1]-p[0]) % (a-p[0])) > 0)
       return 0:
    int pos = lower bound(p.begin() + 2, p.end(), a,
                          [&] (point a, point b) -> bool {
                             return sign((a - p[0]) % (b - p[0])) > 0;
             p.begin();
    assert (pos > 1 && pos < n);
    return isInTriangle(p[0], p[pos - 1], p[pos], a);
Diameter.cpp
Description: Rotating calipers.
Time: \mathcal{O}(n)
                                                              0f341c, 21 lines
ld diameter(vector<point> p) {
    p = hull(p);
    int n = p.size();
    if (n <= 1) {
       return 0;
   if (n == 2) {
       return len(p[0] - p[1]);
   1d ans = 0;
   int i = 0, j = 1;
    while (i < n) {
       while (sign((p[(i + 1) % n] - p[i]) % (p[(j + 1) % n] - p[j])) >=
           chkmax(ans, len(p[i] - p[j]));
           j = (j + 1) % n;
        chkmax(ans, len(p[i] - p[j]));
       ++i;
    return ans;
TangentsAlex.cpp
Description: Find both tangets to the convex polygon.
(Zakaldovany algos mozhet sgonyat za pivom tak zhe).
Time: \mathcal{O}(\log(n))
pair<int, int> tangents_alex(vector<point> &p, point &a) {
   int n = p.size();
    int 1 = __lg(n);
   auto findWithSign = [&] (int val) {
       int i = 0;
        for (int k = 1; k >= 0; --k) {
           int i1 = (i - (1 << k) + n) % n;
           int i2 = (i + (1 << k)) % n;
           if (sign((p[i1] - a) % (p[i] - a)) == val)
               i = i1:
           if (sign((p[i2] - a) % (p[i] - a)) == val)
               i = i2;
       return i:
```

```
2eeea8, 17 lines
    return {findWithSign(1), findWithSign(-1)};
IsHpiEmpty.cpp
Description: Determines is half plane intersectinos.
Time: \mathcal{O}(n) (expected)
                                                                 3b5e69, 42 lines
// all lines must be normed!!!!!, sign > 0
```

bool isHpiEmpty(vector<line> lines) {

// return hpi(lines).empty();

```
// overflow/precision problems?
    shuffle(all(lines), rnd);
    const ld C = 1e9:
    point ans(C, C);
    vector<point> box = {{-C, -C}, {C, -C}, {C, C}, {-C, C}};
    for (int i = 0; i < 4; ++i)</pre>
       lines.push_back(getln(box[i], box[(i + 1) % 4]));
    int n = lines.size();
    for (int i = n - 4; i >= 0; --i) {
        if (lines[i].isIn(ans))
        point up (0, C + 1), down (0, -C - 1), pi = {lines[i].b, -lines[i].
              a};
        for (int j = i + 1; j < n; ++j) {
            if (lines[i] == lines[j])
                continue:
            point p, pj = {lines[j].b, -lines[j].a};
            if (!intersect(lines[i], lines[j], p)) {
                if (sign(pi * pj) != -1)
                     continue;
                if (sign(lines[i].c + lines[j].c) *
                        (!sign(pi.y) ? sign(pi.x) : -1) ==
                     return true:
            } else -
                if ((!sign(pi.y) ? sign(pi.x) : sign(pi.y)) * (sign(pi %
                      pj)) ==
                     1)
                    chkmin(up, p);
                else
                     chkmax(down, p);
        if ((ans = up) < down)</pre>
            return true;
    // \ for \ (int \ i = 0; \ i < n; ++i) \ {}
         assert(lines[i].eval(ans) < EPS);
    return false:
HalfPlaneIntersection.cpp
Description: Find the intersection of the half planes.
Time: \mathcal{O}(n\log(n))
                                                                fdf28f, 62 lines
vec getPoint(line 1) { return {-1.b, 1.a}; }
bool bad(line a, line b, line c) {
   point x:
    assert(intersect(b, c, x) == 1);
    return a.eval(x) < 0;</pre>
// Do not forget about the bounding box
vector<point> hpi(vector<line> lines) {
    sort(all(lines), [](line al, line bl) -> bool {
        point a = getPoint(al);
        point b = getPoint(bl);
        if (half(a) != half(b)) {
            return half(a) < half(b);
        return a % b > 0;
    });
    vector<pair<line, int>> st;
    for (int it = 0; it < 2; it++) {</pre>
        for (int i = 0; i < (int)lines.size(); i++) {</pre>
            bool flag = false;
            while (!st.emptv()) {
                if (len(getPoint(st.back().first) - getPoint(lines[i])) <</pre>
                     if (lines[i].c >= st.back().first.c) {
                         flag = true;
                        break:
                     } else {
                        st.pop_back();
```

```
} else if (getPoint(st.back().first) % getPoint(lines[i])
                       < EPS / 2) {
                    return {};
                } else if (st.size() >= 2 &&
                           bad(st[st.size() - 2].first, st[st.size() -
                                 1].first,
                               lines[i])) {
                    st.pop_back();
                } else {
                    break;
            if (!flag)
                st.push back({lines[i], i});
    vector<int> en(lines.size(), -1);
    vector<point> ans:
    for (int i = 0; i < (int)st.size(); i++) {</pre>
        if (en[st[i].second] == -1) {
            en[st[i].second] = i;
            continue:
        for (int j = en[st[i].second]; j < i; j++) {</pre>
            point I:
            assert(intersect(st[j].first, st[j + 1].first, I) == 1);
            ans.push back(I);
        break:
    return ans:
CHT.cpp
Description: CHT for minimum, k is decreasing, works for equal slopes 34 lines
struct line {
    int k, b;
    int eval(int x) {
        return k * x + b;
struct part {
    line a:
ld intersection(line a, line b) {
    return (ld) (a.b - b.b) / (b.k - a.k);
struct ConvexHullMin {
    vector <part> st;
    void add(line a) {
        if (!st.emptv() && st.back().a.k == a.k) {
            if (st.back().a.b > a.b) st.pop back();
            else return:
        while (st.size() > 1 && intersection(st[st.size() - 2].a, a) <=</pre>
             st[st.size() - 2].x) st.pop_back();
        if (!st.empty()) st.back().x = intersection(st.back().a, a);
        st.push_back({a, INF});
    int get_val(int x) {
        int 1 = -1, r = (int)st.size() - 1;
        while (r - 1 > 1) {
           int m = (1 + r) / 2;
            if (st[m].x < x) l = m;
            else r = m;
        return st[r].a.eval(x);
};
DvnamicCHT.cpp
Description: Dynamic CHT for maximum
                                                              8a0777, 30 lines
struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const {</pre>
```

return 0 ? p < o.p : k < o.k;

MinPlusCony Kinetic GoldenSearch

if (other.k >= k) {

```
struct LineContainer : multiset<Line> {
    const ll inf = LLONG MAX;
    ll div(ll a, ll b) {
        return a / b - ((a ^ b) < 0 && a % b);
   bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= v->p;
    void add(ll k, ll m) {
        auto z = insert(\{k, m, 0\}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() \&\& isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    ll querv(ll x) {
        assert(!empty());
        Q = 1; auto 1 = *lower_bound({0,0,x}); Q = 0;
        return 1.k * x + 1.m;
MinPlusConv.cpp
Description: Min-Plusconv, A is convex down
Time: O(nlognfast)
                                                             5d63d9, 28 lines
// Assumptions: 'a' is convex, 'opt' has size 'n+m-1'
// 'opt[k]' will be equal to 'arg min(a[k-i] + b[i])'
template<typename T>
void convex_arbitrary_min_plus_conv(T *a, int n, T *b, int m, int *opt)
    auto rec = [&] (auto &&self, int lx, int rx, int ly, int ry) -> void {
        if (lx > rx) return;
        int mx = (lx + rx) >> 1;
        opt[mx] = ly;
        for (int i = ly; i <= ry; ++i)</pre>
           if (mx >= i && (mx - opt[mx] >= n || a[mx - opt[mx]] + b[opt[
                 mx]] > a[mx - i] + b[i]))
                opt[mx] = i;
        self(self, lx, mx - 1, ly, opt[mx]);
        self(self, mx + 1, rx, opt[mx], ry);
    };
    rec(rec, 0, n + m - 2, 0, m - 1):
// Assumptions: 'a' is convex down
template<typename T>
std::vector<T> convex_arbitrary_min_plus_conv(const std::vector<T> &a,
     const std::vector<T> &b) {
    int n = a.size(), m = b.size();
    int *opt = (int*) malloc(sizeof(int) * (n + m - 1));
    convex_arbitrary_min_plus_conv(a.data(), n, b.data(), m, opt);
    std::vector<T> ans(n + m - 1);
    for (int i = 0; i < n + m - 1; ++i) ans[i] = a[i - opt[i]] + b[opt[i</pre>
         11:
    free (opt);
    return ans:
Kinetic.cpp
Description: kinetic segment tree
Time: \mathcal{O}(hz)
                                                            49b24c, 127 lines
//vnutrennii functions - poluintervali, vneshnie - otrezki. ishet min
     priamuy
struct line {
   ll k,b,temp;
    ll eval() const {
        return k * temp + b;
    11 melting point(const line& other) const {
        ll val1 = eval();
        11 val2 = other.eval();
        assert (val1 <= val2);
```

```
return INF:
        11 delta_val = val2 - val1;
        ll delta k = k - other.k;
        assert(delta_val >= 0 && delta_k > 0);
        return (delta val + delta k - 1) / delta k;
};
struct kinetic_segtree {
    etruct node (
        11 lazy_b = 0, lazy_temp = 0, melt = INF;
        line hest:
        node(line best = line()) : best(best) {}
    vector<node> tree;
    void update(int v) {
        if (make_pair(tree[v << 1].best.eval(), tree[v << 1].best.k) <</pre>
             make\_pair(tree[v << 1 | 1].best.eval(), tree[v << 1 | 1].
             best.k)) {
            tree[v].best = tree[v << 1].best;
            tree[v].melt = tree[v].best.melting_point(tree[v << 1 | 1].</pre>
                  best):
        } else {
            tree[v].best = tree[v << 1 | 1].best;
            tree[v].melt = tree[v].best.melting point(tree[v << 1].best);</pre>
        tree[v].melt = min({tree[v].melt, tree[v << 1].melt, tree[v << 1]
             | 1].melt});
        assert(tree[v].melt > 0);
    void apply(int v, int vl, int vr, ll delta b, ll delta temp) {
        tree[v].lazy b += delta b;
        tree[v].lazy_temp += delta_temp;
        tree[v].best.b += delta b;
        tree[v].best.temp += delta_temp;
        tree[v].melt -= delta temp;
        if (tree[v].melt <= 0) {</pre>
            push(v, vl, vr);
            update(v);
    void push(int v, int vl, int vr) {
        int vm = (v1 + vr) / 2;
        apply(v << 1, vl, vm, tree[v].lazy_b, tree[v].lazy_temp);
        apply(v << 1 | 1, vm, vr, tree[v].lazy_b, tree[v].lazy_temp);
        tree[v].lazv b = 0;
        tree[v].lazy\_temp = 0;
    void build(int v, int vl, int vr, const vector<line> &lines) {
        if (vr - vl == 1) {
           tree[v] = node(lines[vl]);
            return;
        int vm = (vl + vr) / 2;
        build(v << 1, vl, vm, lines);
        build(v \ll 1 | 1, vm, vr, lines);
        update(v);
    void add(int v, int vl, int vr, int l, int r, ll delta b, ll
         delta temp) {
        if (r <= vl || vr <= l) {
            return:
        if (1 <= v1 && vr <= r) {
            apply(v, vl, vr, delta_b, delta_temp);
            return:
        nush (v. vl. vr):
        int vm = (vl + vr) / 2;
        add(v << 1, v1, vm, 1, r, delta_b, delta_temp);
        add(v << 1 | 1, vm, vr, 1, r, delta_b, delta_temp);
        update(v):
```

```
void change line (int v, int vl, int vr, int pos, const line &new line
    if (vr - vl == 1) {
        tree[v].best = new_line;
        return:
    push(v, vl, vr);
    int vm = (vl + vr) / 2;
    if (nos < vm) {
        change_line(v << 1, vl, vm, pos, new_line);
        change_line(v << 1 | 1, vm, vr, pos, new_line);</pre>
    update(v);
11 query(int v, int v1, int vr, int 1, int r) {
    if (r <= vl || vr <= l) {</pre>
        return INF:
    if (1 <= v1 && vr <= r) {
        return tree[v].best.eval();
    push(v, vl, vr);
    int vm = (vl + vr) / 2;
    return min(query(v << 1, v1, vm, 1, r), query(v << 1 | 1, vm, vr,
          l. r));
kinetic segtree(const vector<line> &lines) : n(lines.size()), tree(4
    build(1, 0, n, lines);
kinetic segtree(int n) : n(n), tree(4 * n) {
    vector <line> lines(n, {0, INF, 0});
    build(1, 0, n, lines);
void add(int 1, int r, 11 delta_b, 11 delta_temp) {
    assert(delta_temp >= 0);
    add(1, 0, n, 1, r + 1, delta_b, delta_temp);
void change line (int pos. const line &new line) {
    assert(0 <= pos && pos < n);
    change_line(1, 0, n, pos, new_line);
ll query(int 1, int r) {
    return query(1, 0, n, 1, r + 1);
```

GoldenSearch.cpp Description: Golden Search

31d45b, 14 lines

```
double gss(double a, double b, double (*f)(double)) {
   double r = (sqrt(5) - 1) / 2, eps = 1e-7;
   double x1 = b - r * (b - a), x2 = a + r * (b - a);
   double f1 = f(x1), f2 = f(x2);
   while (b - a > eps)
    if (f1 < f2) { // change to > to find maximum
        b = x2; x2 = x1; f2 = f1;
        x1 = b - r * (b - a); f1 = f(x1);
   } else {
        a = x1; x1 = x2; f1 = f2;
        x2 = a + r * (b - a); f2 = f(x2);
   }
   return a;
```

Math (6)

```
BerlekampMassey.cpp
```

Description: Find the shortest linear-feedback shift register

Time: $\mathcal{O}\left(n^2\right)$

08eddc, 86 lines

```
vector<int> berlekamp(vector<int> x) {
   vector<int> ls, cur;
   int lf = 0, d = 0;
   for (int i = 0; i < x.size(); ++i) {</pre>
        11 t = 0;
        for (int j = 0; j < cur.size(); ++j) {</pre>
           t = (t + (11) x[i - j - 1] * cur[j]) % MOD;
        if ((t - x[i]) % MOD == 0)
           continue;
        if (cur.empty()) {
           cur.resize(i + 1);
           lf = i:
            d = (t - x[i]) % MOD;
            continue;
        11 k = -(x[i] - t) * powmod(d, MOD - 2) % MOD;
        vector<int> c(i - lf - 1);
        c.push back(k);
        for (auto &j : ls)
            c.push_back(-j * k % MOD);
        if (c.size() < cur.size())</pre>
            c.resize(cur.size());
        for (int j = 0; j < cur.size(); ++j) {</pre>
            c[j] = (c[j] + cur[j]) % MOD;
        if (i - lf + (int)ls.size() >= (int)cur.size()) {
           tie(ls, lf, d) = make_tuple(cur, i, (t - x[i]) % MOD);
        cur = c;
    for (auto &i : cur)
        i = (i % MOD + MOD) % MOD;
   return cur:
// for a_i = 2 * a_i - 1 + a_i - 2 returns \{2, 1\}
// kth element of p/q as fps
int getkfps(vector<int> p, vector<int> q, ll k) {
   assert (q[0] != 0);
    while (k) {
        auto f = q;
        for (int i = 1; i < (int) f.size(); i += 2) {</pre>
           f[i] = sub(0, f[i]);
        auto p2 = conv(p, f);
        auto q2 = conv(q, f);
        p.clear(), q.clear();
        for (int i = k % 2; i < (int) p2.size(); i += 2) {</pre>
           p.pbc(p2[i]);
        for (int i = 0; i < (int)q2.size(); i += 2) {</pre>
           q.pbc(q2[i]);
        k >>= 1;
    return mul(p[0], inv(q[0]));
// vals - initials values of recurrence, c - result of belekamp on vals
int getk(const vector<int> &vals, vector<int> c, ll k) {
    int d = c.size();
    c.insert(c.begin(), MOD-1);
    while (c.back() == 0) {
        c.pop_back();
   for (auto &el : c) {
        el = sub(0, el);
   vector<int> p(d);
   copy(vals.begin(), vals.begin() + d, p.begin());
   p = conv(p, c);
```

```
p.resize(d);
    return getkfps(p, c, k);
vector<int> getmod(vector<int> a, vector<int> md) {
    for (int i = a.size() - 1; i + 1 >= md.size(); --i) {
        int v = mul(a[i], inv(md.back()));
        for (int j = 0; j < md.size(); ++j) {</pre>
            a[i - md.size() + 1 + j] = sub(a[i - md.size() + 1 + j], mul(
        a.pop_back();
    return a;
GoncharFedor.cpp
Description: Calculating number of points x, y > 0, Ax + By < C
Time: \mathcal{O}(\log(C))
                                                               0ef10e, 11 lines
ll solve_triangle(ll A, ll B, ll C) { // x,y>=0, Ax+By \le C
    if (C < 0)
        return 0:
    if (A > B)
        swap(A, B);
    11 p = C / B;
    11 k = B / A;
   11 d = (C - p * B) / A;
    return solve_triangle(B - k * A, A, C - A * (k * p + d + 1)) +
           (p + 1) * (d + 1) + k * p * (p + 1) / 2;
CRT.cpp
Description: CRT for arbitrary modulos
                                                              28309e, 25 lines
int extgcd(int a, int b, int &x, int &y) { // define int ll
    if (a == 0) {
        x = 0, y = 1;
        return b;
    int x1, y1;
    int g = extgcd(b % a, a, x1, y1);
    x = y1 - x1 * (b / a);
    y = x1;
    return q;
int lcm(int a, int b) { return a / __gcd(a, b) * b; }
int crt(int mod1, int mod2, int rem1, int rem2) {
    int r = (rem2 - (rem1 \% mod2) + mod2) \% mod2;
    int x, y;
    int g = extgcd(mod1, mod2, x, y);
    if (r % g) return -1;
    x %= mod2;
    if (x < 0) x += mod2;
    int ans = (x * (r / q)) % mod2;
    ans = ans * mod1 + rem1:
    assert (ans % mod1 == rem1);
    assert (ans % mod2 == rem2):
    return ans % lcm(mod1, mod2);
Fastmod.cpp
Description: Fast multiplication by modulo(in [0;2b))
                                                                38ea39, 7 lines
struct FastMod {
   ull b, m;
    FastMod(ull b) : b(b), m(-1ULL / b) {}
    ull reduce(ull a) { // a \% b + (0 or b)
        return a - (ull) ((__uint128_t(m) * a) >> 64) * b;
};
ModularSqrt.cpp
Description: Calculating sqrt modulo smth
Time: \mathcal{O}\left(\log^2\right)
                                                              19a793, 23 lines
ll sqrt(ll a, ll p) {
 a %= p;
  if (a < 0) a += p;
```

```
assert (modpow(a, (p-1) / 2, p) == 1); // e lse no so lution
  if (p % 4 == 3) return modpow(a, (p + 1) / 4, p);
  // a^{(n+3)/8} \text{ or } 2^{(n+3)/8} * 2^{(n+3)/4} \text{ works } i \text{ f } p \% 8 == 5
  11 s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0) ++r, s /= 2;
  while (modpow(n, (p-1) / 2, p) != p-1) ++n;
  11 x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    11 t = b;
    for (m = 0; m < r && t != 1; ++m) t = t * t % p;
    if (m == 0) return x;
    11 \text{ gs} = \text{modpow}(g, 1LL << (r - m - 1), p);
    q = qs * qs % p;
    x = x * gs % p;
    b = b * q % p;
PrimalityTest.cpp
Description: Checking primality of p
Time: \mathcal{O}(\log(C))
const int iters = 8; // can change
bool isprime(ll p) {
    if (p == 1 | | p == 4)
```

ad2714, 32 lines

```
return 0;
if (p == 2 || p == 3)
    return 1;
for (int it = 0; it < iters; ++it) {</pre>
    11 a = rnd() % (p - 2) + 2;
    11 \text{ nw} = p - 1;
    while (nw % 2 == 0)
        nw /= 2;
    ll x = binpow(a, nw, p); // int128
    if (x == 1)
        continue;
    ll last = x;
    nw *= 2:
    while (nw <= p - 1) {
        x = (\underline{\ }int128\_t)x * x % p;
        if (x == 1) {
             if (last != p - 1) {
                 return 0:
             break:
        last = x:
        nw \star = 2;
    if (x != 1)
        return 0:
return 1:
```

XorConvolution.cpp

Description: Calculating xor-convolution of 2 vectors modulo smth Time: $\mathcal{O}(n \log(n))$

```
454afd, 23 lines
void fwht(vector<int> &a) {
    int n = a.size();
    for (int 1 = 1; 1 < n; 1 <<= 1) {
        for (int i = 0; i < n; i += 2 * 1) {
            for (int j = 0; j < 1; ++j) {
                int u = a[i + j], v = a[i + j + 1];
                a[i + j] = add(u, v), a[i + j + 1] = sub(u, v);
} // https://judge.yosupo.jp/problem/bitwise_xor_convolution
vector<int> xorconvo(vector<int> a, vector<int> b) {
    int n = 1;
    while (n < max(a.size(), b.size()))</pre>
        n \neq 2;
    a.resize(n), b.resize(n);
    fwht(a), fwht(b);
```

Factorization PrimeCount NTT

```
int in = inv(n);
    for (int i = 0; i < n; ++i)</pre>
        a[i] = mul(a[i], mul(b[i], in));
    return a:
Factorization.cpp
Description: Factorizing a number real quick
Time: \mathcal{O}\left(n^{\frac{1}{4}}\right)
                                                                 f0d7c6, 51 lines
ll gcd(ll a, ll b) {
    while (b)
        a %= b, swap(a, b);
ll f(ll a, ll n) { return ((__int128_t)a * a % n + 1) % n; }
vector<ll> factorize(ll n) {
    if (n <= 1e6) { // can add primality check for speed?
        vector<ll> res:
        for (ll i = 2; i * i <= n; ++i) {
            while (n % i == 0) {
                res.pbc(i);
                n /= i:
        if (n != 1)
            res.pbc(n);
        return res;
    11 x = rnd() % (n - 1) + 1;
    11 tries = 10 * sqrt(sqrt(n));
    const int C = 60;
    for (ll i = 0; i < tries; i += C) {
        11 xs = x;
        11 ys = y;
        11 m = 1;
        for (int k = 0; k < C; ++k) {
            x = f(x, n);
            y = f(f(y, n), n);
            m = (\underline{1}nt128_t)m * abs(x - y) % n;
        if (\gcd(n, m) == 1)
            continue;
        x = xs, y = ys;
        for (int k = 0; k < C; ++k) {
            x = f(x, n);
            y = f(f(y, n), n);
            ll res = gcd(n, abs(x - y));
            if (res != 1 && res != n) {
                 vector<ll> v1 = factorize(res), v2 = factorize(n / res);
                 for (auto i : v2)
                     v1.pbc(j);
                 return v1;
    return {n};
PrimeCount.cpp
Description: counting number of primes below N
Time: \mathcal{O}\left(N^2/3\right)
                                                                 a8507c, 53 lines
ll prime_pi(const ll N) {
  if (N <= 1) return 0;
  if (N == 2) return 1;
  const int v = sqrt(N);
  int s = (v + 1) / 2;
  vector<int> smalls(s);
  for (int i = 1; i < s; i++) smalls[i] = i;</pre>
```

```
vector<int> roughs(s);
for (int i = 0; i < s; i++) roughs[i] = 2 * i + 1;</pre>
vector<ll> larges(s);
for (int i = 0; i < s; i++) larges[i] = (N / (2 * i + 1) - 1) / 2;
```

```
vector<bool> skip(v + 1);
  const auto divide = [](ll n, ll d) -> int { return n / d; };
  const auto half = [](int n) -> int { return (n - 1) >> 1; };
  for (int p = 3; p <= v; p += 2)
    if (!skip[p]) {
      int q = p * p;
      if ((11)q * q > N) break;
      skip[p] = true:
      for (int i = q; i <= v; i += 2 * p) skip[i] = true;
      int ne = 0.
      for (int k = 0; k < s; k++) {
        int i = roughs[k];
        if (skip[i]) continue;
        11 d = (11)i * p;
        larges[ns] = larges[k] -
                      (d \le v ? larges[smalls[d >> 1] - pc]
                              : smalls[half(divide(N, d))]) +
        roughs[ns++] = i;
      for (int i = half(v), j = ((v / p) - 1) | 1; j >= p; j -= 2) {
        int c = smalls[j >> 1] - pc;
        for (int e = (j * p) >> 1; i >= e; i--) smalls[i] -= c;
      pc++;
  larges[0] += (11)(s + 2 * (pc - 1)) * (s - 1) / 2;
  for (int k = 1; k < s; k++) larges[0] -= larges[k];</pre>
  for (int 1 = 1; 1 < s; 1++) {</pre>
   ll q = roughs[1];
    11 \dot{M} = N / q;
    int e = smalls[half(M / q)] - pc;
    if (e < 1 + 1) break;
    11 t = 0;
    for (int k = 1 + 1; k \le e; k++)
     t += smalls[half(divide(M, roughs[k]))];
    larges[0] += t - (l1) (e - 1) * (pc + 1 - 1);
  return larges[0] + 1;
NTT.cpp
Description: Calculating FFT modulo MOD
Time: \mathcal{O}(n\log(n))
                                                              3e2f3a, 226 lines
// DONT FORGET TO CALL initNTT() AND CHECK MAXLOG
const int MOD = 998244353;
const int G = 3;
const int MAXLOG = 23;
int W[1 << MAXLOG];</pre>
bool nttinit = false:
vector<int> pws:
int add(int a, int b) {
    a += b:
    if (a >= MOD) {
        return a - MOD;
    return a;
int sub(int a, int b) {
    a -= b;
    if (a < 0) {
        return a + MOD;
    return a:
int mul(int a, int b) {
    return (11) a * b % MOD;
int power(int a, int n) {
    int ans = 1:
    while (n) {
```

```
if (n & 1) {
            ans = mul(ans, a);
        a = mul(a, a);
       n >>= 1;
    return ans:
int inv(int a) {
    return power(a, MOD - 2);
void initNTT() {
   assert((MOD - 1) % (1 << MAXLOG) == 0);
    pws.push_back(power(G, (MOD - 1) / (1 << MAXLOG)));</pre>
    for (int i = 0; i < MAXLOG - 1; ++i) {</pre>
        pws.push back(mul(pws.back(), pws.back()));
    assert(pws.back() == MOD - 1);
    W[0] = 1;
    for (int i = 1; i < (1 << MAXLOG); ++i) {</pre>
        W[i] = mul(W[i-1], pws[0]);
void ntt(int n, vector <int>& a, bool rev) {
    if (!nttinit) {
        initNTT();
        nttinit = 1:
    int lg = log2(n);
    vector<int> rv(n);
    for (int i = 1; i < n; ++i) {</pre>
        rv[i] = (rv[i >> 1] >> 1) ^ ((i & 1) << (lq - 1));
        if (rv[i] > i) swap(a[i], a[rv[i]]);
    int num = MAXLOG - 1;
    for (int len = 1; len < n; len *= 2) {
        for (int i = 0; i < n; i += 2 * len) {
            for (int j = 0; j < len; ++j) {
                int u = a[i + j], v = mul(W[j << num], a[i + j + len]);</pre>
                a[i + j] = add(u, v);
                a[i + j + len] = sub(u, v);
        --num;
        int rev_n = power(n, MOD - 2);
        for (int i = 0; i < n; ++i) a[i] = mul(a[i], rev n);</pre>
        reverse(a.begin() + 1, a.end());
vector<int> conv(vector<int> a, vector<int> b) {
    int lq = 0;
    while ((1 << lg) < a.size() + b.size() + 1)</pre>
        ++la;
    int n = 1 << lq;
    assert(a.size() + b.size() <= n + 1);
    a.resize(n):
   b.resize(n);
   ntt(n, a, false);
   ntt(n, b, false);
    for (int i = 0; i < n; ++i) {
        a[i] = mul(a[i], b[i]);
   ntt(n, a, true);
    while (a.size() > 1 && a.back() == 0) {
        a.pop_back();
    return a:
vector<int> add(vector<int> a, vector<int> b) {
    a.resize(max(a.size(), b.size()));
    for (int i = 0; i < (int) b.size(); ++i) {</pre>
        a[i] = add(a[i], b[i]);
```

FFT AndConvolution SubsetConvolution

```
return a:
vector<int> sub(vector<int> a, vector<int> b) {
    a.resize(max(a.size(), b.size()));
    for (int i = 0; i < (int) b.size(); ++i) {</pre>
       a[i] = sub(a[i], b[i]);
    return a:
vector<int> inv(const vector<int> &a, int need) {
    vector<int> b = {inv(a[0])};
    while ((int) b.size() < need) {</pre>
        vector<int> a1 = a;
        int m = b.size():
        al.resize(min((int) al.size(), 2 * m));
       b = conv(b, sub(\{2\}, conv(al, b)));
       b.resize(2 * m);
    b.resize(need):
    return b:
vector<int> div(vector<int> a, vector<int> b) {
    if (count(all(a), 0) == a.size()) {
        return {0};
    assert(a.back() != 0 && b.back() != 0);
    int n = a.size() - 1;
    int m = b.size() - 1;
    if (n < m) {
       return {0};
    reverse(all(a));
    reverse(all(b));
    a.resize(n - m + 1);
    b.resize(n - m + 1);
    vector<int> c = inv(b, b.size());
    vector<int> q = conv(a, c);
    q.resize(n - m + 1);
    reverse(all(q));
    return a:
vector<int> mod(vector<int> a, vector<int> b) {
    auto res = sub(a, conv(b, div(a, b)));
    while (res.size() > 1 && res.back() == 0) {
        res.pop_back();
    return res;
vector<int> multipoint(vector<int> a, vector<int> x) {
    int n = x size():
    vector<vector<int>> tree(2 * n);
    for (int i = 0; i < n; ++i) {</pre>
        tree[i + n] = \{x[i], MOD - 1\};
    for (int i = n - 1; i; --i) {
        tree[i] = conv(tree[2 * i], tree[2 * i + 1]);
    tree[1] = mod(a, tree[1]);
    for (int i = 2; i < 2 * n; ++i) {</pre>
        tree[i] = mod(tree[i >> 1], tree[i]);
    vector<int> res(n):
    for (int i = 0; i < n; ++i) {
        res[i] = tree[i + n][0];
    return res:
vector<int> deriv(vector<int> a) {
    for (int i = 1; i < (int) a.size(); ++i) {</pre>
       a[i - 1] = mul(i, a[i]);
   a.back() = 0;
```

```
if (a.size() > 1) {
        a.pop_back();
    return a;
vector<int> integ(vector<int> a) {
    a.push_back(0);
    for (int i = (int) a.size() - 1; i; --i) {
        a[i] = mul(a[i - 1], inv(i));
    a[0] = 0:
    return a:
vector<int> log(vector<int> a, int n) {
    assert(a[0] == 1);
    auto res = integ(conv(deriv(a), inv(a, n)));
    res resize(n):
    return res:
vector<int> exp(vector<int> a, int need) {
   assert(a[0] == 0);
    vector<int> b = {1};
    while ((int) b.size() < need) {</pre>
        vector<int> a1 = a;
        int m = b.size();
        al.resize(min((int) al.size(), 2 * m));
        a1[0] = add(a1[0], 1);
        b = conv(b, sub(a1, log(b, 2 * m)));
        b.resize(2 * m):
    b.resize(need);
    return b;
FFT.cpp
Description: Calculating product of two polynomials
Time: \mathcal{O}\left(n\log(n)\right)
                                                               3adba5, 67 lines
const ld PI = acos(-1);
using cd = complex<ld>;
const int MAXLOG = 19, N = (1 << MAXLOG), MAXN = (1 << MAXLOG) + 228;
int rev[MAXN]:
cd w[MAXN];
bool fftInit = false:
void initFFT() {
    for (int i = 0; i < N; ++i) {</pre>
        w[i] = cd(cos(2 * PI * i / N), sin(2 * PI * i / N));
    for (int i = 1; i < N; ++i) {</pre>
        rev[i] = (rev[i >> 1] >> 1) ^ ((i & 1) << (MAXLOG - 1));
void FFT(int n, vector <cd>& a, bool rv = false) {
    if (!fftInit) {
        initFFT();
        fftInit = 1;
    int LOG = ceil(log2(n));
    for (int i = 0; i < n; ++i) {</pre>
        if (i < (rev[i] >> (MAXLOG - LOG))) {
            swap(a[i], a[(rev[i] >> (MAXLOG - LOG))]);
    for (int lvl = 0; lvl < LOG; ++lvl) {
        int len = 1 << lvl;</pre>
        for (int st = 0; st < n; st += len \star 2) {
            for (int i = 0; i < len; ++i) {</pre>
                cd x = a[st + i], y = a[st + len + i] * w[i << (MAXLOG -
                      1 - lvl)];
                 a[st + i] = x + y;
                 a[st + i + len] = x - y;
```

```
if (rv) {
        reverse(a.begin() + 1, a.end());
        for (auto& i : a) i /= n;
vector <11> mul(vector <11> a, vector <11> b) {
    int xd = max(a.size(), b.size()) * 2;
    int cur = 1;
    while (cur < xd) {
        cur *= 2:
    a.resize(cur);
    b.resize(cur):
    vector <cd> ma(cur), mb(cur);
    for (int i = 0; i < cur; ++i) {
        ma[i] += a[i];
        mb[i] += b[i];
    FFT(cur, ma);
    FFT (cur, mb):
    for (int i = 0; i < cur; ++i) ma[i] *= mb[i];</pre>
    FFT(cur, ma, true);
    vector <11> ans(cur);
    for (int i = 0; i < cur; ++i) {
        ans[i] = (ll) (ma[i].real() + 0.5);
    return ans;
AndConvolution.cpp
Description: Calculating and-convolution modulo smth
Time: O(n \log(n))
                                                               5dedf4 24 lines
void conv(vector<int> &a, bool x) {
    int n = a.size();
    for (int j = 0; (1 << j) < n; ++j) {
        for (int i = 0; i < n; ++i) {</pre>
            if (!(i & (1 << j))) {
                if (x)
                     a[i] = add(a[i], a[i | (1 << i)));
                    a[i] = sub(a[i], a[i | (1 << j)]);
vector<int> andcon(vector<int> a, vector<int> b) {
    int n = 1:
    while (n < max(a.size(), b.size()))</pre>
       n *= 2:
    a.resize(n), b.resize(n);
    conv(a, 1), conv(b, 1);
    for (int i = 0; i < n; ++i)</pre>
       a[i] = mul(a[i], b[i]);
    conv(a, 0);
    return a;
SubsetConvolution.cpp
Description: subset convolution
Time: \mathcal{O}\left(2^n * n^2\right) (500 ms n = 20 with pragms)
void transform(int n, int N, vector <int>& b, const vector <int>& a,
     const vector <int>& pc, bool rev) {
    if (!rev) {
        b.assign(N << n, 0);
        for(int i = 0; i < (int)a.size(); ++i) b[pc[i] + i*N] = a[i];</pre>
    for (int w = 1; w \le (1 \le n); ++w) {
        for (int d = 0; ! (w& (1<<d)); ++d) {
            int W = N * (w - (1 << d)), dd = N << d;
            for(int i = N * (w - (2 << d)); i < W; ++i) {
                if (!rev) b[i + dd] = add(b[i + dd], b[i]);
                else b[i + dd] = sub(b[i + dd], b[i]);
```

```
c2ab5a, 19 lines
```

```
while((1 << n) < max(a.size(),b.size())) n++;</pre>
    int N = n+1;
    vector<int> pc(1<<n,0);</pre>
    for(int i = 1; i < (1<<n); ++i) pc[i] = pc[i - (i&-i)] + 1;</pre>
    vector<int> bufA, bufB;
    transform(n, N, bufA, a, pc, false);
    transform(n, N, bufB, b, pc, false);
    for(int i = 0; i < (1<<n); i++) {</pre>
        int I = i * N;
        vector<int> O(N);
        for(int ja = 0; ja <= pc[i]; ++ja) {</pre>
            for (int jb = pc[i] - ja, x = min(n - ja, pc[i]); jb \le x; ++
                O[ja + jb] = add(O[ja + jb], mul(bufA[ja + I], bufB[jb +
                      I]));
        copy(Q.begin(), Q.end(), bufA.begin() + I);
    transform(n, N, bufA, a, pc, true);
    vector<int> res(1<<n);
    for(int i = 0; i<(1<<n); ++i) res[i] = bufA[pc[i] + i*N];</pre>
Simplex.cpp
Description: Simplex
Time: exponential XD(ok for 200-300 variables/bounds)
                                                               4dda3c, 99 lines
/* solver for linear programs of the form
maximize c^T x, subject to A x \le b, x > 0
outputs target function for optimal solution and
the solution by reference
if unbounded above : returns inf, if infeasible : returns -inf
create\ Simplex\_Steep < ld > LP(A, b, c),\ then\ call\ LP.\ Solve\ (x)
template <typename DOUBLE>
struct Simplex Steep {
   using VD = vector<DOUBLE>;
    using VVD = vector<VD>;
    using VI = vector<int>;
    DOUBLE EPS = 1e-12;
   int m, n;
    VI B, N;
    Simplex_Steep(const VVD &A, const VD &b, const VD &c)
        : m(b.size()), n(c.size()), B(m), N(n + 1), D(m + 2, VD(n + 2)) {
        for (int i = 0; i < m; i++)</pre>
            for (int j = 0; j < n; j++) D[i][j] = A[i][j];</pre>
        for (int i = 0; i < m; i++) {</pre>
            B[i] = n + i;
            D[i][n] = -1;
            D[i][n + 1] = b[i];
        for (int j = 0; j < n; j++) {
            N[j] = j;
            D[m][j] = -c[j];
        N[n] = -1;
        D[m + 1][n] = 1;
    void Pivot(int r, int s) {
        for (int i = 0; i < m + 2; i++)</pre>
            if (i != r)
                for (int j = 0; j < n + 2; j++)
                    if (j != s) D[i][j] -= D[r][j] * D[i][s] / D[r][s];
        for (int j = 0; j < n + 2; j++)
            if (j != s) D[r][j] /= D[r][s];
        for (int i = 0; i < m + 2; i++)</pre>
            if (i != r) D[i][s] /= -D[r][s];
        D[r][s] = 1.0 / D[r][s];
        swap(B[r], N[s]);
    bool Simplex(int phase) {
        int x = m + (int) (phase == 1);
```

vector<int> SubsetConvolution(const vector<int>& a. const vector<int>& b)

```
while (true) {
            int s = -1:
            DOUBLE c_val = -1;
            for (int j = 0; j <= n; j++) {</pre>
                if (phase == 2 && N[j] == -1) continue;
                DOUBLE norm_sq = 0;
                for (int k = 0; k <= m; k++) norm_sq += D[k][j] * D[k][j</pre>
                norm_sq = max(norm_sq, EPS);
                DOUBLE c_val_j = D[x][j] / sqrtl(norm_sq);
                if (s == -1 || c_val_j < c_val ||</pre>
                    (c_val == c_val_j && N[j] < N[s])) {
                    s = j;
                    c_val = c_val_j;
            if (D[x][s] >= -EPS) return true;
            int r = -1;
            for (int i = 0; i < m; i++) {</pre>
                if (D[i][s] <= EPS) continue;</pre>
                if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r]
                      1[s] ||
                     (D[i][n + 1] / D[i][s] == D[r][n + 1] / D[r][s] &&
                     B[i] < B[r])
                    r = i;
            if (r == -1) return false;
            Pivot(r, s);
   DOUBLE Solve(VD &x) {
        int r = 0:
        for (int i = 1; i < m; i++)</pre>
            if (D[i][n + 1] < D[r][n + 1]) r = i;
        if (D[r][n + 1] \le -EPS) {
            Pivot(r, n);
            if (!Simplex(1) | | D[m + 1][n + 1] < -EPS)
                return -numeric_limits<DOUBLE>::infinity();
            for (int i = 0; i < m; i++)</pre>
                if (B[i] == -1) {
                    int s = -1;
                    for (int j = 0; j <= n; j++)
                        if (s == -1 || D[i][j] < D[i][s] ||</pre>
                             (D[i][j] == D[i][s] && N[j] < N[s]))
                             s = j;
                    Pivot(i, s);
        if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
        for (int i = 0; i < m; i++)</pre>
           if (B[i] < n) x[B[i]] = D[i][n + 1];</pre>
        return D[m][n + 1];
};
DeterminantLd.cpp
Description: Determinant in ld
                                                               1a6123, 18 lines
double det(vector<vector<double>>& a) {
 int n = sz(a);
 double res = 1;
 for (int i = 0; i < n; ++i) {</pre>
    for (int j = i + 1; j < n; ++j)
     if (fabs(a[i][i]) > fabs(a[b][i])) b = i;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    for (int j = i + 1; j < n; ++j) {
     double v = a[j][i] / a[i][i];
        for (int k = i + 1; k < n; ++k) a[j][k] -= v * a[i][k];</pre>
 return res:
```

```
const 11 mod = 12345;
ll det(vector<vector<ll>>& a) {
  int n = sz(a);
  ll ans = 1;
  for (int i = 0; i < n; ++i) {</pre>
    for (int j = i + 1; j < n; ++j) {
      while (a[j][i] != 0) { // gcd step
        11 t = a[i][i] / a[j][i];
        if (+)
          for (int k = i; k < n; ++k)
           a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  return (ans + mod) % mod;
```

DeterminantInt.cpp

Description: Determinant in ints

6.1 Fun things

$$ClassesCount = \frac{1}{|G|} \sum_{\pi \in G} I(\pi)$$
$$ClassesCount = \frac{1}{|G|} \sum_{\pi \in G} k^{C(\pi)}$$

Stirling 2kind - count of partitions of n objects into k nonempty

sets.
$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,k) = \sum_{j=0}^{n-1} {n-1 \choose j} S(j,k-1)$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k+j} {k \choose j} j^n$$

$$n! \approx \sqrt{2n\pi} (\frac{n}{e})^n$$

$${n \choose k} \equiv \prod_i {n_i \choose k_i}, n_i, k_i - \text{digits of } n, k \text{ in p-adic system}$$

$$\int_{a}^{b} f(x)dx \approx \frac{b-a}{6} (f(a) + 4f(\frac{a+b}{2}) + f(b))$$

$$x_{n+1} = x_n - \frac{\ddot{f}(x_n)}{f'(x_n)}, O(loglog)$$

$$G(n) = n \oplus (n >> 1)$$

$$g(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} g(d)\mu(\frac{n}{d})$$

$$\sum_{d|n} \mu(d) = [n=1], \mu(1) = 1, \mu(p) = -1, \mu(p^k) = 0$$

$$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$$
$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$tg(a \pm b) = \frac{tg \ a \pm tg \ b}{1 \mp tg \ a tg \ b}$$
$$ctg(a \pm b) = \frac{ctg \ a \cot b + 1}{\cot b \pm \cot a}$$

$$\operatorname{ctg}(a \pm b) = \frac{\operatorname{ctg} a \operatorname{ctg} b + 1}{\operatorname{ctg} b \pm \operatorname{ctg} a}$$

$$\sin\frac{a}{2} = \pm\sqrt{\frac{1-\cos a}{2}}$$

$$\cos\frac{a}{2} = \pm\sqrt{\frac{1+\cos a}{2}}$$

$$\begin{array}{ccc}
 & 2 & \sqrt{2} \\
 & \sqrt{2} \\
 & \sqrt{2} \\
 & \sqrt{2} \\
 & \sqrt{2}
\end{array}$$

$$\begin{array}{l}
2 & 1 - \cos a & \sin a \\
\sin a \sin b &= \frac{\cos(a-b) - \cos(a+b)}{\cos(a-b)}
\end{array}$$

$$\sin a \sin b = \frac{2}{\sin(a-b) + \sin(a+b)}$$

$$\sin a \cos b = \frac{\sin(a-b) + \sin(a+b)}{2}$$
$$\cos(a-b) + \cos(a+b)$$

$$tg \frac{a}{2} = \frac{\sin a}{1 - \cos a} = \frac{1 - \cos a}{\sin a}$$

$$\sin a \sin b = \frac{\cos(a - b) - \cos(a + b)}{2}$$

$$\sin a \cos b = \frac{\sin(a - b) + \sin(a + b)}{2}$$

$$\cos a \cos b = \frac{\cos(a - b) + \cos(a + b)}{2}$$

1 jan 2000 - saturday, 1 jan 1900 - monday, 14 apr 1961 - friday Bell numbers: 0:1, 1:1, 2:2, 3:5, 4:15, 5:52, 6:203, 7:877, 8:4140, 9:21147, 10:115975, 11:678570, 12:4213597, 13:27644437, 14:190899322, 15:1382958545, 16:10480142147, 17:82864869804, 18:682076806159, 19:5832742205057, 20:51724158235372, 21:474869816156751,

22:4506715738447323, 23:44152005855084346

Fibonacci: 45:1134903170. 46:1836311903(max int), 91:

4660046610375530309

Highly composite numbers:

 $< 1000 : d(840) = 32, < 10^4 : d(9240) = 64, < 10^5 : d(83160) = 128, <$ $10^6: d(720720) = 240, \le 10^7: d(8648640) = 448, \le 10^8: d(91891800) =$ $768, < 10^9 : d(931170240) = 1344, < 10^{11} : d(97772875200) = 4032, < 10^{11}$ $10^{15}: d(866421317361600) = 26880, < 10^{18}: d(897612484786617600) =$ 103680

BEST Theorem:

$$ec(G) = \#SpanningTrees(G) \cdot \prod_{v \in V} (deg(v) - 1)!$$

Erdos: Graph exists

$$\Leftrightarrow d_1 \ge .. \ge d_n, \forall k \sum_{i=1}^k d_i \le k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

Pick: $Area = Interior + \frac{Bounds}{2} - 1$ Euler: V - E + F = 1 + C

Kirchhoff: put degree on diagonal, −1 for each edge, cut out first row + column, calc det - result is #SpanningTrees

Tree Hash: for vertex v calculate $\prod (c_i + d_{h_i})$, where c_i - hash of ith

child, d_{h_i} - random number associated to depth of current child Get position of Gray Code g: int n = 0; for (; g; g >> = 1) n xor= g; return n:

Table of Basic Integrals (7)

Basic Forms

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \ n \neq -1$$
 (7.1)

$$\int \frac{1}{x} dx = \ln|x| \tag{7.2}$$

$$\int u dv = uv - \int v du \tag{7.3}$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b| \tag{7.4}$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} \tag{7.5}$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1$$
 (7.6)

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)}$$
(7.7)

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x \tag{7.8}$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} \tag{7.9}$$

$$\int \frac{x}{a^2 + x^2} dx = \frac{1}{2} \ln|a^2 + x^2| \tag{7.10}$$

$$\int \frac{x^2}{a^2 + x^2} dx = x - a \tan^{-1} \frac{x}{a}$$
 (7.11)

$$\int \frac{x^3}{a^2 + x^2} dx = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln|a^2 + x^2|$$
 (7.12)

$$\int \frac{1}{ax^2 + bx + c} dx = \frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \frac{2ax + b}{\sqrt{4ac - b^2}}$$
 (7.13)

$$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} \ln \frac{a+x}{b+x}, \ a \neq b$$
 (7.14)

$$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln|a+x| \tag{7.15}$$

$$\int \frac{x}{ax^2 + bx + c} dx = \frac{1}{2a} \ln|ax^2 + bx + c| - \frac{b}{a\sqrt{4ac - b^2}} \tan^{-1} \frac{2ax + b}{\sqrt{4ac - b^2}}$$
 (7.16)

Integrals with Roots

$$\int \sqrt{x-a} \ dx = \frac{2}{3}(x-a)^{3/2} \tag{7.17}$$

$$\int \frac{1}{\sqrt{x \pm a}} \, dx = 2\sqrt{x \pm a} \tag{7.18}$$

$$\int \frac{1}{\sqrt{a-x}} dx = -2\sqrt{a-x} \tag{7.19}$$

$$\int x\sqrt{x-a} \ dx = \begin{cases} \frac{2a}{3} (x-a)^{3/2} + \frac{2}{5} (x-a)^{5/2}, \text{ or } \\ \frac{2}{3} x(x-a)^{3/2} - \frac{4}{15} (x-a)^{5/2}, \text{ or } \\ \frac{2}{15} (2a+3x)(x-a)^{3/2} \end{cases}$$
(7.20)

$$\int \sqrt{ax+b} \ dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right) \sqrt{ax+b} \tag{7.21}$$

$$\int (ax+b)^{3/2} dx = \frac{2}{5a}(ax+b)^{5/2}$$
 (7.22)

$$\int \frac{x}{\sqrt{x \pm a}} dx = \frac{2}{3} (x \mp 2a) \sqrt{x \pm a}$$
 (7.23)

$$\int \sqrt{\frac{x}{a-x}} \, dx = -\sqrt{x(a-x)} - a \tan^{-1} \frac{\sqrt{x(a-x)}}{x-a}$$
 (7.24)

$$\int \sqrt{\frac{x}{a+x}} \, dx = \sqrt{x(a+x)} - a \ln \left[\sqrt{x} + \sqrt{x+a} \right] \tag{7.25}$$

$$\int x\sqrt{ax+b}\ dx = \frac{2}{15a^2}(-2b^2+abx+3a^2x^2)\sqrt{ax+b} \tag{7.26}$$

$$\int \sqrt{x(ax+b)} \, dx = \frac{1}{4a^{3/2}} \left[(2ax+b)\sqrt{ax(ax+b)} - b^2 \ln \left| a\sqrt{x} + \sqrt{a(ax+b)} \right| \right]$$
(7.27)

$$\int \sqrt{x^3(ax+b)} \ dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3} \right] \sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}} \ln \left| a\sqrt{x} + \sqrt{a(ax+b)} \right|$$
(7.28)

$$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \pm \frac{1}{2} a^2 \ln \left| x + \sqrt{x^2 \pm a^2} \right| \tag{7.29}$$

$$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}}$$
 (7.30)

$$\int x\sqrt{x^2 \pm a^2} \ dx = \frac{1}{3} \left(x^2 \pm a^2 \right)^{3/2} \tag{7.31}$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left| x + \sqrt{x^2 \pm a^2} \right| \tag{7.32}$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} \tag{7.33}$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}} \, dx = \sqrt{x^2 \pm a^2} \tag{7.34}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} \, dx = -\sqrt{a^2 - x^2} \tag{7.35}$$

17

(7.38)

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \mp \frac{1}{2} a^2 \ln \left| x + \sqrt{x^2 \pm a^2} \right|$$
 (7.36)

$$\int \sqrt{ax^2 + bx + c} \ dx = \frac{b + 2ax}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a^{3/2}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right|$$

$$(7.37)$$

$$\int x\sqrt{ax^2 + bx + c} \ dx = \frac{1}{48a^{5/2}} \left(2\sqrt{a}\sqrt{ax^2 + bx + c} \ \left(-3b^2 + 2abx + 8a(c + ax^2) \right) + 3(b^3 - 4abc) \ln \left| b + 2ax + 2\sqrt{a}\sqrt{ax^2 + bx + c} \right| \right)$$

$$\int \frac{1}{-} dx = \frac{1}{-} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right|$$
 (7.39)

$$\int \frac{1}{\sqrt{ax^2 + bx + c}} \, dx = \frac{1}{\sqrt{a}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right| \tag{7.39}$$

$$\int \frac{x}{\sqrt{ax^2 + bx + c}} dx = \frac{1}{a} \sqrt{ax^2 + bx + c} - \frac{b}{2a^{3/2}} \ln \left| 2ax + b + 2\sqrt{a(ax^2 + bx + c)} \right|$$
(7.40)

$$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 + x^2}} \tag{7.41}$$

Integrals with Logarithms

$$\int \ln ax \, dx = x \ln ax - x \tag{7.42}$$

$$\int x \ln x \, dx = \frac{1}{2} x^2 \ln x - \frac{x^2}{4} \tag{7.43}$$

$$\int x^2 \ln x \, dx = \frac{1}{3} x^3 \ln x - \frac{x^3}{9} \tag{7.44}$$

$$\int x^n \ln x \, dx = x^{n+1} \left(\frac{\ln x}{n+1} - \frac{1}{(n+1)^2} \right), \quad n \neq -1$$
 (7.45)

$$\int \frac{\ln ax}{x} \, dx = \frac{1}{2} \left(\ln ax \right)^2 \tag{7.46}$$

$$\int \frac{\ln x}{x^2} dx = -\frac{1}{x} - \frac{\ln x}{x} \tag{7.47}$$

$$\int \ln(ax+b) \ dx = \left(x + \frac{b}{a}\right) \ln(ax+b) - x, a \neq 0 \tag{7.48}$$

$$\int \ln(x^2 + a^2) \, dx = x \ln(x^2 + a^2) + 2a \tan^{-1} \frac{x}{a} - 2x \tag{7.49}$$

$$\int \ln(x^2 - a^2) \, dx = x \ln(x^2 - a^2) + a \ln \frac{x+a}{x-a} - 2x \tag{7.50}$$

$$\int \ln\left(ax^2 + bx + c\right) dx = \frac{1}{a}\sqrt{4ac - b^2} \tan^{-1} \frac{2ax + b}{\sqrt{4ac - b^2}} - 2x + \left(\frac{b}{2a} + x\right) \ln\left(ax^2 + bx + c\right)$$
(7.51)

NRU HSE

$$\int x \ln(ax+b) \ dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right) \ln(ax+b) \tag{7.52}$$

$$\int x \ln \left(a^2 - b^2 x^2\right) dx = -\frac{1}{2} x^2 + \frac{1}{2} \left(x^2 - \frac{a^2}{b^2}\right) \ln \left(a^2 - b^2 x^2\right)$$
(7.53)

$$\int (\ln x)^2 dx = 2x - 2x \ln x + x(\ln x)^2$$
 (7.54)

$$\int (\ln x)^3 dx = -6x + x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x$$
 (7.55)

$$\int x(\ln x)^2 dx = \frac{x^2}{4} + \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x$$
 (7.56)

$$\int x^2 (\ln x)^2 dx = \frac{2x^3}{27} + \frac{1}{3}x^3 (\ln x)^2 - \frac{2}{9}x^3 \ln x$$
 (7.57)

Integrals with Exponentials

$$\int e^{ax} dx = \frac{1}{a} e^{ax} \tag{7.58}$$

$$\int \sqrt{x}e^{ax} \ dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\mathrm{erf}\left(i\sqrt{ax}\right), \text{ where } \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2} dt \quad \left(7.59\right)$$

$$\int xe^x dx = (x-1)e^x \tag{7.60}$$

$$\int xe^{ax} dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax} \tag{7.61}$$

$$\int x^2 e^x \, dx = (x^2 - 2x + 2) e^x \tag{7.62}$$

$$\int x^2 e^{ax} dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right) e^{ax} \tag{7.63}$$

$$\int x^3 e^x dx = (x^3 - 3x^2 + 6x - 6) e^x$$
 (7.64)

$$\int x^n e^{ax} \, dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx \tag{7.65}$$

$$\int x^n e^{ax} \ dx = \frac{(-1)^n}{a^{n+1}} \Gamma[1+n, -ax], \text{ where } \Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} \ dt$$
 (7.66)

$$\int e^{ax^2} dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}\left(ix\sqrt{a}\right) \tag{7.67}$$

$$\int e^{-ax^2} dx = \frac{\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}(x\sqrt{a})$$
 (7.68)

$$\int xe^{-ax^2} dx = -\frac{1}{2a}e^{-ax^2} \tag{7.69}$$

$$\int x^2 e^{-ax^2} dx = \frac{1}{4} \sqrt{\frac{\pi}{a^3}} \operatorname{erf}(x\sqrt{a}) - \frac{x}{2a} e^{-ax^2}$$
 (7.70)

Integrals with Trigonometric Functions
$$\int \sin ax \ dx = -\frac{1}{a} \cos ax \tag{7.71}$$

$$\int \sin^2 ax \ dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \tag{7.72}$$

$$\int \sin^3 ax \ dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a} \tag{7.73}$$

$$\int \sin^n ax \ dx = -\frac{1}{a} \cos ax \ _2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right]$$
 (7.74)

$$\int \cos ax \, dx = \frac{1}{a} \sin ax \tag{7.75}$$

$$\int \cos^2 ax \ dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \tag{7.76}$$

$$\int \cos^3 ax dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a}$$
 (7.77)

$$\int \cos^p ax dx = -\frac{1}{a(1+p)} \cos^{1+p} ax \times {}_{2}F_{1}\left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right]$$
(7.78)

$$\int \cos x \sin x \, dx = \frac{1}{2} \sin^2 x + c_1 = -\frac{1}{2} \cos^2 x + c_2 = -\frac{1}{4} \cos 2x + c_3 \tag{7.79}$$

$$\int \cos ax \sin bx \, dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b \tag{7.80}$$

$$\int \sin^2 ax \cos bx \ dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)}$$
 (7.81)

$$\int \sin^2 x \cos x \, dx = \frac{1}{3} \sin^3 x \tag{7.82}$$

$$\int \cos^2 ax \sin bx \ dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)}$$
 (7.83)

$$\int \cos^2 ax \sin ax \ dx = -\frac{1}{3a} \cos^3 ax \tag{7.84}$$

$$\int \sin^2 ax \cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)}$$
 (7.85)

$$\int \sin^2 ax \cos^2 ax \, dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \tag{7.86}$$

$$\int \tan ax \, dx = -\frac{1}{a} \ln \cos ax \tag{7.87}$$

$$\int \tan^2 ax \ dx = -x + \frac{1}{a} \tan ax \tag{7.88}$$

$$\int \tan^n ax \ dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right)$$
 (7.89)

$$\int \tan^3 ax dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax \tag{7.90}$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| = 2 \tanh^{-1} \left(\tan \frac{x}{2}\right) \tag{7.91}$$

$$\int \sec^2 ax \ dx = \frac{1}{a} \tan ax \tag{7.92}$$

$$\int \sec^3 x \ dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| \tag{7.93}$$

$$\int \sec x \tan x \, dx = \sec x \tag{7.94}$$

$$\int \sec^2 x \tan x \, dx = \frac{1}{2} \sec^2 x \tag{7.95}$$

$$\int \sec^n x \tan x \, dx = \frac{1}{n} \sec^n x, n \neq 0 \tag{7.96}$$

$$\int \csc x \, dx = \ln\left|\tan\frac{x}{2}\right| = \ln\left|\csc x - \cot x\right| + C \tag{7.97}$$

$$\int \csc^2 ax \ dx = -\frac{1}{a} \cot ax \tag{7.98}$$

$$\int \csc^3 x \ dx = -\frac{1}{2} \cot x \csc x + \frac{1}{2} \ln|\csc x - \cot x|$$
 (7.99)

$$\int \csc^n x \cot x \, dx = -\frac{1}{n} \csc^n x, n \neq 0 \tag{7.100}$$

$$\int \sec x \csc x \, dx = \ln|\tan x| \tag{7.101}$$

Products of Trigonometric Functions and Monomials

$$\int x \cos x \, dx = \cos x + x \sin x \tag{7.102}$$

$$\int x \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax \tag{7.103}$$

$$\int x^2 \cos x \, dx = 2x \cos x + (x^2 - 2) \sin x \tag{7.104}$$

$$\int x^2 \cos ax \ dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax \tag{7.105}$$

$$\int x^n \cos x dx = -\frac{1}{2} (i)^{n+1} \left[\Gamma(n+1, -ix) + (-1)^n \Gamma(n+1, ix) \right]$$
 (7.106)

$$\int x^n \cos ax \ dx = \frac{1}{2} (ia)^{1-n} \left[(-1)^n \Gamma(n+1, -iax) - \Gamma(n+1, ixa) \right]$$
 (7.107)

$$\int x \sin x \, dx = -x \cos x + \sin x \tag{7.108}$$

$$\int x \sin ax \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \tag{7.109}$$

$$\int x^2 \sin x \, dx = (2 - x^2) \cos x + 2x \sin x \tag{7.110}$$

$$\int x^2 \sin ax \ dx = \frac{2 - a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2}$$
 (7.111)

$$\int x^n \sin x \, dx = -\frac{1}{2} (i)^n \left[\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, -ix) \right]$$
 (7.112)

$$\int x \cos^2 x \, dx = \frac{x^2}{4} + \frac{1}{8} \cos 2x + \frac{1}{4} x \sin 2x \tag{7.113}$$

$$\int x \sin^2 x \, dx = \frac{x^2}{4} - \frac{1}{8} \cos 2x - \frac{1}{4} x \sin 2x \tag{7.114}$$

$$\int x \tan^2 x \, dx = -\frac{x^2}{2} + \ln \cos x + x \tan x \tag{7.115}$$

$$\int x \sec^2 x \, dx = \ln \cos x + x \tan x \tag{7.116}$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x) \tag{7.117}$$

$$\int e^{bx} \sin ax \, dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax)$$
 (7.118)

$$\int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x) \tag{7.119}$$

$$\int e^{bx} \cos ax \, dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax) \tag{7.120}$$

$$\int xe^x \sin x \, dx = \frac{1}{2}e^x(\cos x - x\cos x + x\sin x) \tag{7.121}$$

$$\int xe^x \cos x \, dx = \frac{1}{2}e^x (x \cos x - \sin x + x \sin x) \tag{7.122}$$

Integrals of Hyperbolic Functions

$$\int \cosh ax \ dx = \frac{1}{a} \sinh ax \tag{7.123}$$

$$\int e^{ax} \cosh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [a \cosh bx - b \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} + \frac{x}{2} & a = b \end{cases}$$
 (7.124)

$$\int \sinh ax \, dx = \frac{1}{a} \cosh ax \tag{7.125}$$

$$\int e^{ax} \sinh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [-b \cosh bx + a \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} - \frac{x}{2} & a = b \end{cases}$$
 (7.126)

$$\int \tanh ax \, dx = \frac{1}{a} \ln \cosh ax \tag{7.127}$$

$$\int e^{ax} \tanh bx \ dx = \begin{cases}
\frac{e^{(a+2b)x}}{(a+2b)} {}_{2}F_{1} \left[1 + \frac{a}{2b}, 1, 2 + \frac{a}{2b}, -e^{2bx} \right] \\
-\frac{1}{a} e^{ax} {}_{2}F_{1} \left[1, \frac{a}{2b}, 1 + \frac{a}{2b}, -e^{2bx} \right] & a \neq b \\
\frac{e^{ax} - 2 \tan^{-1} [e^{ax}]}{a} & a = b
\end{cases}$$
(7.128)

$$\int \cos ax \cosh bx \, dx = \frac{1}{a^2 + b^2} \left[a \sin ax \cosh bx + b \cos ax \sinh bx \right] \tag{7.129}$$

$$\int \cos ax \sinh bx \ dx = \frac{1}{a^2 + b^2} \left[b \cos ax \cosh bx + a \sin ax \sinh bx \right] \tag{7.130}$$

$$\int \sin ax \cosh bx \, dx = \frac{1}{a^2 + b^2} \left[-a \cos ax \cosh bx + b \sin ax \sinh bx \right] \tag{7.131}$$

$$\int \sin ax \sinh bx \, dx = \frac{1}{a^2 + b^2} \left[b \cosh bx \sin ax - a \cos ax \sinh bx \right] \tag{7.132}$$

$$\int \sinh ax \cosh ax dx = \frac{1}{4a} \left[-2ax + \sinh 2ax \right] \tag{7.133}$$

$$\int \sinh ax \cosh bx \ dx = \frac{1}{b^2 - a^2} \left[b \cosh bx \sinh ax - a \cosh ax \sinh bx \right] \tag{7.134}$$

Problem	Status	Comment	Iurii	Alex	Leha
A - 1					
B - 2					
C - 3					
D - 4					
E - 5					
F - 6					
G - 7					
H - 8					
I - 9					
J - 10					
K - 11					
L - 12					
M - 13					
N - 14					
O - 15					