National Research University Higher School of Economics

Youthful Passion Fruit

Alexandr Nekrasov, Iurii Pustovalov, Alexey Vasilyev

August 24, 2024

Contest (1)

template.cpp

42 lines

```
#ifdef LOCAL
#define _GLIBCXX_DEBUG
#endif
#include <bits/stdc++.h>
using namespace std;

using ll = long long;
using ld = long double;
using ull = unsigned long long;

#define pbc push_back
#define mp make_pair
#define all(v) (v).begin(), (v).end()
#define vin(v) for (auto &el : a) cin >> el

mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());

template <typename T1, typename T2> inline void chkmin(T1 &x, const T2 &y
    ) {
    if (y < x) {
        x = y;
    }
}

template <typename T1, typename T2> inline void chkmax(T1 &x, const T2 &y
    ) {
    if (x < y) {
        x = y;
    }
}

void solve() {

}

signed main() {
    cin.tie(0)->sync_with_stdio(0);
    cout.precision(20), cout.setf(ios::fixed);
    int t = 1;
    // cin >> t;
    while (t--) {
        solve();
    }
}
```

genfolders.sh

6 lines

```
chmod +x bld*
for f in {A..Z}
do
    mkdir $f
    cp main.cpp bld* $f
done
```

bld

1 lines

```
g++ -std=c++20 -g -DLOCAL -fsanitize=address,bounds,undefined -o $1 $1.
    cpp
```

bldf

1 lines

```
g++ -std=c++20 -g -O2 -o $1 $1.cpp
```

hacks.sh

2 lines

```
UBSAN_OPTIONS=print_stacktrace=1 ./main
gdb rbreak regex
```

hash.sh

3 lines

```
# Hashes a file, ignoring all whitespace and comments.
# Use for verifying that code was correctly typed.
cpp -dD -P -fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

C++ (2)

GpHashtable.cpp

Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

e44914, 11 lines

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

const int RANDOM =
    chrono::high_resolution_clock::now().time_since_epoch().count();
struct hasher {
    int operator()(int x) const { return x ^ RANDOM; }
};

gp_hash_table<int, int, hasher> table;
```

OrderedSet.cpp

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.

Time: $\mathcal{O}(\log(n))$

<bits/extc++.h>, <bits/stdc++.h> dff260, 37 lines

```
using namespace __gnu_pbds;
using namespace std;

template <typename T>
using ordered_set =
    tree<T, null_type, less<>, rb_tree_tag,
          tree_order_statistics_node_update>;

int main() {
    ordered_set<int> X;
    X.insert(1);
    X.insert(2);
    X.insert(4);
    X.insert(8);
    X.insert(16);

    assert(*X.find_by_order(1) == 2);
    assert(*X.find_by_order(2) == 4);
    assert(*X.find_by_order(4) == 16);
    assert(X.find_by_order(6) ==X.end());

    assert(X.order_of_key(-5) == 0);
    assert(X.order_of_key(1) == 0);
    assert(X.order_of_key(3) == 2);
    assert(X.order_of_key(4) == 2);
    assert(X.order_of_key(400) == 5);
    // std::cout << *X.find_by_order(1) << std::endl;              // 2
    // std::cout << *X.find_by_order(2) << std::endl;              // 4
    // std::cout << *X.find_by_order(4) << std::endl;              // 16
    // std::cout << (end(X) == X.find_by_order(6)) << std::endl; // true

    // std::cout << X.order_of_key(-5) << std::endl;  // 0
    // std::cout << X.order_of_key(1) << std::endl;   // 0
    // std::cout << X.order_of_key(3) << std::endl;   // 2
    // std::cout << X.order_of_key(4) << std::endl;   // 2
    // std::cout << X.order_of_key(400) << std::endl; // 5
    return 0;
}
```

bitset.cpp

Description: bitset

521d1f, 2 lines

```
bs._Find_first()
bs._Find_next(idx) - returns right after
```

alloc.cpp

Description: fastalloc

8726b1, 11 lines

```
const int MAX_MEM = 1e8;
int mpos = 0;
char mem[MAX_MEM];
inline void *operator new(size_t n) {
  assert((mpos += n) <= MAX_MEM);
  return (void *)(mem + mpos - n);
}
void operator delete(void *) noexcept {
}  // must have!
void operator delete(void *, size_t) noexcept {
}  // must have!
```

fastio.cpp

Description: fastio

79fd14, 52 lines

```
inline int readChar();
template <class T = int>
inline T readInt();
template <class T>
inline void writeInt(T x, char end = 0);
inline void writeChar(int x);
inline void writeWord(const char *s);
static const int buf_size = 4096;
inline int getChar() {
  static char buf[buf_size];
  static int len = 0, pos = 0;
  if (pos == len) pos = 0, len = fread(buf, 1, buf_size, stdin);
  if (pos == len) return -1;
  return buf[pos++];
}
inline int readChar() {
  int c = getChar();
  while (c <= 32) c = getChar();
  return c;
}
template <class T>
inline T readInt() {
  int s = 1, c = readChar();
  T x = 0;
  if (c == '-') s = -1, c = getChar();
  while ('0' <= c && c <= '9') x = x * 10 + c - '0', c = getChar();
  return s == 1 ? x : -x;
}
static int write_pos = 0;
static char write_buf[buf_size];
inline void writeChar(int x) {
  if (write_pos == buf_size)
    fwrite(write_buf, 1, buf_size, stdout), write_pos = 0;
  write_buf[write_pos++] = x;
}
template <class T>
inline void writeInt(T x, char end) {
  if (x < 0) writeChar('-'), x = -x;
  char s[24];
  int n = 0;
  while (x || !n) s[n++] = '0' + x % 10, x /= 10;
  while (n--) writeChar(s[n]);
  if (end) writeChar(end);
}
inline void writeWord(const char *s) {
  while (*s) writeChar(*s++);
}
struct Flusher {
  ~Flusher() {
    if (write_pos) fwrite(write_buf, 1, write_pos, stdout), write_pos =
        0;
  }
} flusher;
```

Strings (3)

Manacher.cpp

Description: Manacher algorithm
Time: $\mathcal{O}(n)$

a6ddfb, 27 lines

```
vector<int> manacherOdd(string s) {
    int n = s.size();
    vector<int> d1(n);
    int l = 0, r = -1;
    for (int i = 0; i < n; ++i) {
        int k = i > r ? 1 : min(d1[l + r - i], r - i + 1);
        while (i + k < n && i - k >= 0 && s[i + k] == s[i - k])
            ++k;
        d1[i] = k;
        if (i + k - 1 > r)
            l = i - k + 1, r = i + k - 1;
    }
}

vector<int> manacherEven(string s) {
    int n = s.size();
    vector<int> d2(n);
    l = 0, r = -1;
    for (int i = 0; i < n; ++i) {
        int k = i > r ? 0 : min(d2[l + r - i + 1], r - i + 1);
        while (i + k < n && i - k - 1 >= 0 && s[i + k] == s[i - k - 1])
            ++k;
        d2[i] = k;
        if (i + k - 1 > r)
            l = i - k, r = i + k - 1;
    }
}
```

AhoCorasick.cpp

Description: Build aho-corasick automaton.
Time: $\mathcal{O}(n)$

ae5fc2, 19 lines

```
int go(int v, char c);

int get_link(int v) {
    if (t[v].link == -1)
        if (v == 0 || t[v].p == 0)
            t[v].link = 0;
        else
            t[v].link = go(get_link(t[v].p), t[v].pch);
    return t[v].link;
}

int go(int v, char c) {
    if (t[v].go[c] == -1)
        if (t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
        else
            t[v].go[c] = v == 0 ? 0 : go(get_link(v), c);
    return t[v].go[c];
}
```

SuffixArray.cpp

Description: Build suffix array
Time: $\mathcal{O}(n\log(n))$

5bd011, 47 lines

```
vector<int> buildSuffixArray(string &s) {
    // Remove, if you want to sort cyclic shifts
    s += (char)(1);
    int n = s.size();
    vector<int> a(n);
    iota(all(a), 0);
    stable_sort(all(a), [&](int i, int j) { return s[i] < s[j]; });
    vector<int> c(n);
    int cc = 0;
    for (int i = 0; i < n; i++) {
        if (i == 0 || s[a[i]] != s[a[i - 1]]) {
            c[a[i]] = cc++;
        } else {
            c[a[i]] = c[a[i - 1]];
        }
    }
    for (int L = 1; L < n; L *= 2) {
        vector<int> cnt(n);
        for (auto i : c) {
            cnt[i]++;
        }
        vector<int> pref(n);
        for (int i = 1; i < n; i++) {
            pref[i] = pref[i - 1] + cnt[i - 1];
        }
        vector<int> na(n);
        for (int i = 0; i < n; i++) {
            int pos = (a[i] - L + n) % n;
            na[pref[c[pos]]++] = pos;
        }
        a = na;
        vector<int> nc(n);
        cc = 0;
        for (int i = 0; i < n; i++) {
            if (i == 0 || c[a[i]] != c[a[i - 1]] ||
                c[(a[i] + L) % n] != c[(a[i - 1] + L) % n]) {
                nc[a[i]] = cc++;
            } else {
                nc[a[i]] = nc[a[i - 1]];
            }
        }
        c = nc;
    }
    a.erase(a.begin());
    s.pop_back();
    return a;
}
```

Lcp.cpp

Description: lcp array
Time: $\mathcal{O}(n)$

1cc27c, 43 lines

```
vector<int> perm;
vector<int> buildLCP(string &s, vector<int> &a) {
    int n = s.size();
    vector<int> ra(n);
    for (int i = 0; i < n; i++) {
        ra[a[i]] = i;
    }
    vector<int> lcp(n - 1);
    int cur = 0;
    for (int i = 0; i < n; i++) {
        cur--;
        chkmax(cur, 0);
        if (ra[i] == n - 1) {
            cur = 0;
            continue;
        }
        int j = a[ra[i] + 1];
        while (s[i + cur] == s[j + cur]) cur++;
        lcp[ra[i]] = cur;
    }
    perm.resize(a.size());
    for (int i = 0; i < a.size(); ++i) perm[a[i]] = i;
    return lcp;
}
int cntr[MAXN];
int spt[MAXN][lgg];
void build(vector<int> &a) {
    for (int i = 0; i < a.size(); ++i) {
        spt[i][0] = a[i];
    }
    for (int i = 2; i < MAXN; ++i) cntr[i] = cntr[i / 2] + 1;
    for (int h = 1; (1 << (h - 1)) < a.size(); ++h) {
        for (int i = 0; i + (1 << (h - 1)) < a.size(); ++i) {
            spt[i][h] = min(spt[i][h - 1], spt[i + (1 << (h - 1))][h -
                1]);
        }
    }
}
int getLCP(int l, int r) {
    l = perm[l], r = perm[r];
    if (l > r) swap(l, r);
    int xx = cntr[r - l];
    return min(spt[l][xx], spt[r - (1 << xx)][xx]);
}
```

Eertree.cpp

Description: Creates Eertree of string str
Time: $\mathcal{O}(n)$

7924c8, 40 lines

```
struct eertree {
    int len[MAXN], suffLink[MAXN];
    int to[MAXN][26];
    int numV, v;
    void addLetter(int n, string &str) {
        while (str[n - len[v] - 1] != str[n])
            v = suffLink[v];
        int u = suffLink[v];
        while (str[n - len[u] - 1] != str[n])
            u = suffLink[u];
        int u_ = to[u][str[n] - 'a'];
        int v_ = to[v][str[n] - 'a'];
        if (v_ == -1) {
            v_ = to[v][str[n] - 'a'] = numV;
            len[numV++] = len[v] + 2;
            suffLink[v_] = u_;
        }
        v = v_;
    }
    void init() {
        len[0] = -1;
        len[1] = 0;
        suffLink[1] = 0;
        suffLink[0] = 0;
        numV = 2;
        for (int i = 0; i < 26; ++i) {
            to[0][i] = numV++;
            suffLink[numV - 1] = 1;
            len[numV - 1] = 1;
        }
        v = 0;
    }
    void init(int sz) {
        for (int i = 0; i < sz; ++i) {
            len[i] = suffLink[i] = 0;
            for (int j = 0; j < 26; ++j)
                to[i][j] = -1;
        }
    }
};
```

SuffixAutomaton.cpp

Description: Build suffix automaton.
Time: $\mathcal{O}(n)$

662a10, 45 lines

```
struct state {
    int len, link;
    map<char, int> next;
};

const int MAXLEN = 100000;
state st[MAXLEN * 2];
int sz, last;

void sa_init() {
    sz = last = 0;
    st[0].len = 0;
    st[0].link = -1;
    ++sz;
    /*
    // if you want to build an automaton for different strings:
    for (int i=0; i<MAXLEN*2; ++i)
        st[i].next.clear();
    */
}

void sa_extend(char c) {
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    int p;
    for (p = last; p != -1 && !st[p].next.count(c); p = st[p].link)
        st[p].next[c] = cur;
    if (p == -1)
        st[cur].link = 0;
    else {
```

```
        int q = st[p].next[c];
        if (st[p].len + 1 == st[q].len)
            st[cur].link = q;
        else {
            int clone = sz++;
            st[clone].len = st[p].len + 1;
            st[clone].next = st[q].next;
            st[clone].link = st[q].link;
            for (; p != -1 && st[p].next[c] == q; p = st[p].link)
                st[p].next[c] = clone;
            st[q].link = st[cur].link = clone;
        }
    }
    last = cur;
}
```

PrefixZ.cpp

Description: Calculates Prefix,Z-functions
Time: $\mathcal{O}(n)$ 1c4e93, 25 lines

```
vector<int> pf(string s) {
    int k = 0;
    vector<int> p(s.size());
    for (int i = 1; i < s.size(); ++i) {
        while (k && s[i] != s[k])
            k = p[k - 1];
        k += (s[i] == s[k]);
        p[i] = k;
    }
    return p;
}
vector<int> zf(string s) {
    int n = s.size();
    vector<int> z(n, 0);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r)
            z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
            ++z[i];
        if (i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1;
    }
    z[0] = n;
    return z;
}
```

MinShift.cpp

Description: Calculates min-cyclic-shift of s, Duval decomposition
Time: $\mathcal{O}(n)$ 3f0fb9, 20 lines

```
string minshift(string s) {
    int i = 0, ans = 0;
    s += s;
    int n = s.size();
    while (i < n / 2) {
        ans = i;
        int j = i + 1, k = i;
        while (j < n && s[k] <= s[j]) {
            if (s[k] < s[j])
                k = i;
            else
                ++k;
            ++j;
        }
        while (i <= k) {
            i += j - k;
        }
    }
    return s.substr(ans, n / 2);
}
```

SA-IS.cpp

Description: Build suffix array
Time: $\mathcal{O}(n)$ f90ffe, 87 lines

```
void induced_sort(vector<int> &vec, int LIM, vector<int> &sa, vector<bool
    > &sl,
                  vector<int> &fx) {
    vector<int> l(LIM), r(LIM);
    for (int c : vec) {
        if (c + 1 < LIM) {
            ++l[c + 1];
        }
        ++r[c];
    }
    partial_sum(all(l), l.begin());
    partial_sum(all(r), r.begin());
    fill(all(sa), -1);
    for (int i = fx.size() - 1; i >= 0; --i) {
        sa[--r[vec[fx[i]]]] = fx[i];
    }
    for (int i : sa) {
        if (i >= 1 && sl[i - 1]) {
            sa[l[vec[i - 1]]++] = i - 1;
        }
    }
    fill(all(r), 0);
    for (int c : vec) ++r[c];
    partial_sum(all(r), r.begin());
    for (int k = sa.size() - 1, i = sa[k]; k >= 1; --k, i = sa[k])
        if (i >= 1 && !sl[i - 1]) sa[--r[vec[i - 1]]] = i - 1;
}
vector<int> SA_IS(vector<int> &vec, int LIM) {
    const int n = vec.size();
    vector<int> sa(n), fx;
    vector<bool> sl(n);
    sl[n - 1] = false;
    for (int i = n - 2; i >= 0; --i) {
        sl[i] = (vec[i] > vec[i + 1] || (vec[i] == vec[i + 1] && sl[i +
            1]));
        if (sl[i] && !sl[i + 1]) {
            fx.pbc(i + 1);
        }
    }
    reverse(all(fx));
    induced_sort(vec, LIM, sa, sl, fx);
    vector<int> nfx(fx.size()), lmv(fx.size());
    for (int i = 0, k = 0; i < n; ++i) {
        if (!sl[sa[i]] && sa[i] >= 1 && sl[sa[i] - 1]) {
            nfx[k++] = sa[i];
        }
    }
    int cur = 0;
    sa[n - 1] = cur;
    for (int k = 1; k < nfx.size(); ++k) {
        int i = nfx[k - 1], j = nfx[k];
        if (vec[i] != vec[j]) {
            sa[j] = ++cur;
            continue;
        }
        bool flag = false;
        for (int a = i + 1, b = j + 1;; ++a, ++b) {
            if (vec[a] != vec[b]) {
                flag = true;
                break;
            }
            if ((!sl[a] && sl[a - 1]) || (!sl[b] && sl[b - 1])) {
                flag = !((!sl[a] && sl[a - 1]) && (!sl[b] && sl[b - 1]));
                break;
            }
        }
        sa[j] = (flag ? ++cur : cur);
    }
    for (int i = 0; i < fx.size(); ++i) {
        lmv[i] = sa[fx[i]];
    }
    if (cur + 1 < (int)fx.size()) {
        auto lms = SA_IS(lmv, cur + 1);
        for (int i = 0; i < fx.size(); ++i) {
            nfx[i] = fx[lms[i]];
        }
    }
    induced_sort(vec, LIM, sa, sl, nfx);
    return sa;
}
template <typename T>
vector<int> suffix_array(T &s, const int LIM = 128) {
    vector<int> vec(s.size() + 1);
    copy(all(s), begin(vec));
    vec.back() = (char)(1);
    auto ret = SA_IS(vec, LIM);
    ret.erase(ret.begin());
    return ret;
}
```

Graph (4)

Hungarian.cpp

Description: Hungarian algorithm
Time: $\mathcal{O}\left(n^3\right)$

5afee5, 41 lines

```
int n, m;
vector<vector<int>> a;
vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
for (int i = 1; i <= n; ++i) {
    p[0] = i;
    int j0 = 0;
    vector<int> minv(m + 1, INF);
    vector<char> used(m + 1, false);
    do {
        used[j0] = true;
        int i0 = p[j0], delta = INF, j1;
        for (int j = 1; j <= m; ++j)
            if (!used[j]) {
                int cur = a[i0][j] - u[i0] - v[j];
                if (cur < minv[j])
                    minv[j] = cur, way[j] = j0;
                if (minv[j] < delta)
                    delta = minv[j], j1 = j;
            }
        for (int j = 0; j <= m; ++j)
            if (used[j])
                u[p[j]] += delta, v[j] -= delta;
            else
                minv[j] -= delta;
        j0 = j1;
    } while (p[j0] != 0);
    do {
        int j1 = way[j0];
        p[j0] = p[j1];
        j0 = j1;
    } while (j0);
}

// matching
vector<int> ans(n + 1);
for (int j = 1; j <= m; ++j) {
    ans[p[j]] = j;
}

// cost
int cost = -v[0];
```

BlossomShrinking.cpp

Description: Maximum matching in general graph
Time: $\mathcal{O}\left(n^3\right)$

23839d, 118 lines

```
struct Edge {
    int u, v;
};
const int N = 510;
int n, m;
vector<int> g[N];
vector<Edge> perfectMatching;
int match[N], par[N], base[N];
bool used[N], blossom[N], lcaUsed[N];
int lca(int u, int v) {
    fill(lcaUsed, lcaUsed + n, false);
    while (u != -1) {
        u = base[u];
        lcaUsed[u] = true;
        if (match[u] == -1)
            break;
        u = par[match[u]];
    }
    while (v != -1) {
        v = base[v];
        if (lcaUsed[v])
            return v;
        v = par[match[v]];
    }
    assert(false);
    return -1;
}
void markPath(int v, int myBase, int children) {
    while (base[v] != myBase) {
        blossom[v] = blossom[match[v]] = true;
        par[v] = children;
        children = match[v];
        v = par[match[v]];
    }
}
int findPath(int root) {
    iota(base, base + n, 0);
    fill(par, par + n, -1);
    fill(used, used + n, false);
    queue<int> q;
    q.push(root);
    used[root] = true;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (auto to : g[v]) {
            if (match[v] == to)
                continue;
            if (base[v] == base[to])
                continue;
            if (to == root || (match[to] != -1 && par[match[to]] != -1))
                {
                fill(blossom, blossom + n, false);
                int myBase = lca(to, v);
                markPath(v, myBase, to);
                markPath(to, myBase, v);
                for (int u = 0; u < n; ++u) {
                    if (!blossom[base[u]])
                        continue;
                    base[u] = myBase;
                    if (used[u])
                        continue;
                    used[u] = true;
                    q.push(u);
                }
            } else if (par[to] == -1) {
                par[to] = v;
                if (match[to] == -1) {
                    return to;
                }
                used[match[to]] = true;
                q.push(match[to]);
            }
        }
    }
    return -1;
}
void blossomShrinking() {
    fill(match, match + n, -1);
    for (int v = 0; v < n; ++v) {
        if (match[v] != -1)
            continue;
        int nxt = findPath(v);
        while (nxt != -1) {
            int parV = par[nxt];
            int parParV = match[parV];
            match[nxt] = parV;
            match[parV] = nxt;
            nxt = parParV;
        }
    }
    for (int v = 0; v < n; ++v) {
        if (match[v] != -1 && v < match[v]) {
            perfectMatching.push_back({v, match[v]});
        }
    }
}
signed main() {
    cin >> n;
    int u, v;
    set<pair<int, int>> edges;
    while (cin >> u >> v) {
        --u;
        --v;
        if (u > v)
            swap(u, v);
        if (edges.count({u, v}))
            continue;
        edges.insert({u, v});
        g[u].push_back(v);
        g[v].push_back(u);
    }
    blossomShrinking();
    cout << perfectMatching.size() * 2 << '\n';
    for (auto i : perfectMatching) {
        cout << i.u + 1 << " " << i.v + 1 << "\n";
    }
    return 0;
}
```

Lct.cpp

Description: link-cut tree
Time: $\mathcal{O}(n\log(n))$

3d8a3f, 136 lines

```
#include <bits/stdc++.h>
using namespace std;

const int MAXN = 1e5 + 228;

struct node {
    node *ch[2];
    node *p;
    bool rev;
    int sz;

    node() {
        ch[0] = ch[1] = p = NULL;
        rev = false;
        sz = 1;
    }
};

int getsz(node *n) { return (n == NULL) ? 0 : n->sz; }

void pull(node *n) { n->sz = getsz(n->ch[0]) + getsz(n->ch[1]) + 1; }

void push(node *n) {
    if (n->rev) {
        if (n->ch[0]) {
            n->ch[0]->rev ^= 1;
        }
        if (n->ch[1]) {
            n->ch[1]->rev ^= 1;
        }
        swap(n->ch[0], n->ch[1]);
        n->rev = 0;
    }
}

bool isRoot(node *n) {
    return n->p == NULL || (n->p->ch[0] != n && n->p->ch[1] != n);
}

int chnum(node *n) { return n->p->ch[1] == n; }

void attach(node *n, node *p, int num) {
    if (n != NULL)
        n->p = p;
    if (p != NULL)
        p->ch[num] = n;
}

void rotate(node *n) {
    int num = chnum(n);
    node *p = n->p;
    node *b = n->ch[1 - num];
    n->p = p->p;
    if (!isRoot(p)) {
        p->p->ch[chnum(p)] = n;
    }
    attach(p, n, 1 - num);
    attach(b, p, num);
    pull(p);
    pull(n);
```

```
}

node *qq[MAXN];

void splay(node *n) {
    node *nn = n;
    int top = 0;
    qq[top++] = nn;
    while (!isRoot(nn)) {
        nn = nn->p;
        qq[top++] = nn;
    }
    while (top) {
        push(qq[--top]);
    }
    while (!isRoot(n)) {
        if (!isRoot(n->p)) {
            if (chnum(n) == chnum(n->p)) {
                rotate(n->p);
            } else {
                rotate(n);
            }
        }
        rotate(n);
    }
}

void expose(node *n) {
    splay(n);
    n->ch[1] = NULL;
    pull(n);
    while (n->p != NULL) {
        splay(n->p);
        attach(n, n->p, 1);
        pull(n->p);
        splay(n);
    }
}

void makeRoot(node *n) {
    expose(n);
    n->rev ^= 1;
}

node *nodes[MAXN];

int main() {
    int n;
    cin >> n;
    for (int i = 0; i <= n; i++) {
        nodes[i] = new node();
    }
    int q;
    cin >> q;
    while (q--) {
        string s;
        cin >> s;
        int u, v;
        cin >> u >> v;
        makeRoot(nodes[u]);
        makeRoot(nodes[v]);
        if (s == "get") {
            if (isRoot(nodes[u]) && u != v) {
                cout << "-1" << endl;
            } else {
                cout << getsz(nodes[v]) - 1 << endl;
            }
        } else if (s == "link") {
            nodes[v]->p = nodes[u];
        } else {
            push(nodes[v]);
            nodes[v]->ch[1] = NULL;
            nodes[u]->p = NULL;
        }
    }
}
```

MaxFlow.cpp

Description: Dinic
Time: $\mathcal{O}\left(n^2m\right)$

1c1bc8, 72 lines

```
struct MaxFlow {
    const int inf = 1e9 + 20;
    struct edge {
        int a, b, cap;
    };
    int n;
    vector<edge> e;
    vector<vector<int>> g;
    MaxFlow() {}
    int s, t;
    vector<int> d, ptr;
    void init(int n_, int s_, int t_) {
        s = s_, t = t_, n = n_;
        g.resize(n);
        ptr.resize(n);
    }
    void addedge(int a, int b, int cap) {
        g[a].pbc(e.size());
        e.pbc({a, b, cap});
        g[b].pbc(e.size());
        e.pbc({b, a, 0});
    }
    bool bfs() {
        d.assign(n, inf);
        d[s] = 0;
        queue<int> q;
        q.push(s);
        while (q.size()) {
            int v = q.front();
            q.pop();
            for (int i : g[v]) {
                if (e[i].cap > 0) {
                    int b = e[i].b;
                    if (d[b] > d[v] + 1) {
                        d[b] = d[v] + 1;
                        q.push(b);
                    }
                }
            }
        }
        return d[t] != inf;
    }
    int dfs(int v, int flow) {
        if (v == t) return flow;
        if (!flow) return 0;
        int sum = 0;
        for (; ptr[v] < g[v].size(); ++ptr[v]) {
            int b = e[g[v][ptr[v]]].b;
            int cap = e[g[v][ptr[v]]].cap;
            if (cap <= 0) continue;
            if (d[b] != d[v] + 1) continue;
            int x = dfs(b, min(flow, cap));
            int id = g[v][ptr[v]];
            e[id].cap -= x;
            e[id ^ 1].cap += x;
            flow -= x;
            sum += x;
        }
        return sum;
    }
    int dinic() {
        int ans = 0;
        while (1) {
            if (!bfs()) break;
            ptr.assign(n, 0);
            int x = dfs(s, inf);
            if (!x) break;
            ans += x;
        }
        return ans;
    }
};
```

MCMF.cpp

Description: Min cost
Time: $\mathcal{O}\left(?\right)$

32340a, 61 lines

```
struct MCMF {
    struct edge {
        int a, b, cap, cost;
    };
    vector<edge> e;
    vector<vector<int>> g;
    int s, t;
    int n;
    void init(int N, int S, int T) {
        s = S, t = T, n = N;
        g.resize(N);
        e.clear();
    }
    void addedge(int a, int b, int cap, int cost) {
        g[a].pbc(e.size());
        e.pbc({a, b, cap, cost});
        g[b].pbc(e.size());
        e.pbc({b, a, 0, -cost});
    }
    int getcost(int k) {
        int flow = 0;
        int cost = 0;
        while (flow < k) {
            vector<int> d(n, INF);
            vector<int> pr(n);
            d[s] = 0;
            queue<int> q;
            q.push(s);
            while (q.size()) {
                int v = q.front();
                q.pop();
                for (int i : g[v]) {
                    int u = e[i].b;
                    if (e[i].cap && d[u] > d[v] + e[i].cost) {
                        d[u] = d[v] + e[i].cost;
                        q.push(u);
                        pr[u] = i;
                    }
                }
            }
            if (d[t] == INF) return INF;
            int gf = k - flow;
            int v = t;
            while (v != s) {
                int id = pr[v];
                chkmin(gf, e[id].cap);
                v = e[id].a;
            }
            v = t;
            while (v != s) {
                int id = pr[v];
                e[id].cap -= gf;
                e[id ^ 1].cap += gf;
                cost += e[id].cost * gf;
                v = e[id].a;
            }
            flow += gf;
        }
        return cost;
    }
};
```

GlobalMincut.cpp

Description: Global min cut
Time: $\mathcal{O}\left(n^3\right)$

7b8a6b, 35 lines

```
const int MAXN = 500;
int n, g[MAXN][MAXN];
int best_cost = 1000000000;
vector<int> best_cut;
void mincut() {
    vector<int> v[MAXN];
    for (int i = 0; i < n; ++i)
        v[i].assign(1, i);
    int w[MAXN];
```

```
    bool exist[MAXN], in_a[MAXN];
    memset(exist, true, sizeof exist);
    for (int ph = 0; ph < n - 1; ++ph) {
        memset(in_a, false, sizeof in_a);
        memset(w, 0, sizeof w);
        for (int it = 0, prev; it < n - ph; ++it) {
            int sel = -1;
            for (int i = 0; i < n; ++i)
                if (exist[i] && !in_a[i] && (sel == -1 || w[i] > w[sel]))
                    sel = i;
            if (it == n - ph - 1) {
                if (w[sel] < best_cost)
                    best_cost = w[sel], best_cut = v[sel];
                v[prev].insert(v[prev].end(), v[sel].begin(), v[sel].end
                     ());
                for (int i = 0; i < n; ++i)
                    g[prev][i] = g[i][prev] += g[sel][i];
                exist[sel] = false;
            } else {
                in_a[sel] = true;
                for (int i = 0; i < n; ++i)
                    w[i] += g[sel][i];
                prev = sel;
            }
        }
    }
}
```

WeightedMatching.cpp

Description: Max weighted matching

Time: $\mathcal{O}\left(N^3\right)$ or so

c3f149, 193 lines

```
#define Dist(e) (lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2)
const int N = 1023, INF = 1e9;
struct Edge {
    int u, v, w;
} g[N][N];
int n, m, n_x, lab[N], match[N], slack[N], st[N], pa[N], flower_from[N][N
    ], S[N], vis[N];
vector<int> flower[N];
deque<int> q;
void update_slack(int u, int x) {
    if (!slack[x] || Dist(g[u][x]) < Dist(g[slack[x]][x])) slack[x] = u;
}
void set_slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
        if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0) update_slack(u,
             x);
}
void q_push(int x) {
    if (x <= n) return q.push_back(x);
    for (int i = 0; i < flower[x].size(); ++i) q_push(flower[x][i]);
}
void set_st(int x, int b) {
    st[x] = b;
    if (x <= n) return;
    for (int i = 0; i < flower[x].size(); ++i) set_st(flower[x][i], b);
}
int get_pr(int b, int xr) {
    int pr = find(flower[b].begin(), flower[b].end(), xr) - flower[b].
         begin();
    if (pr % 2 == 1) {
        reverse(flower[b].begin() + 1, flower[b].end());
        return (int)flower[b].size() - pr;
    } else return pr;
}
void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    Edge e = g[u][v];
    int xr = flower_from[u][e.u], pr = get_pr(u, xr);
    for (int i = 0; i < pr; ++i) set_match(flower[u][i], flower[u][i ^
         1]);
    set_match(xr, v);
    rotate(flower[u].begin(), flower[u].begin() + pr, flower[u].end());
}
void augment(int u, int v) {
    int xnv = st[match[u]];
    set_match(u, v);
    if (!xnv) return;
    set_match(xnv, st[pa[xnv]]);
    augment(st[pa[xnv]], xnv);
}
int get_lca(int u, int v) {
    static int t = 0;
    for (++t; u || v; swap(u, v)) {
        if (u == 0) continue;
        if (vis[u] == t) return u;
        vis[u] = t;
        u = st[match[u]];
        if (u) u = st[pa[u]];
    }
    return 0;
}
void add_blossom(int u, int lca, int v) {
    int b = n + 1;
    while (b <= n_x && st[b]) ++b;
    if (b > n_x) ++n_x;
    lab[b] = 0, S[b] = 0, match[b] = match[lca];
    flower[b].clear();
    flower[b].push_back(lca);
    for (int x = u, y; x != lca; x = st[pa[y]])
        flower[b].push_back(x), flower[b].push_back(y = st[match[x]]),
            q_push(y);
    reverse(flower[b].begin() + 1, flower[b].end());
    for (int x = v, y; x != lca; x = st[pa[y]])
        flower[b].push_back(x), flower[b].push_back(y = st[match[x]]),
            q_push(y);
    set_st(b, b);
    for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].w = 0;
    for (int x = 1; x <= n; ++x) flower_from[b][x] = 0;
    for (int i = 0; i < flower[b].size(); ++i) {
        int xs = flower[b][i];
        for (int x = 1; x <= n_x; ++x) {
            if (g[b][x].w == 0 || Dist(g[xs][x]) < Dist(g[b][x]))
                g[b][x] = g[xs][x], g[x][b] = g[x][xs];
        }
        for (int x = 1; x <= n; ++x) if (flower_from[xs][x]) flower_from[
             b][x] = xs;
    }
    set_slack(b);
}
void expand_blossom(int b) {
    for (int i = 0; i < flower[b].size(); ++i) set_st(flower[b][i],
         flower[b][i]);
    int xr = flower_from[b][g[b][pa[b]].u], pr = get_pr(b, xr);
    for (int i = 0; i < pr; i += 2) {
        int xs = flower[b][i], xns = flower[b][i + 1];
        pa[xs] = g[xns][xs].u;
        S[xs] = 1, S[xns] = 0;
        slack[xs] = 0, set_slack(xns);
        q_push(xns);
    }
    S[xr] = 1, pa[xr] = pa[b];
    for (int i = pr + 1; i < flower[b].size(); ++i) {
        int xs = flower[b][i];
        S[xs] = -1, set_slack(xs);
    }
    st[b] = 0;
}
bool on_found_Edge(const Edge &e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
        pa[v] = e.u, S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = 0;
        S[nu] = 0, q_push(nu);
    } else if (S[v] == 0) {
        int lca = get_lca(u, v);
        if (!lca) return augment(u, v), augment(v, u), 1;
        else add_blossom(u, lca, v);
    }
    return 0;
}
bool matching() {
    fill(S, S + n_x + 1, -1), fill(slack, slack + n_x + 1, 0);
    q.clear();
    for (int x = 1; x <= n_x; ++x) if (st[x] == x && !match[x]) pa[x] =
         0, S[x] = 0, q_push(x);
    if (q.empty()) return 0;
    while(1) {
        while (q.size()) {
            int u = q.front();
            q.pop_front();
            if (S[st[u]] == 1) continue;
            for (int v = 1; v <= n; ++v) {
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (Dist(g[u][v]) == 0) {
                        if (on_found_Edge(g[u][v])) return 1;
                    } else
                        update_slack(u, st[v]);
                }
            }
        }
        int d = INF;
        for (int b = n + 1; b <= n_x; ++b) if (st[b] == b && S[b] == 1)
            chkmin(d, lab[b] / 2);
        for (int x = 1; x <= n_x; ++x) {
            if (st[x] == x && slack[x]) {
                if (S[x] == -1)
                    d = min(d, Dist(g[slack[x]][x]));
                else if (S[x] == 0)
                    d = min(d, Dist(g[slack[x]][x]) / 2);
            }
        }
        for (int u = 1; u <= n; ++u) {
            if (S[st[u]] == 0) {
                if (lab[u] <= d) return 0;
                lab[u] -= d;
            } else if (S[st[u]] == 1)
                lab[u] += d;
        }
        for (int b = n + 1; b <= n_x; ++b) {
            if (st[b] == b) {
                if (S[st[b]] == 0)
                    lab[b] += d * 2;
                else if (S[st[b]] == 1)
                    lab[b] -= d * 2;
            }
        }
        q.clear();
        for (int x = 1; x <= n_x; ++x) {
            if (st[x] == x && slack[x] && st[slack[x]] != x &&
                Dist(g[slack[x]][x]) == 0)
                if (on_found_Edge(g[slack[x]][x])) return 1;
        }
        for (int b = n + 1; b <= n_x; ++b)
            if (st[b] == b && S[b] == 1 && lab[b] == 0) expand_blossom(b)
                ;
    }
    return 0;
}
pair<ll, int> weight_blossom() {
    fill(match, match + n + 1, 0);
    n_x = n;
    int n_matches = 0;
    ll tot_weight = 0;
    for (int u = 0; u <= n; ++u) st[u] = u, flower[u].clear();
    int w_max = 0;
    for (int u = 1; u <= n; ++u) {
        for (int v = 1; v <= n; ++v) {
            flower_from[u][v] = (u == v ? u : 0);
            w_max = max(w_max, g[u][v].w);
        }
    }
    ll answer = 0;
    for (int u = 1; u <= n; ++u) lab[u] = w_max;
    while (matching()) ++n_matches;
    for(int u=1; u<=n; ++u)
        if(match[u]&&match[u]<u)
            tot_weight+=g[u][match[u]].w;
    return make_pair(tot_weight,n_matches);
}
```

Geometry (5)

Point.cpp

Description: struct Point 80dfd5, 80 lines

```
const ld EPS = 1e-7;

ld sq(ld x) {
    return x * x;
}

int sign(ld x) {
    if (x < -EPS) {
        return -1;
    }
    if (x > EPS) {
        return 1;
    }
    return 0;
}

#define vec point
struct point {//% - cross, * - dot
    ld x, y;
    auto operator<=>(const point&) const = default;
};
ld operator*(const point &a, const point &b) {
    return a.x * b.x + a.y * b.y;
}
ld operator%(const point &a, const point &b) {
    return a.x * b.y - a.y * b.x;
}
point operator-(const point &a, const point &b) {
    return {a.x - b.x, a.y - b.y};
}
point operator+(const point &a, const point &b) {
    return {a.x + b.x, a.y + b.y};
}
point operator*(const point &a, ld b) {
    return {a.x * b, a.y * b};
}
point operator/(const point &a, ld b) {
    return {a.x / b, a.y / b};
}
bool operator<(const point &a, const point &b)  {
    if (sign(a.y - b.y) != 0) {
        return a.y < b.y;
    } else if (sign(a.x - b.x) != 0) {
        return a.x < b.x;
    }
    return 0;
}
ld len2(const point &a) {
    return sq(a.x) + sq(a.y);
}
ld len(const point &a) {
    return sqrt(len2(a));
}
point norm(point a) {
    return a / len(a);
}
int half(point a) {
    return (sign(a.y) == -1 || (sign(a.y) ==0 && a.x < 0));
}
point ort(point a) {
    return {-a.y, a.x};
}
point turn(point a, ld ang) {
    return {a.x * cos(ang) - a.y * sin(ang), a.x * sin(ang) + a.y * cos(
        ang)};
}
ld getAngle(point &a, point &b) {
    return atan2(a % b, a * b);
}
bool cmpHalf(const point &a, const point &b) {
    if (half(a) != half(b)) {
        return half(b);
    } else {
        int sgn = sign(a % b);
        if (!sgn) {
            return len2(a) < len2(b);
        } else {
            return sgn == 1;
        }
    }
}
```

Line.cpp

Description: struct Line 887306, 26 lines

```
struct line {
    ld a, b, c;
    void norm() {
        // for half planes
        ld d = len({a, b});
        assert(sign(d) > 0);
        a /= d;
        b /= d;
        c /= d;
    }
    ld eval(point p) const { return a * p.x + b * p.y + c; }
    bool isIn(point p) const { return sign(eval(p)) >= 0; }
    bool operator==(const line &other) const {
        return sign(a * other.b - b * other.a) == 0 &&
               sign(a * other.c - c * other.a) == 0 &&
               sign(b * other.c - c * other.b) == 0;
    }
};
line getln(point a, point b) {
    line res;
    res.a = a.y - b.y;
    res.b = b.x - a.x;
    res.c = -(res.a * a.x + res.b * a.y);
    res.norm();
    return res;
}
```

Intersections.cpp

Description: Geometry intersections 45d7d9, 75 lines

```
bool isCrossed(ld lx, ld rx, ld ly, ld ry) {
    if (lx > rx)
        swap(lx, rx);
    if (ly > ry)
        swap(ly, ry);
    return sign(min(rx, ry) - max(lx, ly)) >= 0;
}

// if two segments [a, b] and [c, d] has AT LEAST one common point ->
    true
bool intersects(const point &a, const point &b, const point &c, const
    point &d) {
    if (!isCrossed(a.x, b.x, c.x, d.x))
        return false;
    if (!isCrossed(a.y, b.y, c.y, d.y))
        return false;
    if (sign((b - a) % (c - a)) * sign((b - a) % (d - a)) == 1) return 0;
    if (sign((d - c) % (a - c)) * sign((d - c) % (b - c)) == 1) return 0;
    return 1;
}
//intersecting lines
bool intersect(line l, line m, point &I) {
    ld d = l.b * m.a - m.b * l.a;
    if (sign(d) == 0) {
        return false;
    }
    ld dx = m.b * l.c - m.c * l.b;
    ld dy = m.c * l.a - l.c * m.a;
    I = {dx / d, dy / d};
    return true;
}
//intersecting circles
int intersect(point o1, ld r1, point o2, ld r2, point &i1, point &i2) {
    if (r1 < r2) {
        swap(o1, o2);
        swap(r1, r2);
    }
    if (sign(r1 - r2) == 0 && len2(o2 - o1) < EPS) {
        return 3;
    }
    ld ln = len(o1 - o2);
    if (sign(ln - r1 - r2) == 1 || sign(r1 - ln - r2) == 1) {
        return 0;
    }
    ld d = (sq(r1) - sq(r2) + sq(ln)) / 2 / ln;
    vec v = norm(o2 - o1);
    point a = o1 + v * d;
    if (sign(ln - r1 - r2) == 0 || sign(ln + r2 - r1) == 0) {
        i1 = a;
        return 1;
    }
    v = ort(v) * sqrt(sq(r1) - sq(d));
    i1 = a + v;
    i2 = a - v;
    return 2;
}
//intersecting line and circle, line should be normed
int intersect(point o, ld r, line l, point &i1, point &i2) {
    ld len = abs(l.eval(o));
    int sgn = sign(len - r);
    if (sgn == 1) {
        return 0;
    }
    vec v = norm(vec{l.a, l.b}) * len;
    if (sign(l.eval(o + v)) != 0) {
        v = vec{0, 0} - v;
    }
    point a = o + v;
    if (sgn == 0) {
        i1 = a;
        return 1;
    }
    v = norm({-l.b, l.a}) * sqrt(sq(r) - sq(len));
    i1 = a + v;
    i2 = a - v;
    return 2;
}
```

Tangents.cpp

Description: Tangents to circles. c73373, 43 lines

```
// tangents from point to circle
int tangents(point &o, ld r, point &p, point &i1, point &i2) {
    ld ln = len(o - p);
    int sgn = sign(ln - r);
    if (sgn == -1) {
        return 0;
    } else if (sgn == 0) {
        i1 = p;
        return 1;
    } else {
        ld x = sq(r) / ln;
        vec v = norm(p - o) * x;
        point a = o + v;
        v = ort(norm(p - o)) * sqrt(sq(r) - sq(x));
        i1 = a + v;
        i2 = a - v;
        return 2;
    }
}

void _tangents(point c, ld r1, ld r2, vector<line> &ans) {
    ld r = r2 - r1;
    ld z = sq(c.x) + sq(c.y);
    ld d = z - sq(r);
    if (sign(d) == -1)
        return;
    d = sqrt(abs(d));
    line l;
    l.a = (c.x * r + c.y * d) / z;
    l.b = (c.y * r - c.x * d) / z;
    l.c = r1;
    ans.push_back(l);
}
// tangents between two circles
vector<line> tangents(point o1, ld r1, point o2, ld r2) {
    vector<line> ans;
```

```
    for (int i = -1; i <= 1; i += 2)
        for (int j = -1; j <= 1; j += 2)
            _tangents(o2 - o1, r1 * i, r2 * j, ans);
    for (int i = 0; i < (int)ans.size(); ++i)
        ans[i].c -= ans[i].a * o1.x + ans[i].b * o1.y;
    return ans;
}
```

Hull.cpp

Description: Polygon functions fc1928, 16 lines

```
vector<point> hull(vector<point> p, bool need_all=false) {
   sort(all(p));
   p.erase(unique(all(p)), end(p));
   int n = p.size(), k = 0;
   if (n <= 2) return p;
   vector<point> ch(2 * n);
   ld th = need_all ? -EPS : +EPS; // 0 : 1 if int
   for (int i = 0; i < n; ch[k++] = p[i++]) {
    while (k >= 2 && (ch[k - 1] - ch[k - 2]) % (p[i] - ch[k - 1]) < th)
          --k;
   }
   for (int i = n - 2, t = k + 1; i >= 0; ch[k++] = p[i--]) {
    while (k >= t && (ch[k - 1] - ch[k - 2]) % (p[i] - ch[k - 1]) < th)
          --k;
   }
   ch.resize(k - 1);
   return ch;
 }
```

IsInPolygon.cpp

Description: Is in polygon functions f17b31, 65 lines

```
bool isOnSegment(point &a, point &b, point &x) {
    if (sign(len2(a - b)) == 0) {
        return sign(len(a - x)) == 0;
    }
    return sign((b - a) % (x - a)) == 0 && sign((b - x) * (a - x)) <= 0;
    // optional (slower, but works better if there are some precision
    // problems) return sign((b - a).len() - (x - a).len() - (x - b).len
          ())
    // == 0;
}

int isIn(vector<point> &p, point &a) {
    int n = p.size();
    // depends on limitations(2*MAXC + 228)
    point b = a + point{2e9 + 228, 1};
    int cnt = 0;
    for (int i = 0; i < n; ++i) {
        point x = p[i];
        point y = p[i + 1 < n ? i + 1 : 0];
        if (isOnSegment(x, y, a)) {
            // depends on the problem statement
            return 1;
        }
        cnt += intersects(x, y, a, b);
    }
    return 2 * (cnt % 2 == 1);
    /*optional (atan2 is VERY SLOW)!
    ld ans = 0;
    int n = p.size();
    for (int i = 0; i < n; ++i) {
      Point x = p[i];
      Point y = p[i + 1 < n ? i + 1 : 0];
      if (isOnSegment(x, y, a)) {
        // depends on the problem statement
        return true;
      }
      x = x - a;
      y = y - a;
      ans += atan2(x ^ y, x * y);
    }
    return abs(ans) > 1;*/
}

bool isInTriangle(point &a, point &b, point &c, point &x) {
    return sign((b - a) % (x - a)) >= 0 && sign((c - b) % (x - b)) >= 0
          &&
           sign((a - c) % (x - c)) >= 0;
}

// points should be in the counterclockwise order
bool isInConvex(vector<point> &p, point &a) {
    int n = p.size();
    assert(n >= 3);
    // assert(isConvex(p));
    // assert(isCounterclockwise(p));
    if (sign((p[1] - p[0]) % (a - p[0])) < 0)
        return 0;
    if (sign((p[n - 1] - p[0]) % (a - p[0])) > 0)
        return 0;
    int pos = lower_bound(p.begin() + 2, p.end(), a,
                          [&](point a, point b) -> bool {
                              return sign((a - p[0]) % (b - p[0])) > 0;
                          }) -
              p.begin();
    assert(pos > 1 && pos < n);
    return isInTriangle(p[0], p[pos - 1], p[pos], a);
}
```

Diameter.cpp

Description: Rotating calipers.
Time: $\mathcal{O}(n)$ 0f341c, 21 lines

```
ld diameter(vector<point> p) {
    p = hull(p);
    int n = p.size();
    if (n <= 1) {
        return 0;
    }
    if (n == 2) {
        return len(p[0] - p[1]);
    }
    ld ans = 0;
    int i = 0, j = 1;
    while (i < n) {
        while (sign((p[(i + 1) % n] - p[i]) % (p[(j + 1) % n] - p[j])) >=
              0) {
            chkmax(ans, len(p[i] - p[j]));
            j = (j + 1) % n;
        }
        chkmax(ans, len(p[i] - p[j]));
        ++i;
    }
    return ans;
}
```

TangentsAlex.cpp

Description: Find both tangets to the convex polygon. (Zakaldovany algos mozhet sgonyat za pivom tak zhe).
Time: $\mathcal{O}(\log(n))$ 2eeea8, 17 lines

```
pair<int, int> tangents_alex(vector<point> &p, point &a) {
    int n = p.size();
    int l = __lg(n);
    auto findWithSign = [&](int val) {
        int i = 0;
        for (int k = l; k >= 0; --k) {
            int i1 = (i - (1 << k) + n) % n;
            int i2 = (i + (1 << k)) % n;
            if (sign((p[i1] - a) % (p[i] - a)) == val)
                i = i1;
            if (sign((p[i2] - a) % (p[i] - a)) == val)
                i = i2;
        }
        return i;
    };
    return {findWithSign(1), findWithSign(-1)};
}
```

IsHpiEmpty.cpp

Description: Determines is half plane intersectinos.
Time: $\mathcal{O}(n)$ (expected) 3b5e69, 42 lines

```
// all lines must be normed!!!!!, sign > 0
bool isHpiEmpty(vector<line> lines) {
    // return hpi(lines).empty();
    // overflow/precision problems?
    shuffle(all(lines), rnd);
    const ld C = 1e9;
    point ans(C, C);
    vector<point> box = {{-C, -C}, {C, -C}, {C, C}, {-C, C}};
    for (int i = 0; i < 4; ++i)
        lines.push_back(getln(box[i], box[(i + 1) % 4]));
    int n = lines.size();
    for (int i = n - 4; i >= 0; --i) {
        if (lines[i].isIn(ans))
            continue;
        point up(0, C + 1), down(0, -C - 1), pi = {lines[i].b, -lines[i].
              a};
        for (int j = i + 1; j < n; ++j) {
            if (lines[i] == lines[j])
                continue;
            point p, pj = {lines[j].b, -lines[j].a};
            if (!intersect(lines[i], lines[j], p)) {
                if (sign(pi * pj) != -1)
                    continue;
                if (sign(lines[i].c + lines[j].c) *
                        (!sign(pi.y) ? sign(pi.x) : -1) ==
                    1)
                    return true;
            } else {
                if ((!sign(pi.y) ? sign(pi.x) : sign(pi.y)) * (sign(pi %
                      pj)) ==
                    1)
                    chkmin(up, p);
                else
                    chkmax(down, p);
            }
        }
        if ((ans = up) < down)
            return true;
    }
    // for (int i = 0; i < n; ++i) {
    //   assert(lines[i].eval(ans) < EPS);
    // }
    return false;
}
```

HalfPlaneIntersection.cpp

Description: Find the intersection of the half planes.
Time: $\mathcal{O}(n \log(n))$ fdf28f, 62 lines

```
vec getPoint(line l) { return {-l.b, l.a}; }

bool bad(line a, line b, line c) {
    point x;
    assert(intersect(b, c, x) == 1);
    return a.eval(x) < 0;
}

// Do not forget about the bounding box
vector<point> hpi(vector<line> lines) {
    sort(all(lines), [](line al, line bl) -> bool {
        point a = getPoint(al);
        point b = getPoint(bl);
        if (half(a) != half(b)) {
            return half(a) < half(b);
        }
        return a % b > 0;
    });

    vector<pair<line, int>> st;
    for (int it = 0; it < 2; it++) {
        for (int i = 0; i < (int)lines.size(); i++) {
            bool flag = false;
            while (!st.empty()) {
                if (len(getPoint(st.back().first) - getPoint(lines[i])) <
                      EPS) {
                    if (lines[i].c >= st.back().first.c) {
                        flag = true;
                        break;
                    } else {
                        st.pop_back();
                    }
```

```
            } else if (getPoint(st.back().first) % getPoint(lines[i])
                    < EPS / 2) {
                return {};
            } else if (st.size() >= 2 &&
                       bad(st[st.size() - 2].first, st[st.size() -
                           1].first,
                         lines[i])) {
                st.pop_back();
            } else {
                break;
            }
        }
        if (!flag)
            st.push_back({lines[i], i});
        }
    }

    vector<int> en(lines.size(), -1);
    vector<point> ans;
    for (int i = 0; i < (int)st.size(); i++) {
        if (en[st[i].second] == -1) {
            en[st[i].second] = i;
            continue;
        }
        for (int j = en[st[i].second]; j < i; j++) {
            point I;
            assert(intersect(st[j].first, st[j + 1].first, I) == 1);
            ans.push_back(I);
        }
        break;
    }
    return ans;
}
```

CHT.cpp

Description: CHT for minimum, k is decreasing, works for equal slopes 30baab, 34 lines

```
struct line {
    int k, b;
    int eval(int x) {
        return k * x + b;
    }
};
struct part {
    line a;
    ld x;
};
ld intersection(line a, line b) {
    return (ld) (a.b - b.b) / (b.k - a.k);
}
struct ConvexHullMin {
    vector <part> st;
    void add(line a) {
        if (!st.empty() && st.back().a.k == a.k) {
            if (st.back().a.b > a.b) st.pop_back();
            else return;
        }
        while (st.size() > 1 && intersection(st[st.size() - 2].a, a) <=
               st[st.size() - 2].x) st.pop_back();
        if (!st.empty()) st.back().x = intersection(st.back().a, a);
        st.push_back({a, INF});
    }
    int get_val(int x) {
        int l = -1, r = (int)st.size() - 1;
        while (r - l > 1) {
            int m = (l + r) / 2;
            if (st[m].x < x) l = m;
            else r = m;
        }
        return st[r].a.eval(x);
    }
};
```

DynamicCHT.cpp

Description: Dynamic CHT for maximum 8a0777, 30 lines

```
struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const {
        return Q ? p < o.p : k < o.k;
    }
};
struct LineContainer : multiset<Line> {
    const ll inf = LLONG_MAX;
    ll div(ll a, ll b){
        return a / b - ((a ^ b) < 0 && a % b);
    }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        Q = 1; auto l = *lower_bound({0,0,x}); Q = 0;
        return l.k * x + l.m;
    }
};
```

MinPlusConv.cpp

Description: Min-Plusconv, A is convex down
Time: $\mathcal{O}(nlognfast)$ 5d63d9, 28 lines

```
// Assumptions: 'a' is convex, 'opt' has size 'n+m-1'
// 'opt[k]' will be equal to 'arg min(a[k-i] + b[i])'
template<typename T>
void convex_arbitrary_min_plus_conv(T *a, int n, T *b, int m, int *opt) {
    auto rec = [&](auto &&self, int lx, int rx, int ly, int ry) -> void {
        if (lx > rx) return;
        int mx = (lx + rx) >> 1;
        opt[mx] = ly;
        for (int i = ly; i <= ry; ++i)
            if (mx >= i && (mx - opt[mx] >= n || a[mx - opt[mx]] + b[opt[
                mx]] > a[mx - i] + b[i]))
                opt[mx] = i;
        self(self, lx, mx - 1, ly, opt[mx]);
        self(self, mx + 1, rx, opt[mx], ry);
    };
    rec(rec, 0, n + m - 2, 0, m - 1);
}

// Assumptions: 'a' is convex down
template<typename T>
std::vector<T> convex_arbitrary_min_plus_conv(const std::vector<T> &a,
    const std::vector<T> &b) {
    int n = a.size(), m = b.size();
    int *opt = (int*) malloc(sizeof(int) * (n + m - 1));
    convex_arbitrary_min_plus_conv(a.data(), n, b.data(), m, opt);
    std::vector<T> ans(n + m - 1);
    for (int i = 0; i < n + m - 1; ++i) ans[i] = a[i - opt[i]] + b[opt[i
        ]];
    free(opt);
    return ans;
}
```

Kinetic.cpp

Description: kinetic segment tree
Time: $\mathcal{O}(hz)$ 49b24c, 127 lines

```
//vnutrennii functions - poluintervali, vneshnie - otrezki. ishet min
    priamuy
struct line {
    ll k,b,temp;
    ll eval() const {
        return k * temp + b;
    }
    ll melting_point(const line& other) const {
        ll val1 = eval();
        ll val2 = other.eval();
        assert(val1 <= val2);
        if (other.k >= k) {
            return INF;
        }
        ll delta_val = val2 - val1;
        ll delta_k = k - other.k;
        assert(delta_val >= 0 && delta_k > 0);
        return (delta_val + delta_k - 1) / delta_k;
    }
};
struct kinetic_segtree {
    struct node {
        ll lazy_b = 0, lazy_temp = 0, melt = INF;
        line best;
        node(line best = line()) : best(best) {}
    };
    int n;
    vector<node> tree;
    void update(int v) {
        if (make_pair(tree[v << 1].best.eval(), tree[v << 1].best.k) <
             make_pair(tree[v << 1 | 1].best.eval(), tree[v << 1 | 1].
             best.k)) {
            tree[v].best = tree[v << 1].best;
            tree[v].melt = tree[v].best.melting_point(tree[v << 1 | 1].
                best);
        } else {
            tree[v].best = tree[v << 1 | 1].best;
            tree[v].melt = tree[v].best.melting_point(tree[v << 1].best);
        }

        tree[v].melt = min({tree[v].melt, tree[v << 1].melt, tree[v << 1
              | 1].melt});
        assert(tree[v].melt > 0);
    }
    void apply(int v, int vl, int vr, ll delta_b, ll delta_temp) {
        tree[v].lazy_b += delta_b;
        tree[v].lazy_temp += delta_temp;

        tree[v].best.b += delta_b;
        tree[v].best.temp += delta_temp;

        tree[v].melt -= delta_temp;
        if (tree[v].melt <= 0) {
            push(v, vl, vr);
            update(v);
        }
    }
    void push(int v, int vl, int vr) {
        int vm = (vl + vr) / 2;
        apply(v << 1, vl, vm, tree[v].lazy_b, tree[v].lazy_temp);
        apply(v << 1 | 1, vm, vr, tree[v].lazy_b, tree[v].lazy_temp);
        tree[v].lazy_b = 0;
        tree[v].lazy_temp = 0;
    }
    void build(int v, int vl, int vr, const vector<line> &lines) {
        if (vr - vl == 1) {
            tree[v] = node(lines[vl]);
            return;
        }
        int vm = (vl + vr) / 2;
        build(v << 1, vl, vm, lines);
        build(v << 1 | 1, vm, vr, lines);
        update(v);
    }
    void add(int v, int vl, int vr, int l, int r, ll delta_b, ll
         delta_temp) {
        if (r <= vl || vr <= l) {
            return;
        }
        if (l <= vl && vr <= r) {
            apply(v, vl, vr, delta_b, delta_temp);
            return;
        }
        push(v, vl, vr);
        int vm = (vl + vr) / 2;
        add(v << 1, vl, vm, l, r, delta_b, delta_temp);
        add(v << 1 | 1, vm, vr, l, r, delta_b, delta_temp);
        update(v);
    }
```

```
    void change_line(int v, int vl, int vr, int pos, const line &new_line
        ) {
        if (vr - vl == 1) {
            tree[v].best = new_line;
            return;
        }
        push(v, vl, vr);
        int vm = (vl + vr) / 2;
        if (pos < vm) {
            change_line(v << 1, vl, vm, pos, new_line);
        } else {
            change_line(v << 1 | 1, vm, vr, pos, new_line);
        }
        update(v);
    }
    ll query(int v, int vl, int vr, int l, int r) {
        if (r <= vl || vr <= l) {
            return INF;
        }
        if (l <= vl && vr <= r) {
            return tree[v].best.eval();
        }
        push(v, vl, vr);
        int vm = (vl + vr) / 2;
        return min(query(v << 1, vl, vm, l, r), query(v << 1 | 1, vm, vr,
            l, r));
    }
    kinetic_segtree(const vector<line> &lines) : n(lines.size()), tree(4
        * n) {
        build(1, 0, n, lines);
    }
    kinetic_segtree(int n) : n(n), tree(4 * n) {
        vector <line> lines(n, {0, INF, 0});
        build(1, 0, n, lines);
    }
    void add(int l, int r, ll delta_b, ll delta_temp) {
        assert(delta_temp >= 0);
        add(1, 0, n, l, r + 1, delta_b, delta_temp);
    }
    void change_line(int pos, const line &new_line) {
        assert(0 <= pos && pos < n);
        change_line(1, 0, n, pos, new_line);
    }
    ll query(int l, int r) {
        return query(1, 0, n, l, r + 1);
    }
}
```

Math (6)

BerlekampMassey.cpp

Description: Find the shortest linear-feedback shift register
Time: $\mathcal{O}\left(n^2\right)$

08eddc, 86 lines

```
vector<int> berlekamp(vector<int> x) {
    vector<int> ls, cur;
    int lf = 0, d = 0;
    for (int i = 0; i < x.size(); ++i) {
        ll t = 0;
        for (int j = 0; j < cur.size(); ++j) {
            t = (t + (ll) x[i - j - 1] * cur[j]) % MOD;
        }
        if ((t - x[i]) % MOD == 0)
            continue;
        if (cur.empty()) {
            cur.resize(i + 1);
            lf = i;
            d = (t - x[i]) % MOD;
            continue;
        }
        ll k = -(x[i] - t) * powmod(d, MOD - 2) % MOD;
        vector<int> c(i - lf - 1);
        c.push_back(k);
        for (auto &j : ls)
            c.push_back(-j * k % MOD);
        if (c.size() < cur.size())
            c.resize(cur.size());
        for (int j = 0; j < cur.size(); ++j) {
            c[j] = (c[j] + cur[j]) % MOD;
        }
        if (i - lf + (int)ls.size() >= (int)cur.size()) {
            tie(ls, lf, d) = make_tuple(cur, i, (t - x[i]) % MOD);
        }
        cur = c;
    }
    for (auto &i : cur)
        i = (i % MOD + MOD) % MOD;
    return cur;
}
// for a_i = 2 * a_i-1 + a_i-2 returns {2, 1}

// kth element of p/q as fps
int getkfps(vector<int> p, vector<int> q, ll k) {
    assert(q[0] != 0);
    while (k) {
        auto f = q;
        for (int i = 1; i < (int) f.size(); i += 2) {
            f[i] = sub(0, f[i]);
        }
        auto p2 = conv(p, f);
        auto q2 = conv(q, f);
        p.clear(), q.clear();
        for (int i = k % 2; i < (int) p2.size(); i += 2) {
            p.pbc(p2[i]);
        }
        for (int i = 0; i < (int)q2.size(); i += 2) {
            q.pbc(q2[i]);
        }
        k >>= 1;
    }
    return mul(p[0], inv(q[0]));
}

// vals - initials values of reccurence, c - result of belekamp on vals
int getk(const vector<int> &vals, vector<int> c, ll k) {
    int d = c.size();
    c.insert(c.begin(), MOD-1);
    while (c.back() == 0) {
        c.pop_back();
    }
    for (auto &el : c) {
        el = sub(0, el);
    }
    vector<int> p(d);
    copy(vals.begin(), vals.begin() + d, p.begin());
    p = conv(p, c);
    p.resize(d);
    return getkfps(p, c, k);
}

vector<int> getmod(vector<int> a, vector<int> md) {
    for (int i = a.size() - 1; i + 1 >= md.size(); --i) {
        int v = mul(a[i], inv(md.back()));
        for (int j = 0; j < md.size(); ++j) {
            a[i - md.size() + 1 + j] = sub(a[i - md.size() + 1 + j], mul(
                md[j], v));
        }
        a.pop_back();
    }
    return a;
}
```

GoncharFedor.cpp

Description: Calculating number of points $x, y \geq 0, Ax + By \leq C$
Time: $\mathcal{O}(\log(C))$

0ef10e, 11 lines

```
ll solve_triangle(ll A, ll B, ll C) { // x,y >=0, Ax+By<=C
    if (C < 0)
        return 0;
    if (A > B)
        swap(A, B);
    ll p = C / B;
    ll k = B / A;
    ll d = (C - p * B) / A;
    return solve_triangle(B - k * A, A, C - A * (k * p + d + 1)) +
           (p + 1) * (d + 1) + k * p * (p + 1) / 2;
}
```

CRT.cpp

Description: CRT for arbitrary modulos

28309e, 25 lines

```
int extgcd(int a, int b, int &x, int &y) {  // define int ll
    if (a == 0) {
        x = 0, y = 1;
        return b;
    }
    int x1, y1;
    int g = extgcd(b % a, a, x1, y1);
    x = y1 - x1 * (b / a);
    y = x1;
    return g;
}
int lcm(int a, int b) { return a / __gcd(a, b) * b; }
int crt(int mod1, int mod2, int rem1, int rem2) {
    int r = (rem2 - (rem1 % mod2) + mod2) % mod2;
    int x, y;
    int g = extgcd(mod1, mod2, x, y);
    if (r % g) return -1;
    x %= mod2;
    if (x < 0) x += mod2;
    int ans = (x * (r / g)) % mod2;
    ans = ans * mod1 + rem1;
    assert(ans % mod1 == rem1);
    assert(ans % mod2 == rem2);
    return ans % lcm(mod1, mod2);
}
```

Fastmod.cpp

Description: Fast multiplication by modulo(in [0;2b))

38ea39, 7 lines

```
struct FastMod {
    ull b, m;
    FastMod(ull b) : b(b), m(-1ULL / b) {}
    ull reduce(ull a) { // a % b + (0 or b)
        return a - (ull)((__uint128_t(m) * a) >> 64) * b;
    }
};
```

PrimalityTest.cpp

Description: Checking primality of p
Time: $\mathcal{O}(\log(C))$

ad2714, 32 lines

```
const int iters = 8; // can change
bool isprime(ll p) {
    if (p == 1 || p == 4)
```

```
        return 0;
    if (p == 2 || p == 3)
        return 1;
    for (int it = 0; it < iters; ++it) {
        ll a = rnd() % (p - 2) + 2;
        ll nw = p - 1;
        while (nw % 2 == 0)
            nw /= 2;
        ll x = binpow(a, nw, p); // int128
        if (x == 1)
            continue;
        ll last = x;
        nw *= 2;
        while (nw <= p - 1) {
            x = (__int128_t)x * x % p;
            if (x == 1) {
                if (last != p - 1) {
                    return 0;
                }
                break;
            }
            last = x;
            nw *= 2;
        }
        if (x != 1)
            return 0;
    }
    return 1;
}
```

XorConvolution.cpp

Description: Calculating xor-convolution of 2 vectors modulo smth
Time: $\mathcal{O}(n\log(n))$ 454afd, 23 lines

```
void fwht(vector<int> &a) {
    int n = a.size();
    for (int l = 1; l < n; l <<= 1) {
        for (int i = 0; i < n; i += 2 * l) {
            for (int j = 0; j < l; ++j) {
                int u = a[i + j], v = a[i + j + l];
                a[i + j] = add(u, v), a[i + j + l] = sub(u, v);
            }
        }
    }
} // https://judge.yosupo.jp/problem/bitwise_xor_convolution
vector<int> xorconvo(vector<int> a, vector<int> b) {
    int n = 1;
    while (n < max(a.size(), b.size()))
        n *= 2;
    a.resize(n), b.resize(n);
    fwht(a), fwht(b);
    int in = inv(n);
    for (int i = 0; i < n; ++i)
        a[i] = mul(a[i], mul(b[i], in));
    fwht(a);
    return a;
}
```

Factorization.cpp

Description: Factorizing a number real quick
Time: $\mathcal{O}\left(n^{\frac{1}{4}}\right)$ f0d7c6, 51 lines

```
ll gcd(ll a, ll b) {
    while (b)
        a %= b, swap(a, b);
    return a;
}

ll f(ll a, ll n) { return ((__int128_t)a * a % n + 1) % n; }

vector<ll> factorize(ll n) {
    if (n <= 1e6) { // can add primality check for speed?
        vector<ll> res;
        for (ll i = 2; i * i <= n; ++i) {
            while (n % i == 0) {
                res.pbc(i);
                n /= i;
            }
        }
        if (n != 1)
            res.pbc(n);
        return res;
    }
    ll x = rnd() % (n - 1) + 1;
    ll y = x;
    ll tries = 10 * sqrt(sqrt(n));
    const int C = 60;
    for (ll i = 0; i < tries; i += C) {
        ll xs = x;
        ll ys = y;
        ll m = 1;
        for (int k = 0; k < C; ++k) {
            x = f(x, n);
            y = f(f(y, n), n);
            m = (__int128_t)m * abs(x - y) % n;
        }
        if (gcd(n, m) == 1)
            continue;
        x = xs, y = ys;
        for (int k = 0; k < C; ++k) {
            x = f(x, n);
            y = f(f(y, n), n);
            ll res = gcd(n, abs(x - y));
            if (res != 1 && res != n) {
                vector<ll> v1 = factorize(res), v2 = factorize(n / res);
                for (auto j : v2)
                    v1.pbc(j);
                return v1;
            }
        }
    }
    return {n};
}
```

NTT.cpp

Description: Calculating FFT modulo MOD
Time: $\mathcal{O}(n\log(n))$ 3e2f3a, 226 lines

```
// DONT FORGET TO CALL initNTT() AND CHECK MAXLOG

const int MOD = 998244353;
const int G = 3;
const int MAXLOG = 23;
int W[1 << MAXLOG];
bool nttinit = false;
vector<int> pws;

int add(int a, int b) {
    a += b;
    if (a >= MOD) {
        return a - MOD;
    }
    return a;
}

int sub(int a, int b) {
    a -= b;
    if (a < 0) {
        return a + MOD;
    }
    return a;
}

int mul(int a, int b) {
    return (ll) a * b % MOD;
}

int power(int a, int n) {
    int ans = 1;
    while (n) {
        if (n & 1) {
            ans = mul(ans, a);
        }
        a = mul(a, a);
        n >>= 1;
    }
    return ans;
}

int inv(int a) {
    return power(a, MOD - 2);
}

void initNTT() {
    assert((MOD - 1) % (1 << MAXLOG) == 0);
    pws.push_back(power(G, (MOD - 1) / (1 << MAXLOG)));
    for (int i = 0; i < MAXLOG - 1; ++i) {
        pws.push_back(mul(pws.back(), pws.back()));
    }
    assert(pws.back() == MOD - 1);
    W[0] = 1;
    for (int i = 1; i < (1 << MAXLOG); ++i) {
        W[i] = mul(W[i - 1], pws[0]);
    }
}

void ntt(int n, vector <int>& a, bool rev) {
    if (!nttinit) {
        initNTT();
        nttinit = 1;
    }
    int lg = log2(n);
    vector<int> rv(n);
    for (int i = 1; i < n; ++i) {
        rv[i] = (rv[i >> 1] >> 1) ^ ((i & 1) << (lg - 1));
        if (rv[i] > i) swap(a[i], a[rv[i]]);
    }
    int num = MAXLOG - 1;
    for (int len = 1; len < n; len *= 2) {
        for (int i = 0; i < n; i += 2 * len) {
            for (int j = 0; j < len; ++j) {
                int u = a[i + j], v = mul(W[j << num], a[i + j + len]);
                a[i + j] = add(u, v);
                a[i + j + len] = sub(u, v);
            }
        }
        --num;
    }
    if (rev) {
        int rev_n = power(n, MOD - 2);
        for (int i = 0; i < n; ++i) a[i] = mul(a[i], rev_n);
        reverse(a.begin() + 1, a.end());
    }
}

vector<int> conv(vector<int> a, vector<int> b) {
    int lg = 0;
    while ((1 << lg) < a.size() + b.size() + 1)
        ++lg;
    int n = 1 << lg;
    assert(a.size() + b.size() <= n + 1);
    a.resize(n);
    b.resize(n);
    ntt(n, a, false);
    ntt(n, b, false);
    for (int i = 0; i < n; ++i) {
        a[i] = mul(a[i], b[i]);
    }
    ntt(n, a, true);
    while (a.size() > 1 && a.back() == 0) {
        a.pop_back();
    }
    return a;
}

vector<int> add(vector<int> a, vector<int> b) {
    a.resize(max(a.size(), b.size()));
    for (int i = 0; i < (int) b.size(); ++i) {
        a[i] = add(a[i], b[i]);
    }
    return a;
}

vector<int> sub(vector<int> a, vector<int> b) {
    a.resize(max(a.size(), b.size()));
    for (int i = 0; i < (int) b.size(); ++i) {
```

```cpp
        a[i] = sub(a[i], b[i]);
    }
    return a;
}

vector<int> inv(const vector<int> &a, int need) {
    vector<int> b = {inv(a[0])};
    while ((int) b.size() < need) {
        vector<int> a1 = a;
        int m = b.size();
        a1.resize(min((int) a1.size(), 2 * m));
        b = conv(b, sub({2}, conv(a1, b)));
        b.resize(2 * m);
    }
    b.resize(need);
    return b;
}

vector<int> div(vector<int> a, vector<int> b) {
    if (count(all(a), 0) == a.size()) {
        return {0};
    }
    assert(a.back() != 0 && b.back() != 0);
    int n = a.size() - 1;
    int m = b.size() - 1;
    if (n < m) {
        return {0};
    }
    reverse(all(a));
    reverse(all(b));
    a.resize(n - m + 1);
    b.resize(n - m + 1);
    vector<int> c = inv(b, b.size());
    vector<int> q = conv(a, c);
    q.resize(n - m + 1);
    reverse(all(q));
    return q;
}

vector<int> mod(vector<int> a, vector<int> b) {
    auto res = sub(a, conv(b, div(a, b)));
    while (res.size() > 1 && res.back() == 0) {
        res.pop_back();
    }
    return res;
}

vector<int> multipoint(vector<int> a, vector<int> x) {
    int n = x.size();
    vector<vector<int>> tree(2 * n);
    for (int i = 0; i < n; ++i) {
        tree[i + n] = {x[i], MOD - 1};
    }
    for (int i = n - 1; i; --i) {
        tree[i] = conv(tree[2 * i], tree[2 * i + 1]);
    }
    tree[1] = mod(a, tree[1]);
    for (int i = 2; i < 2 * n; ++i) {
        tree[i] = mod(tree[i >> 1], tree[i]);
    }
    vector<int> res(n);
    for (int i = 0; i < n; ++i) {
        res[i] = tree[i + n][0];
    }
    return res;
}

vector<int> deriv(vector<int> a) {
    for (int i = 1; i < (int) a.size(); ++i) {
        a[i - 1] = mul(i, a[i]);
    }
    a.back() = 0;
    if (a.size() > 1) {
        a.pop_back();
    }
    return a;
}

vector<int> integ(vector<int> a) {
    a.push_back(0);
    for (int i = (int) a.size() - 1; i; --i) {
        a[i] = mul(a[i - 1], inv(i));
    }
    a[0] = 0;
    return a;
}

vector<int> log(vector<int> a, int n) {
    assert(a[0] == 1);
    auto res = integ(conv(deriv(a), inv(a, n)));
    res.resize(n);
    return res;
}

vector<int> exp(vector<int> a, int need) {
    assert(a[0] == 0);
    vector<int> b = {1};
    while ((int) b.size() < need) {
        vector<int> a1 = a;
        int m = b.size();
        a1.resize(min((int) a1.size(), 2 * m));
        a1[0] = add(a1[0], 1);
        b = conv(b, sub(a1, log(b, 2 * m)));
        b.resize(2 * m);
    }
    b.resize(need);
    return b;
}
```

FFT.cpp

Description: Calculating product of two polynomials
Time: $\mathcal{O}(n\log(n))$

3adba5, 67 lines

```cpp
const ld PI = acos(-1);
using cd = complex<ld>;
const int MAXLOG = 19, N = (1 << MAXLOG), MAXN = (1 << MAXLOG) + 228;
int rev[MAXN];
cd w[MAXN];
bool fftInit = false;

void initFFT() {
    for (int i = 0; i < N; ++i) {
        w[i] = cd(cos(2 * PI * i / N), sin(2 * PI * i / N));
    }
    rev[0] = 0;
    for (int i = 1; i < N; ++i) {
        rev[i] = (rev[i >> 1] >> 1) ^ ((i & 1) << (MAXLOG - 1));
    }
}

void FFT(int n, vector <cd>& a, bool rv = false) {
    if (!fftInit) {
        initFFT();
        fftInit = 1;
    }
    int LOG = ceil(log2(n));
    for (int i = 0; i < n; ++i) {
        if (i < (rev[i] >> (MAXLOG - LOG))) {
            swap(a[i], a[(rev[i] >> (MAXLOG - LOG))]);
        }
    }
    for (int lvl = 0; lvl < LOG; ++lvl) {
        int len = 1 << lvl;
        for (int st = 0; st < n; st += len * 2) {
            for (int i = 0; i < len; ++i) {
                cd x = a[st + i], y = a[st + len + i] * w[i << (MAXLOG -
                    1 - lvl)];
                a[st + i] = x + y;
                a[st + i + len] = x - y;
            }
        }
    }
    if (rv) {
        reverse(a.begin() + 1, a.end());
        for (auto& i : a) i /= n;
    }
}

vector <ll> mul(vector <ll> a, vector <ll> b) {
    int xd = max(a.size(), b.size()) * 2;
    int cur = 1;
    while (cur < xd) {
        cur *= 2;
    }
    a.resize(cur);
    b.resize(cur);
    vector <cd> ma(cur), mb(cur);
    for (int i = 0; i < cur; ++i) {
        ma[i] += a[i];
        mb[i] += b[i];
    }
    FFT(cur, ma);
    FFT(cur, mb);
    for (int i = 0; i < cur; ++i) ma[i] *= mb[i];
    FFT(cur, ma, true);
    vector <ll> ans(cur);
    for (int i = 0; i < cur; ++i) {
        ans[i] = (ll)(ma[i].real() + 0.5);
    }
    return ans;
}
```

AndConvolution.cpp

Description: Calculating and-convolution modulo smth
Time: $\mathcal{O}(n\log(n))$

5dedf4, 24 lines

```cpp
void conv(vector<int> &a, bool x) {
    int n = a.size();
    for (int j = 0; (1 << j) < n; ++j) {
        for (int i = 0; i < n; ++i) {
            if (!(i & (1 << j))) {
                if (x)
                    a[i] = add(a[i], a[i | (1 << j)]);
                else
                    a[i] = sub(a[i], a[i | (1 << j)]);
            }
        }
    }
}
vector<int> andcon(vector<int> a, vector<int> b) {
    int n = 1;
    while (n < max(a.size(), b.size()))
        n *= 2;
    a.resize(n), b.resize(n);
    conv(a, 1), conv(b, 1);
    for (int i = 0; i < n; ++i)
        a[i] = mul(a[i], b[i]);
    conv(a, 0);
    return a;
}
```

Simplex.cpp

Description: Simplex
Time: exponential XD(ok for 200-300 variables/bounds)

4dda3c, 99 lines

```cpp
/* solver for linear programs of the form
maximize c^T x, subject to A x <= b, x >= 0
outputs target function for optimal solution and
the solution by reference
if unbounded above : returns inf, if infeasible : returns -inf
create Simplex_Steep <ld > LP(A, b, c), then call LP. Solve (x)
*/
template <typename DOUBLE>
struct Simplex_Steep {
    using VD = vector<DOUBLE>;
    using VVD = vector<VD>;
    using VI = vector<int>;
    DOUBLE EPS = 1e-12;
    int m, n;
    VI B, N;
    VVD D;
    Simplex_Steep(const VVD &A, const VD &b, const VD &c)
        : m(b.size()), n(c.size()), B(m), N(n + 1), D(m + 2, VD(n + 2)) {
        for (int i = 0; i < m; i++)
            for (int j = 0; j < n; j++) D[i][j] = A[i][j];
        for (int i = 0; i < m; i++) {
            B[i] = n + i;
```

```cpp
            D[i][n] = -1;
            D[i][n + 1] = b[i];
        }
        for (int j = 0; j < n; j++) {
            N[j] = j;
            D[m][j] = -c[j];
        }
        N[n] = -1;
        D[m + 1][n] = 1;
    }
    void Pivot(int r, int s) {
        for (int i = 0; i < m + 2; i++)
            if (i != r)
                for (int j = 0; j < n + 2; j++)
                    if (j != s) D[i][j] -= D[r][j] * D[i][s] / D[r][s];
        for (int j = 0; j < n + 2; j++)
            if (j != s) D[r][j] /= D[r][s];
        for (int i = 0; i < m + 2; i++)
            if (i != r) D[i][s] /= -D[r][s];
        D[r][s] = 1.0 / D[r][s];
        swap(B[r], N[s]);
    }
    bool Simplex(int phase) {
        int x = m + (int)(phase == 1);
        while (true) {
            int s = -1;
            DOUBLE c_val = -1;
            for (int j = 0; j <= n; j++) {
                if (phase == 2 && N[j] == -1) continue;
                DOUBLE norm_sq = 0;
                for (int k = 0; k <= m; k++) norm_sq += D[k][j] * D[k][j
                    ];
                norm_sq = max(norm_sq, EPS);
                DOUBLE c_val_j = D[x][j] / sqrtl(norm_sq);
                if (s == -1 || c_val_j < c_val ||
                    (c_val == c_val_j && N[j] < N[s])) {
                    s = j;
                    c_val = c_val_j;
                }
            }
            if (D[x][s] >= -EPS) return true;
            int r = -1;
            for (int i = 0; i < m; i++) {
                if (D[i][s] <= EPS) continue;
                if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r
                    ][s] ||
                    (D[i][n + 1] / D[i][s] == D[r][n + 1] / D[r][s] &&
                     B[i] < B[r]))
                    r = i;
            }
            if (r == -1) return false;
            Pivot(r, s);
        }
    }
    DOUBLE Solve(VD &x) {
        int r = 0;
        for (int i = 1; i < m; i++)
            if (D[i][n + 1] < D[r][n + 1]) r = i;
        if (D[r][n + 1] <= -EPS) {
            Pivot(r, n);
            if (!Simplex(1) || D[m + 1][n + 1] < -EPS)
                return -numeric_limits<DOUBLE>::infinity();
            for (int i = 0; i < m; i++)
                if (B[i] == -1) {
                    int s = -1;
                    for (int j = 0; j <= n; j++)
                        if (s == -1 || D[i][j] < D[i][s] ||
                            (D[i][j] == D[i][s] && N[j] < N[s]))
                            s = j;
                    Pivot(i, s);
                }
        }
        if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
        x = VD(n);
        for (int i = 0; i < m; i++)
            if (B[i] < n) x[B[i]] = D[i][n + 1];
        return D[m][n + 1];
    }
};
```

6.1 Fun things

$ClassesCount = \frac{1}{|G|}\sum_{\pi \in G} I(\pi)$

$ClassesCount = \frac{1}{|G|}\sum_{\pi \in G} k^{C(\pi)}$

Stirling 2kind - count of partitions of n objects into k nonempty sets:

$S(n,k) = S(n-1,k-1) + kS(n-1,k)$

$S(n,k) = \sum_{j=0}^{n-1}\binom{n-1}{j}S(j,k-1)$

$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k+j}\binom{k}{j}j^n$

$n! \approx \sqrt{2n\pi}(\frac{n}{e})^n$

$\binom{n}{k} \equiv \prod_i \binom{n_i}{k_i}$, n_i, k_i - digits of n, k in p-adic system

$\int_a^b f(x)dx \approx \frac{b-a}{6}(f(a) + 4f(\frac{a+b}{2}) + f(b))$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, O(loglog)$

$G(n) = n \oplus (n >> 1)$

$g(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} g(d)\mu(\frac{n}{d})$

$\sum_{d|n}\mu(d) = [n = 1], \mu(1) = 1, \mu(p) = -1, \mu(p^k) = 0$

$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

$\mathrm{tg}(a \pm b) = \frac{\mathrm{tg}\, a \pm \mathrm{tg}\, b}{1 \mp \mathrm{tg}\, a\, \mathrm{tg}\, b}$

$\mathrm{ctg}(a \pm b) = \frac{\mathrm{ctg}\, a\, \mathrm{ctg}\, b \mp 1}{\mathrm{ctg}\, b \pm \mathrm{ctg}\, a}$

$\sin\frac{a}{2} = \pm\sqrt{\frac{1-\cos a}{2}}$

$\cos\frac{a}{2} = \pm\sqrt{\frac{1+\cos a}{2}}$

$\mathrm{tg}\frac{a}{2} = \frac{\sin a}{1-\cos a} = \frac{1-cosa}{sina}$

$\sin a \sin b = \frac{\cos(a-b) - \cos(a+b)}{2}$

$\sin a \cos b = \frac{\sin(a-b) + \sin(a+b)}{2}$

$\cos a \cos b = \frac{\cos(a-b) + \cos(a+b)}{2}$

1 jan 2000 - saturday, 1 jan 1900 - monday, 14 apr 1961 - friday

Bell numbers: 0:1, 1:1, 2:2, 3:5, 4:15, 5:52, 6:203, 7:877, 8:4140, 9:21147, 10:115975, 11:678570, 12:4213597, 13:27644437, 14:190899322, 15:1382958545, 16:10480142147, 17:82864869804, 18:682076806159, 19:5832742205057, 20:51724158235372, 21:474869816156751, 22:4506715738447323, 23:44152005855084346

Fibonacci: 45:1134903170. 46:1836311903(max int), 91: 4660046610375530309

Highly composite numbers:

$\leq 1000 : d(840) = 32, \leq 10^4 : d(9240) = 64, \leq 10^5 : d(83160) = 128, \leq 10^6 : d(720720) = 240, \leq 10^7 : d(8648640) = 448, \leq 10^8 : d(91891800) = 768, \leq 10^9 : d(931170240) = 1344, \leq 10^{11} : d(97772875200) = 4032, \leq 10^{15} : d(866421317361600) = 26880, \leq 10^{18} : d(897612484786617600) = 103680$

BEST Theorem:

$ec(G) = \#SpanningTrees(G) \cdot \prod_{v \in V}(deg(v) - 1)!$

Erdos: Graph exists

$\Leftrightarrow d_1 \geq .. \geq d_n, \forall k \sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} min(d_i, k)$

Pick: $Area = Interior + \frac{Bounds}{2} - 1$

Euler: $V - E + F = 1 + C$

Kirchhoff: put degree on diagonal, -1 for each edge, cut out first row + column, calc det - result is #SpanningTrees

Table of Basic Integrals (7)

Basic Forms

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ \ n \neq -1 \tag{7.1}$$

$$\int \frac{1}{x} dx = \ln|x| \tag{7.2}$$

$$\int u dv = uv - \int v du \tag{7.3}$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a}\ln|ax+b| \tag{7.4}$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} \tag{7.5}$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1 \tag{7.6}$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \tag{7.7}$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x \tag{7.8}$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} \tag{7.9}$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2}\ln|a^2+x^2| \tag{7.10}$$

$$\int \frac{x^2}{a^2+x^2} dx = x - a\tan^{-1}\frac{x}{a} \tag{7.11}$$

$$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln|a^2+x^2| \tag{7.12}$$

$$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{7.13}$$

$$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a}\ln\frac{a+x}{b+x},\ \ a \neq b \tag{7.14}$$

$$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln|a+x| \tag{7.15}$$

$$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{7.16}$$

Integrals with Roots

$$\int \sqrt{x-a}\ dx = \frac{2}{3}(x-a)^{3/2} \tag{7.17}$$

$$\int \frac{1}{\sqrt{x\pm a}}\ dx = 2\sqrt{x \pm a} \tag{7.18}$$

$$\int \frac{1}{\sqrt{a-x}}\ dx = -2\sqrt{a-x} \tag{7.19}$$

$$\int x\sqrt{x-a}\ dx = \begin{cases} \frac{2a}{3}(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2},\ \text{or} \\ \frac{2}{3}x(x-a)^{3/2} - \frac{4}{15}(x-a)^{5/2},\ \text{or} \\ \frac{2}{15}(2a+3x)(x-a)^{3/2} \end{cases} \tag{7.20}$$

$$\int \sqrt{ax+b}\ dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b} \tag{7.21}$$

$$\int (ax+b)^{3/2}\ dx = \frac{2}{5a}(ax+b)^{5/2} \tag{7.22}$$

$$\int \frac{x}{\sqrt{x\pm a}}\ dx = \frac{2}{3}(x \mp 2a)\sqrt{x \pm a} \tag{7.23}$$

$$\int \sqrt{\frac{x}{a-x}}\ dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a} \tag{7.24}$$

$$\int \sqrt{\frac{x}{a+x}}\ dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x} + \sqrt{x+a}\right] \tag{7.25}$$

$$\int x\sqrt{ax+b}\ dx = \frac{2}{15a^2}(-2b^2+abx+3a^2x^2)\sqrt{ax+b} \tag{7.26}$$

$$\int \sqrt{x(ax+b)}\ dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right|\right] \tag{7.27}$$

$$\int \sqrt{x^3(ax+b)}\ dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}}\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right| \tag{7.28}$$

$$\int \sqrt{x^2 \pm a^2}\ dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2\ln\left|x + \sqrt{x^2 \pm a^2}\right| \tag{7.29}$$

$$\int \sqrt{a^2 - x^2}\ dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2-x^2}} \tag{7.30}$$

$$\int x\sqrt{x^2 \pm a^2}\ dx = \frac{1}{3}\left(x^2 \pm a^2\right)^{3/2} \tag{7.31}$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}}\ dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| \tag{7.32}$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\ dx = \sin^{-1}\frac{x}{a} \tag{7.33}$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}}\ dx = \sqrt{x^2 \pm a^2} \tag{7.34}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}}\ dx = -\sqrt{a^2 - x^2} \tag{7.35}$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}}\ dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2\ln\left|x + \sqrt{x^2 \pm a^2}\right| \tag{7.36}$$

$$\int \sqrt{ax^2+bx+c}\ dx = \frac{b+2ax}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx^+c)}\right| \tag{7.37}$$

$$\int x\sqrt{ax^2+bx+c}\ dx = \frac{1}{48a^{5/2}}\left(2\sqrt{a}\sqrt{ax^2+bx+c}\left(-3b^2+2abx+8a(c+ax^2)\right)\right.$$
$$\left. +3(b^3-4abc)\ln\left|b+2ax+2\sqrt{a}\sqrt{ax^2+bx+c}\right|\right) \tag{7.38}$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}}\ dx = \frac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| \tag{7.39}$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}}\ dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| \tag{7.40}$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} \tag{7.41}$$

Integrals with Logarithms

$$\int \ln ax\ dx = x\ln ax - x \tag{7.42}$$

$$\int x\ln x\ dx = \frac{1}{2}x^2\ln x - \frac{x^2}{4} \tag{7.43}$$

$$\int x^2\ln x\ dx = \frac{1}{3}x^3\ln x - \frac{x^3}{9} \tag{7.44}$$

$$\int x^n\ln x\ dx = x^{n+1}\left(\frac{\ln x}{n+1} - \frac{1}{(n+1)^2}\right),\quad n \neq -1 \tag{7.45}$$

$$\int \frac{\ln ax}{x}\ dx = \frac{1}{2}(\ln ax)^2 \tag{7.46}$$

$$\int \frac{\ln x}{x^2}\ dx = -\frac{1}{x} - \frac{\ln x}{x} \tag{7.47}$$

$$\int \ln(ax+b)\ dx = \left(x + \frac{b}{a}\right)\ln(ax+b) - x, a \neq 0 \tag{7.48}$$

$$\int \ln(x^2+a^2)\ dx = x\ln(x^2+a^2) + 2a\tan^{-1}\frac{x}{a} - 2x \tag{7.49}$$

$$\int \ln(x^2-a^2)\ dx = x\ln(x^2-a^2) + a\ln\frac{x+a}{x-a} - 2x \tag{7.50}$$

$$\int \ln\left(ax^2+bx+c\right)\ dx = \frac{1}{a}\sqrt{4ac-b^2}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left(\frac{b}{2a} + x\right)\ln\left(ax^2+bx+c\right) \tag{7.51}$$

$$\int x\ln(ax+b)\ dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax+b) \tag{7.52}$$

$$\int x\ln\left(a^2-b^2x^2\right)\ dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 - \frac{a^2}{b^2}\right)\ln\left(a^2-b^2x^2\right) \tag{7.53}$$

$$\int(\ln x)^2\ dx = 2x - 2x\ln x + x(\ln x)^2 \tag{7.54}$$

$$\int(\ln x)^3\ dx = -6x + x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x \tag{7.55}$$

$$\int x(\ln x)^2\ dx = \frac{x^2}{4} + \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x \tag{7.56}$$

$$\int x^2(\ln x)^2\ dx = \frac{2x^3}{27} + \frac{1}{3}x^3(\ln x)^2 - \frac{2}{9}x^3\ln x \tag{7.57}$$

Integrals with Exponentials

$$\int e^{ax}\ dx = \frac{1}{a}e^{ax} \tag{7.58}$$

$$\int \sqrt{x}e^{ax}\ dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\operatorname{erf}\left(i\sqrt{ax}\right),\ \text{where } \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \tag{7.59}$$

$$\int xe^x\ dx = (x-1)e^x \tag{7.60}$$

$$\int xe^{ax}\ dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax} \tag{7.61}$$

$$\int x^2e^x\ dx = \left(x^2-2x+2\right)e^x \tag{7.62}$$

$$\int x^2e^{ax}\ dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax} \tag{7.63}$$

$$\int x^3e^x\ dx = \left(x^3-3x^2+6x-6\right)e^x \tag{7.64}$$

$$\int x^ne^{ax}\ dx = \frac{x^ne^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,\mathrm{d}x \tag{7.65}$$

$$\int x^ne^{ax}\ dx = \frac{(-1)^n}{a^{n+1}}\Gamma[1+n,-ax],\ \text{where } \Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}\ \mathrm{d}t \tag{7.66}$$

$$\int e^{ax^2}\ dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}}\operatorname{erf}\left(ix\sqrt{a}\right) \tag{7.67}$$

$$\int e^{-ax^2}\ dx = \frac{\sqrt{\pi}}{2\sqrt{a}}\operatorname{erf}\left(x\sqrt{a}\right) \tag{7.68}$$

$$\int xe^{-ax^2}\ dx = -\frac{1}{2a}e^{-ax^2} \tag{7.69}$$

$$\int x^2e^{-ax^2}\ dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}\operatorname{erf}(x\sqrt{a}) - \frac{x}{2a}e^{-ax^2} \tag{7.70}$$

Integrals with Trigonometric Functions

$$\int \sin ax\ dx = -\frac{1}{a}\cos ax \tag{7.71}$$

$$\int \sin^2 ax\ dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \tag{7.72}$$

$$\int \sin^3 ax\ dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a} \tag{7.73}$$

$$\int \sin^n ax\ dx = -\frac{1}{a}\cos ax\ \ {}_2F_1\left[\frac{1}{2},\frac{1-n}{2},\frac{3}{2},\cos^2 ax\right] \tag{7.74}$$

$$\int \cos ax\ dx = \frac{1}{a}\sin ax \tag{7.75}$$

$$\int \cos^2 ax\ dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \tag{7.76}$$

$$\int \cos^3 ax dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a} \tag{7.77}$$

$$\int \cos^p ax dx = -\frac{1}{a(1+p)}\cos^{1+p}ax \times {}_2F_1\left[\frac{1+p}{2},\frac{1}{2},\frac{3+p}{2},\cos^2 ax\right] \tag{7.78}$$

$$\int \cos x\sin x\ dx = \frac{1}{2}\sin^2 x + c_1 = -\frac{1}{2}\cos^2 x + c_2 = -\frac{1}{4}\cos 2x + c_3 \tag{7.79}$$

$$\int \cos ax\sin bx\ dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a\neq b \tag{7.80}$$

$$\int \sin^2 ax\cos bx\ dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)} \tag{7.81}$$

$$\int \sin^2 x\cos x\ dx = \frac{1}{3}\sin^3 x \tag{7.82}$$

$$\int \cos^2 ax\sin bx\ dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)} \tag{7.83}$$

$$\int \cos^2 ax\sin ax\ dx = -\frac{1}{3a}\cos^3 ax \tag{7.84}$$

$$\int \sin^2 ax\cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} \tag{7.85}$$

$$\int \sin^2 ax\cos^2 ax\ dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \tag{7.86}$$

$$\int \tan ax\ dx = -\frac{1}{a}\ln\cos ax \tag{7.87}$$

$$\int \tan^2 ax\ dx = -x + \frac{1}{a}\tan ax \tag{7.88}$$

$$\int \tan^n ax\ dx = \frac{\tan^{n+1}ax}{a(1+n)} \times {}_2F_1\left(\frac{n+1}{2},1,\frac{n+3}{2},-\tan^2 ax\right) \tag{7.89}$$

$$\int \tan^3 ax dx = \frac{1}{a}\ln\cos ax + \frac{1}{2a}\sec^2 ax \tag{7.90}$$

$$\int \sec x\ dx = \ln|\sec x + \tan x| = 2\tanh^{-1}\left(\tan\frac{x}{2}\right) \tag{7.91}$$

$$\int \sec^2 ax\ dx = \frac{1}{a}\tan ax \tag{7.92}$$

$$\int \sec^3 x\ dx = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x + \tan x| \tag{7.93}$$

$$\int \sec x\tan x\ dx = \sec x \tag{7.94}$$

$$\int \sec^2 x\tan x\ dx = \frac{1}{2}\sec^2 x \tag{7.95}$$

$$\int \sec^n x\tan x\ dx = \frac{1}{n}\sec^n x, n\neq 0 \tag{7.96}$$

$$\int \csc x\ dx = \ln\left|\tan\frac{x}{2}\right| = \ln|\csc x - \cot x| + C \tag{7.97}$$

$$\int \csc^2 ax\ dx = -\frac{1}{a}\cot ax \tag{7.98}$$

$$\int \csc^3 x\ dx = -\frac{1}{2}\cot x\csc x + \frac{1}{2}\ln|\csc x - \cot x| \tag{7.99}$$

$$\int \csc^n x\cot x\ dx = -\frac{1}{n}\csc^n x, n\neq 0 \tag{7.100}$$

$$\int \sec x\csc x\ dx = \ln|\tan x| \tag{7.101}$$

Products of Trigonometric Functions and Monomials

$$\int x \cos x \; dx = \cos x + x \sin x \tag{7.102}$$

$$\int x \cos ax \; dx = \tfrac{1}{a^2} \cos ax + \tfrac{x}{a} \sin ax \tag{7.103}$$

$$\int x^2 \cos x \; dx = 2x \cos x + \left(x^2 - 2\right) \sin x \tag{7.104}$$

$$\int x^2 \cos ax \; dx = \tfrac{2x \cos ax}{a^2} + \tfrac{a^2x^2-2}{a^3} \sin ax \tag{7.105}$$

$$\int x^n \cos x dx = -\tfrac{1}{2}(i)^{n+1} \left[\Gamma(n+1,-ix) + (-1)^n \Gamma(n+1,ix)\right] \tag{7.106}$$

$$\int x^n \cos ax \; dx = \tfrac{1}{2}(ia)^{1-n} \left[(-1)^n \Gamma(n+1,-iax) - \Gamma(n+1,ixa)\right] \tag{7.107}$$

$$\int x \sin x \; dx = -x \cos x + \sin x \tag{7.108}$$

$$\int x \sin ax \; dx = -\tfrac{x \cos ax}{a} + \tfrac{\sin ax}{a^2} \tag{7.109}$$

$$\int x^2 \sin x \; dx = \left(2 - x^2\right) \cos x + 2x \sin x \tag{7.110}$$

$$\int x^2 \sin ax \; dx = \tfrac{2-a^2x^2}{a^3} \cos ax + \tfrac{2x \sin ax}{a^2} \tag{7.111}$$

$$\int x^n \sin x \; dx = -\tfrac{1}{2}(i)^n \left[\Gamma(n+1,-ix) - (-1)^n \Gamma(n+1,-ix)\right] \tag{7.112}$$

$$\int x \cos^2 x \; dx = \tfrac{x^2}{4} + \tfrac{1}{8} \cos 2x + \tfrac{1}{4} x \sin 2x \tag{7.113}$$

$$\int x \sin^2 x \; dx = \tfrac{x^2}{4} - \tfrac{1}{8} \cos 2x - \tfrac{1}{4} x \sin 2x \tag{7.114}$$

$$\int x \tan^2 x \; dx = -\tfrac{x^2}{2} + \ln \cos x + x \tan x \tag{7.115}$$

$$\int x \sec^2 x \; dx = \ln \cos x + x \tan x \tag{7.116}$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x \; dx = \tfrac{1}{2} e^x (\sin x - \cos x) \tag{7.117}$$

$$\int e^{bx} \sin ax \; dx = \tfrac{1}{a^2+b^2} e^{bx} (b \sin ax - a \cos ax) \tag{7.118}$$

$$\int e^x \cos x \; dx = \tfrac{1}{2} e^x (\sin x + \cos x) \tag{7.119}$$

$$\int e^{bx} \cos ax \; dx = \tfrac{1}{a^2+b^2} e^{bx} (a \sin ax + b \cos ax) \tag{7.120}$$

$$\int x e^x \sin x \; dx = \tfrac{1}{2} e^x (\cos x - x \cos x + x \sin x) \tag{7.121}$$

$$\int x e^x \cos x \; dx = \tfrac{1}{2} e^x (x \cos x - \sin x + x \sin x) \tag{7.122}$$

Integrals of Hyperbolic Functions

$$\int \cosh ax \; dx = \tfrac{1}{a} \sinh ax \tag{7.123}$$

$$\int e^{ax} \cosh bx \; dx = \begin{cases} \dfrac{e^{ax}}{a^2 - b^2} [a \cosh bx - b \sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} + \dfrac{x}{2} & a = b \end{cases} \tag{7.124}$$

$$\int \sinh ax \; dx = \tfrac{1}{a} \cosh ax \tag{7.125}$$

$$\int e^{ax} \sinh bx \; dx = \begin{cases} \dfrac{e^{ax}}{a^2 - b^2} [-b \cosh bx + a \sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} - \dfrac{x}{2} & a = b \end{cases} \tag{7.126}$$

$$\int \tanh ax \, dx = \tfrac{1}{a} \ln \cosh ax \tag{7.127}$$

$$\int e^{ax} \tanh bx \; dx = \begin{cases} \dfrac{e^{(a+2b)x}}{(a+2b)} {}_2F_1 \left[1 + \dfrac{a}{2b}, 1, 2 + \dfrac{a}{2b}, -e^{2bx}\right] \\ \qquad -\dfrac{1}{a} e^{ax} {}_2F_1 \left[1, \dfrac{a}{2b}, 1 + \dfrac{a}{2b}, -e^{2bx}\right] & a \neq b \\ \dfrac{e^{ax} - 2 \tan^{-1}[e^{ax}]}{a} & a = b \end{cases} \tag{7.128}$$

$$\int \cos ax \cosh bx \; dx = \tfrac{1}{a^2+b^2} \left[a \sin ax \cosh bx + b \cos ax \sinh bx\right] \tag{7.129}$$

$$\int \cos ax \sinh bx \; dx = \tfrac{1}{a^2+b^2} \left[b \cos ax \cosh bx + a \sin ax \sinh bx\right] \tag{7.130}$$

$$\int \sin ax \cosh bx \; dx = \tfrac{1}{a^2+b^2} \left[-a \cos ax \cosh bx + b \sin ax \sinh bx\right] \tag{7.131}$$

$$\int \sin ax \sinh bx \; dx = \tfrac{1}{a^2+b^2} \left[b \cosh bx \sin ax - a \cos ax \sinh bx\right] \tag{7.132}$$

$$\int \sinh ax \cosh ax dx = \tfrac{1}{4a} \left[-2ax + \sinh 2ax\right] \tag{7.133}$$

$$\int \sinh ax \cosh bx \; dx = \tfrac{1}{b^2-a^2} \left[b \cosh bx \sinh ax - a \cosh ax \sinh bx\right] \tag{7.134}$$

Problem	Status	Comment	Iurii	Alex	Leha
A - 1					
B - 2					
C - 3					
D - 4					
E - 5					
F - 6					
G - 7					
H - 8					
I - 9					
J - 10					
K - 11					
L - 12					
M - 13					
N - 14					
O - 15					