



National Research University Higher School of Economics

# Youthful Passion Fruit

Alexandr Nekrasov, Iurii Pustovalov, Alexey Vasilyev

August 22, 2024

Contest (1)

template.cpp	42 lines
#ifndef LOCAL #define _GLIBCXX_DEBUG #endif #include <bits/stdc++.h> using namespace std;  using ll = long long; using ld = long double; using ull = unsigned long long;  #define pbc push_back #define mp make_pair #define all(v) (v).begin(), (v).end() #define vin(v) for (auto &el : a) cin >> el  mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());  template <typename T1, typename T2> inline void chkmin(T1 &x, const T2 &y) { if (y < x) { x = y; } }  template <typename T1, typename T2> inline void chkmax(T1 &x, const T2 &y) { if (x < y) { x = y; } }  void solve() {  }  signed main() { cin.tie(0)->sync_with_stdio(0); cout.precision(20), cout.setf(ios::fixed); int t = 1; // cin >> t; while (t--) { solve(); } }	

genfolders.sh	6 lines
chmod +x bld* for f in {A..Z} do mkdir \$f cp main.cpp bld* \$f done	

bld	1 lines
g++ -std=c++20 -g -DLOCAL -fsanitize=address,bounds,undefined -o \$1 \$1. cpp	

bldf	1 lines
g++ -std=c++20 -g -O2 -o \$1 \$1.cpp	

hacks.sh	2 lines
UBSAN_OPTIONS=print_stacktrace=1 ./main gdb rbreak regex	

hash.sh	3 lines
# Hashes a file, ignoring all whitespace and comments. # Use for verifying that code was correctly typed. cpp -dD -P -fpreprocessed   tr -d '[:space:]'  md5sum  cut -c-6	

C++ (2)

GpHashtable.cpp	24914, 14 lines
<b>Description:</b> Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).  #include <ext/pb_ds/assoc_container.hpp> #include <ext/pb_ds/tree_policy.hpp> using namespace __gnu_pbds;  const int RANDOM = chrono::high_resolution_clock::now().time_since_epoch().count(); struct hasher { int operator() (int x) const { return x ^ RANDOM; } };  gp_hash_table<int, int, hasher> table;  OrderedSet.cpp <b>Description:</b> A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null.type. <b>Time:</b> $\mathcal{O}(\log(n))$ <bits/extc++.h>, <bits/stdc++.h> dff260, 37 lines using namespace __gnu_pbds; using namespace std;  template <typename T> using ordered_set = tree<T, null_type, less<>, rb_tree_tag, tree_order_statistics_node_update>;  int main() { ordered_set<int> X; X.insert(1); X.insert(2); X.insert(4); X.insert(8); X.insert(16);  assert(*X.find_by_order(1) == 2); assert(*X.find_by_order(2) == 4); assert(*X.find_by_order(4) == 16); assert(X.find_by_order(6) ==X.end());  assert(X.order_of_key(-5) == 0); assert(X.order_of_key(1) == 0); assert(X.order_of_key(3) == 2); assert(X.order_of_key(4) == 2); assert(X.order_of_key(400) == 5); // std::cout << *X.find_by_order(1) << std::endl; // 2 // std::cout << *X.find_by_order(2) << std::endl; // 4 // std::cout << *X.find_by_order(4) << std::endl; // 16 // std::cout << (end(X) == X.find_by_order(6)) << std::endl; // true  // std::cout << X.order_of_key(-5) << std::endl; // 0 // std::cout << X.order_of_key(1) << std::endl; // 0 // std::cout << X.order_of_key(3) << std::endl; // 2 // std::cout << X.order_of_key(4) << std::endl; // 2 // std::cout << X.order_of_key(400) << std::endl; // 5 return 0; }	

bitset.cpp	521d1f, 2 lines
<b>Description:</b> bitset  bs._Find_first() bs._Find_next(idx) - returns right after	

Strings (3)

Manacher.cpp	a6ddfb, 27 lines
<b>Description:</b> Manacher algorithm <b>Time:</b> $\mathcal{O}(n)$  vector<int> manacherOdd(string s) { int n = s.size(); vector<int> d1(n); int l = 0, r = -1; for (int i = 0; i < n; ++i) { int k = i > r ? 1 : min(d1[l + r - i], r - i + 1); while (i + k < n && i - k >= 0 && s[i + k] == s[i - k]) ++k; d1[i] = k; if (i + k - 1 > r) l = i - k + 1, r = i + k - 1; } }  vector<int> manacherEven(string s) { int n = s.size(); vector<int> d2(n); l = 0, r = -1; for (int i = 0; i < n; ++i) { int k = i > r ? 0 : min(d2[l + r - i + 1], r - i + 1); while (i + k < n && i - k - 1 >= 0 && s[i + k] == s[i - k - 1]) ++k; d2[i] = k; if (i + k - 1 > r) l = i - k, r = i + k - 1; } }  AhoCorasick.cpp <b>Description:</b> Build aho-corasick automaton. <b>Time:</b> $\mathcal{O}(n)$ ae5fc2, 19 lines  int go(int v, char c);  int get_link(int v) { if (t[v].link == -1) if (v == 0    t[v].p == 0) t[v].link = 0; else t[v].link = go(get_link(t[v].p), t[v].pch); return t[v].link; }  int go(int v, char c) { if (t[v].go[c] == -1) if (t[v].next[c] != -1) t[v].go[c] = t[v].next[c]; else t[v].go[c] = v == 0 ? 0 : go(get_link(v), c); return t[v].go[c]; }  SuffixArray.cpp <b>Description:</b> Build suffix array <b>Time:</b> $\mathcal{O}(n \log(n))$ 5bd011, 47 lines  vector<int> buildSuffixArray(string &s) { // Remove, if you want to sort cyclic shifts s += (char)(1); int n = s.size(); vector<int> a(n); iota(all(a), 0); stable_sort(all(a), [&](int i, int j) { return s[i] < s[j]; }); vector<int> c(n); int cc = 0; for (int i = 0; i < n; i++) { if (i == 0    s[a[i]] != s[a[i - 1]]) { c[a[i]] = cc++; } else { c[a[i]] = c[a[i - 1]]; } } for (int L = 1; L < n; L *= 2) {	

```
vector<int> cnt(n);
for (auto i : c) {
    cnt[i]++;
}
vector<int> pref(n);
for (int i = 1; i < n; i++) {
    pref[i] = pref[i - 1] + cnt[i - 1];
}
vector<int> na(n);
for (int i = 0; i < n; i++) {
    int pos = (a[i] - L + n) % n;
    na[pref[c[pos]]++] = pos;
}
a = na;
vector<int> nc(n);
cc = 0;
for (int i = 0; i < n; i++) {
    if (i == 0 || c[a[i]] != c[a[i - 1]] ||
        c[(a[i] + L) % n] != c[(a[i - 1] + L) % n]) {
        nc[a[i]] = cc++;
    } else {
        nc[a[i]] = nc[a[i - 1]];
    }
}
c = nc;
}
a.erase(a.begin());
s.pop_back();
return a;
}
```

Lcp.cpp  
Description: lcp array  
Time:  $\mathcal{O}(n)$

1cc27c, 43 lines

```
vector<int> perm;
vector<int> buildLCP(string &s, vector<int> &a) {
    int n = s.size();
    vector<int> ra(n);
    for (int i = 0; i < n; i++) {
        ra[a[i]] = i;
    }
    vector<int> lcp(n - 1);
    int cur = 0;
    for (int i = 0; i < n; i++) {
        cur--;
        chkmax(cur, 0);
        if (ra[i] == n - 1) {
            cur = 0;
            continue;
        }
        int j = a[ra[i] + 1];
        while (s[i + cur] == s[j + cur]) cur++;
        lcp[ra[i]] = cur;
    }
    perm.resize(a.size());
    for (int i = 0; i < a.size(); ++i) perm[a[i]] = i;
    return lcp;
}
int cntr[MAXN];
int spt[MAXN][lgg];
void build(vector<int> &a) {
    for (int i = 0; i < a.size(); ++i) {
        spt[i][0] = a[i];
    }
    for (int i = 2; i < MAXN; ++i) cntr[i] = cntr[i / 2] + 1;
    for (int h = 1; (1 << (h - 1)) < a.size(); ++h) {
        for (int i = 0; i + (1 << (h - 1)) < a.size(); ++i) {
            spt[i][h] = min(spt[i][h - 1], spt[i + (1 << (h - 1))][h - 1]);
        }
    }
}
int getLCP(int l, int r) {
    l = perm[l], r = perm[r];
    if (l > r) swap(l, r);
    int xx = cntr[r - l];
    return min(spt[l][xx], spt[r - (1 << xx)][xx]);
}
```

Eertree.cpp  
Description: Creates Eertree of string str  
Time:  $\mathcal{O}(n)$

7924c8, 40 lines

```
struct eertree {
    int len[MAXN], suffLink[MAXN];
    int to[MAXN][26];
    int numV, v;
    void addLetter(int n, string &str) {
        while (str[n - len[v] - 1] != str[n])
            v = suffLink[v];
        int u = suffLink[v];
        while (str[n - len[u] - 1] != str[n])
            u = suffLink[u];
        int u_ = to[u][str[n] - 'a'];
        int v_ = to[v][str[n] - 'a'];
        if (v_ == -1) {
            v_ = to[v][str[n] - 'a'] = numV;
            len[numV++] = len[v] + 2;
            suffLink[v_] = u_;
        }
        v = v_;
    }
    void init() {
        len[0] = -1;
        len[1] = 0;
        suffLink[1] = 0;
        suffLink[0] = 0;
        numV = 2;
        for (int i = 0; i < 26; ++i) {
            to[0][i] = numV++;
            suffLink[numV - 1] = 1;
            len[numV - 1] = 1;
        }
        v = 0;
    }
    void init(int sz) {
        for (int i = 0; i < sz; ++i) {
            len[i] = suffLink[i] = 0;
            for (int j = 0; j < 26; ++j)
                to[i][j] = -1;
        }
    }
};
```

SuffixAutomaton.cpp  
Description: Build suffix automaton.  
Time:  $\mathcal{O}(n)$

662a10, 45 lines

```
struct state {
    int len, link;
    map<char, int> next;
};

const int MAXLEN = 100000;
state st[MAXLEN * 2];
int sz, last;

void sa_init() {
    sz = last = 0;
    st[0].len = 0;
    st[0].link = -1;
    ++sz;
    /*
    // if you want to build an automaton for different strings:
    for (int i=0; i<MAXLEN*2; ++i)
        st[i].next.clear();
    */
}

void sa_extend(char c) {
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    int p;
    for (p = last; p != -1 && !st[p].next.count(c); p = st[p].link)
        st[p].next[c] = cur;
    if (p == -1)
        st[cur].link = 0;
    else {
```

```
int q = st[p].next[c];
if (st[p].len + 1 == st[q].len)
    st[cur].link = q;
else {
    int clone = sz++;
    st[clone].len = st[p].len + 1;
    st[clone].next = st[q].next;
    st[clone].link = st[q].link;
    for (; p != -1 && st[p].next[c] == q; p = st[p].link)
        st[p].next[c] = clone;
    st[q].link = st[cur].link = clone;
}
last = cur;
}
```

PrefixZ.cpp  
Description: Calculates Prefix,Z-functions  
Time:  $\mathcal{O}(n)$

1c4e93, 25 lines

```
vector<int> pf(string s) {
    int k = 0;
    vector<int> p(s.size());
    for (int i = 1; i < s.size(); ++i) {
        while (k && s[i] != s[k])
            k = p[k - 1];
        k += (s[i] == s[k]);
        p[i] = k;
    }
    return p;
}
vector<int> zf(string s) {
    int n = s.size();
    vector<int> z(n, 0);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r)
            z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
            ++z[i];
        if (i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1;
    }
    z[0] = n;
    return z;
}
```

MinShift.cpp  
Description: Calculates min-cyclic-shift of s, Duval decomposition  
Time:  $\mathcal{O}(n)$

3f0fb9, 20 lines

```
string minshift(string s) {
    int i = 0, ans = 0;
    s += s;
    int n = s.size();
    while (i < n / 2) {
        ans = i;
        int j = i + 1, k = i;
        while (j < n && s[k] <= s[j]) {
            if (s[k] < s[j])
                k = i;
            else
                ++k;
            ++j;
        }
        while (i <= k) {
            i += j - k;
        }
    }
    return s.substr(ans, n / 2);
}
```

SA-IS.cpp  
Description: Build suffix array  
Time:  $\mathcal{O}(n)$

f90ffe, 87 lines

```
void induced_sort(vector<int> &vec, int LIM, vector<int> &sa, vector<bool>
    > &sl,
    vector<int> &fx) {
    vector<int> l(LIM), r(LIM);
```

```
for (int c : vec) {
    if (c + 1 < LIM) {
        ++l[c + 1];
    }
    ++r[c];
}
partial_sum(all(l), l.begin());
partial_sum(all(r), r.begin());
fill(all(sa), -1);
for (int i = fx.size() - 1; i >= 0; --i) {
    sa[--r[vec[fx[i]]]] = fx[i];
}
for (int i : sa) {
    if (i >= 1 && sl[i - 1]) {
        sa[l[vec[i - 1]]++] = i - 1;
    }
}
fill(all(r), 0);
for (int c : vec) ++r[c];
partial_sum(all(r), r.begin());
for (int k = sa.size() - 1, i = sa[k]; k >= 1; --k, i = sa[k])
    if (i >= 1 && !sl[i - 1]) sa[--r[vec[i - 1]]] = i - 1;
}
vector<int> SA_IS(vector<int> &vec, int LIM) {
    const int n = vec.size();
    vector<int> sa(n), fx;
    vector<bool> sl(n);
    sl[n - 1] = false;
    for (int i = n - 2; i >= 0; --i) {
        sl[i] = (vec[i] > vec[i + 1] || (vec[i] == vec[i + 1] && sl[i + 1]));
        if (sl[i] && !sl[i + 1]) {
            fx.pbc(i + 1);
        }
    }
    reverse(all(fx));
    induced_sort(vec, LIM, sa, sl, fx);
    vector<int> nfx(fx.size()), lmv(fx.size());
    for (int i = 0, k = 0; i < n; ++i) {
        if (!sl[sa[i]] && sa[i] >= 1 && sl[sa[i] - 1]) {
            nfx[k++] = sa[i];
        }
    }
    int cur = 0;
    sa[n - 1] = cur;
    for (int k = 1; k < nfx.size(); ++k) {
        int i = nfx[k - 1], j = nfx[k];
        if (vec[i] != vec[j]) {
            sa[j] = ++cur;
            continue;
        }
        bool flag = false;
        for (int a = i + 1, b = j + 1;; ++a, ++b) {
            if (vec[a] != vec[b]) {
                flag = true;
                break;
            }
            if ((!sl[a] && sl[a - 1]) || (!sl[b] && sl[b - 1])) {
                flag = !((!sl[a] && sl[a - 1]) && (!sl[b] && sl[b - 1]));
                break;
            }
        }
        sa[j] = (flag ? ++cur : cur);
    }
    for (int i = 0; i < fx.size(); ++i) {
        lmv[i] = sa[fx[i]];
    }
    if (cur + 1 < (int)fx.size()) {
        auto lms = SA_IS(lmv, cur + 1);
        for (int i = 0; i < fx.size(); ++i) {
            nfx[i] = fx[lms[i]];
        }
    }
    induced_sort(vec, LIM, sa, sl, nfx);
    return sa;
}
template <typename T>
vector<int> suffix_array(T &s, const int LIM = 128) {
    vector<int> vec(s.size() + 1);
```

```
copy(all(s), begin(vec));
vec.back() = (char) (1);
auto ret = SA_IS(vec, LIM);
ret.erase(ret.begin());
return ret;
}
```

Graph (4)

Hungarian.cpp  
Description: Hungarian algorithm  
Time:  $\mathcal{O}\left(n^3\right)$

5afee5, 41 lines

```
int n, m;
vector<vector<int>>> a;
vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
for (int i = 1; i <= n; ++i) {
    p[0] = i;
    int j0 = 0;
    vector<int> minv(m + 1, INF);
    vector<char> used(m + 1, false);
    do {
        used[j0] = true;
        int i0 = p[j0], delta = INF, j1;
        for (int j = 1; j <= m; ++j)
            if (!used[j]) {
                int cur = a[i0][j] - u[i0] - v[j];
                if (cur < minv[j])
                    minv[j] = cur, way[j] = j0;
                if (minv[j] < delta)
                    delta = minv[j], j1 = j;
            }
        for (int j = 0; j <= m; ++j)
            if (used[j])
                u[p[j]] += delta, v[j] -= delta;
            else
                minv[j] -= delta;
        j0 = j1;
    } while (p[j0] != 0);
    do {
        int j1 = way[j0];
        p[j0] = p[j1];
        j0 = j1;
    } while (j0);
}

// matching
vector<int> ans(n + 1);
for (int j = 1; j <= m; ++j) {
    ans[p[j]] = j;
}

// cost
int cost = -v[0];
```

BlossomShrinking.cpp  
Description: Maximum matching in general graph  
Time:  $\mathcal{O}\left(n^3\right)$

23839d, 118 lines

```
struct Edge {
    int u, v;
};
const int N = 510;
int n, m;
vector<int> g[N];
vector<Edge> perfectMatching;
int match[N], par[N], base[N];
bool used[N], blossom[N], lcaUsed[N];
int lca(int u, int v) {
    fill(lcaUsed, lcaUsed + n, false);
    while (u != -1) {
        u = base[u];
        lcaUsed[u] = true;
        if (match[u] == -1)
            break;
        u = par[match[u]];
    }
    while (v != -1) {
        v = base[v];
        if (lcaUsed[v])
            return v;
        v = par[match[v]];
    }
    assert(false);
    return -1;
```

```

}
void markPath(int v, int myBase, int children) {
    while (base[v] != myBase) {
        blossom[v] = blossom[match[v]] = true;
        par[v] = children;
        children = match[v];
        v = par[match[v]];
    }
}

int findPath(int root) {
    iota(base, base + n, 0);
    fill(par, par + n, -1);
    fill(used, used + n, false);
    queue<int> q;
    q.push(root);
    used[root] = true;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (auto to : g[v]) {
            if (match[v] == to)
                continue;
            if (base[v] == base[to])
                continue;
            if (to == root || (match[to] != -1 && par[match[to]] != -1))
            {
                fill(blossom, blossom + n, false);
                int myBase = lca(to, v);
                markPath(v, myBase, to);
                markPath(to, myBase, v);
                for (int u = 0; u < n; ++u) {
                    if (!blossom[base[u]])
                        continue;
                    base[u] = myBase;
                    if (used[u])
                        continue;
                    used[u] = true;
                    q.push(u);
                }
            }
            else if (par[to] == -1) {
                par[to] = v;
                if (match[to] == -1) {
                    return to;
                }
                used[match[to]] = true;
                q.push(match[to]);
            }
        }
    }
    return -1;
}

void blossomShrinking() {
    fill(match, match + n, -1);
    for (int v = 0; v < n; ++v) {
        if (match[v] != -1)
            continue;
        int nxt = findPath(v);
        while (nxt != -1) {
            int parV = par[nxt];
            int parParV = match[parV];
            match[nxt] = parV;
            match[parV] = nxt;
            nxt = parParV;
        }
    }
    for (int v = 0; v < n; ++v) {
        if (match[v] != -1 && v < match[v]) {
            perfectMatching.push_back({v, match[v]});
        }
    }
}

signed main() {
    cin >> n;
    int u, v;
    set<pair<int, int>> edges;
    while (cin >> u >> v) {
        --u;
        --v;
        if (u > v)

```

```

        swap(u, v);
        if (edges.count({u, v}))
            continue;
        edges.insert({u, v});
        g[u].push_back(v);
        g[v].push_back(u);
    }
    blossomShrinking();
    cout << perfectMatching.size() * 2 << '\n';
    for (auto i : perfectMatching) {
        cout << i.u + 1 << " " << i.v + 1 << "\n";
    }
    return 0;
}

```

## Lct.cpp

Description: link-cut tree

Time:  $\mathcal{O}(n \log(n))$

3d8a3f, 136 lines

```

#include <bits/stdc++.h>
using namespace std;

const int MAXN = 1e5 + 228;

struct node {
    node *ch[2];
    node *p;
    bool rev;
    int sz;

    node() {
        ch[0] = ch[1] = p = NULL;
        rev = false;
        sz = 1;
    }
};

int getsz(node *n) { return (n == NULL) ? 0 : n->sz; }

void pull(node *n) { n->sz = getsz(n->ch[0]) + getsz(n->ch[1]) + 1; }

void push(node *n) {
    if (n->rev) {
        if (n->ch[0]) {
            n->ch[0]->rev ^= 1;
        }
        if (n->ch[1]) {
            n->ch[1]->rev ^= 1;
        }
        swap(n->ch[0], n->ch[1]);
        n->rev = 0;
    }
}

bool isRoot(node *n) {
    return n->p == NULL || (n->p->ch[0] != n && n->p->ch[1] != n);
}

int chnum(node *n) { return n->p->ch[1] == n; }

void attach(node *n, node *p, int num) {
    if (n != NULL)
        n->p = p;
    if (p != NULL)
        p->ch[num] = n;
}

void rotate(node *n) {
    int num = chnum(n);
    node *p = n->p;
    node *b = n->ch[1 - num];
    n->p = p->p;
    if (!isRoot(p)) {
        p->p->ch[chnum(p)] = n;
    }
    attach(p, n, 1 - num);
    attach(b, p, num);
    pull(p);
    pull(n);
}

```

```

}

node *qq[MAXN];

void splay(node *n) {
    node *nn = n;
    int top = 0;
    qq[top++] = nn;
    while (!isRoot(nn)) {
        nn = nn->p;
        qq[top++] = nn;
    }
    while (top) {
        push(qq[--top]);
    }
    while (!isRoot(n)) {
        if (!isRoot(n->p)) {
            if (chnum(n) == chnum(n->p)) {
                rotate(n->p);
            } else {
                rotate(n);
            }
        }
        rotate(n);
    }
}

void expose(node *n) {
    splay(n);
    n->ch[1] = NULL;
    pull(n);
    while (n->p != NULL) {
        splay(n->p);
        attach(n, n->p, 1);
        pull(n->p);
        splay(n);
    }
}

void makeRoot(node *n) {
    expose(n);
    n->rev ^= 1;
}

node *nodes[MAXN];

int main() {
    int n;
    cin >> n;
    for (int i = 0; i <= n; i++) {
        nodes[i] = new node();
    }
    int q;
    cin >> q;
    while (q--) {
        string s;
        cin >> s;
        int u, v;
        cin >> u >> v;
        makeRoot(nodes[u]);
        makeRoot(nodes[v]);
        if (s == "get") {
            if (isRoot(nodes[u]) && u != v) {
                cout << "-1" << endl;
            } else {
                cout << getsz(nodes[v]) - 1 << endl;
            }
        } else if (s == "link") {
            nodes[v]->p = nodes[u];
        } else {
            push(nodes[v]);
            nodes[v]->ch[1] = NULL;
            nodes[u]->p = NULL;
        }
    }
}

```

<div>MaxFlow.cpp</div> <div>Description: Dinic</div> <div>Time: <math>\mathcal{O}\left(n^2m\right)</math></div>	1c1be8, 72 lines
<pre> <b>struct</b> MaxFlow {     <b>const int</b> inf = 1e9 + 20;     <b>struct</b> edge {         <b>int</b> a, b, cap;     };     <b>int</b> n;     <b>vector</b>&lt;edge&gt; e;     <b>vector</b>&lt;<b>vector</b>&lt;<b>int</b>&gt;&gt; g;     MaxFlow() {}     <b>int</b> s, t;     <b>vector</b>&lt;<b>int</b>&gt; d, ptr;     <b>void</b> init(<b>int</b> n_, <b>int</b> s_, <b>int</b> t_) {         s = s_, t = t_, n = n_;         g.resize(n);         ptr.resize(n);     }     <b>void</b> addedge(<b>int</b> a, <b>int</b> b, <b>int</b> cap) {         g[a].pbc(e.size());         e.pbc({a, b, cap});         g[b].pbc(e.size());         e.pbc({b, a, 0});     }      <b>bool</b> bfs() {         d.assign(n, inf);         d[s] = 0;         <b>queue</b>&lt;<b>int</b>&gt; q;         q.push(s);         <b>while</b> (q.size()) {             <b>int</b> v = q.front();             q.pop();             <b>for</b> (<b>int</b> i : g[v]) {                 <b>if</b> (e[i].cap &gt; 0) {                     <b>int</b> b = e[i].b;                     <b>if</b> (d[b] &gt; d[v] + 1) {                         d[b] = d[v] + 1;                         q.push(b);                     }                 }             }         }         <b>return</b> d[t] != inf;     }      <b>int</b> dfs(<b>int</b> v, <b>int</b> flow) {         <b>if</b> (v == t) <b>return</b> flow;         <b>if</b> (!flow) <b>return</b> 0;         <b>int</b> sum = 0;         <b>for</b> (; ptr[v] &lt; g[v].size(); ++ptr[v]) {             <b>int</b> b = e[g[v][ptr[v]]].b;             <b>int</b> cap = e[g[v][ptr[v]]].cap;             <b>if</b> (cap &lt;= 0) <b>continue</b>;             <b>if</b> (d[b] != d[v] + 1) <b>continue</b>;             <b>int</b> x = dfs(b, min(flow, cap));             <b>int</b> id = g[v][ptr[v]];             e[id].cap -= x;             e[id ^ 1].cap += x;             flow -= x;             sum += x;         }         <b>return</b> sum;     }      <b>int</b> dinic() {         <b>int</b> ans = 0;         <b>while</b> (1) {             <b>if</b> (!bfs()) <b>break</b>;             ptr.assign(n, 0);             <b>int</b> x = dfs(s, inf);             <b>if</b> (!x) <b>break</b>;             ans += x;         }         <b>return</b> ans;     } }; </pre>	

<div>MCMF.cpp</div> <div>Description: Min cost</div> <div>Time: <math>\mathcal{O}\left(?\right)</math></div>	32340a, 61 lines
<pre> <b>struct</b> MCMF {     <b>struct</b> edge {         <b>int</b> a, b, cap, cost;     };     <b>vector</b>&lt;edge&gt; e;     <b>vector</b>&lt;<b>vector</b>&lt;<b>int</b>&gt;&gt; g;     <b>int</b> s, t;     <b>int</b> n;     <b>void</b> init(<b>int</b> N, <b>int</b> S, <b>int</b> T) {         s = S, t = T, n = N;         g.resize(N);         e.clear();     }     <b>void</b> addedge(<b>int</b> a, <b>int</b> b, <b>int</b> cap, <b>int</b> cost) {         g[a].pbc(e.size());         e.pbc({a, b, cap, cost});         g[b].pbc(e.size());         e.pbc({b, a, 0, -cost});     }     <b>int</b> getcost(<b>int</b> k) {         <b>int</b> flow = 0;         <b>int</b> cost = 0;         <b>while</b> (flow &lt; k) {             <b>vector</b>&lt;<b>int</b>&gt; d(n, INF);             <b>vector</b>&lt;<b>int</b>&gt; pr(n);             d[s] = 0;             <b>queue</b>&lt;<b>int</b>&gt; q;             q.push(s);             <b>while</b> (q.size()) {                 <b>int</b> v = q.front();                 q.pop();                 <b>for</b> (<b>int</b> i : g[v]) {                     <b>int</b> u = e[i].b;                     <b>if</b> (e[i].cap &amp;&amp; d[u] &gt; d[v] + e[i].cost) {                         d[u] = d[v] + e[i].cost;                         q.push(u);                         pr[u] = i;                     }                 }             }             <b>if</b> (d[t] == INF) <b>return</b> INF;             <b>int</b> gf = k - flow;             <b>int</b> v = t;             <b>while</b> (v != s) {                 <b>int</b> id = pr[v];                 chkmin(gf, e[id].cap);                 v = e[id].a;             }             v = t;             <b>while</b> (v != s) {                 <b>int</b> id = pr[v];                 e[id].cap -= gf;                 e[id ^ 1].cap += gf;                 cost += e[id].cost * gf;                 v = e[id].a;             }             flow += gf;         }         <b>return</b> cost;     } }; </pre>	

<div>GlobalMincut.cpp</div> <div>Description: Global min cut</div> <div>Time: <math>\mathcal{O}\left(n^3\right)</math></div>	7b8a6b, 35 lines
<pre> <b>const int</b> MAXN = 500; <b>int</b> n, g[MAXN][MAXN]; <b>int</b> best_cost = 1000000000; <b>vector</b>&lt;<b>int</b>&gt; best_cut; <b>void</b> mincut() {     <b>vector</b>&lt;<b>int</b>&gt; v[MAXN];     <b>for</b> (<b>int</b> i = 0; i &lt; n; ++i)         v[i].assign(1, i);     <b>int</b> w[MAXN]; </pre>	

<pre> <b>bool</b> exist[MAXN], in_a[MAXN]; memset(exist, <b>true</b>, sizeof exist); <b>for</b> (<b>int</b> ph = 0; ph &lt; n - 1; ++ph) {     memset(in_a, <b>false</b>, sizeof in_a);     memset(w, 0, sizeof w);     <b>for</b> (<b>int</b> it = 0, prev; it &lt; n - ph; ++it) {         <b>int</b> sel = -1;         <b>for</b> (<b>int</b> i = 0; i &lt; n; ++i)             <b>if</b> (exist[i] &amp;&amp; !in_a[i] &amp;&amp; (sel == -1    w[i] &gt; w[sel]))                 sel = i;         <b>if</b> (it == n - ph - 1) {             <b>if</b> (w[sel] &lt; best_cost)                 best_cost = w[sel], best_cut = v[sel];             v[prev].insert(v[prev].end(), v[sel].begin(), v[sel].end());         }         <b>for</b> (<b>int</b> i = 0; i &lt; n; ++i)             g[prev][i] = g[i][prev] += g[sel][i];         exist[sel] = <b>false</b>;     } <b>else</b> {         in_a[sel] = <b>true</b>;         <b>for</b> (<b>int</b> i = 0; i &lt; n; ++i)             w[i] += g[sel][i];         prev = sel;     } } </pre>	
--	--

<div>WeightedMatching.cpp</div> <div>Description: Max weighted matching</div> <div>Time: <math>\mathcal{O}\left(N^3\right)</math> or so</div>	c3f149, 193 lines
<pre> <b>#define</b> Dist(e) (lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2) <b>const int</b> N = 1023, INF = 1e9; <b>struct</b> Edge {     <b>int</b> u, v, w; } g[N][N]; <b>int</b> n, m, n_x, lab[N], match[N], slack[N], st[N], pa[N], flower_from[N][N], S[N], vis[N]; <b>vector</b>&lt;<b>int</b>&gt; flower[N]; <b>deque</b>&lt;<b>int</b>&gt; q; <b>void</b> update_slack(<b>int</b> u, <b>int</b> x) {     <b>if</b> (!slack[x]    Dist(g[u][x]) &lt; Dist(g[slack[x]][x])) slack[x] = u; } <b>void</b> set_slack(<b>int</b> x) {     slack[x] = 0;     <b>for</b> (<b>int</b> u = 1; u &lt;= n; ++u)         <b>if</b> (g[u][x].w &gt; 0 &amp;&amp; st[u] != x &amp;&amp; S[st[u]] == 0) update_slack(u, x); } <b>void</b> q_push(<b>int</b> x) {     <b>if</b> (x &lt;= n) <b>return</b> q.push_back(x);     <b>for</b> (<b>int</b> i = 0; i &lt; flower[x].size(); ++i) q_push(flower[x][i]); } <b>void</b> set_st(<b>int</b> x, <b>int</b> b) {     st[x] = b;     <b>if</b> (x &lt;= n) <b>return</b>;     <b>for</b> (<b>int</b> i = 0; i &lt; flower[x].size(); ++i) set_st(flower[x][i], b); } <b>int</b> get_pr(<b>int</b> b, <b>int</b> xr) {     <b>int</b> pr = find(flower[b].begin(), flower[b].end(), xr) - flower[b].begin();     <b>if</b> (pr % 2 == 1) {         reverse(flower[b].begin() + 1, flower[b].end());         <b>return</b> (int)flower[b].size() - pr;     } <b>else</b> <b>return</b> pr; } <b>void</b> set_match(<b>int</b> u, <b>int</b> v) {     match[u] = g[u][v].v;     <b>if</b> (u &lt;= n) <b>return</b>;     Edge e = g[u][v];     <b>int</b> xr = flower_from[u][e.u], pr = get_pr(u, xr);     <b>for</b> (<b>int</b> i = 0; i &lt; pr; ++i) set_match(flower[u][i], flower[u][i ^ 1]);     set_match(xr, v);     rotate(flower[u].begin(), flower[u].begin() + pr, flower[u].end()); } <b>void</b> augment(<b>int</b> u, <b>int</b> v) { </pre>	

```
int xnv = st[match[u]];
set_match(u, v);
if (!xnv) return;
set_match(xnv, st[pa[xnv]]);
augment(st[pa[xnv]], xnv);
}

int get_lca(int u, int v) {
    static int t = 0;
    for (++t; u || v; swap(u, v)) {
        if (u == 0) continue;
        if (vis[u] == t) return u;
        vis[u] = t;
        u = st[match[u]];
        if (u) u = st[pa[u]];
    }
    return 0;
}

void add_blossom(int u, int lca, int v) {
    int b = n + 1;
    while (b <= n_x && st[b]) ++b;
    if (b > n_x) ++n_x;
    lab[b] = 0, S[b] = 0, match[b] = match[lca];
    flower[b].clear();
    flower[b].push_back(lca);
    for (int x = u, y; x != lca; x = st[pa[y]])
        flower[b].push_back(x), flower[b].push_back(y = st[match[x]]),
            q_push(y);
    reverse(flower[b].begin() + 1, flower[b].end());
    for (int x = v, y; x != lca; x = st[pa[y]])
        flower[b].push_back(x), flower[b].push_back(y = st[match[x]]),
            q_push(y);
    set_st(b, b);
    for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].w = 0;
    for (int x = 1; x <= n; ++x) flower_from[b][x] = 0;
    for (int i = 0; i < flower[b].size(); ++i) {
        int xs = flower[b][i];
        for (int x = 1; x <= n_x; ++x) {
            if (g[b][x].w == 0 || Dist(g[xs][x]) < Dist(g[b][x]))
                g[b][x] = g[xs][x], g[x][b] = g[x][xs];
        }
        for (int x = 1; x <= n; ++x) if (flower_from[xs][x]) flower_from[
            b][x] = xs;
    }
    set_slack(b);
}

void expand_blossom(int b) {
    for (int i = 0; i < flower[b].size(); ++i) set_st(flower[b][i],
        flower[b][i]);
    int xr = flower_from[b][g[b][pa[b]].u], pr = get_pr(b, xr);
    for (int i = 0; i < pr; i += 2) {
        int xs = flower[b][i], xns = flower[b][i + 1];
        pa[xs] = g[xns][xs].u;
        S[xs] = 1, S[xns] = 0;
        slack[xs] = 0, set_slack(xns);
        q_push(xns);
    }
    S[xr] = 1, pa[xr] = pa[b];
    for (int i = pr + 1; i < flower[b].size(); ++i) {
        int xs = flower[b][i];
        S[xs] = -1, set_slack(xs);
    }
    st[b] = 0;
}

bool on_found_Edge(const Edge &e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
        pa[v] = e.u, S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = 0;
        S[nu] = 0, q_push(nu);
    } else if (S[v] == 0) {
        int lca = get_lca(u, v);
        if (!lca) return augment(u, v), augment(v, u), 1;
        else add_blossom(u, lca, v);
    }
    return 0;
}

bool matching() {
    fill(S, S + n_x + 1, -1), fill(slack, slack + n_x + 1, 0);
```

```
q.clear();
for (int x = 1; x <= n_x; ++x) if (st[x] == x && !match[x]) pa[x] =
    0, S[x] = 0, q_push(x);
if (q.empty()) return 0;
while(1) {
    while (q.size()) {
        int u = q.front();
        q.pop_front();
        if (S[st[u]] == 1) continue;
        for (int v = 1; v <= n; ++v) {
            if (g[u][v].w > 0 && st[u] != st[v]) {
                if (Dist(g[u][v]) == 0) {
                    if (on_found_Edge(g[u][v])) return 1;
                } else
                    update_slack(u, st[v]);
            }
        }
    }
    int d = INF;
    for (int b = n + 1; b <= n_x; ++b) if (st[b] == b && S[b] == 1)
        chkmin(d, lab[b] / 2);
    for (int x = 1; x <= n_x; ++x) {
        if (st[x] == x && slack[x]) {
            if (S[x] == -1)
                d = min(d, Dist(g[slack[x]][x]));
            else if (S[x] == 0)
                d = min(d, Dist(g[slack[x]][x]) / 2);
        }
    }
    for (int u = 1; u <= n; ++u) {
        if (S[st[u]] == 0) {
            if (lab[u] <= d) return 0;
            lab[u] -= d;
        } else if (S[st[u]] == 1)
            lab[u] += d;
    }
    for (int b = n + 1; b <= n_x; ++b) {
        if (st[b] == b) {
            if (S[st[b]] == 0)
                lab[b] += d * 2;
            else if (S[st[b]] == 1)
                lab[b] -= d * 2;
        }
    }
    q.clear();
    for (int x = 1; x <= n_x; ++x) {
        if (st[x] == x && slack[x] && st[slack[x]] != x &&
            Dist(g[slack[x]][x]) == 0)
            if (on_found_Edge(g[slack[x]][x])) return 1;
    }
    for (int b = n + 1; b <= n_x; ++b)
        if (st[b] == b && S[b] == 1 && lab[b] == 0) expand_blossom(b);
}
return 0;
}

pair<ll, int> weight_blossom() {
    fill(match, match + n + 1, 0);
    n_x = n;
    int n_matches = 0;
    ll tot_weight = 0;
    for (int u = 0; u <= n; ++u) st[u] = u, flower[u].clear();
    int w_max = 0;
    for (int u = 1; u <= n; ++u) {
        for (int v = 1; v <= n; ++v) {
            flower_from[u][v] = (u == v ? u : 0);
            w_max = max(w_max, g[u][v].w);
        }
    }
    ll answer = 0;
    for (int u = 1; u <= n; ++u) lab[u] = w_max;
    while (matching()) ++n_matches;
    for(int u=1; u<=n; ++u)
        if (match[u] && match[u] < u)
            tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}
```

## Geometry (5)

### Point.cpp

Description: struct Point

80dfd5, 80 lines

```
const ld EPS = 1e-7;

ld sq(ld x) {
    return x * x;
}

int sign(ld x) {
    if (x < -EPS) {
        return -1;
    }
    if (x > EPS) {
        return 1;
    }
    return 0;
}

#define vec point
struct point { // % - cross, * - dot
    ld x, y;
    auto operator<=>(const point&) const = default;
};
ld operator*(const point &a, const point &b) {
    return a.x * b.x + a.y * b.y;
}
ld operator%(const point &a, const point &b) {
    return a.x * b.y - a.y * b.x;
}
point operator-(const point &a, const point &b) {
    return {a.x - b.x, a.y - b.y};
}
point operator+(const point &a, const point &b) {
    return {a.x + b.x, a.y + b.y};
}
point operator*(const point &a, ld b) {
    return {a.x * b, a.y * b};
}
point operator/(const point &a, ld b) {
    return {a.x / b, a.y / b};
}
bool operator<(const point &a, const point &b) {
    if (sign(a.y - b.y) != 0) {
        return a.y < b.y;
    } else if (sign(a.x - b.x) != 0) {
        return a.x < b.x;
    }
    return 0;
}
ld len2(const point &a) {
    return sq(a.x) + sq(a.y);
}
ld len(const point &a) {
    return sqrt(len2(a));
}
point norm(point a) {
    return a / len(a);
}
int half(point a) {
    return (sign(a.y) == -1 || (sign(a.y) == 0 && a.x < 0));
}
point ort(point a) {
    return {-a.y, a.x};
}
point turn(point a, ld ang) {
    return {a.x * cos(ang) - a.y * sin(ang), a.x * sin(ang) + a.y * cos(
        ang)};
}
ld getAngle(point &a, point &b) {
    return atan2(a % b, a * b);
}
bool cmpHalf(const point &a, const point &b) {
    if (half(a) != half(b)) {
        return half(b);
    } else {
        int sgn = sign(a % b);
```

```
        if (!sgn) {
            return len2(a) < len2(b);
        } else {
            return sgn == 1;
        }
    }
}
```

Line.cpp

Description: struct Line887306, 26 lines

```
struct line {
    ld a, b, c;
    void norm() {
        // for half planes
        ld d = len({a, b});
        assert(sign(d) > 0);
        a /= d;
        b /= d;
        c /= d;
    }
    ld eval(point p) const { return a * p.x + b * p.y + c; }
    bool isIn(point p) const { return sign(eval(p)) >= 0; }
    bool operator==(const line &other) const {
        return sign(a * other.b - b * other.a) == 0 &&
            sign(a * other.c - c * other.a) == 0 &&
            sign(b * other.c - c * other.b) == 0;
    }
};

line getln(point a, point b) {
    line res;
    res.a = a.y - b.y;
    res.b = b.x - a.x;
    res.c = -(res.a * a.x + res.b * a.y);
    res.norm();
    return res;
}
```

Intersections.cpp

Description: Geometry intersections45d7d9, 75 lines

```
bool isCrossed(ld lx, ld rx, ld ly, ld ry) {
    if (lx > rx)
        swap(lx, rx);
    if (ly > ry)
        swap(ly, ry);
    return sign(min(rx, ry) - max(lx, ly)) >= 0;
}

// if two segments [a, b] and [c, d] has AT LEAST one common point ->
// true
bool intersects(const point &a, const point &b, const point &c, const
point &d) {
    if (!isCrossed(a.x, b.x, c.x, d.x))
        return false;
    if (!isCrossed(a.y, b.y, c.y, d.y))
        return false;
    if (sign((b - a) % (c - a)) * sign((b - a) % (d - a)) == 1) return 0;
    if (sign((d - c) % (a - c)) * sign((d - c) % (b - c)) == 1) return 0;
    return 1;
}

//intersecting lines
bool intersect(line l, line m, point &I) {
    ld d = l.b * m.a - m.b * l.a;
    if (sign(d) == 0) {
        return false;
    }
    ld dx = m.b * l.c - m.c * l.b;
    ld dy = m.c * l.a - l.c * m.a;
    I = {dx / d, dy / d};
    return true;
}

//intersecting circles
int intersect(point o1, ld r1, point o2, ld r2, point &i1, point &i2) {
    if (r1 < r2) {
        swap(o1, o2);
        swap(r1, r2);
    }
    if (sign(r1 - r2) == 0 && len2(o2 - o1) < EPS) {
```

```
        return 3;
    }
    ld ln = len(o1 - o2);
    if (sign(ln - r1 - r2) == 1 || sign(r1 - ln - r2) == 1) {
        return 0;
    }
    ld d = (sq(r1) - sq(r2) + sq(ln)) / 2 / ln;
    vec v = norm(o2 - o1);
    point a = o1 + v * d;
    if (sign(ln - r1 - r2) == 0 || sign(ln + r2 - r1) == 0) {
        i1 = a;
        return 1;
    }
    v = ort(v) * sqrt(sq(r1) - sq(d));
    i1 = a + v;
    i2 = a - v;
    return 2;
}

//intersecting line and circle, line should be normed
int intersect(point o, ld r, line l, point &i1, point &i2) {
    ld len = abs(l.eval(o));
    int sgn = sign(len - r);
    if (sgn == 1) {
        return 0;
    }
    vec v = norm(vec{l.a, l.b}) * len;
    if (sign(l.eval(o + v)) != 0) {
        v = vec{0, 0} - v;
    }
    point a = o + v;
    if (sgn == 0) {
        i1 = a;
        return 1;
    }
    v = norm({-l.b, l.a}) * sqrt(sq(r) - sq(len));
    i1 = a + v;
    i2 = a - v;
    return 2;
}
```

Tangents.cpp

Description: Tangents to circles.c73373, 43 lines

```
// tangents from point to circle
int tangents(point &o, ld r, point &p, point &i1, point &i2) {
    ld ln = len(o - p);
    int sgn = sign(ln - r);
    if (sgn == -1) {
        return 0;
    } else if (sgn == 0) {
        i1 = p;
        return 1;
    } else {
        ld x = sq(r) / ln;
        vec v = norm(p - o) * x;
        point a = o + v;
        v = ort(norm(p - o)) * sqrt(sq(r) - sq(x));
        i1 = a + v;
        i2 = a - v;
        return 2;
    }
}

void _tangents(point c, ld r1, ld r2, vector<line> &ans) {
    ld r = r2 - r1;
    ld z = sq(c.x) + sq(c.y);
    ld d = z - sq(r);
    if (sign(d) == -1)
        return;
    d = sqrt(abs(d));
    line l;
    l.a = (c.x * r + c.y * d) / z;
    l.b = (c.y * r - c.x * d) / z;
    l.c = r1;
    ans.push_back(l);
}

// tangents between two circles
vector<line> tangents(point o1, ld r1, point o2, ld r2) {
    vector<line> ans;
```

```
    for (int i = -1; i <= 1; i += 2)
        for (int j = -1; j <= 1; j += 2)
            _tangents(o2 - o1, r1 * i, r2 * j, ans);
    for (int i = 0; i < (int)ans.size(); ++i)
        ans[i].c -= ans[i].a * o1.x + ans[i].b * o1.y;
    return ans;
}
```

Hull.cpp

Description: Polygon functionsfc1928, 16 lines

```
vector<point> hull(vector<point> p, bool need_all=false) {
    sort(all(p));
    p.erase(unique(all(p)), end(p));
    int n = p.size(), k = 0;
    if (n <= 2) return p;
    vector<point> ch(2 * n);
    ld th = need_all ? -EPS : +EPS; // 0 : 1 if int
    for (int i = 0; i < n; ch[k++] = p[i++]) {
        while (k >= 2 && (ch[k - 1] - ch[k - 2]) % (p[i] - ch[k - 1]) < th)
            --k;
    }
    for (int i = n - 2, t = k + 1; i >= 0; ch[k++] = p[i--]) {
        while (k >= t && (ch[k - 1] - ch[k - 2]) % (p[i] - ch[k - 1]) < th)
            --k;
    }
    ch.resize(k - 1);
    return ch;
}
```

IsInPolygon.cpp

Description: Is in polygon functionsf17b31, 65 lines

```
bool isOnSegment(point &a, point &b, point &x) {
    if (sign(len2(a - b)) == 0) {
        return sign(len(a - x)) == 0;
    }
    return sign((b - a) % (x - a)) == 0 && sign((b - x) * (a - x)) <= 0;
    // optional (slower, but works better if there are some precision
    // problems) return sign((b - a).len() - (x - a).len() - (x - b).len
    // ())
    // == 0;
}

int isIn(vector<point> &p, point &a) {
    int n = p.size();
    // depends on limitations(2*MAXC + 228)
    point b = a + point{2e9 + 228, 1};
    int cnt = 0;
    for (int i = 0; i < n; ++i) {
        point x = p[i];
        point y = p[i + 1 < n ? i + 1 : 0];
        if (isOnSegment(x, y, a)) {
            // depends on the problem statement
            return 1;
        }
        cnt += intersects(x, y, a, b);
    }
    return 2 * (cnt % 2 == 1);
    /*optional (atan2 is VERY SLOW)!
    ld ans = 0;
    int n = p.size();
    for (int i = 0; i < n; ++i) {
        Point x = p[i];
        Point y = p[i + 1 < n ? i + 1 : 0];
        if (isOnSegment(x, y, a)) {
            // depends on the problem statement
            return true;
        }
        x = x - a;
        y = y - a;
        ans += atan2(x ^ y, x * y);
    }
    return abs(ans) > 1;*/
}
```

```
bool isInTriangle(point &a, point &b, point &c, point &x) {
    return sign((b - a) % (x - a)) >= 0 && sign((c - b) % (x - b)) >= 0
        &&
```



```
        sign((a - c) % (x - c)) >= 0;
    }

    // points should be in the counterclockwise order
    bool isInConvex(vector<point> &p, point &a) {
        int n = p.size();
        assert(n >= 3);
        // assert(isConvex(p));
        // assert(isCounterclockwise(p));
        if (sign((p[1] - p[0]) % (a - p[0])) < 0)
            return 0;
        if (sign((p[n - 1] - p[0]) % (a - p[0])) > 0)
            return 0;
        int pos = lower_bound(p.begin() + 2, p.end(), a,
                               [&](point a, point b) -> bool {
                                   return sign((a - p[0]) % (b - p[0])) > 0;
                               }) -
            p.begin();
        assert(pos > 1 && pos < n);
        return isInTriangle(p[0], p[pos - 1], p[pos], a);
    }
}
```

Diameter.cpp  
Description: Rotating calipers.  
Time:  $\mathcal{O}(n)$  Of341c, 21 lines

```
ld diameter(vector<point> p) {
    p = hull(p);
    int n = p.size();
    if (n <= 1) {
        return 0;
    }
    if (n == 2) {
        return len(p[0] - p[1]);
    }
    ld ans = 0;
    int i = 0, j = 1;
    while (i < n) {
        while (sign((p[(i + 1) % n] - p[i]) % (p[(j + 1) % n] - p[j]))) >=
            0) {
            chkmax(ans, len(p[i] - p[j]));
            j = (j + 1) % n;
        }
        chkmax(ans, len(p[i] - p[j]));
        ++i;
    }
    return ans;
}
```

TangentsAlex.cpp  
Description: Find both tangents to the convex polygon.  
(Zakaldovany algos mozhet sgonyat za pivom tak zhe).  
Time:  $\mathcal{O}(\log(n))$  2eeea8, 17 lines

```
pair<int, int> tangents_alex(vector<point> &p, point &a) {
    int n = p.size();
    int l = __lg(n);
    auto findWithSign = [&](int val) {
        int i = 0;
        for (int k = 1; k >= 0; --k) {
            int i1 = (i - (1 << k) + n) % n;
            int i2 = (i + (1 << k)) % n;
            if (sign((p[i1] - a) % (p[i] - a)) == val)
                i = i1;
            if (sign((p[i2] - a) % (p[i] - a)) == val)
                i = i2;
        }
        return i;
    };
    return {findWithSign(1), findWithSign(-1)};
}
```

IsHpiEmpty.cpp  
Description: Determines is half plane intersectinos.  
Time:  $\mathcal{O}(n)$  (expected) 3b5e69, 42 lines

```
// all lines must be normed!!!!, sign > 0
bool isHpiEmpty(vector<line> lines) {
    // return hpi(lines).empty();
}
```

```
// overflow/precision problems?
shuffle(all(lines), rnd);
const ld C = 1e9;
point ans(C, C);
vector<point> box = {{-C, -C}, {C, -C}, {C, C}, {-C, C}};
for (int i = 0; i < 4; ++i)
    lines.push_back(getln(box[i], box[(i + 1) % 4]));
int n = lines.size();
for (int i = n - 4; i >= 0; --i) {
    if (lines[i].isIn(ans))
        continue;
    point up(0, C + 1), down(0, -C - 1), pi = {lines[i].b, -lines[i].a};
    for (int j = i + 1; j < n; ++j) {
        if (lines[j] == lines[i])
            continue;
        point p, pj = {lines[j].b, -lines[j].a};
        if (!intersect(lines[i], lines[j], p)) {
            if (sign(pi * pj) != -1)
                continue;
            if (sign(lines[i].c + lines[j].c) *
                (!sign(pi.y) ? sign(pi.x) : -1) ==
                1)
                return true;
        } else {
            if ((!sign(pi.y) ? sign(pi.x) : sign(pi.y)) * (sign(pi %
                pj)) ==
                1)
                chkmin(up, p);
            else
                chkmax(down, p);
        }
    }
    if ((ans = up) < down)
        return true;
}
// for (int i = 0; i < n; ++i) {
//     assert(lines[i].eval(ans) < EPS);
// }
return false;
}
```

HalfPlaneIntersection.cpp  
Description: Find the intersection of the half planes.  
Time:  $\mathcal{O}(n \log(n))$  fdf28f, 62 lines

```
vec getPoint(line l) { return {-l.b, l.a}; }

bool bad(line a, line b, line c) {
    point x;
    assert(intersect(b, c, x) == 1);
    return a.eval(x) < 0;
}

// Do not forget about the bounding box
vector<point> hpi(vector<line> lines) {
    sort(all(lines), [](line al, line bl) -> bool {
        point a = getPoint(al);
        point b = getPoint(bl);
        if (half(a) != half(b)) {
            return half(a) < half(b);
        }
        return a % b > 0;
    });
    vector<pair<line, int>> st;
    for (int it = 0; it < 2; it++) {
        for (int i = 0; i < (int)lines.size(); i++) {
            bool flag = false;
            while (!st.empty()) {
                if (len(getPoint(st.back().first) - getPoint(lines[i])) <
                    EPS) {
                    if (lines[i].c >= st.back().first.c) {
                        flag = true;
                        break;
                    } else {
                        st.pop_back();
                    }
                }
            }
        }
    }
}
```

```
    } else if (getPoint(st.back().first) % getPoint(lines[i])
        < EPS / 2) {
        return {};
    } else if (st.size() >= 2 &&
        bad(st[st.size() - 2].first, st[st.size() -
            1].first,
            lines[i])) {
        st.pop_back();
    } else {
        break;
    }
}
if (!flag)
    st.push_back({lines[i], i});
}

vector<int> en(lines.size(), -1);
vector<point> ans;
for (int i = 0; i < (int)st.size(); i++) {
    if (en[st[i].second] == -1) {
        en[st[i].second] = i;
        continue;
    }
    for (int j = en[st[i].second]; j < i; j++) {
        point I;
        assert(intersect(st[j].first, st[j + 1].first, I) == 1);
        ans.push_back(I);
    }
    break;
}
return ans;
}
```

CHT.cpp  
Description: CHT for minimum, k is decreasing, works for equal slopes. 800120, 34 lines

```
struct line {
    int k, b;
    int eval(int x) {
        return k * x + b;
    }
};
struct part {
    line a;
    double x;
};
double intersection(line a, line b) {
    return (a.b - b.b) / (b.k - a.k);
}
struct ConvexHullMin {
    vector <part> st;
    void add(line a) {
        if (!st.empty() && st.back().a.k == a.k) {
            if (st.back().a.b > a.b) st.pop_back();
            else return;
        }
        while (st.size() > 1 && intersection(st[st.size() - 2].a, a) <=
            st[st.size() - 2].x) st.pop_back();
        if (!st.empty()) st.back().x = intersection(st.back().a, a);
        st.push_back({a, INF});
    }
    int get_val(int x) {
        int l = -1, r = (int)st.size() - 1;
        while (r - l > 1) {
            int m = (l + r) / 2;
            if (st[m].x < x) l = m;
            else r = m;
        }
        return st[r].a.eval(x);
    }
};
```

DynamicCHT.cpp  
Description: Dynamic CHT for maximum 8a0777, 30 lines

```
struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const {
```

```
        return Q ? p < o.p : k < o.k;
    }
};

struct LineContainer : multiset<Line> {
    const ll inf = LLONG_MAX;
    ll div(ll a, ll b) {
        return a / b - ((a ^ b) < 0 && a % b);
    }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        Q = 1; auto l = *lower_bound({0,0,x}); Q = 0;
        return l.k * x + l.m;
    }
};
```

MinPlusConv.cpp

Description: Min-Plusconv, A is convex down  
Time:  $\mathcal{O}(n\log n\text{fast})$

5d63d9, 28 lines

```
// Assumptions: 'a' is convex, 'opt' has size 'n+m-1'
// 'opt[k]' will be equal to 'arg min(a[k-i] + b[i])'
template<typename T>
void convex_arbitrary_min_plus_conv(T *a, int n, T *b, int m, int *opt) {
    auto rec = [&](auto &&self, int lx, int rx, int ly, int ry) -> void {
        if (lx > rx) return;
        int mx = (lx + rx) >> 1;
        opt[mx] = ly;
        for (int i = ly; i <= ry; ++i)
            if (mx >= i && (mx - opt[mx]) >= n || a[mx - opt[mx]] + b[opt[mx]] > a[mx - i] + b[i])
                opt[mx] = i;
        self(self, lx, mx - 1, ly, opt[mx]);
        self(self, mx + 1, rx, opt[mx], ry);
    };
    rec(rec, 0, n + m - 2, 0, m - 1);
}

// Assumptions: 'a' is convex down
template<typename T>
std::vector<T> convex_arbitrary_min_plus_conv(const std::vector<T> &a,
    const std::vector<T> &b) {
    int n = a.size(), m = b.size();
    int *opt = (int*) malloc(sizeof(int) * (n + m - 1));
    convex_arbitrary_min_plus_conv(a.data(), n, b.data(), m, opt);
    std::vector<T> ans(n + m - 1);
    for (int i = 0; i < n + m - 1; ++i) ans[i] = a[i - opt[i]] + b[opt[i]
        1];
    free(opt);
    return ans;
}
```

Kinetic.cpp

Description: kinetic segment tree  
Time:  $\mathcal{O}(hz)$

49b24c, 127 lines

```
//vnutrennii functions - poluintervali, vneshnie - otrezki. ishet min priamuy
struct line {
    ll k,b,temp;
    ll eval() const {
        return k * temp + b;
    }
    ll melting_point(const line& other) const {
        ll val1 = eval();
        ll val2 = other.eval();
        assert(val1 <= val2);
    }
};
```

```
        if (other.k >= k) {
            return INF;
        }
        ll delta_val = val2 - val1;
        ll delta_k = k - other.k;
        assert(delta_val >= 0 && delta_k > 0);
        return (delta_val + delta_k - 1) / delta_k;
    }
};

struct kinetic_segtree {
    struct node {
        ll lazy_b = 0, lazy_temp = 0, melt = INF;
        line best;
        node(line best = line()) : best(best) {}
    };
    int n;
    vector<node> tree;
    void update(int v) {
        if (make_pair(tree[v << 1].best.eval(), tree[v << 1].best.k) <
            make_pair(tree[v << 1 | 1].best.eval(), tree[v << 1 | 1].
                best.k)) {
            tree[v].best = tree[v << 1].best;
            tree[v].melt = tree[v].best.melting_point(tree[v << 1 | 1].
                best);
        } else {
            tree[v].best = tree[v << 1 | 1].best;
            tree[v].melt = tree[v].best.melting_point(tree[v << 1].best);
        }

        tree[v].melt = min({tree[v].melt, tree[v << 1].melt, tree[v << 1
            | 1].melt});
        assert(tree[v].melt > 0);
    }
    void apply(int v, int vl, int vr, ll delta_b, ll delta_temp) {
        tree[v].lazy_b += delta_b;
        tree[v].lazy_temp += delta_temp;

        tree[v].best.b += delta_b;
        tree[v].best.temp += delta_temp;

        tree[v].melt -= delta_temp;
        if (tree[v].melt <= 0) {
            push(v, vl, vr);
            update(v);
        }
    }
    void push(int v, int vl, int vr) {
        int vm = (vl + vr) / 2;
        apply(v << 1, vl, vm, tree[v].lazy_b, tree[v].lazy_temp);
        apply(v << 1 | 1, vm, vr, tree[v].lazy_b, tree[v].lazy_temp);
        tree[v].lazy_b = 0;
        tree[v].lazy_temp = 0;
    }
    void build(int v, int vl, int vr, const vector<line> &lines) {
        if (vr - vl == 1) {
            tree[v] = node(lines[vl]);
            return;
        }
        int vm = (vl + vr) / 2;
        build(v << 1, vl, vm, lines);
        build(v << 1 | 1, vm, vr, lines);
        update(v);
    }
    void add(int v, int vl, int vr, int l, int r, ll delta_b, ll
        delta_temp) {
        if (r <= vl || vr <= l) {
            return;
        }
        if (l <= vl && vr <= r) {
            apply(v, vl, vr, delta_b, delta_temp);
            return;
        }
        push(v, vl, vr);
        int vm = (vl + vr) / 2;
        add(v << 1, vl, vm, l, r, delta_b, delta_temp);
        add(v << 1 | 1, vm, vr, l, r, delta_b, delta_temp);
        update(v);
    }
};
```

```
void change_line(int v, int vl, int vr, int pos, const line &new_line)
    ) {
        if (vr - vl == 1) {
            tree[v].best = new_line;
            return;
        }
        push(v, vl, vr);
        int vm = (vl + vr) / 2;
        if (pos < vm) {
            change_line(v << 1, vl, vm, pos, new_line);
        } else {
            change_line(v << 1 | 1, vm, vr, pos, new_line);
        }
        update(v);
    }
    ll query(int v, int vl, int vr, int l, int r) {
        if (r <= vl || vr <= l) {
            return INF;
        }
        if (l <= vl && vr <= r) {
            return tree[v].best.eval();
        }
        push(v, vl, vr);
        int vm = (vl + vr) / 2;
        return min(query(v << 1, vl, vm, l, r), query(v << 1 | 1, vm, vr,
            l, r));
    }
    kinetic_segtree(const vector<line> &lines) : n(lines.size()), tree(4
        * n) {
        build(1, 0, n, lines);
    }
    kinetic_segtree(int n) : n(n), tree(4 * n) {
        vector <line> lines(n, {0, INF, 0});
        build(1, 0, n, lines);
    }
    void add(int l, int r, ll delta_b, ll delta_temp) {
        assert(delta_temp >= 0);
        add(1, 0, n, l, r + 1, delta_b, delta_temp);
    }
    void change_line(int pos, const line &new_line) {
        assert(0 <= pos && pos < n);
        change_line(1, 0, n, pos, new_line);
    }
    ll query(int l, int r) {
        return query(1, 0, n, l, r + 1);
    }
};
```

Math (6)

BerlekampMassey.cpp

Description: Find the shortest linear-feedback shift register

Time:  $\mathcal{O}(n^2)$  505033, 36 lines

```
vector<int> berlekamp_massey(vector<int> x) {
    vector<int> ls, cur;
    int lf = 0, d = 0;
    for (int i = 0; i < x.size(); ++i) {
        ll t = 0;
        for (int j = 0; j < cur.size(); ++j) {
            t = (t + 1ll * x[i - j - 1] * cur[j]) % MOD;
        }
        if ((t - x[i]) % MOD == 0)
            continue;
        if (cur.empty()) {
            cur.resize(i + 1);
            lf = i;
            d = (t - x[i]) % MOD;
            continue;
        }
        ll k = -(x[i] - t) * pw(d, MOD - 2) % MOD;
        vector<int> c(i - lf - 1);
        c.push_back(k);
        for (auto &j : ls)
            c.push_back(-j * k % MOD);
        if (c.size() < cur.size())
            c.resize(cur.size());
        for (int j = 0; j < cur.size(); ++j) {
            c[j] = (c[j] + cur[j]) % MOD;
        }
        if (i - lf + (int)ls.size() >= (int)cur.size()) {
            tie(ls, lf, d) = make_tuple(cur, i, (t - x[i]) % MOD);
        }
        cur = c;
    }
    for (auto &i : cur)
        i = (i % MOD + MOD) % MOD;
    return cur;
}
// for a_i = 2 * a_{i-1} + a_{i-1} returns {2, 1}
```

GoncharFedor.cpp

Description: Calculating number of points  $x, y \geq 0, Ax + By \leq C$

Time:  $\mathcal{O}(\log(C))$  0ef10e, 11 lines

```
ll solve_triangle(ll A, ll B, ll C) { // x,y >=0, Ax+By<=C
    if (C < 0)
        return 0;
    if (A > B)
        swap(A, B);
    ll p = C / B;
    ll k = B / A;
    ll d = (C - p * B) / A;
    return solve_triangle(B - k * A, A, C - A * (k * p + d + 1)) +
        (p + 1) * (d + 1) + k * p * (p + 1) / 2;
}
```

CRT.cpp

Description: CRT for arbitrary modulus

28309e, 25 lines

```
int extgcd(int a, int b, int &x, int &y) { // define int ll
    if (a == 0) {
        x = 0, y = 1;
        return b;
    }
    int x1, y1;
    int g = extgcd(b % a, a, x1, y1);
    x = y1 - x1 * (b / a);
    y = x1;
    return g;
}
lcm(int a, int b) { return a / __gcd(a, b) * b; }
int crt(int mod1, int mod2, int rem1, int rem2) {
    int r = (rem2 - (rem1 % mod2) + mod2) % mod2;
    int x, y;
    int g = extgcd(mod1, mod2, x, y);
```

```
    if (r % g) return -1;
    x %= mod2;
    x += mod2;
    if (x < 0) x += mod2;
    int ans = (x * (r / g)) % mod2;
    ans = ans * mod1 + rem1;
    assert(ans % mod1 == rem1);
    assert(ans % mod2 == rem2);
    return ans % lcm(mod1, mod2);
}
```

Fastmod.cpp

Description: Fast multiplication by modulo(in [0;2b))

38ea39, 7 lines

```
struct FastMod {
    ull b, m;
    FastMod(ull b) : b(b), m(-1ULL / b) {}
    ull reduce(ull a) { // a % b + (0 or b)
        return a - (ull)((__uint128_t(m) * a) >> 64) * b;
    }
};
```

PrimalityTest.cpp

Description: Checking primality of p

Time:  $\mathcal{O}(\log(C))$  ad2714, 32 lines

```
const int iters = 8; // can change
bool isprime(ll p) {
    if (p == 1 || p == 4)
        return 0;
    if (p == 2 || p == 3)
        return 1;
    for (int it = 0; it < iters; ++it) {
        ll a = rnd() % (p - 2) + 2;
        ll nw = p - 1;
        while (nw % 2 == 0)
            nw /= 2;
        ll x = binpow(a, nw, p); // int128
        if (x == 1)
            continue;
        ll last = x;
        nw += 2;
        while (nw <= p - 1) {
            x = (__int128_t)x * x % p;
            if (x == 1) {
                if (last != p - 1) {
                    return 0;
                }
                break;
            }
            last = x;
            nw += 2;
        }
        if (x != 1)
            return 0;
    }
    return 1;
}
```

XorConvolution.cpp

Description: Calculating xor-convolution of 2 vectors modulo smth

Time:  $\mathcal{O}(n \log(n))$  454afd, 23 lines

```
void fwht(vector<int> &a) {
    int n = a.size();
    for (int l = 1; l < n; l <= 1) {
        for (int i = 0; i < n; i <= 2 * l) {
            for (int j = 0; j < l; ++j) {
                int u = a[i + j], v = a[i + j + l];
                a[i + j] = add(u, v), a[i + j + l] = sub(u, v);
            }
        }
    }
}
// https://judge.yosupo.jp/problem/bitwise_xor_convolution
vector<int> xorconvo(vector<int> a, vector<int> b) {
    int n = 1;
    while (n < max(a.size(), b.size()))
        n *= 2;
    a.resize(n), b.resize(n);
    fwht(a), fwht(b);
```

```
    int in = inv(n);
    for (int i = 0; i < n; ++i)
        a[i] = mul(a[i], mul(b[i], in));
    fwht(a);
    return a;
}
```

Factorization.cpp

Description: Factorizing a number real quick

Time:  $\mathcal{O}(n^{\frac{1}{4}})$  f0d7c6, 51 lines

```
ll gcd(ll a, ll b) {
    while (b)
        a %= b, swap(a, b);
    return a;
}

ll f(ll a, ll n) { return ((__int128_t)a * a % n + 1) % n; }

vector<ll> factorize(ll n) {
    if (n <= 1e6) { // can add primality check for speed?
        vector<ll> res;
        for (ll i = 2; i * i <= n; ++i) {
            while (n % i == 0) {
                res.pb(i);
                n /= i;
            }
        }
        if (n != 1)
            res.pb(n);
        return res;
    }
    ll x = rnd() % (n - 1) + 1;
    ll y = x;
    ll tries = 10 * sqrt(sqrt(n));
    const int C = 60;
    for (ll i = 0; i < tries; i += C) {
        ll xs = x;
        ll ys = y;
        ll m = 1;
        for (int k = 0; k < C; ++k) {
            x = f(x, n);
            y = f(y, n, n);
            m = (__int128_t)m * abs(x - y) % n;
        }
        if (gcd(n, m) == 1)
            continue;
        x = xs, y = ys;
        for (int k = 0; k < C; ++k) {
            x = f(x, n);
            y = f(y, n, n);
            ll res = gcd(n, abs(x - y));
            if (res != 1 && res != n) {
                vector<ll> v1 = factorize(res), v2 = factorize(n / res);
                for (auto j : v2)
                    v1.pb(j);
                return v1;
            }
        }
    }
    return {n};
}
```

NTT.cpp

Description: Calculating FFT modulo MOD

Time:  $\mathcal{O}(n \log(n))$  3e2f3a, 226 lines

// DONT FORGET TO CALL initNTT() AND CHECK MAXLOG

```
const int MOD = 998244353;
const int G = 3;
const int MAXLOG = 23;
int W[1 << MAXLOG];
bool nttinit = false;
vector<int> pws;

int add(int a, int b) {
    a += b;
```

```

    if (a >= MOD) {
        return a - MOD;
    }
    return a;
}

int sub(int a, int b) {
    a -= b;
    if (a < 0) {
        return a + MOD;
    }
    return a;
}

int mul(int a, int b) {
    return (ll) a * b % MOD;
}

int power(int a, int n) {
    int ans = 1;
    while (n) {
        if (n & 1) {
            ans = mul(ans, a);
        }
        a = mul(a, a);
        n >>= 1;
    }
    return ans;
}

int inv(int a) {
    return power(a, MOD - 2);
}

void initNTT() {
    assert((MOD - 1) % (1 << MAXLOG) == 0);
    pws.push_back(power(G, (MOD - 1) / (1 << MAXLOG)));
    for (int i = 0; i < MAXLOG - 1; ++i) {
        pws.push_back(mul(pws.back(), pws.back()));
    }
    assert(pws.back() == MOD - 1);
    W[0] = 1;
    for (int i = 1; i < (1 << MAXLOG); ++i) {
        W[i] = mul(W[i - 1], pws[0]);
    }
}

void ntt(int n, vector<int>& a, bool rev) {
    if (!nttinit) {
        initNTT();
        nttinit = 1;
    }
    int lg = log2(n);
    vector<int> rv(n);
    for (int i = 1; i < n; ++i) {
        rv[i] = (rv[i >> 1] >> 1) ^ ((i & 1) << (lg - 1));
        if (rv[i] > i) swap(a[i], a[rv[i]]);
    }
    int num = MAXLOG - 1;
    for (int len = 1; len < n; len *= 2) {
        for (int i = 0; i < n; i += 2 * len) {
            for (int j = 0; j < len; ++j) {
                int u = a[i + j], v = mul(W[j << num], a[i + j + len]);
                a[i + j] = add(u, v);
                a[i + j + len] = sub(u, v);
            }
        }
        --num;
    }
    if (rev) {
        int rev_n = power(n, MOD - 2);
        for (int i = 0; i < n; ++i) a[i] = mul(a[i], rev_n);
        reverse(a.begin() + 1, a.end());
    }
}

vector<int> conv(vector<int> a, vector<int> b) {
    int lg = 0;
    while ((1 << lg) < a.size() + b.size() + 1)

```

```

        ++lg;
        int n = 1 << lg;
        assert(a.size() + b.size() <= n + 1);
        a.resize(n);
        b.resize(n);
        ntt(n, a, false);
        ntt(n, b, false);
        for (int i = 0; i < n; ++i) {
            a[i] = mul(a[i], b[i]);
        }
        ntt(n, a, true);
        while (a.size() > 1 && a.back() == 0) {
            a.pop_back();
        }
        return a;
    }

    vector<int> add(vector<int> a, vector<int> b) {
        a.resize(max(a.size(), b.size()));
        for (int i = 0; i < (int) b.size(); ++i) {
            a[i] = add(a[i], b[i]);
        }
        return a;
    }

    vector<int> sub(vector<int> a, vector<int> b) {
        a.resize(max(a.size(), b.size()));
        for (int i = 0; i < (int) b.size(); ++i) {
            a[i] = sub(a[i], b[i]);
        }
        return a;
    }

    vector<int> inv(const vector<int> &a, int need) {
        vector<int> b = {inv(a[0])};
        while ((int) b.size() < need) {
            vector<int> al = a;
            int m = b.size();
            al.resize(min((int) al.size(), 2 * m));
            b = conv(b, sub({2}, conv(al, b)));
            b.resize(2 * m);
        }
        b.resize(need);
        return b;
    }

    vector<int> div(vector<int> a, vector<int> b) {
        if (count(all(a), 0) == a.size()) {
            return {0};
        }
        assert(a.back() != 0 && b.back() != 0);
        int n = a.size() - 1;
        int m = b.size() - 1;
        if (n < m) {
            return {0};
        }
        reverse(all(a));
        reverse(all(b));
        a.resize(n - m + 1);
        b.resize(n - m + 1);
        vector<int> c = inv(b, b.size());
        vector<int> q = conv(a, c);
        q.resize(n - m + 1);
        reverse(all(q));
        return q;
    }

    vector<int> mod(vector<int> a, vector<int> b) {
        auto res = sub(a, conv(b, div(a, b)));
        while (res.size() > 1 && res.back() == 0) {
            res.pop_back();
        }
        return res;
    }

    vector<int> multipoint(vector<int> a, vector<int> x) {
        int n = x.size();
        vector<vector<int>> tree(2 * n);
        for (int i = 0; i < n; ++i) {

```

```

            tree[i + n] = {x[i], MOD - 1};
        }
        for (int i = n - 1; i; --i) {
            tree[i] = conv(tree[2 * i], tree[2 * i + 1]);
        }
        tree[1] = mod(a, tree[1]);
        for (int i = 2; i < 2 * n; ++i) {
            tree[i] = mod(tree[i >> 1], tree[i]);
        }
        vector<int> res(n);
        for (int i = 0; i < n; ++i) {
            res[i] = tree[i + n][0];
        }
        return res;
    }

    vector<int> deriv(vector<int> a) {
        for (int i = 1; i < (int) a.size(); ++i) {
            a[i - 1] = mul(i, a[i]);
        }
        a.back() = 0;
        if (a.size() > 1) {
            a.pop_back();
        }
        return a;
    }

    vector<int> integ(vector<int> a) {
        a.push_back(0);
        for (int i = (int) a.size() - 1; i; --i) {
            a[i] = mul(a[i - 1], inv(i));
        }
        a[0] = 0;
        return a;
    }

    vector<int> log(vector<int> a, int n) {
        assert(a[0] == 1);
        auto res = integ(conv(deriv(a), inv(a, n)));
        res.resize(n);
        return res;
    }

    vector<int> exp(vector<int> a, int need) {
        assert(a[0] == 0);
        vector<int> b = {1};
        while ((int) b.size() < need) {
            vector<int> al = a;
            int m = b.size();
            al.resize(min((int) al.size(), 2 * m));
            al[0] = add(al[0], 1);
            b = conv(b, sub(al, log(b, 2 * m)));
            b.resize(2 * m);
        }
        b.resize(need);
        return b;
    }
}

FFT.cpp
Description: Calculating product of two polynomials
Time:  $\mathcal{O}(n \log(n))$ 
3adba5, 67 lines

const ld PI = acos(-1);
using cd = complex<ld>;
const int MAXLOG = 19, N = (1 << MAXLOG), MAXN = (1 << MAXLOG) + 228;
int rev[MAXN];
cd w[MAXN];
bool fftInit = false;

void initFFT() {
    for (int i = 0; i < N; ++i) {
        w[i] = cd(cos(2 * PI * i / N), sin(2 * PI * i / N));
    }
    rev[0] = 0;
    for (int i = 1; i < N; ++i) {
        rev[i] = (rev[i >> 1] >> 1) ^ ((i & 1) << (MAXLOG - 1));
    }
}

```

```
void FFT(int n, vector<cd>& a, bool rv = false) {
    if (!fftInit) {
        initFFT();
        fftInit = 1;
    }
    int LOG = ceil(log2(n));
    for (int i = 0; i < n; ++i) {
        if (i < (rev[i] >> (MAXLOG - LOG))) {
            swap(a[i], a[(rev[i] >> (MAXLOG - LOG))]);
        }
    }
    for (int lvl = 0; lvl < LOG; ++lvl) {
        int len = 1 << lvl;
        for (int st = 0; st < n; st += len * 2) {
            for (int i = 0; i < len; ++i) {
                cd x = a[st + i], y = a[st + len + i] * w[i << (MAXLOG - 1 - lvl)];
                a[st + i] = x + y;
                a[st + i + len] = x - y;
            }
        }
    }
    if (rv) {
        reverse(a.begin() + 1, a.end());
        for (auto& i : a) i /= n;
    }
}

vector<ll> mul(vector<ll> a, vector<ll> b) {
    int xd = max(a.size(), b.size()) * 2;
    int cur = 1;
    while (cur < xd) {
        cur *= 2;
    }
    a.resize(cur);
    b.resize(cur);
    vector<cd> ma(cur), mb(cur);
    for (int i = 0; i < cur; ++i) {
        ma[i] += a[i];
        mb[i] += b[i];
    }
    FFT(cur, ma);
    FFT(cur, mb);
    for (int i = 0; i < cur; ++i) ma[i] *= mb[i];
    FFT(cur, ma, true);
    vector<ll> ans(cur);
    for (int i = 0; i < cur; ++i) {
        ans[i] = (ll)(ma[i].real() + 0.5);
    }
    return ans;
}
```

AndConvolution.cpp

Description: Calculating and-convolution modulo smth  
Time:  $\mathcal{O}(n \log(n))$  5dedf4, 24 lines

```
void conv(vector<int> &a, bool x) {
    int n = a.size();
    for (int j = 0; (1 << j) < n; ++j) {
        for (int i = 0; i < n; ++i) {
            if (!(i & (1 << j))) {
                if (x)
                    a[i] = add(a[i], a[i | (1 << j)]);
                else
                    a[i] = sub(a[i], a[i | (1 << j)]);
            }
        }
    }
}

vector<int> andcon(vector<int> a, vector<int> b) {
    int n = 1;
    while (n < max(a.size(), b.size()))
        n *= 2;
    a.resize(n), b.resize(n);
    conv(a, 1), conv(b, 1);
    for (int i = 0; i < n; ++i)
        a[i] = mul(a[i], b[i]);
    conv(a, 0);
    return a;
}
```

```
}

Simplex.cpp
Description: Simplex
Time: exponential XD(ok for 200-300 variables/bounds) 4dda3c, 99 lines

/* solver for linear programs of the form
maximize c^T x, subject to A x <= b, x >= 0
outputs target function for optimal solution and
the solution by reference
if unbounded above : returns inf, if infeasible : returns -inf
create Simplex_Steep<ld> LP(A, b, c), then call LP. Solve (x)
*/
template<typename DOUBLE>
struct Simplex_Steep {
    using VD = vector<DOUBLE>;
    using VVD = vector<VD>;
    using VI = vector<int>;
    DOUBLE EPS = 1e-12;
    int m, n;
    VI B, N;
    VVD D;
    Simplex_Steep(const VVD &A, const VD &b, const VD &c)
        : m(b.size()), n(c.size()), B(m), N(n + 1), D(m + 2, VD(n + 2)) {
        for (int i = 0; i < m; i++)
            for (int j = 0; j < n; j++) D[i][j] = A[i][j];
        for (int i = 0; i < m; i++) {
            B[i] = n + i;
            D[i][n] = -1;
            D[i][n + 1] = b[i];
        }
        for (int j = 0; j < n; j++) {
            N[j] = j;
            D[m][j] = -c[j];
        }
        N[n] = -1;
        D[m + 1][n] = 1;
    }
    void Pivot(int r, int s) {
        for (int i = 0; i < m + 2; i++)
            if (i != r)
                for (int j = 0; j < n + 2; j++)
                    if (j != s) D[i][j] -= D[r][j] * D[i][s] / D[r][s];
        for (int j = 0; j < n + 2; j++)
            if (j != s) D[r][j] /= D[r][s];
        for (int i = 0; i < m + 2; i++)
            if (i != r) D[i][s] /= -D[r][s];
        D[r][s] = 1.0 / D[r][s];
        swap(B[r], N[s]);
    }
    bool Simplex(int phase) {
        int x = m + (int)(phase == 1);
        while (true) {
            int s = -1;
            DOUBLE c_val = -1;
            for (int j = 0; j <= n; j++) {
                if (phase == 2 && N[j] == -1) continue;
                DOUBLE norm_sq = 0;
                for (int k = 0; k <= m; k++) norm_sq += D[k][j] * D[k][j];
                norm_sq = max(norm_sq, EPS);
                DOUBLE c_val_j = D[x][j] / sqrt1(norm_sq);
                if (s == -1 || c_val_j < c_val || (c_val == c_val_j && N[j] < N[s])) {
                    s = j;
                    c_val = c_val_j;
                }
            }
            if (D[x][s] >= -EPS) return true;
            int r = -1;
            for (int i = 0; i < m; i++) {
                if (D[i][s] <= EPS) continue;
                if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] || (D[i][n + 1] / D[i][s] == D[r][n + 1] / D[r][s] && B[i] < B[r]))
                    r = i;
            }
            if (r == -1) return false;
        }
    }
};
```

```
Pivot(r, s);
}
}
DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++)
        if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] <= -EPS) {
        Pivot(r, n);
        if (!Simplex(1) || D[m + 1][n + 1] < -EPS)
            return -numeric_limits<DOUBLE>::infinity();
        for (int i = 0; i < m; i++)
            if (B[i] == -1) {
                int s = -1;
                for (int j = 0; j <= n; j++)
                    if (s == -1 || D[i][j] < D[i][s] || (D[i][j] == D[i][s] && N[j] < N[s]))
                        s = j;
                Pivot(i, s);
            }
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++)
        if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];
};
```

6.1 Fun things

$ClassesCount = \frac{1}{|G|} \sum_{\pi \in G} I(\pi)$

$ClassesCount = \frac{1}{|G|} \sum_{\pi \in G} k^{C(\pi)}$

Stirling 2kind - count of partitions of n objects into k nonempty sets:

$S(n, k) = S(n - 1, k - 1) + kS(n - 1, k)$

$S(n, k) = \sum_{j=0}^{n-1} \binom{n-1}{j} S(j, k - 1)$

$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k+j} \binom{k}{j} j^n$

$n! \approx \sqrt{2n\pi} (\frac{n}{e})^n$   
 $\binom{n}{k} \equiv \prod_i \binom{n_i}{k_i}, n_i, k_i$  - digits of  $n, k$  in p-adic system

$\int_a^b f(x)dx \approx \frac{b-a}{6} (f(a) + 4f(\frac{a+b}{2}) + f(b))$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, O(loglog)$

$G(n) = n \oplus (n >> 1)$

$g(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} g(d) \mu(\frac{n}{d})$

$\sum_{d|n} \mu(d) = [n = 1], \mu(1) = 1, \mu(p) = -1, \mu(p^k) = 0$

$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

$tg(a \pm b) = \frac{tg a \pm tg b}{1 \mp tg a tg b}$

$ctg(a \pm b) = \frac{ctg a ctg b \mp 1}{ctg b \pm ctg a}$

$\sin \frac{a}{2} = \pm \sqrt{\frac{1 - \cos a}{2}}$

$\cos \frac{a}{2} = \pm \sqrt{\frac{1 + \cos a}{2}}$

$tg \frac{a}{2} = \frac{\sin a}{1 - \cos a} = \frac{1 - \cos a}{sina}$

$\sin a \sin b = \frac{\cos(a - b) - \cos(a + b)}{2}$

$\sin a \cos b = \frac{\sin(a - b) + \sin(a + b)}{2}$

$\cos a \cos b = \frac{\cos(a - b) + \cos(a + b)}{2}$

1 jan 2000 - saturday, 1 jan 1900 - monday, 14 apr 1961 - friday

Bell numbers: 0:1, 1:1, 2:2, 3:5, 4:15, 5:52, 6:203, 7:877, 8:4140,

9:21147, 10:115975, 11:678570, 12:4213597, 13:27644437, 14:190899322,

15:1382958545, 16:10480142147, 17:82864869804, 18:682076806159,

19:5832742205057, 20:51724158235372, 21:474869816156751,

22:4506715738447323, 23:44152005855084346

Fibonacci: 45:1134903170. 46:1836311903(max int), 91:

4660046610375530309

Highly composite numbers:

$\leq 1000 : d(840) = 32, \leq 10^4 : d(9240) = 64, \leq 10^5 : d(83160) = 128, \leq 10^6 : d(720720) = 240, \leq 10^7 : d(8648640) = 448, \leq 10^8 : d(91891800) = 768, \leq 10^9 : d(931170240) = 1344, \leq 10^{11} : d(97772875200) = 4032, \leq 10^{15} : d(866421317361600) = 26880, \leq 10^{18} : d(897612484786617600) = 103680$

BEST Theorem:

$ec(G) = \#SpanningTrees(G) \cdot \prod_{v \in V} (deg(v) - 1)!$

Erdos: Graph exists

$\Leftrightarrow d_1 \geq .. \geq d_n, \forall k \sum_{i=1}^k d_i \leq k(k - 1) + \sum_{i=k+1}^n min(d_i, k)$

Pick:  $Area = Interior + \frac{Bounds}{2} - 1$

Euler:  $V - E + F = 1 + C$

Kirchhoff: put degree on diagonal,  $-1$  for each edge, cut out first row + column, calc det - result is  $\#SpanningTrees$

Table of Basic Integrals (7)

Basic Forms

∫ x^n dx = 1/(n+1) x^{n+1}, n ≠ -1 (7.1)

∫ 1/x dx = ln |x| (7.2)

∫ u dv = uv - ∫ v du (7.3)

∫ 1/(ax+b) dx = 1/a ln |ax+b| (7.4)

Integrals of Rational Functions

∫ 1/(x+a)^2 dx = -1/(x+a) (7.5)

∫ (x+a)^n dx = (x+a)^{n+1}/(n+1), n ≠ -1 (7.6)

∫ x(x+a)^n dx = (x+a)^{n+1}((n+1)x-a)/((n+1)(n+2)) (7.7)

∫ 1/(1+x^2) dx = tan^-1 x (7.8)

∫ 1/(a^2+x^2) dx = 1/a tan^-1 x/a (7.9)

∫ x/(a^2+x^2) dx = 1/2 ln |a^2+x^2| (7.10)

∫ x^2/(a^2+x^2) dx = x - a tan^-1 x/a (7.11)

∫ x^3/(a^2+x^2) dx = 1/2 x^2 - 1/2 a^2 ln |a^2+x^2| (7.12)

∫ 1/(ax^2+bx+c) dx = 2/(√(4ac-b^2)) tan^-1 (2ax+b)/√(4ac-b^2) (7.13)

∫ 1/((x+a)(x+b)) dx = 1/(b-a) ln (a+x)/(b+x), a ≠ b (7.14)

∫ x/(x+a)^2 dx = a/(a+x) + ln |a+x| (7.15)

∫ x/(ax^2+bx+c) dx = 1/2a ln |ax^2+bx+c| - b/(a√(4ac-b^2)) tan^-1 (2ax+b)/√(4ac-b^2) (7.16)

Integrals with Roots

∫ √(x-a) dx = 2/3 (x-a)^{3/2} (7.17)

∫ 1/√(x±a) dx = 2√(x±a) (7.18)

∫ 1/√(a-x) dx = -2√(a-x) (7.19)

∫ x√(x-a) dx = { 2a/3 (x-a)^{3/2} + 2/5 (x-a)^{5/2}, or 2/3 x(x-a)^{3/2} - 4/15 (x-a)^{5/2}, or 2/15 (2a+3x)(x-a)^{3/2} } (7.20)

∫ √(ax+b) dx = (2b/3a + 2x/3) √(ax+b) (7.21)

∫ (ax+b)^{3/2} dx = 2/5a (ax+b)^{5/2} (7.22)

∫ x/√(x±a) dx = 2/3 (x∓2a)√(x±a) (7.23)

∫ √(x/(a-x)) dx = -√(x(a-x)) - a tan^-1 √(x(a-x))/(x-a) (7.24)

∫ √(x/(a+x)) dx = √(x(a+x)) - a ln [√x + √(x+a)] (7.25)

∫ x√(ax+b) dx = 2/15a^2 (-2b^2+abx+3a^2x^2)√(ax+b) (7.26)

∫ √(x(ax+b)) dx = 1/(4a^{3/2}) [(2ax+b)√(ax(ax+b)) - b^2 ln |a√x + √(a(ax+b))|] (7.27)

∫ √(x^3(ax+b)) dx = [b/12a - b^2/(8a^2x) + x/3] √(x^3(ax+b)) + b^3/(8a^{5/2}) ln |a√x + √(a(ax+b))| (7.28)

∫ √(x^2±a^2) dx = 1/2 x√(x^2±a^2) ± 1/2 a^2 ln |x+√(x^2±a^2)| (7.29)

∫ √(a^2-x^2) dx = 1/2 x√(a^2-x^2) + 1/2 a^2 tan^-1 x/√(a^2-x^2) (7.30)

∫ x√(x^2±a^2) dx = 1/3 (x^2±a^2)^{3/2} (7.31)

∫ 1/√(x^2±a^2) dx = ln |x+√(x^2±a^2)| (7.32)

∫ 1/√(a^2-x^2) dx = sin^-1 x/a (7.33)

∫ x/√(x^2±a^2) dx = √(x^2±a^2) (7.34)

∫ x/√(a^2-x^2) dx = -√(a^2-x^2) (7.35)

∫ x^2/√(x^2±a^2) dx = 1/2 x√(x^2±a^2) ± 1/2 a^2 ln |x+√(x^2±a^2)| (7.36)

∫ √(ax^2+bx+c) dx = (b+2ax)/(4a) √(ax^2+bx+c) + 4ac-b^2/(8a^{3/2}) ln |2ax+b+2√(a(ax^2+bx+c))| (7.37)

∫ x√(ax^2+bx+c) dx = 1/(48a^{5/2}) (2√a√(ax^2+bx+c) (-3b^2+2abx+8a(c+ax^2)) + 3(b^3-4abc) ln |b+2ax+2√a√(ax^2+bx+c)|) (7.38)

∫ 1/√(ax^2+bx+c) dx = 1/√a ln |2ax+b+2√(a(ax^2+bx+c))| (7.39)

∫ x/√(ax^2+bx+c) dx = 1/a √(ax^2+bx+c) - b/(2a^{3/2}) ln |2ax+b+2√(a(ax^2+bx+c))| (7.40)

∫ dx/(a^2+x^2)^{3/2} = x/(a^2√(a^2+x^2)) (7.41)

Integrals with Logarithms

∫ ln ax dx = x ln ax - x (7.42)

∫ x ln x dx = 1/2 x^2 ln x - x^2/4 (7.43)

∫ x^2 ln x dx = 1/3 x^3 ln x - x^3/9 (7.44)

∫ x^n ln x dx = x^{n+1} (ln x/(n+1) - 1/((n+1)^2)), n ≠ -1 (7.45)

∫ ln x/x dx = 1/2 (ln ax)^2 (7.46)

∫ ln x/x^2 dx = -1/x - ln x/x (7.47)

∫ ln(ax+b) dx = (x+b/a) ln(ax+b) - x, a ≠ 0 (7.48)

∫ ln(x^2+a^2) dx = x ln(x^2+a^2) + 2a tan^-1 x/a - 2x (7.49)

∫ ln(x^2-a^2) dx = x ln(x^2-a^2) + a ln (x+a)/(x-a) - 2x (7.50)

∫ ln (ax^2+bx+c) dx = 1/a √(4ac-b^2) tan^-1 (2ax+b)/√(4ac-b^2) - 2x + (b/2a+x) ln (ax^2+bx+c) (7.51)

Integrals with Exponentials

Integrals with Trigonometric Functions

∫ x ln(ax + b) dx = (bx)/(2a) - 1/4 x^2 + 1/2 (x^2 - b^2/a^2) ln(ax + b) (7.52)

∫ x ln(a^2 - b^2 x^2) dx = -1/2 x^2 + 1/2 (x^2 - a^2/b^2) ln(a^2 - b^2 x^2) (7.53)

∫ (ln x)^2 dx = 2x - 2x ln x + x(ln x)^2 (7.54)

∫ (ln x)^3 dx = -6x + x(ln x)^3 - 3x(ln x)^2 + 6x ln x (7.55)

∫ x(ln x)^2 dx = x^2/4 + 1/2 x^2 (ln x)^2 - 1/2 x^2 ln x (7.56)

∫ x^2(ln x)^2 dx = 2x^3/27 + 1/3 x^3 (ln x)^2 - 2/9 x^3 ln x (7.57)

∫ e^{ax} dx = 1/a e^{ax} (7.58)

∫ √x e^{ax} dx = 1/a √x e^{ax} + i√π/(2a^{3/2}) erf(i√ax), where erf(x) = 2/√π ∫\_0^x e^{-t^2} dt (7.59)

∫ x e^x dx = (x - 1)e^x (7.60)

∫ x e^{ax} dx = (x/a - 1/a^2) e^{ax} (7.61)

∫ x^2 e^x dx = (x^2 - 2x + 2) e^x (7.62)

∫ x^2 e^{ax} dx = (x^2/a - 2x/a^2 + 2/a^3) e^{ax} (7.63)

∫ x^3 e^x dx = (x^3 - 3x^2 + 6x - 6) e^x (7.64)

∫ x^n e^{ax} dx = (x^n e^{ax})/a - n/a ∫ x^{n-1} e^{ax} dx (7.65)

∫ x^n e^{ax} dx = ((-1)^n)/(a^{n+1}) Γ[1 + n, -ax], where Γ(a, x) = ∫\_x^∞ t^{a-1} e^{-t} dt (7.66)

∫ e^{ax^2} dx = -i√π/(2√a) erf(ix√a) (7.67)

∫ e^{-ax^2} dx = √π/(2√a) erf(x√a) (7.68)

∫ x e^{-ax^2} dx = -1/(2a) e^{-ax^2} (7.69)

∫ x^2 e^{-ax^2} dx = 1/4 √(π/a^3) erf(x√a) - x/(2a) e^{-ax^2} (7.70)

∫ sin ax dx = -1/a cos ax (7.71)

∫ sin^2 ax dx = x/2 - (sin 2ax)/(4a) (7.72)

∫ sin^3 ax dx = -3 cos ax/(4a) + cos 3ax/(12a) (7.73)

∫ sin^n ax dx = -1/a cos ax {}\_2F\_1[1/2, (1-n)/2, 3/2, cos^2 ax] (7.74)

∫ cos ax dx = 1/a sin ax (7.75)

∫ cos^2 ax dx = x/2 + (sin 2ax)/(4a) (7.76)

∫ cos^3 ax dx = 3 sin ax/(4a) + sin 3ax/(12a) (7.77)

∫ cos^p ax dx = -1/(a(1+p)) cos^{1+p} ax × {}\_2F\_1[1/2+p, 1/2, 3/2+p, cos^2 ax] (7.78)

∫ cos x sin x dx = 1/2 sin^2 x + c\_1 = -1/2 cos^2 x + c\_2 = -1/4 cos 2x + c\_3 (7.79)

∫ cos ax sin bx dx = (cos[(a-b)x])/2(a-b) - (cos[(a+b)x])/2(a+b), a ≠ b (7.80)

∫ sin^2 ax cos bx dx = -sin[(2a-b)x]/4(2a-b) + sin bx/2b - sin[(2a+b)x]/4(2a+b) (7.81)

∫ sin^2 x cos x dx = 1/3 sin^3 x (7.82)

∫ cos^2 ax sin bx dx = (cos[(2a-b)x])/4(2a-b) - cos bx/2b - (cos[(2a+b)x])/4(2a+b) (7.83)

∫ cos^2 ax sin ax dx = -1/3a cos^3 ax (7.84)

∫ sin^2 ax cos^2 bxdx = x/4 - sin 2ax/8a - sin[2(a-b)x]/16(a-b) + sin 2bx/8b - sin[2(a+b)x]/16(a+b) (7.85)

∫ sin^2 ax cos^2 ax dx = x/8 - (sin 4ax)/(32a) (7.86)

∫ tan ax dx = -1/a ln cos ax (7.87)

∫ tan^2 ax dx = -x + 1/a tan ax (7.88)

∫ tan^n ax dx = (tan^{n+1} ax)/(a(1+n)) × {}\_2F\_1((n+1)/2, 1, (n+3)/2, -tan^2 ax) (7.89)

∫ tan^3 ax dx = 1/a ln cos ax + 1/(2a) sec^2 ax (7.90)

∫ sec x dx = ln |sec x + tan x| = 2 tanh^{-1} (tan x/2) (7.91)

∫ sec^2 ax dx = 1/a tan ax (7.92)

∫ sec^3 x dx = 1/2 sec x tan x + 1/2 ln |sec x + tan x| (7.93)

∫ sec x tan x dx = sec x (7.94)

∫ sec^2 x tan x dx = 1/2 sec^2 x (7.95)

∫ sec^n x tan x dx = 1/n sec^n x, n ≠ 0 (7.96)

∫ csc x dx = ln |tan x/2| = ln |csc x - cot x| + C (7.97)

∫ csc^2 ax dx = -1/a cot ax (7.98)

∫ csc^3 x dx = -1/2 cot x csc x + 1/2 ln |csc x - cot x| (7.99)

∫ csc^n x cot x dx = -1/n csc^n x, n ≠ 0 (7.100)

∫ sec x csc x dx = ln |tan x| (7.101)



## Products of Trigonometric Functions and Monomials

$$\int x \cos x \, dx = \cos x + x \sin x \quad (7.102)$$

$$\int x \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax \quad (7.103)$$

$$\int x^2 \cos x \, dx = 2x \cos x + (x^2 - 2) \sin x \quad (7.104)$$

$$\int x^2 \cos ax \, dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax \quad (7.105)$$

$$\int x^n \cos x \, dx = -\frac{1}{2}(i)^{n+1} [\Gamma(n+1, -ix) + (-1)^n \Gamma(n+1, ix)] \quad (7.106)$$

$$\int x^n \cos ax \, dx = \frac{1}{2}(ia)^{1-n} [(-1)^n \Gamma(n+1, -iax) - \Gamma(n+1, ixa)] \quad (7.107)$$

$$\int x \sin x \, dx = -x \cos x + \sin x \quad (7.108)$$

$$\int x \sin ax \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \quad (7.109)$$

$$\int x^2 \sin x \, dx = (2 - x^2) \cos x + 2x \sin x \quad (7.110)$$

$$\int x^2 \sin ax \, dx = \frac{2 - a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2} \quad (7.111)$$

$$\int x^n \sin x \, dx = -\frac{1}{2}(i)^n [\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, -ix)] \quad (7.112)$$

$$\int x \cos^2 x \, dx = \frac{x^2}{4} + \frac{1}{8} \cos 2x + \frac{1}{4} x \sin 2x \quad (7.113)$$

$$\int x \sin^2 x \, dx = \frac{x^2}{4} - \frac{1}{8} \cos 2x - \frac{1}{4} x \sin 2x \quad (7.114)$$

$$\int x \tan^2 x \, dx = -\frac{x^2}{2} + \ln \cos x + x \tan x \quad (7.115)$$

$$\int x \sec^2 x \, dx = \ln \cos x + x \tan x \quad (7.116)$$

## Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x) \quad (7.117)$$

$$\int e^{bx} \sin ax \, dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax) \quad (7.118)$$

$$\int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x) \quad (7.119)$$

$$\int e^{bx} \cos ax \, dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax) \quad (7.120)$$

$$\int x e^x \sin x \, dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x) \quad (7.121)$$

$$\int x e^x \cos x \, dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x) \quad (7.122)$$

## Integrals of Hyperbolic Functions

$$\int \cosh ax \, dx = \frac{1}{a} \sinh ax \quad (7.123)$$

$$\int e^{ax} \cosh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [a \cosh bx - b \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} + \frac{x}{2} & a = b \end{cases} \quad (7.124)$$

$$\int \sinh ax \, dx = \frac{1}{a} \cosh ax \quad (7.125)$$

$$\int e^{ax} \sinh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [-b \cosh bx + a \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} - \frac{x}{2} & a = b \end{cases} \quad (7.126)$$

$$\int \tanh ax \, dx = \frac{1}{a} \ln \cosh ax \quad (7.127)$$

$$\int e^{ax} \tanh bx \, dx = \begin{cases} \frac{e^{(a+2b)x}}{(a+2b)} {}_2F_1 \left[ 1 + \frac{a}{2b}, 1, 2 + \frac{a}{2b}, -e^{2bx} \right] \\ \quad - \frac{1}{a} e^{ax} {}_2F_1 \left[ 1, \frac{a}{2b}, 1 + \frac{a}{2b}, -e^{2bx} \right] & a \neq b \\ \frac{e^{ax} - 2 \tan^{-1}[e^{ax}]}{a} & a = b \end{cases} \quad (7.128)$$

$$\int \cos ax \cosh bx \, dx = \frac{1}{a^2 + b^2} [a \sin ax \cosh bx + b \cos ax \sinh bx] \quad (7.129)$$

$$\int \cos ax \sinh bx \, dx = \frac{1}{a^2 + b^2} [b \cos ax \cosh bx + a \sin ax \sinh bx] \quad (7.130)$$

$$\int \sin ax \cosh bx \, dx = \frac{1}{a^2 + b^2} [-a \cos ax \cosh bx + b \sin ax \sinh bx] \quad (7.131)$$

$$\int \sin ax \sinh bx \, dx = \frac{1}{a^2 + b^2} [b \cosh bx \sin ax - a \cos ax \sinh bx] \quad (7.132)$$

$$\int \sinh ax \cosh ax \, dx = \frac{1}{4a} [-2ax + \sinh 2ax] \quad (7.133)$$

$$\int \sinh ax \cosh bx \, dx = \frac{1}{b^2 - a^2} [b \cosh bx \sinh ax - a \cosh ax \sinh bx] \quad (7.134)$$

Problem	Status	Comment	Iurii	Alex	Leha
A - 1					
B - 2					
C - 3					
D - 4					
E - 5					
F - 6					
G - 7					
H - 8					
I - 9					
J - 10					
K - 11					
L - 12					
M - 13					
N - 14					
O - 15					