



National Research University Higher School of Economics

# Youthful Passion Fruit

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## Contest (1)

template.cpp42 lines

```
#ifndef LOCAL
#define _GLIBCXX_DEBUG
#endif
#include <bits/stdc++.h>
using namespace std;

using ll = long long;
using ld = long double;
using ull = unsigned long long;

#define pbc push_back
#define mp make_pair
#define all(v) (v).begin(), (v).end()
#define vin(v) for (auto &el : a) cin >> el

mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());

template <typename T1, typename T2> inline void chkmin(T1 &x, const T2 &y)
{
    if (y < x) {
        x = y;
    }
}

template <typename T1, typename T2> inline void chkmax(T1 &x, const T2 &y)
{
    if (x < y) {
        x = y;
    }
}

void solve() {

}

signed main() {
    cin.tie(0)->sync_with_stdio(0);
    cout.precision(20), cout.setf(ios::fixed);
    int t = 1;
    // cin >> t;
    while (t--) {
        solve();
    }
}
```

genfolders.sh6 lines

```
chmod +x bld*
for f in {A..Z}
do
    mkdir $f
    cp main.cpp bld* $f
done
```

bld1 lines

```
g++ -std=c++20 -g -DLOCAL -fsanitize=address,bounds,undefined -o $1 $1.cpp
```

bldf1 lines

```
g++ -std=c++20 -g -O2 -o $1 $1.cpp
```

hacks.sh2 lines

```
UBSAN_OPTIONS=print_stacktrace=1 ./main
gdb rbreak regex
```

hash.sh3 lines

```
# Hashes a file, ignoring all whitespace and comments.
# Use for verifying that code was correctly typed.
cpp -dD -P -fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

clion.cpp2 lines

```
set(CMAKE_CXX_STANDARD 20)
set(CMAKE_CXX_FLAGS "-DLOCAL")
```

## C++ (2)

GpHashtable.cppDescription: Hash map with mostly the same API as unordered\_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).41 lines

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

const int RANDOM =
    chrono::high_resolution_clock::now().time_since_epoch().count();
struct hasher {
    int operator()(int x) const { return x ^ RANDOM; }
};

gp_hash_table<int, int, hasher> table;
```

OrderedSet.cppDescription: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null\_type. Time: O(log(n))<bits/extc++.h>, <bits/stdc++.h>dff260, 37 lines

```
using namespace __gnu_pbds;
using namespace std;

template <typename T>
using ordered_set =
    tree<T, null_type, less<>, rb_tree_tag,
        tree_order_statistics_node_update>;

int main() {
    ordered_set<int> X;
    X.insert(1);
    X.insert(2);
    X.insert(4);
    X.insert(8);
    X.insert(16);

    assert(*X.find_by_order(1) == 2);
    assert(*X.find_by_order(2) == 4);
    assert(*X.find_by_order(4) == 16);
    assert(X.find_by_order(6) ==X.end());

    assert(X.order_of_key(-5) == 0);
    assert(X.order_of_key(1) == 0);
    assert(X.order_of_key(3) == 2);
    assert(X.order_of_key(4) == 2);
    assert(X.order_of_key(400) == 5);
    // std::cout << *X.find_by_order(1) << std::endl; // 2
    // std::cout << *X.find_by_order(2) << std::endl; // 4
    // std::cout << *X.find_by_order(4) << std::endl; // 16
    // std::cout << (end(X) == X.find_by_order(6)) << std::endl; // true

    // std::cout << X.order_of_key(-5) << std::endl; // 0
    // std::cout << X.order_of_key(1) << std::endl; // 0
    // std::cout << X.order_of_key(3) << std::endl; // 2
    // std::cout << X.order_of_key(4) << std::endl; // 2
    // std::cout << X.order_of_key(400) << std::endl; // 5
    return 0;
}
```

bitset.cppDescription: bitset521d1f, 2 lines

```
bs._Find_first()
bs._Find_next(idx) - returns right after
```

alloc.cppDescription: fastalloc8726b1, 11 lines

```
const int MAX_MEM = 1e8;
int mpos = 0;
char mem[MAX_MEM];
inline void *operator new(size_t n) {
    assert((mpos += n) <= MAX_MEM);
    return (void *) (mem + mpos - n);
}
```

```
void operator delete(void *) noexcept {
} // must have!
void operator delete(void *, size_t) noexcept {
} // must have!
```

fastio.cpp

Description: fastio79fd14, 52 lines

```
inline int readChar();
template <class T = int>
inline T readInt();
template <class T>
inline void writeInt(T x, char end = 0);
inline void writeChar(int x);
inline void writeWord(const char *s);
static const int buf_size = 4096;
inline int getChar() {
    static char buf[buf_size];
    static int len = 0, pos = 0;
    if (pos == len) pos = 0, len = fread(buf, 1, buf_size, stdin);
    if (pos == len) return -1;
    return buf[pos++];
}
inline int readChar() {
    int c = getChar();
    while (c <= 32) c = getChar();
    return c;
}
template <class T>
inline T readInt() {
    int s = 1, c = readChar();
    T x = 0;
    if (c == '-') s = -1, c = getChar();
    while ('0' <= c && c <= '9') x = x * 10 + c - '0', c = getChar();
    return s == 1 ? x : -x;
}
static int write_pos = 0;
static char write_buf[buf_size];
inline void writeChar(int x) {
    if (write_pos == buf_size)
        fwrite(write_buf, 1, buf_size, stdout), write_pos = 0;
    write_buf[write_pos++] = x;
}
template <class T>
inline void writeInt(T x, char end) {
    if (x < 0) writeChar('-'), x = -x;
    char s[24];
    int n = 0;
    while (x || !n) s[n++] = '0' + x % 10, x /= 10;
    while (n--) writeChar(s[n]);
    if (end) writeChar(end);
}
inline void writeWord(const char *s) {
    while (*s) writeChar(*s++);
}
struct Flusher {
    ~Flusher() {
        if (write_pos) fwrite(write_buf, 1, write_pos, stdout), write_pos = 0;
    }
} flusher;
```

Strings (3)

Manacher.cpp

Description: Manacher algorithmTime:  $\mathcal{O}(n)$ a6ddf8, 27 lines

```
vector<int> manacherOdd(string s) {
    int n = s.size();
    vector<int> d1(n);
    int l = 0, r = -1;
    for (int i = 0; i < n; ++i) {
        int k = i > r ? 1 : min(d1[l + r - i], r - i + 1);
        while (i + k < n && i - k >= 0 && s[i + k] == s[i - k])
            ++k;
        d1[i] = k;
        if (i + k - 1 > r)
            l = i - k + 1, r = i + k - 1;
    }
}

vector<int> manacherEven(string s) {
    int n = s.size();
    vector<int> d2(n);
    l = 0, r = -1;
    for (int i = 0; i < n; ++i) {
        int k = i > r ? 0 : min(d2[l + r - i + 1], r - i + 1);
        while (i + k < n && i - k - 1 >= 0 && s[i + k] == s[i - k - 1])
            ++k;
        d2[i] = k;
        if (i + k - 1 > r)
            l = i - k, r = i + k - 1;
    }
}
```

AhoCorasick.cpp

Description: Build aho-corasick automaton.Time:  $\mathcal{O}(n)$ ae5fc2, 19 lines

```
int go(int v, char c);

int get_link(int v) {
    if (t[v].link == -1)
        if (v == 0 || t[v].p == 0)
            t[v].link = 0;
        else
            t[v].link = go(get_link(t[v].p), t[v].pch);
    return t[v].link;
}

int go(int v, char c) {
    if (t[v].go[c] == -1)
        if (t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
        else
            t[v].go[c] = v == 0 ? 0 : go(get_link(v), c);
    return t[v].go[c];
}
```

SuffixArray.cpp

Description: Build suffix arrayTime:  $\mathcal{O}(n \log(n))$ 5bd011, 47 lines

```
vector<int> buildSuffixArray(string &s) {
    // Remove, if you want to sort cyclic shifts
    s += (char) (1);
    int n = s.size();
    vector<int> a(n);
    iota(all(a), 0);
    stable_sort(all(a), [&](int i, int j) { return s[i] < s[j]; });
    vector<int> c(n);
    int cc = 0;
    for (int i = 0; i < n; ++i) {
        if (i == 0 || s[a[i]] != s[a[i - 1]]) {
            c[a[i]] = cc++;
        } else {
            c[a[i]] = c[a[i - 1]];
        }
    }
    for (int L = 1; L < n; L *= 2) {
```

```
vector<int> cnt(n);
for (auto i : c) {
    cnt[i]++;
}
vector<int> pref(n);
for (int i = 1; i < n; ++i) {
    pref[i] = pref[i - 1] + cnt[i - 1];
}
vector<int> na(n);
for (int i = 0; i < n; ++i) {
    int pos = (a[i] - L + n) % n;
    na[pref[c[pos]]++] = pos;
}
a = na;
vector<int> nc(n);
cc = 0;
for (int i = 0; i < n; ++i) {
    if (i == 0 || c[a[i]] != c[a[i - 1]] ||
        c[(a[i] + L) % n] != c[(a[i - 1] + L) % n]) {
        nc[a[i]] = cc++;
    } else {
        nc[a[i]] = nc[a[i - 1]];
    }
}
c = nc;
}
a.erase(a.begin());
s.pop_back();
return a;
}
```

Lcp.cpp

Description: lcp arrayTime:  $\mathcal{O}(n)$ 1cc27c, 43 lines

```
vector<int> perm;
vector<int> buildLCP(string &s, vector<int> &a) {
    int n = s.size();
    vector<int> ra(n);
    for (int i = 0; i < n; ++i) {
        ra[a[i]] = i;
    }
    vector<int> lcp(n - 1);
    int cur = 0;
    for (int i = 0; i < n; ++i) {
        cur--;
        chkmax(cur, 0);
        if (ra[i] == n - 1) {
            cur = 0;
            continue;
        }
        int j = a[ra[i] + 1];
        while (s[i + cur] == s[j + cur]) cur++;
        lcp[ra[i]] = cur;
    }
    perm.resize(a.size());
    for (int i = 0; i < a.size(); ++i) perm[a[i]] = i;
    return lcp;
}

int cntr[MAXN];
int spt[MAXN][lgg];
void build(vector<int> &a) {
    for (int i = 0; i < a.size(); ++i) {
        spt[i][0] = a[i];
    }
    for (int i = 2; i < MAXN; ++i) cntr[i] = cntr[i / 2] + 1;
    for (int h = 1; (1 << (h - 1)) < a.size(); ++h) {
        for (int i = 0; i + (1 << (h - 1)) < a.size(); ++i) {
            spt[i][h] = min(spt[i][h - 1], spt[i + (1 << (h - 1))][h - 1]);
        }
    }
}

int getLCP(int l, int r) {
    l = perm[l], r = perm[r];
    if (l > r) swap(l, r);
    int xx = cntr[r - l];
    return min(spt[l][xx], spt[r - (1 << xx)][xx]);
}
```

Eertree.cpp

**Description:** Creates Eertree of string str  
**Time:**  $\mathcal{O}(n)$

```
7924c8, 40 lines

struct eertree {
    int len[MAXN], suffLink[MAXN];
    int to[MAXN][26];
    int numV, v;
    void addLetter(int n, string &str) {
        while (str[n - len[v] - 1] != str[n])
            v = suffLink[v];
        int u = suffLink[v];
        while (str[n - len[u] - 1] != str[n])
            u = suffLink[u];
        int u_ = to[u][str[n] - 'a'];
        int v_ = to[v][str[n] - 'a'];
        if (v_ == -1) {
            v_ = to[v][str[n] - 'a'] = numV;
            len[numV++] = len[v] + 2;
            suffLink[v_] = u_;
        }
        v = v_;
    }
    void init() {
        len[0] = -1;
        len[1] = 0;
        suffLink[1] = 0;
        suffLink[0] = 0;
        numV = 2;
        for (int i = 0; i < 26; ++i) {
            to[0][i] = numV++;
            suffLink[numV - 1] = 1;
            len[numV - 1] = 1;
        }
        v = 0;
    }
    void init(int sz) {
        for (int i = 0; i < sz; ++i) {
            len[i] = suffLink[i] = 0;
            for (int j = 0; j < 26; ++j)
                to[i][j] = -1;
        }
    }
};
```

SuffixAutomaton.cpp

**Description:** Build suffix automaton.  
**Time:**  $\mathcal{O}(n)$

```
662a10, 45 lines

struct state {
    int len, link;
    map<char, int> next;
};

const int MAXLEN = 100000;
state st[MAXLEN * 2];
int sz, last;

void sa_init() {
    sz = last = 0;
    st[0].len = 0;
    st[0].link = -1;
    ++sz;
    /*
    // if you want to build an automaton for different strings:
    for (int i=0; i<MAXLEN*2; ++i)
        st[i].next.clear();
    */
}

void sa_extend(char c) {
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    int p;
    for (p = last; p != -1 && !st[p].next.count(c); p = st[p].link)
        st[p].next[c] = cur;
    if (p == -1)
        st[cur].link = 0;
    else {
```

```
int q = st[p].next[c];
if (st[p].len + 1 == st[q].len)
    st[cur].link = q;
else {
    int clone = sz++;
    st[clone].len = st[p].len + 1;
    st[clone].next = st[q].next;
    st[clone].link = st[q].link;
    for (; p != -1 && st[p].next[c] == q; p = st[p].link)
        st[p].next[c] = clone;
    st[q].link = st[cur].link = clone;
}
last = cur;
}
```

PrefixZ.cpp

**Description:** Calculates Prefix,Z-functions  
**Time:**  $\mathcal{O}(n)$

```
1c4e93, 25 lines

vector<int> pf(string s) {
    int k = 0;
    vector<int> p(s.size());
    for (int i = 1; i < s.size(); ++i) {
        while (k && s[i] != s[k])
            k = p[k - 1];
        k += (s[i] == s[k]);
        p[i] = k;
    }
    return p;
}

vector<int> zf(string s) {
    int n = s.size();
    vector<int> z(n, 0);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r)
            z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
            ++z[i];
        if (i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1;
    }
    z[0] = n;
    return z;
}
```

MinShift.cpp

**Description:** Calculates min-cyclic-shift of s, Duval decomposition  
**Time:**  $\mathcal{O}(n)$

```
3f0fb9, 20 lines

string minshift(string s) {
    int i = 0, ans = 0;
    s += s;
    int n = s.size();
    while (i < n / 2) {
        ans = i;
        int j = i + 1, k = i;
        while (j < n && s[k] <= s[j]) {
            if (s[k] < s[j])
                k = i;
            else
                ++k;
            ++j;
        }
        while (i <= k) {
            i += j - k;
        }
    }
    return s.substr(ans, n / 2);
}
```

SA-IS.cpp

**Description:** Build suffix array  
**Time:**  $\mathcal{O}(n)$

```
f90ffe, 87 lines

void induced_sort(vector<int> &vec, int LIM, vector<int> &sa, vector<bool>
    > &sl,
                    vector<int> &fx) {
    vector<int> l(LIM), r(LIM);
```

```
for (int c : vec) {
    if (c + 1 < LIM) {
        ++l[c + 1];
    }
    ++r[c];
}
partial_sum(all(l), l.begin());
partial_sum(all(r), r.begin());
fill(all(sa), -1);
for (int i = fx.size() - 1; i >= 0; --i) {
    sa[--r[vec[fx[i]]]] = fx[i];
}
for (int i : sa) {
    if (i >= 1 && sl[i - 1]) {
        sa[l[vec[i - 1]]++] = i - 1;
    }
}
fill(all(r), 0);
for (int c : vec) ++r[c];
partial_sum(all(r), r.begin());
for (int k = sa.size() - 1, i = sa[k]; k >= 1; --k, i = sa[k])
    if (i >= 1 && !sl[i - 1]) sa[--r[vec[i - 1]]] = i - 1;
}

vector<int> SA_IS(vector<int> &vec, int LIM) {
    const int n = vec.size();
    vector<int> sa(n), fx;
    vector<bool> sl(n);
    sl[n - 1] = false;
    for (int i = n - 2; i >= 0; --i) {
        sl[i] = (vec[i] > vec[i + 1] || (vec[i] == vec[i + 1] && sl[i + 1]));
        if (sl[i] && !sl[i + 1]) {
            fx.pb(c(i + 1));
        }
    }
    reverse(all(fx));
    induced_sort(vec, LIM, sa, sl, fx);
    vector<int> nfx(fx.size()), lmv(fx.size());
    for (int i = 0, k = 0; i < n; ++i) {
        if (!sl[sa[i]] && sa[i] >= 1 && sl[sa[i] - 1]) {
            nfx[k++] = sa[i];
        }
    }
    int cur = 0;
    sa[n - 1] = cur;
    for (int k = 1; k < nfx.size(); ++k) {
        int i = nfx[k - 1], j = nfx[k];
        if (vec[i] != vec[j]) {
            sa[j] = ++cur;
            continue;
        }
        bool flag = false;
        for (int a = i + 1, b = j + 1; ++a, ++b) {
            if (vec[a] != vec[b]) {
                flag = true;
                break;
            }
            if ((!sl[a] && sl[a - 1]) || (!sl[b] && sl[b - 1])) {
                flag = !((!sl[a] && sl[a - 1]) && (!sl[b] && sl[b - 1]));
                break;
            }
        }
        sa[j] = (flag ? ++cur : cur);
    }
    for (int i = 0; i < fx.size(); ++i) {
        lmv[i] = sa[fx[i]];
    }
    if (cur + 1 < (int)fx.size()) {
        auto lms = SA_IS(lmv, cur + 1);
        for (int i = 0; i < fx.size(); ++i) {
            nfx[i] = fx[lms[i]];
        }
    }
    induced_sort(vec, LIM, sa, sl, nfx);
    return sa;
}

template <typename T>
vector<int> suffix_array(T &s, const int LIM = 128) {
    vector<int> vec(s.size() + 1);
```

```
copy(all(s), begin(vec));
vec.back() = (char)(1);
auto ret = SA_IS(vec, LIM);
ret.erase(ret.begin());
return ret;
}
```

## Graph (4)

### Hungarian.cpp

Description: Hungarian algorithm

Time:  $\mathcal{O}(n^3)$

5afee5, 41 lines

```
int n, m;
vector<vector<int>> a;
vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
for (int i = 1; i <= n; ++i) {
    p[0] = i;
    int j0 = 0;
    vector<int> minv(m + 1, INF);
    vector<char> used(m + 1, false);
    do {
        used[j0] = true;
        int i0 = p[j0], delta = INF, j1;
        for (int j = 1; j <= m; ++j)
            if (!used[j]) {
                int cur = a[i0][j] - u[i0] - v[j];
                if (cur < minv[j])
                    minv[j] = cur, way[j] = j0;
                if (minv[j] < delta)
                    delta = minv[j], j1 = j;
            }
        for (int j = 0; j <= m; ++j)
            if (used[j])
                u[p[j]] += delta, v[j] -= delta;
            else
                minv[j] -= delta;
        j0 = j1;
    } while (p[j0] != 0);
    do {
        int j1 = way[j0];
        p[j0] = p[j1];
        j0 = j1;
    } while (j0);
}

// matching
vector<int> ans(n + 1);
for (int j = 1; j <= m; ++j) {
    ans[p[j]] = j;
}

// cost
int cost = -v[0];
```

### BlossomShrinking.cpp

Description: Maximum matching in general graph

Time:  $\mathcal{O}(n^3)$

23839d, 118 lines

```
struct Edge {
    int u, v;
};
const int N = 510;
int n, m;
vector<int> g[N];
vector<Edge> perfectMatching;
int match[N], par[N], base[N];
bool used[N], blossom[N], lcaUsed[N];
int lca(int u, int v) {
    fill(lcaUsed, lcaUsed + n, false);
    while (u != -1) {
        u = base[u];
        lcaUsed[u] = true;
        if (match[u] == -1)
            break;
        u = par[match[u]];
    }
    while (v != -1) {
        v = base[v];
        if (lcaUsed[v])
            return v;
        v = par[match[v]];
    }
    assert(false);
    return -1;
}
```

```
}
void markPath(int v, int myBase, int children) {
    while (base[v] != myBase) {
        blossom[v] = blossom[match[v]] = true;
        par[v] = children;
        children = match[v];
        v = par[match[v]];
    }
}
int findPath(int root) {
    iota(base, base + n, 0);
    fill(par, par + n, -1);
    fill(used, used + n, false);
    queue<int> q;
    q.push(root);
    used[root] = true;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (auto to : g[v]) {
            if (match[v] == to)
                continue;
            if (base[v] == base[to])
                continue;
            if (to == root || (match[to] != -1 && par[match[to]] != -1))
                {
                    fill(blossom, blossom + n, false);
                    int myBase = lca(to, v);
                    markPath(v, myBase, to);
                    markPath(to, myBase, v);
                    for (int u = 0; u < n; ++u) {
                        if (!blossom[base[u]])
                            continue;
                        base[u] = myBase;
                        if (used[u])
                            continue;
                        used[u] = true;
                        q.push(u);
                    }
                }
            } else if (par[to] == -1) {
                par[to] = v;
                if (match[to] == -1) {
                    return to;
                }
                used[match[to]] = true;
                q.push(match[to]);
            }
        }
    }
    return -1;
}
void blossomShrinking() {
    fill(match, match + n, -1);
    for (int v = 0; v < n; ++v) {
        if (match[v] != -1)
            continue;
        int nxt = findPath(v);
        while (nxt != -1) {
            int parV = par[nxt];
            int parParV = match[parV];
            match[nxt] = parV;
            match[parV] = nxt;
            nxt = parParV;
        }
    }
    for (int v = 0; v < n; ++v) {
        if (match[v] != -1 && v < match[v]) {
            perfectMatching.push_back({v, match[v]});
        }
    }
}
signed main() {
    cin >> n;
    int u, v;
    set<pair<int, int>> edges;
    while (cin >> u >> v) {
        --u;
        --v;
        if (u > v)
```

```
        swap(u, v);
        if (edges.count({u, v}))
            continue;
        edges.insert({u, v});
        g[u].push_back(v);
        g[v].push_back(u);
    }
    blossomShrinking();
    cout << perfectMatching.size() * 2 << '\n';
    for (auto i : perfectMatching) {
        cout << i.u + 1 << " " << i.v + 1 << "\n";
    }
    return 0;
}
```

Lct.cpp  
Description: link-cut tree  
Time:  $\mathcal{O}(n \log(n))$  3d8a3f, 136 lines

```
#include <bits/stdc++.h>
using namespace std;

const int MAXN = 1e5 + 228;

struct node {
    node *ch[2];
    node *p;
    bool rev;
    int sz;

    node() {
        ch[0] = ch[1] = p = NULL;
        rev = false;
        sz = 1;
    }
};

int getsz(node *n) { return (n == NULL) ? 0 : n->sz; }

void pull(node *n) { n->sz = getsz(n->ch[0]) + getsz(n->ch[1]) + 1; }

void push(node *n) {
    if (n->rev) {
        if (n->ch[0]) {
            n->ch[0]->rev ^= 1;
        }
        if (n->ch[1]) {
            n->ch[1]->rev ^= 1;
        }
        swap(n->ch[0], n->ch[1]);
        n->rev = 0;
    }
}

bool isRoot(node *n) {
    return n->p == NULL || (n->p->ch[0] != n && n->p->ch[1] != n);
}

int chnum(node *n) { return n->p->ch[1] == n; }

void attach(node *n, node *p, int num) {
    if (n != NULL)
        n->p = p;
    if (p != NULL)
        p->ch[num] = n;
}

void rotate(node *n) {
    int num = chnum(n);
    node *p = n->p;
    node *b = n->ch[1 - num];
    n->p = p->p;
    if (!isRoot(p)) {
        p->p->ch[chnum(p)] = n;
    }
    attach(p, n, 1 - num);
    attach(b, p, num);
    pull(p);
    pull(n);
}
```

```
}

node *qq[MAXN];

void splay(node *n) {
    node *nn = n;
    int top = 0;
    qq[top++] = nn;
    while (!isRoot(nn)) {
        nn = nn->p;
        qq[top++] = nn;
    }
    while (top) {
        push(qq[--top]);
    }
    while (!isRoot(n)) {
        if (!isRoot(n->p)) {
            if (chnum(n) == chnum(n->p)) {
                rotate(n->p);
            } else {
                rotate(n);
            }
        }
        rotate(n);
    }
}

void expose(node *n) {
    splay(n);
    n->ch[1] = NULL;
    pull(n);
    while (n->p != NULL) {
        splay(n->p);
        attach(n, n->p, 1);
        pull(n->p);
        splay(n);
    }
}

void makeRoot(node *n) {
    expose(n);
    n->rev ^= 1;
}

node *nodes[MAXN];

int main() {
    int n;
    cin >> n;
    for (int i = 0; i <= n; i++) {
        nodes[i] = new node();
    }
    int q;
    cin >> q;
    while (q--) {
        string s;
        cin >> s;
        int u, v;
        cin >> u >> v;
        makeRoot(nodes[u]);
        makeRoot(nodes[v]);
        if (s == "get") {
            if (isRoot(nodes[u]) && u != v) {
                cout << "-1" << endl;
            } else {
                cout << getsz(nodes[v]) - 1 << endl;
            }
        } else if (s == "link") {
            nodes[v]->p = nodes[u];
        } else {
            push(nodes[v]);
            nodes[v]->ch[1] = NULL;
            nodes[u]->p = NULL;
        }
    }
}
```

```
MaxFlow.cpp
Description: Dinic
Time:  $\mathcal{O}(n^2m)$  1c1bc8, 72 lines

struct MaxFlow {
    const int inf = 1e9 + 20;
    struct edge {
        int a, b, cap;
    };
    int n;
    vector<edge> e;
    vector<vector<int>> g;
    MaxFlow() {}
    int s, t;
    vector<int> d, ptr;
    void init(int n_, int s_, int t_) {
        s = s_, t = t_, n = n_;
        g.resize(n);
        ptr.resize(n);
    }
    void addedge(int a, int b, int cap) {
        g[a].pbce(e.size());
        e.pbce({a, b, cap});
        g[b].pbce(e.size());
        e.pbce({b, a, 0});
    }
    bool bfs() {
        d.assign(n, inf);
        d[s] = 0;
        queue<int> q;
        q.push(s);
        while (q.size()) {
            int v = q.front();
            q.pop();
            for (int i : g[v]) {
                if (e[i].cap > 0) {
                    int b = e[i].b;
                    if (d[b] > d[v] + 1) {
                        d[b] = d[v] + 1;
                        q.push(b);
                    }
                }
            }
        }
        return d[t] != inf;
    }
    int dfs(int v, int flow) {
        if (v == t) return flow;
        if (!flow) return 0;
        int sum = 0;
        for (; ptr[v] < g[v].size(); ++ptr[v]) {
            int b = e[g[v][ptr[v]]].b;
            int cap = e[g[v][ptr[v]]].cap;
            if (cap <= 0) continue;
            if (d[b] != d[v] + 1) continue;
            int x = dfs(b, min(flow, cap));
            int id = g[v][ptr[v]];
            e[id].cap -= x;
            e[id ^ 1].cap += x;
            flow -= x;
            sum += x;
        }
        return sum;
    }
    int dinic() {
        int ans = 0;
        while (1) {
            if (!bfs()) break;
            ptr.assign(n, 0);
            int x = dfs(s, inf);
            if (!x) break;
            ans += x;
        }
        return ans;
    }
};
```

MCMF.cpp

Description: Min cost

Time:  $\mathcal{O}(?)$

32340a, 61 lines

```
struct MCMF {
    struct edge {
        int a, b, cap, cost;
    };
    vector<edge> e;
    vector<vector<int>>> g;
    int s, t;
    int n;
    void init(int N, int S, int T) {
        s = S, t = T, n = N;
        g.resize(N);
        e.clear();
    }
    void addedge(int a, int b, int cap, int cost) {
        g[a].pbcc(e.size());
        e.pbcc({a, b, cap, cost});
        g[b].pbcc(e.size());
        e.pbcc({b, a, 0, -cost});
    }
    int getcost(int k) {
        int flow = 0;
        int cost = 0;
        while (flow < k) {
            vector<int> d(n, INF);
            vector<int> pr(n);
            d[s] = 0;
            queue<int> q;
            q.push(s);
            while (q.size()) {
                int v = q.front();
                q.pop();
                for (int i : g[v]) {
                    int u = e[i].b;
                    if (e[i].cap && d[u] > d[v] + e[i].cost) {
                        d[u] = d[v] + e[i].cost;
                        q.push(u);
                        pr[u] = i;
                    }
                }
            }
            if (d[t] == INF) return INF;
            int gf = k - flow;
            int v = t;
            while (v != s) {
                int id = pr[v];
                chkmin(gf, e[id].cap);
                v = e[id].a;
            }
            v = t;
            while (v != s) {
                int id = pr[v];
                e[id].cap -= gf;
                e[id ^ 1].cap += gf;
                cost += e[id].cost * gf;
                v = e[id].a;
            }
            flow += gf;
        }
        return cost;
    }
};
```

MCMFfast.cpp

Description: Min cost with potentials

Time:  $\mathcal{O}(?)$

363228, 86 lines

```
struct MCMF {
    struct edge {
        int a, b, cap, cost;
    };
    vector<edge> e;
    vector<vector<int>>> g;
    vector<ll> po;
    int s, t;
    int n;
```

```
void init(int N, int S, int T) {
    s = S, t = T, n = N;
    g.resize(N);
    e.clear();
}
void addedge(int a, int b, int cap, int cost) {
    g[a].pbcc(e.size());
    e.pbcc({a, b, cap, cost});
    g[b].pbcc(e.size());
    e.pbcc({b, a, 0, -cost});
}
void calc_p() {
    po.assign(n, INF);
    vector<int> inq(n);
    queue<int> q;
    q.push(s);
    po[s] = 0;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        inq[v] = 0;
        for (auto i : g[v]) {
            if (po[e[i].b] > po[v] + e[i].cost && e[i].cap) {
                po[e[i].b] = po[v] + e[i].cost;
                if (!inq[e[i].b]) q.push(e[i].b);
                inq[e[i].b] = 1;
            }
        }
    }
}
ll getcost(int k) {
    calc_p();
    int flow = 0;
    ll cost = 0;
    while (flow < k) {
        vector<ll> d(n, INF);
        vector<int> pr(n);
        d[s] = 0;
        set<pair<ll, int>> q;
        q.insert(mp(0ll, s));
        while (q.size()) {
            int v = q.begin()->second;
            q.erase(q.begin());
            for (int i : g[v]) {
                int u = e[i].b;
                if (e[i].cap && d[u] > d[v] + e[i].cost + po[v] - po[e[i].b]) {
                    q.erase(mp(d[u], u));
                    d[u] = d[v] + e[i].cost + po[v] - po[e[i].b];
                    q.insert(mp(d[u], u));
                    pr[u] = i;
                }
            }
        }
        if (d[t] == INF) return INF;
        for (int i = 0; i < n; ++i) {
            if (d[i] != INF) po[i] += d[i];
        }
        int gf = k - flow;
        int v = t;
        while (v != s) {
            int id = pr[v];
            chkmin(gf, e[id].cap);
            v = e[id].a;
        }
        v = t;
        while (v != s) {
            int id = pr[v];
            e[id].cap -= gf;
            e[id ^ 1].cap += gf;
            cost += 1ll * e[id].cost * gf;
            v = e[id].a;
        }
        flow += gf;
    }
    return cost;
}
```

GlobalMincut.cpp

Description: Global min cut

Time:  $\mathcal{O}(n^3)$

7b8a6b, 35 lines

```
const int MAXN = 500;
int n, g[MAXN][MAXN];
int best_cost = 1000000000;
vector<int> best_cut;
void mincut() {
    vector<int> v[MAXN];
    for (int i = 0; i < n; ++i)
        v[i].assign(1, i);
    int w[MAXN];
    bool exist[MAXN], in_a[MAXN];
    memset(exist, true, sizeof exist);
    for (int ph = 0; ph < n - 1; ++ph) {
        memset(in_a, false, sizeof in_a);
        memset(w, 0, sizeof w);
        for (int it = 0, prev; it < n - ph; ++it) {
            int sel = -1;
            for (int i = 0; i < n; ++i)
                if (exist[i] && !in_a[i] && (sel == -1 || w[i] > w[sel]))
                    sel = i;
            if (it == n - ph - 1) {
                if (w[sel] < best_cost)
                    best_cost = w[sel], best_cut = v[sel];
                v[prev].insert(v[prev].end(), v[sel].begin(), v[sel].end());
            }
            for (int i = 0; i < n; ++i)
                g[prev][i] = g[i][prev] += g[sel][i];
            exist[sel] = false;
        } else {
            in_a[sel] = true;
            for (int i = 0; i < n; ++i)
                w[i] += g[sel][i];
            prev = sel;
        }
    }
}
```

WeightedMatching.cpp

Description: Max weighted matching

Time:  $\mathcal{O}(N^3)$  or so

c3f149, 193 lines

```
#define Dist(e) (lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2)
const int N = 1023, INF = 1e9;
struct Edge {
    int u, v, w;
} g[N][N];
int n, m, n_x, lab[N], match[N], slack[N], st[N], pa[N], flower_from[N][N], S[N], vis[N];
vector<int> flower[N];
deque<int> q;
void update_slack(int u, int x) {
    if (!slack[x] || Dist(g[u][x]) < Dist(g[slack[x]][x])) slack[x] = u;
}
void set_slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
        if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0) update_slack(u, x);
}
void q_push(int x) {
    if (x <= n) return q.push_back(x);
    for (int i = 0; i < flower[x].size(); ++i) q_push(flower[x][i]);
}
void set_st(int x, int b) {
    st[x] = b;
    if (x <= n) return;
    for (int i = 0; i < flower[x].size(); ++i) set_st(flower[x][i], b);
}
int get_pr(int b, int xr) {
    int pr = find(flower[b].begin(), flower[b].end(), xr) - flower[b].begin();
    if (pr % 2 == 1) {
        reverse(flower[b].begin() + 1, flower[b].end());
        return (int)flower[b].size() - pr;
    }
```

```

    } else return pr;
}

void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    Edge e = g[u][v];
    int xr = flower_from[u][e.u], pr = get_pr(u, xr);
    for (int i = 0; i < pr; ++i) set_match(flower[u][i], flower[u][i ^ 1]);
    set_match(xr, v);
    rotate(flower[u].begin(), flower[u].begin() + pr, flower[u].end());
}

void augment(int u, int v) {
    int xnv = st[match[u]];
    set_match(u, v);
    if (!xnv) return;
    set_match(xnv, st[pa[xnv]]);
    augment(st[pa[xnv]], xnv);
}

int get_lca(int u, int v) {
    static int t = 0;
    for (++t; u || v; swap(u, v)) {
        if (u == 0) continue;
        if (vis[u] == t) return u;
        vis[u] = t;
        u = st[match[u]];
        if (u) u = st[pa[u]];
    }
    return 0;
}

void add_blossom(int u, int lca, int v) {
    int b = n + 1;
    while (b <= n_x && st[b]) ++b;
    if (b > n_x) ++n_x;
    lab[b] = 0, S[b] = 0, match[b] = match[lca];
    flower[b].clear();
    flower[b].push_back(lca);
    for (int x = u, y; x != lca; x = st[pa[y]])
        flower[b].push_back(x), flower[b].push_back(y = st[match[x]]),
        q_push(y);
    reverse(flower[b].begin() + 1, flower[b].end());
    for (int x = v, y; x != lca; x = st[pa[y]])
        flower[b].push_back(x), flower[b].push_back(y = st[match[x]]),
        q_push(y);
    set_st(b, b);
    for (int x = 1; x <= n_x; ++x) g[b][x].w = g[x][b].w = 0;
    for (int x = 1; x <= n; ++x) flower_from[b][x] = 0;
    for (int i = 0; i < flower[b].size(); ++i) {
        int xs = flower[b][i];
        for (int x = 1; x <= n_x; ++x) {
            if (g[b][x].w == 0 || Dist(g[xs][x]) < Dist(g[b][x]))
                g[b][x] = g[xs][x], g[x][b] = g[x][xs];
        }
        for (int x = 1; x <= n; ++x) if (flower_from[xs][x]) flower_from[b][x] = xs;
    }
    set_slack(b);
}

void expand_blossom(int b) {
    for (int i = 0; i < flower[b].size(); ++i) set_st(flower[b][i], flower[b][i]);
    int xr = flower_from[b][g[b][pa[b]].u], pr = get_pr(b, xr);
    for (int i = 0; i < pr; i += 2) {
        int xs = flower[b][i], xns = flower[b][i + 1];
        pa[xs] = g[xns][xs].u;
        S[xs] = 1, S[xns] = 0;
        slack[xs] = 0, set_slack(xns);
        q_push(xns);
    }
    S[xr] = 1, pa[xr] = pa[b];
    for (int i = pr + 1; i < flower[b].size(); ++i) {
        int xs = flower[b][i];
        S[xs] = -1, set_slack(xs);
    }
    st[b] = 0;
}

bool on_found_Edge(const Edge &e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {

```

```

        pa[v] = e.u, S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = 0;
        S[nu] = 0, q_push(nu);
    } else if (S[v] == 0) {
        int lca = get_lca(u, v);
        if (!lca) return augment(u, v), augment(v, u), 1;
        else add_blossom(u, lca, v);
    }
    return 0;
}

bool matching() {
    fill(S, S + n_x + 1, -1), fill(slack, slack + n_x + 1, 0);
    q.clear();
    for (int x = 1; x <= n_x; ++x) if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0, q_push(x);
    if (q.empty()) return 0;
    while(1) {
        while (q.size()) {
            int u = q.front();
            q.pop_front();
            if (S[st[u]] == 1) continue;
            for (int v = 1; v <= n; ++v) {
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (Dist(g[u][v]) == 0) {
                        if (on_found_Edge(g[u][v])) return 1;
                    } else
                        update_slack(u, st[v]);
                }
            }
        }
        int d = INF;
        for (int b = n + 1; b <= n_x; ++b) if (st[b] == b && S[b] == 1)
            chkmin(d, lab[b] / 2);
        for (int x = 1; x <= n_x; ++x) {
            if (st[x] == x && slack[x]) {
                if (S[x] == -1)
                    d = min(d, Dist(g[slack[x]][x]));
                else if (S[x] == 0)
                    d = min(d, Dist(g[slack[x]][x]) / 2);
            }
        }
        for (int u = 1; u <= n; ++u) {
            if (S[st[u]] == 0) {
                if (lab[u] <= d) return 0;
                lab[u] -= d;
            } else if (S[st[u]] == 1)
                lab[u] += d;
        }
        for (int b = n + 1; b <= n_x; ++b) {
            if (st[b] == b) {
                if (S[st[b]] == 0)
                    lab[b] += d * 2;
                else if (S[st[b]] == 1)
                    lab[b] -= d * 2;
            }
        }
        q.clear();
        for (int x = 1; x <= n_x; ++x) {
            if (st[x] == x && slack[x] && st[slack[x]] != x && Dist(g[slack[x]][x]) == 0)
                if (on_found_Edge(g[slack[x]][x])) return 1;
        }
        for (int b = n + 1; b <= n_x; ++b)
            if (st[b] == b && S[b] == 1 && lab[b] == 0) expand_blossom(b);
    }
    return 0;
}

pair<ll, int> weight_blossom() {
    fill(match, match + n + 1, 0);
    n_x = n;
    int n_matches = 0;
    ll tot_weight = 0;
    for (int u = 0; u <= n; ++u) st[u] = u, flower[u].clear();
    int w_max = 0;
    for (int u = 1; u <= n; ++u) {
        for (int v = 1; v <= n; ++v) {
            flower_from[u][v] = (u == v ? u : 0);

```

```

                w_max = max(w_max, g[u][v].w);
            }
        }
    }
    ll answer = 0;
    for (int u = 1; u <= n; ++u) lab[u] = w_max;
    while (matching()) ++n_matches;
    for (int u=1; u<=n; ++u)
        if (match[u]&&match[u]<u)
            tot_weight+=g[u][match[u]].w;
    return make_pair(tot_weight,n_matches);
}

```



Geometry (5)

Point.cpp

Description: struct Point 80dfd5, 80 lines

```
const ld EPS = 1e-7;

ld sq(ld x) {
    return x * x;
}

int sign(ld x) {
    if (x < -EPS) {
        return -1;
    }
    if (x > EPS) {
        return 1;
    }
    return 0;
}

#define vec point
struct point { //%- cross, *- dot
    ld x, y;
    auto operator<=>(const point&) const = default;
};
ld operator*(const point &a, const point &b) {
    return a.x * b.x + a.y * b.y;
}
ld operator%(const point &a, const point &b) {
    return a.x * b.y - a.y * b.x;
}
point operator-(const point &a, const point &b) {
    return {a.x - b.x, a.y - b.y};
}
point operator+(const point &a, const point &b) {
    return {a.x + b.x, a.y + b.y};
}
point operator*(const point &a, ld b) {
    return {a.x * b, a.y * b};
}
point operator/(const point &a, ld b) {
    return {a.x / b, a.y / b};
}
bool operator<(const point &a, const point &b) {
    if (sign(a.y - b.y) != 0) {
        return a.y < b.y;
    } else if (sign(a.x - b.x) != 0) {
        return a.x < b.x;
    }
    return 0;
}
ld len2(const point &a) {
    return sq(a.x) + sq(a.y);
}
ld len(const point &a) {
    return sqrt(len2(a));
}
point norm(point a) {
    return a / len(a);
}
int half(point a) {
    return (sign(a.y) == -1 || (sign(a.y) == 0 && a.x < 0));
}
point ort(point a) {
    return {-a.y, a.x};
}
point turn(point a, ld ang) {
    return {a.x * cos(ang) - a.y * sin(ang), a.x * sin(ang) + a.y * cos(ang)};
}
ld getAngle(point &a, point &b) {
    return atan2(a % b, a * b);
}
bool cmpHalf(const point &a, const point &b) {
    if (half(a) != half(b)) {
        return half(b);
    } else {
        int sgn = sign(a % b);
```

```
        if (!sgn) {
            return len2(a) < len2(b);
        } else {
            return sgn == 1;
        }
    }
}
```

Line.cpp

Description: struct Line 887306, 26 lines

```
struct line {
    ld a, b, c;
    void norm() {
        // for half planes
        ld d = len({a, b});
        assert(sign(d) > 0);
        a /= d;
        b /= d;
        c /= d;
    }
    ld eval(point p) const { return a * p.x + b * p.y + c; }
    bool isIn(point p) const { return sign(eval(p)) >= 0; }
    bool operator==(const line &other) const {
        return sign(a * other.b - b * other.a) == 0 &&
            sign(a * other.c - c * other.a) == 0 &&
            sign(b * other.c - c * other.b) == 0;
    }
};
line getln(point a, point b) {
    line res;
    res.a = a.y - b.y;
    res.b = b.x - a.x;
    res.c = -(res.a * a.x + res.b * a.y);
    res.norm();
    return res;
}
```

Intersections.cpp

Description: Geometry intersections 45d7d9, 75 lines

```
bool isCrossed(ld lx, ld rx, ld ly, ld ry) {
    if (lx > rx)
        swap(lx, rx);
    if (ly > ry)
        swap(ly, ry);
    return sign(min(rx, ry) - max(lx, ly)) >= 0;
}

// if two segments [a, b] and [c, d] has AT LEAST one common point ->
// true
bool intersects(const point &a, const point &b, const point &c, const
    point &d) {
    if (!isCrossed(a.x, b.x, c.x, d.x))
        return false;
    if (!isCrossed(a.y, b.y, c.y, d.y))
        return false;
    if (sign((b - a) % (c - a)) * sign((b - a) % (d - a)) == 1) return 0;
    if (sign((d - c) % (a - c)) * sign((d - c) % (b - c)) == 1) return 0;
    return 1;
}

//intersecting lines
bool intersect(line l, line m, point &I) {
    ld d = l.b * m.a - m.b * l.a;
    if (sign(d) == 0) {
        return false;
    }
    ld dx = m.b * l.c - m.c * l.b;
    ld dy = m.c * l.a - l.c * m.a;
    I = {dx / d, dy / d};
    return true;
}

//intersecting circles
int intersect(point o1, ld r1, point o2, ld r2, point &i1, point &i2) {
    if (r1 < r2) {
        swap(o1, o2);
        swap(r1, r2);
    }
    if (sign(r1 - r2) == 0 && len2(o2 - o1) < EPS) {
```

```
        return 3;
    }
    ld ln = len(o1 - o2);
    if (sign(ln - r1 - r2) == 1 || sign(r1 - ln - r2) == 1) {
        return 0;
    }
    ld d = (sq(r1) - sq(r2) + sq(ln)) / 2 / ln;
    vec v = norm(o2 - o1);
    point a = o1 + v * d;
    if (sign(ln - r1 - r2) == 0 || sign(ln + r2 - r1) == 0) {
        i1 = a;
        return 1;
    }
    v = ort(v) * sqrt(sq(r1) - sq(d));
    i1 = a + v;
    i2 = a - v;
    return 2;
}

//intersecting line and circle, line should be normed
int intersect(point o, ld r, line l, point &i1, point &i2) {
    ld len = abs(l.eval(o));
    int sgn = sign(len - r);
    if (sgn == 1) {
        return 0;
    }
    vec v = norm(vec{l.a, l.b}) * len;
    if (sign(l.eval(o + v)) != 0) {
        v = vec{0, 0} - v;
    }
    point a = o + v;
    if (sgn == 0) {
        i1 = a;
        return 1;
    }
    v = norm({-l.b, l.a}) * sqrt(sq(r) - sq(len));
    i1 = a + v;
    i2 = a - v;
    return 2;
}
}
```

Tangents.cpp

Description: Tangents to circles. c73373, 43 lines

```
// tangents from point to circle
int tangents(point &o, ld r, point &p, point &i1, point &i2) {
    ld ln = len(o - p);
    int sgn = sign(ln - r);
    if (sgn == -1) {
        return 0;
    } else if (sgn == 0) {
        i1 = p;
        return 1;
    } else {
        ld x = sq(r) / ln;
        vec v = norm(p - o) * x;
        point a = o + v;
        v = ort(norm(p - o)) * sqrt(sq(r) - sq(x));
        i1 = a + v;
        i2 = a - v;
        return 2;
    }
}

void _tangents(point c, ld r1, ld r2, vector<line> &ans) {
    ld r = r2 - r1;
    ld z = sq(c.x) + sq(c.y);
    ld d = z - sq(r);
    if (sign(d) == -1)
        return;
    d = sqrt(abs(d));
    line l;
    l.a = (c.x * r + c.y * d) / z;
    l.b = (c.y * r - c.x * d) / z;
    l.c = r1;
    ans.push_back(l);
}

// tangents between two circles
vector<line> tangents(point o1, ld r1, point o2, ld r2) {
    vector<line> ans;
```

```
for (int i = -1; i <= 1; i += 2)
    for (int j = -1; j <= 1; j += 2)
        _tangents(o2 - o1, r1 * i, r2 * j, ans);
for (int i = 0; i < (int)ans.size(); ++i)
    ans[i].c -= ans[i].a * o1.x + ans[i].b * o1.y;
return ans;
```

Hull.cpp

Description: Polygon functions fc1928, 16 lines

```
vector<point> hull(vector<point> p, bool need_all=false) {
    sort(all(p));
    p.erase(unique(all(p)), end(p));
    int n = p.size(), k = 0;
    if (n <= 2) return p;
    vector<point> ch(2 * n);
    ld th = need_all ? -EPS : +EPS; // 0 : 1 if int
    for (int i = 0; i < n; ch[k++] = p[i++]) {
        while (k >= 2 && (ch[k - 1] - ch[k - 2]) % (p[i] - ch[k - 1]) < th)
            --k;
    }
    for (int i = n - 2, t = k + 1; i >= 0; ch[k++] = p[i--]) {
        while (k >= t && (ch[k - 1] - ch[k - 2]) % (p[i] - ch[k - 1]) < th)
            --k;
    }
    ch.resize(k - 1);
    return ch;
}
```

IsInPolygon.cpp

Description: Is in polygon functions f17b31, 65 lines

```
bool isOnSegment(point &a, point &b, point &x) {
    if (sign(len2(a - b)) == 0) {
        return sign(len(a - x)) == 0;
    }
    return sign((b - a) % (x - a)) == 0 && sign((b - x) * (a - x)) <= 0;
    // optional (slower, but works better if there are some precision
    // problems) return sign((b - a).len() - (x - a).len() - (x - b).len
    // ())
    // == 0;
}

int isIn(vector<point> &p, point &a) {
    int n = p.size();
    // depends on limitations(2*MAXC + 228)
    point b = a + point(2e9 + 228, 1);
    int cnt = 0;
    for (int i = 0; i < n; ++i) {
        point x = p[i];
        point y = p[i + 1 < n ? i + 1 : 0];
        if (isOnSegment(x, y, a)) {
            // depends on the problem statement
            return 1;
        }
        cnt += intersects(x, y, a, b);
    }
    return 2 * (cnt % 2 == 1);
    /*optional (atan2 is VERY SLOW)!
    ld ans = 0;
    int n = p.size();
    for (int i = 0; i < n; ++i) {
        Point x = p[i];
        Point y = p[i + 1 < n ? i + 1 : 0];
        if (isOnSegment(x, y, a)) {
            // depends on the problem statement
            return true;
        }
        x = x - a;
        y = y - a;
        ans += atan2(x ^ y, x * y);
    }
    return abs(ans) > 1;*/
}
```

```
bool isInTriangle(point &a, point &b, point &c, point &x) {
    return sign((b - a) % (x - a)) >= 0 && sign((c - b) % (x - b)) >= 0
    &&
```

```
sign((a - c) % (x - c)) >= 0;
}

// points should be in the counterclockwise order
bool isInConvex(vector<point> &p, point &a) {
    int n = p.size();
    assert(n >= 3);
    // assert(isConvex(p));
    // assert(isCounterclockwise(p));
    if (sign((p[1] - p[0]) % (a - p[0])) < 0)
        return 0;
    if (sign((p[n - 1] - p[0]) % (a - p[0])) > 0)
        return 0;
    int pos = lower_bound(p.begin() + 2, p.end(), a,
        [&](point a, point b) -> bool {
            return sign((a - p[0]) % (b - p[0])) > 0;
        }) -
        p.begin();
    assert(pos > 1 && pos < n);
    return isInTriangle(p[0], p[pos - 1], p[pos], a);
}
```

Diameter.cpp

Description: Rotating calipers. 0f341c, 21 lines

```
ld diameter(vector<point> p) {
    p = hull(p);
    int n = p.size();
    if (n <= 1) {
        return 0;
    }
    if (n == 2) {
        return len(p[0] - p[1]);
    }
    ld ans = 0;
    int i = 0, j = 1;
    while (i < n) {
        while (sign((p[(i + 1) % n] - p[i]) % (p[(j + 1) % n] - p[j])) >=
            0) {
            chkmax(ans, len(p[i] - p[j]));
            j = (j + 1) % n;
        }
        chkmax(ans, len(p[i] - p[j]));
        ++i;
    }
    return ans;
}
```

TangentsAlex.cpp

Description: Find both tangets to the convex polygon. 2eeea8, 17 lines

```
pair<int, int> tangents_alex(vector<point> &p, point &a) {
    int n = p.size();
    int l = __lg(n);
    auto findWithSign = [&](int val) {
        int i = 0;
        for (int k = 1; k >= 0; --k) {
            int i1 = (i - (1 << k) + n) % n;
            int i2 = (i + (1 << k)) % n;
            if (sign((p[i1] - a) % (p[i1] - a)) == val)
                i = i1;
            if (sign((p[i2] - a) % (p[i1] - a)) == val)
                i = i2;
        }
        return i;
    };
    return {findWithSign(1), findWithSign(-1)};
}
```

IsHpiEmpty.cpp

Description: Determines is half plane intersectinos. 3b5e69, 42 lines

```
// all lines must be normed!!!!, sign > 0
bool isHpiEmpty(vector<line> lines) {
    // return hpi(lines).empty();
```

```
// overflow/precision problems?
shuffle(all(lines), rnd);
const ld C = 1e9;
point ans(C, C);
vector<point> box = {{-C, -C}, {C, -C}, {C, C}, {-C, C}};
for (int i = 0; i < 4; ++i)
    lines.push_back(getln(box[i], box[(i + 1) % 4]));
int n = lines.size();
for (int i = n - 4; i >= 0; --i) {
    if (lines[i].isIn(ans))
        continue;
    point up(0, C + 1), down(0, -C - 1), pi = {lines[i].b, -lines[i].
        a};
    for (int j = i + 1; j < n; ++j) {
        if (lines[i] == lines[j])
            continue;
        point p, pj = {lines[j].b, -lines[j].a};
        if (!intersect(lines[i], lines[j], p)) {
            if (sign(pi * pj) != -1)
                continue;
            if (sign(lines[i].c + lines[j].c) *
                (!sign(pi.y) ? sign(pi.x) : -1) ==
                1)
                return true;
        } else {
            if ((!sign(pi.y) ? sign(pi.x) : sign(pi.y)) * (sign(pi %
                pj)) ==
                1)
                chkmin(up, p);
            else
                chkmax(down, p);
        }
    }
    if ((ans = up) < down)
        return true;
}
// for (int i = 0; i < n; ++i) {
//     assert(lines[i].eval(ans) < EPS);
// }
return false;
}
```

HalfPlaneIntersection.cpp

Description: Find the intersection of the half planes. fdf28f, 62 lines

```
vec getPoint(line l) { return {-l.b, l.a}; }
```

```
bool bad(line a, line b, line c) {
    point x;
    assert(intersect(b, c, x) == 1);
    return a.eval(x) < 0;
}
```

```
// Do not forget about the bounding box
vector<point> hpi(vector<line> lines) {
    sort(all(lines), [](line al, line bl) -> bool {
        point a = getPoint(al);
        point b = getPoint(bl);
        if (half(a) != half(b)) {
            return half(a) < half(b);
        }
        return a % b > 0;
    });
```

```
vector<pair<line, int>> st;
for (int it = 0; it < 2; it++) {
    for (int i = 0; i < (int)lines.size(); i++) {
        bool flag = false;
        while (!st.empty()) {
            if (len(getPoint(st.back().first) - getPoint(lines[i])) <
                EPS) {
                if (lines[i].c >= st.back().first.c) {
                    flag = true;
                    break;
                } else {
                    st.pop_back();
                }
            }
```

```
    } else if (getPoint(st.back().first) % getPoint(lines[i])
    < EPS / 2) {
        return {};
    } else if (st.size() >= 2 &&
        bad(st[st.size() - 2].first, st[st.size() -
        1].first,
        lines[i])) {
        st.pop_back();
    } else {
        break;
    }
}
if (!flag)
    st.push_back({lines[i], i});
}

vector<int> en(lines.size(), -1);
vector<point> ans;
for (int i = 0; i < (int)st.size(); i++) {
    if (en[st[i].second] == -1) {
        en[st[i].second] = i;
        continue;
    }
    for (int j = en[st[i].second]; j < i; j++) {
        point I;
        assert(intersect(st[j].first, st[j + 1].first, I) == 1);
        ans.push_back(I);
    }
    break;
}
return ans;
}
```

CHT.cpp

Description: CHT for minimum, k is decreasing, works for equal slopes

3085b, 34 lines

```
struct line {
    int k, b;
    int eval(int x) {
        return k * x + b;
    }
};

struct part {
    line a;
    ld x;
};

ld intersection(line a, line b) {
    return (ld) (a.b - b.b) / (b.k - a.k);
}

struct ConvexHullMin {
    vector<part> st;
    void add(line a) {
        if (!st.empty() && st.back().a.k == a.k) {
            if (st.back().a.b > a.b) st.pop_back();
            else return;
        }
        while (st.size() > 1 && intersection(st[st.size() - 2].a, a) <=
            st[st.size() - 2].x) st.pop_back();
        if (!st.empty()) st.back().x = intersection(st.back().a, a);
        st.push_back({a, INF});
    }

    int get_val(int x) {
        int l = -1, r = (int)st.size() - 1;
        while (r - l > 1) {
            int m = (l + r) / 2;
            if (st[m].x < x) l = m;
            else r = m;
        }
        return st[r].a.eval(x);
    }
};
```

DynamicCHT.cpp

Description: Dynamic CHT for maximum

8a0777, 30 lines

```
struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const {
```

```
        return Q ? p < o.p : k < o.k;
    }
};

struct LineContainer : multiset<Line> {
    const ll inf = LLONG_MAX;
    ll div(ll a, ll b){
        return a / b - ((a ^ b) < 0 && a % b);
    }

    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }

    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }

    ll query(ll x) {
        assert(!empty());
        Q = 1; auto l = *lower_bound({0,0,x}); Q = 0;
        return l.k * x + l.m;
    }
};
```

MinPlusConv.cpp

Description: Min-Plusconv, A is convex down

Time:  $\mathcal{O}(n\log n\text{fast})$

5d63d9, 28 lines

```
// Assumptions: 'a' is convex, 'opt' has size 'n+m-1'
// 'opt[k]' will be equal to 'arg min(a[k-i] + b[i]}'
template<typename T>
void convex_arbitrary_min_plus_conv(T *a, int n, T *b, int m, int *opt) {
    auto rec = [&](auto &&self, int lx, int rx, int ly, int ry) -> void {
        if (lx > rx) return;
        int mx = (lx + rx) >> 1;
        opt[mx] = ly;
        for (int i = ly; i <= ry; ++i)
            if (mx >= i && (mx - opt[mx] >= n || a[mx - opt[mx]] + b[opt[
                mx]] > a[mx - i] + b[i]))
                opt[mx] = i;
        self(self, lx, mx - 1, ly, opt[mx]);
        self(self, mx + 1, rx, opt[mx], ry);
    };
    rec(rec, 0, n + m - 2, 0, m - 1);
}
```

```
// Assumptions: 'a' is convex down
template<typename T>
std::vector<T> convex_arbitrary_min_plus_conv(const std::vector<T> &a,
    const std::vector<T> &b) {
    int n = a.size(), m = b.size();
    int *opt = (int*) malloc(sizeof(int) * (n + m - 1));
    convex_arbitrary_min_plus_conv(a.data(), n, b.data(), m, opt);
    std::vector<T> ans(n + m - 1);
    for (int i = 0; i < n + m - 1; ++i) ans[i] = a[i - opt[i]] + b[opt[i
        ]];
    free(opt);
    return ans;
}
```

Kinetic.cpp

Description: kinetic segment tree

Time:  $\mathcal{O}(hz)$

49b24c, 127 lines

```
//vnutrennii functions — poluintervali, vneshnie — otrezki. ishet min
priamuy
struct line {
    ll k,b,temp;
    ll eval() const {
        return k * temp + b;
    }
    ll melting_point(const line& other) const {
        ll val1 = eval();
        ll val2 = other.eval();
        assert(val1 <= val2);
```

```
        if (other.k >= k) {
            return INF;
        }
        ll delta_val = val2 - val1;
        ll delta_k = k - other.k;
        assert(delta_val >= 0 && delta_k > 0);
        return (delta_val + delta_k - 1) / delta_k;
    }
};

struct kinetic_segtree {
    struct node {
        ll lazy_b = 0, lazy_temp = 0, melt = INF;
        line best;
        node(line best = line()) : best(best) {}
    };
    int n;
    vector<node> tree;
    void update(int v) {
        if (make_pair(tree[v << 1].best.eval(), tree[v << 1].best.k) <
            make_pair(tree[v << 1 | 1].best.eval(), tree[v << 1 | 1].
                best.k)) {
            tree[v].best = tree[v << 1].best;
            tree[v].melt = tree[v].best.melting_point(tree[v << 1 | 1].
                best);
        } else {
            tree[v].best = tree[v << 1 | 1].best;
            tree[v].melt = tree[v].best.melting_point(tree[v << 1].best);
        }

        tree[v].melt = min({tree[v].melt, tree[v << 1].melt, tree[v << 1
            | 1].melt});
        assert(tree[v].melt > 0);
    }

    void apply(int v, int vl, int vr, ll delta_b, ll delta_temp) {
        tree[v].lazy_b += delta_b;
        tree[v].lazy_temp += delta_temp;

        tree[v].best.b += delta_b;
        tree[v].best.temp += delta_temp;

        tree[v].melt -= delta_temp;
        if (tree[v].melt <= 0) {
            push(v, vl, vr);
            update(v);
        }
    }

    void push(int v, int vl, int vr) {
        int vm = (vl + vr) / 2;
        apply(v << 1, vl, vm, tree[v].lazy_b, tree[v].lazy_temp);
        apply(v << 1 | 1, vm, vr, tree[v].lazy_b, tree[v].lazy_temp);
        tree[v].lazy_b = 0;
        tree[v].lazy_temp = 0;
    }

    void build(int v, int vl, int vr, const vector<line> &lines) {
        if (vr - vl == 1) {
            tree[v] = node(lines[vl]);
            return;
        }
        int vm = (vl + vr) / 2;
        build(v << 1, vl, vm, lines);
        build(v << 1 | 1, vm, vr, lines);
        update(v);
    }

    void add(int v, int vl, int vr, int l, int r, ll delta_b, ll
        delta_temp) {
        if (r <= vl || vr <= l) {
            return;
        }
        if (l <= vl && vr <= r) {
            apply(v, vl, vr, delta_b, delta_temp);
            return;
        }
        push(v, vl, vr);
        int vm = (vl + vr) / 2;
        add(v << 1, vl, vm, l, r, delta_b, delta_temp);
        add(v << 1 | 1, vm, vr, l, r, delta_b, delta_temp);
        update(v);
    }
}
```

```
void change_line(int v, int vl, int vr, int pos, const line &new_line)
{
    if (vr - vl == 1) {
        tree[v].best = new_line;
        return;
    }
    push(v, vl, vr);
    int vm = (vl + vr) / 2;
    if (pos < vm) {
        change_line(v << 1, vl, vm, pos, new_line);
    } else {
        change_line(v << 1 | 1, vm, vr, pos, new_line);
    }
    update(v);
}

ll query(int v, int vl, int vr, int l, int r) {
    if (r <= vl || vr <= l) {
        return INF;
    }
    if (l <= vl && vr <= r) {
        return tree[v].best.eval();
    }
    push(v, vl, vr);
    int vm = (vl + vr) / 2;
    return min(query(v << 1, vl, vm, l, r), query(v << 1 | 1, vm, vr, l, r));
}

kinetic_segtree(const vector<line> &lines) : n(lines.size()), tree(4 * n) {
    build(1, 0, n, lines);
}

kinetic_segtree(int n) : n(n), tree(4 * n) {
    vector <line> lines(n, {0, INF, 0});
    build(1, 0, n, lines);
}

void add(int l, int r, ll delta_b, ll delta_temp) {
    assert(delta_temp >= 0);
    add(1, 0, n, l, r + 1, delta_b, delta_temp);
}

void change_line(int pos, const line &new_line) {
    assert(0 <= pos && pos < n);
    change_line(1, 0, n, pos, new_line);
}

ll query(int l, int r) {
    return query(1, 0, n, l, r + 1);
}

}
```

GoldenSearch.cpp

Description: Golden Search 31d45b, 14 lines

```
double gss(double a, double b, double (*f)(double)) {
    double r = (sqrt(5) - 1) / 2, eps = 1e-7;
    double x1 = b - r * (b - a), x2 = a + r * (b - a);
    double f1 = f(x1), f2 = f(x2);
    while (b - a > eps)
        if (f1 < f2) { // change to > to find maximum
            b = x2; x2 = x1; f2 = f1;
            x1 = b - r * (b - a); f1 = f(x1);
        } else {
            a = x1; x1 = x2; f1 = f2;
            x2 = a + r * (b - a); f2 = f(x2);
        }
    return a;
}
```

Math (6)

BerlekampMassey.cpp

Description: Find the shortest linear-feedback shift register

Time:  $O(n^2)$  08eddc, 86 lines

```
vector<int> berlekamp(vector<int> x) {
    vector<int> ls, cur;
    int lf = 0, d = 0;
    for (int i = 0; i < x.size(); ++i) {
        ll t = 0;
        for (int j = 0; j < cur.size(); ++j) {
            t = (t + (ll) x[i - j - 1] * cur[j]) % MOD;
        }
        if ((t - x[i]) % MOD == 0)
            continue;
        if (cur.empty()) {
            cur.resize(i + 1);
            lf = i;
            d = (t - x[i]) % MOD;
            continue;
        }
        ll k = -(x[i] - t) * powmod(d, MOD - 2) % MOD;
        vector<int> c(i - lf - 1);
        c.push_back(k);
        for (auto &j : ls)
            c.push_back(-j * k % MOD);
        if (c.size() < cur.size())
            c.resize(cur.size());
        for (int j = 0; j < cur.size(); ++j) {
            c[j] = (c[j] + cur[j]) % MOD;
        }
        if (i - lf + (int)ls.size() >= (int)cur.size()) {
            tie(ls, lf, d) = make_tuple(cur, i, (t - x[i]) % MOD);
        }
        cur = c;
    }
    for (auto &i : cur)
        i = (i % MOD + MOD) % MOD;
    return cur;
}

// for a_i = 2 * a_{i-1} + a_{i-2} returns {2, 1}

// kth element of p/q as fps
int getkfps(vector<int> p, vector<int> q, ll k) {
    assert(q[0] != 0);
    while (k) {
        auto f = q;
        for (int i = 1; i < (int) f.size(); i += 2) {
            f[i] = sub(0, f[i]);
        }
        auto p2 = conv(p, f);
        auto q2 = conv(q, f);
        p.clear(), q.clear();
        for (int i = k % 2; i < (int) p2.size(); i += 2) {
            p.pbc(p2[i]);
        }
        for (int i = 0; i < (int) q2.size(); i += 2) {
            q.pbc(q2[i]);
        }
        k >>= 1;
    }
    return mul(p[0], inv(q[0]));
}
```

// vals - initials values of reccurence, c - result of belekamp on vals

```
int getk(const vector<int> &vals, vector<int> c, ll k) {
    int d = c.size();
    c.insert(c.begin(), MOD-1);
    while (c.back() == 0) {
        c.pop_back();
    }
    for (auto &el : c) {
        el = sub(0, el);
    }
    vector<int> p(d);
    copy(vals.begin(), vals.begin() + d, p.begin());
    p = conv(p, c);
}
```

```
p.resize(d);
return getkfps(p, c, k);
}

vector<int> getmod(vector<int> a, vector<int> md) {
    for (int i = a.size() - 1; i + 1 >= md.size(); --i) {
        int v = mul(a[i], inv(md.back()));
        for (int j = 0; j < md.size(); ++j) {
            a[i - md.size() + 1 + j] = sub(a[i - md.size() + 1 + j], mul(
                md[j], v));
        }
        a.pop_back();
    }
    return a;
}
```

GoncharFedor.cpp

Description: Calculating number of points  $x, y \geq 0, Ax + By \leq C$

Time:  $O(\log(C))$  0ef10e, 11 lines

```
ll solve_triangle(ll A, ll B, ll C) { // x,y >=0, Ax+By<=C
    if (C < 0)
        return 0;
    if (A > B)
        swap(A, B);
    ll p = C / B;
    ll k = B / A;
    ll d = (C - p * B) / A;
    return solve_triangle(B - k * A, A, C - A * (k * p + d + 1)) +
        (p + 1) * (d + 1) + k * p * (p + 1) / 2;
}
```

CRT.cpp

Description: CRT for arbitrary modulus

28309e, 25 lines

```
int extgcd(int a, int b, int &x, int &y) { // define int ll
    if (a == 0) {
        x = 0, y = 1;
        return b;
    }
    int x1, y1;
    int g = extgcd(b % a, a, x1, y1);
    x = y1 - x1 * (b / a);
    y = x1;
    return g;
}

int lcm(int a, int b) { return a / __gcd(a, b) * b; }
int crt(int mod1, int mod2, int rem1, int rem2) {
    int r = (rem2 - (rem1 % mod2) + mod2) % mod2;
    int x, y;
    int g = extgcd(mod1, mod2, x, y);
    if (r % g) return -1;
    x %= mod2;
    if (x < 0) x += mod2;
    int ans = (x * (r / g)) % mod2;
    ans = ans * mod1 + rem1;
    assert(ans % mod1 == rem1);
    assert(ans % mod2 == rem2);
    return ans % lcm(mod1, mod2);
}
```

Fastmod.cpp

Description: Fast multiplication by modulo(in [0;2b))

38ea39, 7 lines

```
struct FastMod {
    ull b, m;
    FastMod(ull b) : b(b), m(~1ULL / b) {}
    ull reduce(ull a) { // a % b + (0 or b)
        return a - (ull)((__uint128_t(m) * a) >> 64) * b;
    }
};
```

ModularSqrt.cpp

Description: Calculating sqrt modulo smth

Time:  $O(\log^2)$  19a793, 23 lines

```
ll sqrt(ll a, ll p) {
    a %= p;
    if (a < 0) a += p;
```

```
if (a == 0) return 0;
assert(modpow(a, (p - 1) / 2, p) == 1); // e lse no so lution
if (p % 4 == 3) return modpow(a, (p + 1) / 4, p);
// a^(n+3)/8 or 2^(n+3)/8 * 2^(n=1)/4 works i f p % 8 == 5
ll s = p - 1, n = 2;
int r = 0, m;
while (s % 2 == 0) ++r, s /= 2;
while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
ll x = modpow(a, (s + 1) / 2, p);
ll b = modpow(a, s, p), g = modpow(n, s, p);
for (; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m) t = t * t % p;
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
}
}
```

PrimalityTest.cpp

Description: Checking primality of p  
Time:  $\mathcal{O}(\log(C))$  ad2714, 32 lines

```
const int iters = 8; // can change
bool isprime(ll p) {
    if (p == 1 || p == 4) return 0;
    if (p == 2 || p == 3) return 1;
    for (int it = 0; it < iters; ++it) {
        ll a = rnd() % (p - 2) + 2;
        ll nw = p - 1;
        while (nw % 2 == 0) nw /= 2;
        ll x = binpow(a, nw, p); // int128
        if (x == 1) continue;
        ll last = x;
        nw *= 2;
        while (nw <= p - 1) {
            x = (__int128_t)x * x % p;
            if (x == 1) {
                if (last != p - 1) {
                    return 0;
                }
                break;
            }
            last = x;
            nw *= 2;
        }
        if (x != 1) return 0;
    }
    return 1;
}
```

XorConvolution.cpp

Description: Calculating xor-convolution of 2 vectors modulo smth  
Time:  $\mathcal{O}(n \log(n))$  454afd, 23 lines

```
void fwht(vector<int> &a) {
    int n = a.size();
    for (int l = 1; l < n; l <= 1) {
        for (int i = 0; i < n; i += 2 * l) {
            for (int j = 0; j < l; ++j) {
                int u = a[i + j], v = a[i + j + l];
                a[i + j] = add(u, v), a[i + j + l] = sub(u, v);
            }
        }
    }
}
// https://judge.yosupo.jp/problem/bitwise_xor_convolution
vector<int> xorconvo(vector<int> a, vector<int> b) {
    int n = 1;
    while (n < max(a.size(), b.size()))
        n *= 2;
    a.resize(n), b.resize(n);
    fwht(a), fwht(b);
```

PrimalityTest XorConvolution Factorization NTT

```
int inv = inv(n);
for (int i = 0; i < n; ++i)
    a[i] = mul(a[i], mul(b[i], inv));
fwht(a);
return a;
}
```

Factorization.cpp

Description: Factorizing a number real quick  
Time:  $\mathcal{O}\left(n^{\frac{1}{4}}\right)$  f0d7c6, 51 lines

```
ll gcd(ll a, ll b) {
    while (b)
        a %= b, swap(a, b);
    return a;
}

ll f(ll a, ll n) { return ((__int128_t)a * a % n + 1) % n; }

vector<ll> factorize(ll n) {
    if (n <= 1e6) { // can add primality check for speed?
        vector<ll> res;
        for (ll i = 2; i * i <= n; ++i) {
            while (n % i == 0) {
                res.pb(i);
                n /= i;
            }
        }
        if (n != 1) res.pb(n);
        return res;
    }
    ll x = rnd() % (n - 1) + 1;
    ll y = x;
    ll tries = 10 * sqrt(sqrt(n));
    const int C = 60;
    for (ll i = 0; i < tries; i += C) {
        ll xs = x;
        ll ys = y;
        ll m = 1;
        for (int k = 0; k < C; ++k) {
            x = f(x, n);
            y = f(f(y, n), n);
            m = (__int128_t)m * abs(x - y) % n;
        }
        if (gcd(n, m) == 1) continue;
        x = xs, y = ys;
        for (int k = 0; k < C; ++k) {
            x = f(x, n);
            y = f(f(y, n), n);
            ll res = gcd(n, abs(x - y));
            if (res != 1 && res != n) {
                vector<ll> v1 = factorize(res), v2 = factorize(n / res);
                for (auto j : v2) v1.pb(j);
                return v1;
            }
        }
    }
    return {n};
}
```

NTT.cpp

Description: Calculating FFT modulo MOD  
Time:  $\mathcal{O}(n \log(n))$  3e2f3a, 226 lines

```
// DONT FORGET TO CALL initNTT() AND CHECK MAXLOG

const int MOD = 998244353;
const int G = 3;
const int MAXLOG = 23;
int W[1 << MAXLOG];
bool nttinit = false;
vector<int> pws;

int add(int a, int b) {
    a += b;
```

```
if (a >= MOD) {
    return a - MOD;
}
return a;
}

int sub(int a, int b) {
    a -= b;
    if (a < 0) {
        return a + MOD;
    }
    return a;
}

int mul(int a, int b) {
    return (ll) a * b % MOD;
}

int power(int a, int n) {
    int ans = 1;
    while (n) {
        if (n & 1) {
            ans = mul(ans, a);
        }
        a = mul(a, a);
        n >>= 1;
    }
    return ans;
}

int inv(int a) {
    return power(a, MOD - 2);
}

void initNTT() {
    assert((MOD - 1) % (1 << MAXLOG) == 0);
    pws.push_back(power(G, (MOD - 1) / (1 << MAXLOG)));
    for (int i = 0; i < MAXLOG - 1; ++i) {
        pws.push_back(mul(pws.back(), pws.back()));
    }
    assert(pws.back() == MOD - 1);
    W[0] = 1;
    for (int i = 1; i < (1 << MAXLOG); ++i) {
        W[i] = mul(W[i - 1], pws[0]);
    }
}

void ntt(int n, vector<int>& a, bool rev) {
    if (!nttinit) {
        initNTT();
        nttinit = 1;
    }
    int lg = log2(n);
    vector<int> rv(n);
    for (int i = 1; i < n; ++i) {
        rv[i] = (rv[i >> 1] >> 1) ^ ((i & 1) << (lg - 1));
        if (rv[i] > i) swap(a[i], a[rv[i]]);
    }
    int num = MAXLOG - 1;
    for (int len = 1; len < n; len *= 2) {
        for (int i = 0; i < n; i += 2 * len) {
            for (int j = 0; j < len; ++j) {
                int u = a[i + j], v = mul(W[j << num], a[i + j + len]);
                a[i + j] = add(u, v);
                a[i + j + len] = sub(u, v);
            }
        }
        --num;
    }
    if (rev) {
        int rev_n = power(n, MOD - 2);
        for (int i = 0; i < n; ++i) a[i] = mul(a[i], rev_n);
        reverse(a.begin() + 1, a.end());
    }
}

vector<int> conv(vector<int> a, vector<int> b) {
    int lg = 0;
    while ((1 << lg) < a.size() + b.size() + 1)
```

```
        ++lg;
        int n = 1 << lg;
        assert(a.size() + b.size() <= n + 1);
        a.resize(n);
        b.resize(n);
        ntt(n, a, false);
        ntt(n, b, false);
        for (int i = 0; i < n; ++i) {
            a[i] = mul(a[i], b[i]);
        }
        ntt(n, a, true);
        while (a.size() > 1 && a.back() == 0) {
            a.pop_back();
        }
        return a;
}

vector<int> add(vector<int> a, vector<int> b) {
    a.resize(max(a.size(), b.size()));
    for (int i = 0; i < (int) b.size(); ++i) {
        a[i] = add(a[i], b[i]);
    }
    return a;
}

vector<int> sub(vector<int> a, vector<int> b) {
    a.resize(max(a.size(), b.size()));
    for (int i = 0; i < (int) b.size(); ++i) {
        a[i] = sub(a[i], b[i]);
    }
    return a;
}

vector<int> inv(const vector<int> &a, int need) {
    vector<int> b = {inv(a[0])};
    while ((int) b.size() < need) {
        vector<int> a1 = a;
        int m = b.size();
        a1.resize(min((int) a1.size(), 2 * m));
        b = conv(b, sub({2}, conv(a1, b)));
        b.resize(2 * m);
    }
    b.resize(need);
    return b;
}

vector<int> div(vector<int> a, vector<int> b) {
    if (count(all(a), 0) == a.size()) {
        return {0};
    }
    assert(a.back() != 0 && b.back() != 0);
    int n = a.size() - 1;
    int m = b.size() - 1;
    if (n < m) {
        return {0};
    }
    reverse(all(a));
    reverse(all(b));
    a.resize(n - m + 1);
    b.resize(n - m + 1);
    vector<int> c = inv(b, b.size());
    vector<int> q = conv(a, c);
    q.resize(n - m + 1);
    reverse(all(q));
    return q;
}

vector<int> mod(vector<int> a, vector<int> b) {
    auto res = sub(a, conv(b, div(a, b)));
    while (res.size() > 1 && res.back() == 0) {
        res.pop_back();
    }
    return res;
}

vector<int> multipoint(vector<int> a, vector<int> x) {
    int n = x.size();
    vector<vector<int>> tree(2 * n);
    for (int i = 0; i < n; ++i) {
```

```
        tree[i + n] = {x[i], MOD - 1};
    }
    for (int i = n - 1; i; --i) {
        tree[i] = conv(tree[2 * i], tree[2 * i + 1]);
    }
    tree[1] = mod(a, tree[1]);
    for (int i = 2; i < 2 * n; ++i) {
        tree[i] = mod(tree[i >> 1], tree[i]);
    }
    vector<int> res(n);
    for (int i = 0; i < n; ++i) {
        res[i] = tree[i + n][0];
    }
    return res;
}

vector<int> deriv(vector<int> a) {
    for (int i = 1; i < (int) a.size(); ++i) {
        a[i - 1] = mul(i, a[i]);
    }
    a.back() = 0;
    if (a.size() > 1) {
        a.pop_back();
    }
    return a;
}

vector<int> integ(vector<int> a) {
    a.push_back(0);
    for (int i = (int) a.size() - 1; i; --i) {
        a[i] = mul(a[i - 1], inv(i));
    }
    a[0] = 0;
    return a;
}

vector<int> log(vector<int> a, int n) {
    assert(a[0] == 1);
    auto res = integ(conv(deriv(a), inv(a, n)));
    res.resize(n);
    return res;
}

vector<int> exp(vector<int> a, int need) {
    assert(a[0] == 0);
    vector<int> b = {1};
    while ((int) b.size() < need) {
        vector<int> a1 = a;
        int m = b.size();
        a1.resize(min((int) a1.size(), 2 * m));
        a1[0] = add(a1[0], 1);
        b = conv(b, sub(a1, log(b, 2 * m)));
        b.resize(2 * m);
    }
    b.resize(need);
    return b;
}


```

FFT.cpp

Description: Calculating product of two polynomials

Time:  $\mathcal{O}(n \log(n))$

---

```
const ld PI = acos(-1);
using cd = complex<ld>;
const int MAXLOG = 19, N = (1 << MAXLOG), MAXN = (1 << MAXLOG) + 228;
int rev[MAXN];
cd w[MAXN];
bool fftInit = false;

void initFFT() {
    for (int i = 0; i < N; ++i) {
        w[i] = cd(cos(2 * PI * i / N), sin(2 * PI * i / N));
    }
    rev[0] = 0;
    for (int i = 1; i < N; ++i) {
        rev[i] = (rev[i >> 1] >> 1) ^ ((i & 1) << (MAXLOG - 1));
    }
}


```

```
void FFT(int n, vector<cd>& a, bool rv = false) {
    if (!fftInit) {
        initFFT();
        fftInit = 1;
    }
    int LOG = ceil(log2(n));
    for (int i = 0; i < n; ++i) {
        if (i < (rev[i] >> (MAXLOG - LOG))) {
            swap(a[i], a[(rev[i] >> (MAXLOG - LOG))]);
        }
    }
    for (int lvl = 0; lvl < LOG; ++lvl) {
        int len = 1 << lvl;
        for (int st = 0; st < n; st += len * 2) {
            for (int i = 0; i < len; ++i) {
                cd x = a[st + i], y = a[st + len + i] * w[i << (MAXLOG - 1 - lvl)];
                a[st + i] = x + y;
                a[st + i + len] = x - y;
            }
        }
    }
    if (rv) {
        reverse(a.begin() + 1, a.end());
        for (auto& i : a) i /= n;
    }
}

vector<ll> mul(vector<ll> a, vector<ll> b) {
    int xd = max(a.size(), b.size()) * 2;
    int cur = 1;
    while (cur < xd) {
        cur *= 2;
    }
    a.resize(cur);
    b.resize(cur);
    vector<cd> ma(cur), mb(cur);
    for (int i = 0; i < cur; ++i) {
        ma[i] += a[i];
        mb[i] += b[i];
    }
    FFT(cur, ma);
    FFT(cur, mb);
    for (int i = 0; i < cur; ++i) ma[i] *= mb[i];
    FFT(cur, ma, true);
    vector<ll> ans(cur);
    for (int i = 0; i < cur; ++i) {
        ans[i] = (ll) (ma[i].real() + 0.5);
    }
    return ans;
}


```

AndConvolution.cpp

Description: Calculating and-convolution modulo smth

Time:  $\mathcal{O}(n \log(n))$

---

5dedf4, 24 lines

```
void conv(vector<int> &a, bool x) {
    int n = a.size();
    for (int j = 0; (1 << j) < n; ++j) {
        for (int i = 0; i < n; ++i) {
            if (!(i & (1 << j))) {
                if (x)
                    a[i] = add(a[i], a[i | (1 << j)]);
                else
                    a[i] = sub(a[i], a[i | (1 << j)]);
            }
        }
    }
}

vector<int> andcon(vector<int> a, vector<int> b) {
    int n = 1;
    while (n < max(a.size(), b.size()))
        n *= 2;
    a.resize(n), b.resize(n);
    conv(a, 1), conv(b, 1);
    for (int i = 0; i < n; ++i)
        a[i] = mul(a[i], b[i]);
    conv(a, 0);
    return a;
}


```

```
}

```

Simplex.cpp  
Description: Simplex  
Time: exponential XD(ok for 200-300 variables/bounds) 4dda3c, 99 lines

```
/* solver for linear programs of the form
maximize c^T x, subject to A x <= b, x >= 0
outputs target function for optimal solution and
the solution by reference
if unbounded above : returns inf, if infeasible : returns -inf
create Simplex_Steep <ld> LP(A, b, c), then call LP. Solve(x)
*/
template <typename DOUBLE>
struct Simplex_Steep {
    using VD = vector<DOUBLE>;
    using VVD = vector<VD>;
    using VI = vector<int>;
    DOUBLE EPS = 1e-12;
    int m, n;
    VI B, N;
    VVD D;
    Simplex_Steep(const VVD &A, const VD &b, const VD &c)
        : m(b.size()), n(c.size()), B(m), N(n + 1), D(m + 2, VD(n + 2)) {
        for (int i = 0; i < m; i++)
            for (int j = 0; j < n; j++) D[i][j] = A[i][j];
        for (int i = 0; i < m; i++) {
            B[i] = n + i;
            D[i][n] = -1;
            D[i][n + 1] = b[i];
        }
        for (int j = 0; j < n; j++) {
            N[j] = j;
            D[m][j] = -c[j];
        }
        N[n] = -1;
        D[m + 1][n] = 1;
    }
    void Pivot(int r, int s) {
        for (int i = 0; i < m + 2; i++)
            if (i != r)
                for (int j = 0; j < n + 2; j++)
                    if (j != s) D[i][j] -= D[r][j] * D[i][s] / D[r][s];
        for (int j = 0; j < n + 2; j++)
            if (j != s) D[r][j] /= D[r][s];
        for (int i = 0; i < m + 2; i++)
            if (i != r) D[i][s] /= -D[r][s];
        D[r][s] = 1.0 / D[r][s];
        swap(B[r], N[s]);
    }
    bool Simplex(int phase) {
        int x = m + (int)(phase == 1);
        while (true) {
            int s = -1;
            DOUBLE c_val = -1;
            for (int j = 0; j <= n; j++) {
                if (phase == 2 && N[j] == -1) continue;
                DOUBLE norm_sq = 0;
                for (int k = 0; k <= m; k++) norm_sq += D[k][j] * D[k][j];
                norm_sq = max(norm_sq, EPS);
                DOUBLE c_val_j = D[x][j] / sqrt1(norm_sq);
                if (s == -1 || c_val_j < c_val ||
                    (c_val == c_val_j && N[j] < N[s])) {
                    s = j;
                    c_val = c_val_j;
                }
            }
            if (D[x][s] >= -EPS) return true;
            int r = -1;
            for (int i = 0; i < m; i++) {
                if (D[i][s] <= EPS) continue;
                if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
                    (D[i][n + 1] / D[i][s] == D[r][n + 1] / D[r][s] &&
                     B[i] < B[r]))
                    r = i;
            }
            if (r == -1) return false;
        }
    }
};
```

Simplex

```
        Pivot(r, s);
    }
}
DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++)
        if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] <= -EPS) {
        Pivot(r, n);
        if (!Simplex(1) || D[m + 1][n + 1] < -EPS)
            return -numeric_limits<DOUBLE>::infinity();
        for (int i = 0; i < m; i++)
            if (B[i] == -1) {
                int s = -1;
                for (int j = 0; j <= n; j++)
                    if (s == -1 || D[i][j] < D[i][s] ||
                        (D[i][j] == D[i][s] && N[j] < N[s]))
                        s = j;
                Pivot(i, s);
            }
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++)
        if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];
}
```

6.1 Fun things

$ClassesCount = \frac{1}{|G|} \sum_{\pi \in G} I(\pi)$

$ClassesCount = \frac{1}{|G|} \sum_{\pi \in G} k^{C(\pi)}$

Stirling 2kind - count of partitions of n objects into k nonempty sets:

$S(n,k) = S(n-1,k-1) + kS(n-1,k)$

$S(n,k) = \sum_{j=0}^{n-1} \binom{n-1}{j} S(j,k-1)$

$S(n,k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k+j} \binom{k}{j} j^n$

$n! \approx \sqrt{2n\pi} (\frac{n}{e})^n$

$\binom{n}{k} \equiv \prod_i \binom{n_i}{k_i}, n_i, k_i$  - digits of  $n, k$  in p-adic system

$\int_a^b f(x)dx \approx \frac{b-a}{6} (f(a) + 4f(\frac{a+b}{2}) + f(b))$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, O(loglog)$

$G(n) = n \oplus (n \gg 1)$

$g(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} g(d) \mu(\frac{n}{d})$

$\sum_{d|n} \mu(d) = [n = 1], \mu(1) = 1, \mu(p) = -1, \mu(p^k) = 0$

$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

$tg(a \pm b) = \frac{tg a \pm tg b}{1 \mp tg a tg b}$

$ctg(a \pm b) = \frac{ctg a ctg b \mp 1}{ctg b \pm ctg a}$

$\sin \frac{a}{2} = \pm \sqrt{\frac{1 - \cos a}{2}}$

$\cos \frac{a}{2} = \pm \sqrt{\frac{1 + \cos a}{2}}$

$tg \frac{a}{2} = \frac{\sin a}{1 - \cos a} = \frac{1 - \cos a}{\sin a}$

$\sin a \sin b = \frac{\cos(a - b) - \cos(a + b)}{2}$

$\sin a \cos b = \frac{\sin(a - b) + \sin(a + b)}{2}$

$\cos a \cos b = \frac{\cos(a - b) + \cos(a + b)}{2}$

1 jan 2000 - saturday, 1 jan 1900 - monday, 14 apr 1961 - friday  
Bell numbers: 0:1, 1:1, 2:2, 3:5, 4:15, 5:52, 6:203, 7:877, 8:4140, 9:21147, 10:115975, 11:678570, 12:4213597, 13:27644437, 14:190899322, 15:1382958545, 16:10480142147, 17:82864869804, 18:682076806159, 19:5832742205057, 20:51724158235372, 21:474869816156751, 22:4506715738447323, 23:44152005855084346  
Fibonacci: 45:1134903170. 46:1836311903(max int), 91: 4660046610375530309

Highly composite numbers:  
 $\leq 1000 : d(840) = 32, \leq 10^4 : d(9240) = 64, \leq 10^5 : d(83160) = 128, \leq 10^6 : d(720720) = 240, \leq 10^7 : d(8648640) = 448, \leq 10^8 : d(91891800) = 768, \leq 10^9 : d(931170240) = 1344, \leq 10^{11} : d(97772875200) = 4032, \leq 10^{15} : d(866421317361600) = 26880, \leq 10^{18} : d(897612484786617600) = 103680$

BEST Theorem:  
 $ec(G) = \#SpanningTrees(G) \cdot \prod_{v \in V} (deg(v) - 1)!$

Erdos: Graph exists

$$\Leftrightarrow d_1 \geq \dots \geq d_n, \forall k \sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$$

Pick:  $Area = Interior + \frac{Bounds}{2} - 1$

Euler:  $V - E + F = 1 + C$

Kirchhoff: put degree on diagonal, -1 for each edge, cut out first row + column, calc det - result is #SpanningTrees



Table of Basic Integrals (7)

Basic Forms

∫ x^n dx = 1/(n+1) x^{n+1}, n ≠ -1 (7.1)

∫ 1/x dx = ln |x| (7.2)

∫ u dv = uv - ∫ v du (7.3)

∫ 1/(ax+b) dx = 1/a ln |ax+b| (7.4)

Integrals of Rational Functions

∫ 1/(x+a)^2 dx = -1/(x+a) (7.5)

∫ (x+a)^n dx = (x+a)^{n+1}/(n+1), n ≠ -1 (7.6)

∫ x(x+a)^n dx = (x+a)^{n+1}((n+1)x-a)/((n+1)(n+2)) (7.7)

∫ 1/(1+x^2) dx = tan^-1 x (7.8)

∫ 1/(a^2+x^2) dx = 1/a tan^-1 x/a (7.9)

∫ x/(a^2+x^2) dx = 1/2 ln |a^2+x^2| (7.10)

∫ x^2/(a^2+x^2) dx = x - a tan^-1 x/a (7.11)

∫ x^3/(a^2+x^2) dx = 1/2 x^2 - 1/2 a^2 ln |a^2+x^2| (7.12)

∫ 1/(ax^2+bx+c) dx = 2/(sqrt(4ac-b^2)) tan^-1 (2ax+b)/sqrt(4ac-b^2) (7.13)

∫ 1/((x+a)(x+b)) dx = 1/(b-a) ln (a+x)/(b+x), a ≠ b (7.14)

∫ x/(x+a)^2 dx = a/(a+x) + ln |a+x| (7.15)

∫ x/(ax^2+bx+c) dx = 1/2a ln |ax^2+bx+c| - b/(a sqrt(4ac-b^2)) tan^-1 (2ax+b)/sqrt(4ac-b^2) (7.16)

Integrals with Roots

∫ sqrt(x-a) dx = 2/3 (x-a)^{3/2} (7.17)

∫ 1/sqrt(x±a) dx = 2 sqrt(x±a) (7.18)

∫ 1/sqrt(a-x) dx = -2 sqrt(a-x) (7.19)

∫ x sqrt(x-a) dx = { 2a/3 (x-a)^{3/2} + 2/5 (x-a)^{5/2}, or 2/3 x(x-a)^{3/2} - 4/15 (x-a)^{5/2}, or 2/15 (2a+3x)(x-a)^{3/2} } (7.20)

∫ sqrt(ax+b) dx = (2b/3a + 2x/3) sqrt(ax+b) (7.21)

∫ (ax+b)^{3/2} dx = 2/5a (ax+b)^{5/2} (7.22)

∫ x/sqrt(x±a) dx = 2/3 (x∓2a) sqrt(x±a) (7.23)

∫ sqrt(x/(a-x)) dx = -sqrt(x(a-x)) - a tan^-1 sqrt(x(a-x))/(x-a) (7.24)

∫ sqrt(x/(a+x)) dx = sqrt(x(a+x)) - a ln [sqrt(x)+sqrt(x+a)] (7.25)

∫ x sqrt(ax+b) dx = 2/15a^2 (-2b^2+abx+3a^2x^2) sqrt(ax+b) (7.26)

∫ sqrt(x(ax+b)) dx = 1/(4a^{3/2}) [(2ax+b) sqrt(ax(ax+b)) - b^2 ln |a sqrt(x)+sqrt(a(ax+b))|] (7.27)

∫ sqrt(x^3(ax+b)) dx = [b/12a - b^2/(8a^2x) + x/3] sqrt(x^3(ax+b)) + b^3/(8a^{5/2}) ln |a sqrt(x)+sqrt(a(ax+b))| (7.28)

∫ sqrt(x^2±a^2) dx = 1/2 x sqrt(x^2±a^2) ± 1/2 a^2 ln |x+sqrt(x^2±a^2)| (7.29)

∫ sqrt(a^2-x^2) dx = 1/2 x sqrt(a^2-x^2) + 1/2 a^2 tan^-1 x/sqrt(a^2-x^2) (7.30)

∫ x sqrt(x^2±a^2) dx = 1/3 (x^2±a^2)^{3/2} (7.31)

∫ 1/sqrt(x^2±a^2) dx = ln |x+sqrt(x^2±a^2)| (7.32)

∫ 1/sqrt(a^2-x^2) dx = sin^-1 x/a (7.33)

∫ x/sqrt(x^2±a^2) dx = sqrt(x^2±a^2) (7.34)

∫ x/sqrt(a^2-x^2) dx = -sqrt(a^2-x^2) (7.35)

∫ x^2/sqrt(x^2±a^2) dx = 1/2 x sqrt(x^2±a^2) ± 1/2 a^2 ln |x+sqrt(x^2±a^2)| (7.36)

∫ sqrt(ax^2+bx+c) dx = (b+2ax)/(4a) sqrt(ax^2+bx+c) + 4ac-b^2/(8a^{3/2}) ln |2ax+b+2 sqrt(a(ax^2+bx+c))| (7.37)

∫ x sqrt(ax^2+bx+c) dx = 1/(48a^{5/2}) (2 sqrt(a) sqrt(ax^2+bx+c) (-3b^2+2abx+8a(c+ax^2)) + 3(b^3-4abc) ln |b+2ax+2 sqrt(a) sqrt(ax^2+bx+c)|) (7.38)

∫ 1/sqrt(ax^2+bx+c) dx = 1/sqrt(a) ln |2ax+b+2 sqrt(a(ax^2+bx+c))| (7.39)

∫ x/sqrt(ax^2+bx+c) dx = 1/a sqrt(ax^2+bx+c) - b/(2a^{3/2}) ln |2ax+b+2 sqrt(a(ax^2+bx+c))| (7.40)

∫ dx/((a^2+x^2)^{3/2}) = x/(a^2 sqrt(a^2+x^2)) (7.41)

Integrals with Logarithms

∫ ln ax dx = x ln ax - x (7.42)

∫ x ln x dx = 1/2 x^2 ln x - x^2/4 (7.43)

∫ x^2 ln x dx = 1/3 x^3 ln x - x^3/9 (7.44)

∫ x^n ln x dx = x^{n+1} (ln x/(n+1) - 1/((n+1)^2)), n ≠ -1 (7.45)

∫ ln x/x dx = 1/2 (ln ax)^2 (7.46)

∫ ln x/x^2 dx = -1/x - ln x/x (7.47)

∫ ln(ax+b) dx = (x+b/a) ln(ax+b) - x, a ≠ 0 (7.48)

∫ ln(x^2+a^2) dx = x ln(x^2+a^2) + 2a tan^-1 x/a - 2x (7.49)

∫ ln(x^2-a^2) dx = x ln(x^2-a^2) + a ln (x+a)/(x-a) - 2x (7.50)

∫ ln (ax^2+bx+c) dx = 1/a sqrt(4ac-b^2) tan^-1 (2ax+b)/sqrt(4ac-b^2) - 2x + (b/2a+x) ln (ax^2+bx+c) (7.51)

Integrals with Exponentials

Integrals with Trigonometric Functions

$$\int x \ln(ax + b) \, dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2} \left( x^2 - \frac{b^2}{a^2} \right) \ln(ax + b) \tag{7.52}$$

$$\int x \ln(a^2 - b^2x^2) \, dx = -\frac{1}{2}x^2 + \frac{1}{2} \left( x^2 - \frac{a^2}{b^2} \right) \ln(a^2 - b^2x^2) \tag{7.53}$$

$$\int (\ln x)^2 \, dx = 2x - 2x \ln x + x(\ln x)^2 \tag{7.54}$$

$$\int (\ln x)^3 \, dx = -6x + x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x \tag{7.55}$$

$$\int x(\ln x)^2 \, dx = \frac{x^2}{4} + \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x \tag{7.56}$$

$$\int x^2(\ln x)^2 \, dx = \frac{2x^3}{27} + \frac{1}{3}x^3(\ln x)^2 - \frac{2}{9}x^3 \ln x \tag{7.57}$$

$$\int e^{ax} \, dx = \frac{1}{a}e^{ax} \tag{7.58}$$

$$\int \sqrt{x}e^{ax} \, dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\operatorname{erf}(i\sqrt{ax}), \text{ where } \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{7.59}$$

$$\int xe^x \, dx = (x - 1)e^x \tag{7.60}$$

$$\int xe^{ax} \, dx = \left( \frac{x}{a} - \frac{1}{a^2} \right) e^{ax} \tag{7.61}$$

$$\int x^2e^x \, dx = (x^2 - 2x + 2) e^x \tag{7.62}$$

$$\int x^2e^{ax} \, dx = \left( \frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3} \right) e^{ax} \tag{7.63}$$

$$\int x^3e^x \, dx = (x^3 - 3x^2 + 6x - 6) e^x \tag{7.64}$$

$$\int x^ne^{ax} \, dx = \frac{x^ne^{ax}}{a} - \frac{n}{a} \int x^{n-1}e^{ax} \, dx \tag{7.65}$$

$$\int x^ne^{ax} \, dx = \frac{(-1)^n}{a^{n+1}}\Gamma[1+n, -ax], \text{ where } \Gamma(a, x) = \int_x^\infty t^{a-1}e^{-t} \, dt \tag{7.66}$$

$$\int e^{ax^2} \, dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}}\operatorname{erf}(i x \sqrt{a}) \tag{7.67}$$

$$\int e^{-ax^2} \, dx = \frac{\sqrt{\pi}}{2\sqrt{a}}\operatorname{erf}(x \sqrt{a}) \tag{7.68}$$

$$\int xe^{-ax^2} \, dx = -\frac{1}{2a}e^{-ax^2} \tag{7.69}$$

$$\int x^2e^{-ax^2} \, dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}\operatorname{erf}(x \sqrt{a}) - \frac{x}{2a}e^{-ax^2} \tag{7.70}$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax \tag{7.71}$$

$$\int \sin^2 ax \, dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \tag{7.72}$$

$$\int \sin^3 ax \, dx = -\frac{3 \cos ax}{4a} + \frac{\cos 3ax}{12a} \tag{7.73}$$

$$\int \sin^n ax \, dx = -\frac{1}{a} \cos ax \, {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right] \tag{7.74}$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax \tag{7.75}$$

$$\int \cos^2 ax \, dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \tag{7.76}$$

$$\int \cos^3 ax dx = \frac{3 \sin ax}{4a} + \frac{\sin 3ax}{12a} \tag{7.77}$$

$$\int \cos^p ax dx = -\frac{1}{a(1+p)} \cos^{1+p} ax \times {}_2F_1\left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right] \tag{7.78}$$

$$\int \cos x \sin x \, dx = \frac{1}{2} \sin^2 x + c_1 = -\frac{1}{2} \cos^2 x + c_2 = -\frac{1}{4} \cos 2x + c_3 \tag{7.79}$$

$$\int \cos ax \sin bx \, dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b \tag{7.80}$$

$$\int \sin^2 ax \cos bx \, dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)} \tag{7.81}$$

$$\int \sin^2 x \cos x \, dx = \frac{1}{3} \sin^3 x \tag{7.82}$$

$$\int \cos^2 ax \sin bx \, dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)} \tag{7.83}$$

$$\int \cos^2 ax \sin ax \, dx = -\frac{1}{3a} \cos^3 ax \tag{7.84}$$

$$\int \sin^2 ax \cos^2 bxdx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} \tag{7.85}$$

$$\int \sin^2 ax \cos^2 ax \, dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \tag{7.86}$$

$$\int \tan ax \, dx = -\frac{1}{a} \ln \cos ax \tag{7.87}$$

$$\int \tan^2 ax \, dx = -x + \frac{1}{a} \tan ax \tag{7.88}$$

$$\int \tan^n ax \, dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right) \tag{7.89}$$

$$\int \tan^3 ax dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax \tag{7.90}$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| = 2 \tanh^{-1} \left( \tan \frac{x}{2} \right) \tag{7.91}$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax \tag{7.92}$$

$$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| \tag{7.93}$$

$$\int \sec x \tan x \, dx = \sec x \tag{7.94}$$

$$\int \sec^2 x \tan x \, dx = \frac{1}{2} \sec^2 x \tag{7.95}$$

$$\int \sec^n x \tan x \, dx = \frac{1}{n} \sec^n x, n \neq 0 \tag{7.96}$$

$$\int \csc x \, dx = \ln \left| \tan \frac{x}{2} \right| = \ln |\csc x - \cot x| + C \tag{7.97}$$

$$\int \csc^2 ax \, dx = -\frac{1}{a} \cot ax \tag{7.98}$$

$$\int \csc^3 x \, dx = -\frac{1}{2} \cot x \csc x + \frac{1}{2} \ln |\csc x - \cot x| \tag{7.99}$$

$$\int \csc^n x \cot x \, dx = -\frac{1}{n} \csc^n x, n \neq 0 \tag{7.100}$$

$$\int \sec x \csc x \, dx = \ln |\tan x| \tag{7.101}$$

## Products of Trigonometric Functions and Monomials

$$\int x \cos x \, dx = \cos x + x \sin x \quad (7.102)$$

$$\int x \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax \quad (7.103)$$

$$\int x^2 \cos x \, dx = 2x \cos x + (x^2 - 2) \sin x \quad (7.104)$$

$$\int x^2 \cos ax \, dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax \quad (7.105)$$

$$\int x^n \cos x \, dx = -\frac{1}{2}(i)^{n+1} [\Gamma(n+1, -ix) + (-1)^n \Gamma(n+1, ix)] \quad (7.106)$$

$$\int x^n \cos ax \, dx = \frac{1}{2}(ia)^{1-n} [(-1)^n \Gamma(n+1, -iax) - \Gamma(n+1, ixa)] \quad (7.107)$$

$$\int x \sin x \, dx = -x \cos x + \sin x \quad (7.108)$$

$$\int x \sin ax \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \quad (7.109)$$

$$\int x^2 \sin x \, dx = (2 - x^2) \cos x + 2x \sin x \quad (7.110)$$

$$\int x^2 \sin ax \, dx = \frac{2 - a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2} \quad (7.111)$$

$$\int x^n \sin x \, dx = -\frac{1}{2}(i)^n [\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, -ix)] \quad (7.112)$$

$$\int x \cos^2 x \, dx = \frac{x^2}{4} + \frac{1}{8} \cos 2x + \frac{1}{4} x \sin 2x \quad (7.113)$$

$$\int x \sin^2 x \, dx = \frac{x^2}{4} - \frac{1}{8} \cos 2x - \frac{1}{4} x \sin 2x \quad (7.114)$$

$$\int x \tan^2 x \, dx = -\frac{x^2}{2} + \ln \cos x + x \tan x \quad (7.115)$$

$$\int x \sec^2 x \, dx = \ln \cos x + x \tan x \quad (7.116)$$

## Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x) \quad (7.117)$$

$$\int e^{bx} \sin ax \, dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax) \quad (7.118)$$

$$\int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x) \quad (7.119)$$

$$\int e^{bx} \cos ax \, dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax) \quad (7.120)$$

$$\int x e^x \sin x \, dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x) \quad (7.121)$$

$$\int x e^x \cos x \, dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x) \quad (7.122)$$

## Integrals of Hyperbolic Functions

$$\int \cosh ax \, dx = \frac{1}{a} \sinh ax \quad (7.123)$$

$$\int e^{ax} \cosh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [a \cosh bx - b \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} + \frac{x}{2} & a = b \end{cases} \quad (7.124)$$

$$\int \sinh ax \, dx = \frac{1}{a} \cosh ax \quad (7.125)$$

$$\int e^{ax} \sinh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [-b \cosh bx + a \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} - \frac{x}{2} & a = b \end{cases} \quad (7.126)$$

$$\int \tanh ax \, dx = \frac{1}{a} \ln \cosh ax \quad (7.127)$$

$$\int e^{ax} \tanh bx \, dx = \begin{cases} \frac{e^{(a+2b)x}}{(a+2b)} {}_2F_1 \left[ 1 + \frac{a}{2b}, 1, 2 + \frac{a}{2b}, -e^{2bx} \right] \\ \quad - \frac{1}{a} e^{ax} {}_2F_1 \left[ 1, \frac{a}{2b}, 1 + \frac{a}{2b}, -e^{2bx} \right] & a \neq b \\ \frac{e^{ax} - 2 \tan^{-1}[e^{ax}]}{a} & a = b \end{cases} \quad (7.128)$$

$$\int \cos ax \cosh bx \, dx = \frac{1}{a^2 + b^2} [a \sin ax \cosh bx + b \cos ax \sinh bx] \quad (7.129)$$

$$\int \cos ax \sinh bx \, dx = \frac{1}{a^2 + b^2} [b \cos ax \cosh bx + a \sin ax \sinh bx] \quad (7.130)$$

$$\int \sin ax \cosh bx \, dx = \frac{1}{a^2 + b^2} [-a \cos ax \cosh bx + b \sin ax \sinh bx] \quad (7.131)$$

$$\int \sin ax \sinh bx \, dx = \frac{1}{a^2 + b^2} [b \cosh bx \sin ax - a \cos ax \sinh bx] \quad (7.132)$$

$$\int \sinh ax \cosh ax \, dx = \frac{1}{4a} [-2ax + \sinh 2ax] \quad (7.133)$$

$$\int \sinh ax \cosh bx \, dx = \frac{1}{b^2 - a^2} [b \cosh bx \sinh ax - a \cosh ax \sinh bx] \quad (7.134)$$

| Problem | Status | Comment | Iurii | Alex | Leha |
|---------|--------|---------|-------|------|------|
| A - 1   |        |         |       |      |      |
| B - 2   |        |         |       |      |      |
| C - 3   |        |         |       |      |      |
| D - 4   |        |         |       |      |      |
| E - 5   |        |         |       |      |      |
| F - 6   |        |         |       |      |      |
| G - 7   |        |         |       |      |      |
| H - 8   |        |         |       |      |      |
| I - 9   |        |         |       |      |      |
| J - 10  |        |         |       |      |      |
| K - 11  |        |         |       |      |      |
| L - 12  |        |         |       |      |      |
| M - 13  |        |         |       |      |      |
| N - 14  |        |         |       |      |      |
| O - 15  |        |         |       |      |      |