National Research University Higher School of Economics

Youthful Passion Fruit

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Contest (1)

template.cpp

35 lines

```
#include <bits/stdc++.h>

using namespace std;

using ll = long long;
using ld = long double;
using ull = unsigned long long;

#define pbc push_back
#define mp make_pair
#define all(a) (a).begin(), (a).end()
#define vin(a)                                                        \
    for (auto &i : a)                                                 \
    cin >> i

mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());

template <typename T1, typename T2> inline void chkmin(T1 &x, const T2 &y) {
    if (y < x) {
        x = y;
    }
}

template <typename T1, typename T2> inline void chkmax(T1 &x, const T2 &y) {
    if (x < y) {
        x = y;
    }
}

signed main() {
    cin.tie(0)->sync_with_stdio(0);
    cout.precision(20), cout.setf(ios::fixed);

    return 0;
}
```

genfolders.sh

6 lines

```
for f in {a..z}
do
    mkdir $f
    cp template.cpp $f/$f.cpp
    touch $f/in
done
```

hash.sh

3 lines

```
# Hashes a file, ignoring all whitespace and comments.
# Use for verifying that code was correctly typed.
cpp -dD -P -fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

C++ (2)

GpHashtable.cpp

Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

e44914, 11 lines

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

const int RANDOM =
    chrono::high_resolution_clock::now().time_since_epoch().count();
struct hasher {
    int operator()(int x) const { return x ^ RANDOM; }
};

gp_hash_table<int, int, hasher> table;
```

OrderedSet.cpp

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.

Time: $\mathcal{O}(\log(n))$

<bits/extc++.h>, <bits/stdc++.h>, <bits/extc++.h>, <bits/stdc++.h> e6a5ae, 26 lines

```
using namespace __gnu_pbds;
using namespace std;

template <typename T>
using ordered_set =
    tree<T, null_type, less<>, rb_tree_tag, tree_order_statistics_node_update>;

int main() {
    ordered_set<int> X;
    X.insert(1);
    X.insert(2);
    X.insert(4);
    X.insert(8);
    X.insert(16);

    std::cout << *X.find_by_order(1) << std::endl;             // 2
    std::cout << *X.find_by_order(2) << std::endl;             // 4
    std::cout << *X.find_by_order(4) << std::endl;             // 16
    std::cout << (end(X) == X.find_by_order(6)) << std::endl; // true

    std::cout << X.order_of_key(-5) << std::endl;  // 0
    std::cout << X.order_of_key(1) << std::endl;   // 0
    std::cout << X.order_of_key(3) << std::endl;   // 2
    std::cout << X.order_of_key(4) << std::endl;   // 2
    std::cout << X.order_of_key(400) << std::endl; // 5
}
```

Strings (3)

Manacher.cpp

Description: Manacher algorithm
Time: $\mathcal{O}(n)$

a6ddfb, 27 lines

```
vector<int> manacherOdd(string s) {
    int n = s.size();
    vector<int> d1(n);
    int l = 0, r = -1;
    for (int i = 0; i < n; ++i) {
        int k = i > r ? 1 : min(d1[l + r - i], r - i + 1);
        while (i + k < n && i - k >= 0 && s[i + k] == s[i - k])
            ++k;
        d1[i] = k;
        if (i + k - 1 > r)
            l = i - k + 1, r = i + k - 1;
    }
}

vector<int> manacherEven(string s) {
    int n = s.size();
    vector<int> d2(n);
    l = 0, r = -1;
    for (int i = 0; i < n; ++i) {
        int k = i > r ? 0 : min(d2[l + r - i + 1], r - i + 1);
        while (i + k < n && i - k - 1 >= 0 && s[i + k] == s[i - k - 1])
            ++k;
        d2[i] = k;
        if (i + k - 1 > r)
            l = i - k, r = i + k - 1;
    }
}
```

AhoCorasick.cpp

Description: Build aho-corasick automaton.
Time: $\mathcal{O}(n)$

ae5fc2, 19 lines

```
int go(int v, char c);

int get_link(int v) {
    if (t[v].link == -1)
        if (v == 0 || t[v].p == 0)
            t[v].link = 0;
        else
            t[v].link = go(get_link(t[v].p), t[v].pch);
    return t[v].link;
}

int go(int v, char c) {
    if (t[v].go[c] == -1)
        if (t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
        else
            t[v].go[c] = v == 0 ? 0 : go(get_link(v), c);
    return t[v].go[c];
}
```

SuffixArray.cpp

Description: Build suffix array
Time: $\mathcal{O}(n \log(n))$

3caefc, 45 lines

```
vector<int> buildSuffixArray(string &s) {
    // Remove, if you want to sort cyclic shifts
    s += "$";
    int n = s.size();
    vector<int> a(n);
    iota(all(a), 0);
    stable_sort(all(a), [&](int i, int j) { return s[i] < s[j]; });
    vector<int> c(n);
    int cc = 0;
    for (int i = 0; i < n; i++) {
        if (i == 0 || s[a[i]] != s[a[i - 1]]) {
            c[a[i]] = cc++;
        } else {
            c[a[i]] = c[a[i - 1]];
        }
    }
    for (int l = 1; l < n; l *= 2) {
        vector<int> cnt(n);
        for (auto i : c) {
            cnt[i]++;
        }
        vector<int> pref(n);
        for (int i = 1; i < n; i++) {
            pref[i] = pref[i - 1] + cnt[i - 1];
        }
        vector<int> na(n);
        for (int i = 0; i < n; i++) {
            int pos = (a[i] - l + n) % n;
            na[pref[c[pos]]++] = pos;
        }
        a = na;
        vector<int> nc(n);
        cc = 0;
        for (int i = 0; i < n; i++) {
            if (i == 0 || c[a[i]] != c[a[i - 1]] ||
                c[(a[i] + l) % n] != c[(a[i - 1] + l) % n]) {
                nc[a[i]] = cc++;
            } else {
                nc[a[i]] = nc[a[i - 1]];
            }
        }
        c = nc;
    }
    return a;
}
```

Lcp.cpp

Description: lcp array
Time: $\mathcal{O}(n)$

fa8216, 26 lines

```
vector<int> buildLCP(string &s, vector<int> &a) {
    int n = s.size();
    vector<int> ra(n);
    for (int i = 0; i < n; i++) {
        ra[a[i]] = i;
```

```
    }
    vector<int> lcp(n - 1);
    int cur = 0;
    for (int i = 0; i < n; i++) {
        cur--;
        chkmax(cur, 0);
        if (ra[i] == n - 1) {
            cur = 0;
            continue;
        }
        int j = a[ra[i] + 1];
        while (s[i + cur] == s[j + cur])
            cur++;
        lcp[ra[i]] = cur;
    }
    // for suffixes!!!
    s.pop_back();
    a.erase(a.begin());
    lcp.erase(lcp.begin());
    return lcp;
}
```

Eertree.cpp

Description: Creates Eertree of string str
Time: $\mathcal{O}(n)$

7924c8, 40 lines

```
struct eertree {
    int len[MAXN], suffLink[MAXN];
    int to[MAXN][26];
    int numV, v;
    void addLetter(int n, string &str) {
        while (str[n - len[v] - 1] != str[n])
            v = suffLink[v];
        int u = suffLink[v];
        while (str[n - len[u] - 1] != str[n])
            u = suffLink[u];
        int u_ = to[u][str[n] - 'a'];
        int v_ = to[v][str[n] - 'a'];
        if (v_ == -1) {
            v_ = to[v][str[n] - 'a'] = numV;
            len[numV++] = len[v] + 2;
            suffLink[v_] = u_;
        }
        v = v_;
    }
    void init() {
        len[0] = -1;
        len[1] = 0;
        suffLink[1] = 0;
        suffLink[0] = 0;
        numV = 2;
        for (int i = 0; i < 26; ++i) {
            to[0][i] = numV++;
            suffLink[numV - 1] = 1;
            len[numV - 1] = 1;
        }
        v = 0;
    }
    void init(int sz) {
        for (int i = 0; i < sz; ++i) {
            len[i] = suffLink[i] = 0;
            for (int j = 0; j < 26; ++j)
                to[i][j] = -1;
        }
    }
};
```

SuffixAutomaton.cpp

Description: Build suffix automaton.
Time: $\mathcal{O}(n)$

662a10, 45 lines

```
struct state {
    int len, link;
    map<char, int> next;
};

const int MAXLEN = 100000;
state st[MAXLEN * 2];
int sz, last;

void sa_init() {
    sz = last = 0;
    st[0].len = 0;
    st[0].link = -1;
    ++sz;
    /*
    // if you want to build an automaton for different strings:
    for (int i=0; i<MAXLEN*2; ++i)
            st[i].next.clear();
    */
}

void sa_extend(char c) {
    int cur = sz++;
    st[cur].len = st[last].len + 1;
    int p;
    for (p = last; p != -1 && !st[p].next.count(c); p = st[p].link)
        st[p].next[c] = cur;
    if (p == -1)
        st[cur].link = 0;
    else {
        int q = st[p].next[c];
        if (st[p].len + 1 == st[q].len)
            st[cur].link = q;
        else {
            int clone = sz++;
            st[clone].len = st[p].len + 1;
            st[clone].next = st[q].next;
            st[clone].link = st[q].link;
            for (; p != -1 && st[p].next[c] == q; p = st[p].link)
                st[p].next[c] = clone;
            st[q].link = st[cur].link = clone;
        }
    }
    last = cur;
}
```

PrefixZ.cpp

Description: Calculates Prefix,Z-functions

Time: $\mathcal{O}(n)$

1c4e93, 25 lines

```
vector<int> pf(string s) {
    int k = 0;
    vector<int> p(s.size());
    for (int i = 1; i < s.size(); ++i) {
        while (k && s[i] != s[k])
            k = p[k - 1];
        k += (s[i] == s[k]);
        p[i] = k;
    }
    return p;
}
vector<int> zf(string s) {
    int n = s.size();
    vector<int> z(n, 0);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r)
            z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
            ++z[i];
        if (i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1;
    }
    z[0] = n;
    return z;
}
```

MinShift.cpp

Description: Calculates min-cyclic-shift of s, Duval decomposition

Time: $\mathcal{O}(n)$

3f0fb9, 20 lines

```
string minshift(string s) {
    int i = 0, ans = 0;
    s += s;
    int n = s.size();
    while (i < n / 2) {
        ans = i;
        int j = i + 1, k = i;
        while (j < n && s[k] <= s[j]) {
            if (s[k] < s[j])
                k = i;
            else
                ++k;
            ++j;
        }
        while (i <= k) {
            i += j - k;
        }
    }
    return s.substr(ans, n / 2);
}
```

Graph (4)

Hungarian.cpp

Description: Hungarian algorithm

Time: $\mathcal{O}(n^3)$

5afee5, 41 lines

```
int n, m;
vector<vector<int>> a;
vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
for (int i = 1; i <= n; ++i) {
    p[0] = i;
    int j0 = 0;
    vector<int> minv(m + 1, INF);
    vector<char> used(m + 1, false);
    do {
        used[j0] = true;
        int i0 = p[j0], delta = INF, j1;
        for (int j = 1; j <= m; ++j)
            if (!used[j]) {
                int cur = a[i0][j] - u[i0] - v[j];
                if (cur < minv[j])
                    minv[j] = cur, way[j] = j0;
                if (minv[j] < delta)
                    delta = minv[j], j1 = j;
            }
        for (int j = 0; j <= m; ++j)
            if (used[j])
                u[p[j]] += delta, v[j] -= delta;
            else
                minv[j] -= delta;
        j0 = j1;
    } while (p[j0] != 0);
    do {
        int j1 = way[j0];
        p[j0] = p[j1];
        j0 = j1;
    } while (j0);
}

// matching
vector<int> ans(n + 1);
for (int j = 1; j <= m; ++j) {
    ans[p[j]] = j;
}

// cost
int cost = -v[0];
```

BlossomShrinking.cpp

Description: Maximum matching in general graph

Time: $\mathcal{O}(n^3)$

23839d, 118 lines

```
struct Edge {
    int u, v;
};
const int N = 510;
int n, m;
vector<int> g[N];
```

```
vector<Edge> perfectMatching;
int match[N], par[N], base[N];
bool used[N], blossom[N], lcaUsed[N];
int lca(int u, int v) {
    fill(lcaUsed, lcaUsed + n, false);
    while (u != -1) {
        u = base[u];
        lcaUsed[u] = true;
        if (match[u] == -1)
            break;
        u = par[match[u]];
    }
    while (v != -1) {
        v = base[v];
        if (lcaUsed[v])
            return v;
        v = par[match[v]];
    }
    assert(false);
    return -1;
}
void markPath(int v, int myBase, int children) {
    while (base[v] != myBase) {
        blossom[v] = blossom[match[v]] = true;
        par[v] = children;
        children = match[v];
        v = par[match[v]];
    }
}
int findPath(int root) {
    iota(base, base + n, 0);
    fill(par, par + n, -1);
    fill(used, used + n, false);
    queue<int> q;
    q.push(root);
    used[root] = true;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (auto to : g[v]) {
            if (match[v] == to)
                continue;
            if (base[v] == base[to])
                continue;
            if (to == root || (match[to] != -1 && par[match[to]] != -1)) {
                fill(blossom, blossom + n, false);
                int myBase = lca(to, v);
                markPath(v, myBase, to);
                markPath(to, myBase, v);
                for (int u = 0; u < n; ++u) {
                    if (!blossom[base[u]])
                        continue;
                    base[u] = myBase;
                    if (used[u])
                        continue;
                    used[u] = true;
                    q.push(u);
                }
            } else if (par[to] == -1) {
                par[to] = v;
                if (match[to] == -1) {
                    return to;
                }
                used[match[to]] = true;
                q.push(match[to]);
            }
        }
    }
    return -1;
}
void blossomShrinking() {
    fill(match, match + n, -1);
    for (int v = 0; v < n; ++v) {
        if (match[v] != -1)
            continue;
        int nxt = findPath(v);
        while (nxt != -1) {
            int parV = par[nxt];
            int parParV = match[parV];
            match[nxt] = parV;
            match[parV] = nxt;
            nxt = parParV;
        }
    }
    for (int v = 0; v < n; ++v) {
        if (match[v] != -1 && v < match[v]) {
            perfectMatching.push_back({v, match[v]});
        }
    }
}
signed main() {
    cin >> n;
    int u, v;
    set<pair<int, int>> edges;
    while (cin >> u >> v) {
        --u;
        --v;
        if (u > v)
            swap(u, v);
        if (edges.count({u, v}))
            continue;
        edges.insert({u, v});
        g[u].push_back(v);
        g[v].push_back(u);
    }
    blossomShrinking();
    cout << perfectMatching.size() * 2 << '\n';
    for (auto i : perfectMatching) {
        cout << i.u + 1 << " " << i.v + 1 << "\n";
    }
    return 0;
}
```

Lct.cpp

Description: link-cut tree
Time: $\mathcal{O}(n\log(n))$

3d8a3f, 136 lines

```cpp
#include <bits/stdc++.h>
using namespace std;

const int MAXN = 1e5 + 228;

struct node {
    node *ch[2];
    node *p;
    bool rev;
    int sz;

    node() {
        ch[0] = ch[1] = p = NULL;
        rev = false;
        sz = 1;
    }
};

int getsz(node *n) { return (n == NULL) ? 0 : n->sz; }

void pull(node *n) { n->sz = getsz(n->ch[0]) + getsz(n->ch[1]) + 1; }

void push(node *n) {
    if (n->rev) {
        if (n->ch[0]) {
            n->ch[0]->rev ^= 1;
        }
        if (n->ch[1]) {
            n->ch[1]->rev ^= 1;
        }
        swap(n->ch[0], n->ch[1]);
        n->rev = 0;
    }
}

bool isRoot(node *n) {
    return n->p == NULL || (n->p->ch[0] != n && n->p->ch[1] != n);
}

int chnum(node *n) { return n->p->ch[1] == n; }

void attach(node *n, node *p, int num) {
    if (n != NULL)
        n->p = p;
    if (p != NULL)
        p->ch[num] = n;
}

void rotate(node *n) {
    int num = chnum(n);
    node *p = n->p;
    node *b = n->ch[1 - num];
    n->p = p->p;
    if (!isRoot(p)) {
        p->p->ch[chnum(p)] = n;
    }
    attach(p, n, 1 - num);
    attach(b, p, num);
    pull(p);
    pull(n);
}

node *qq[MAXN];

void splay(node *n) {
    node *nn = n;
    int top = 0;
    qq[top++] = nn;
    while (!isRoot(nn)) {
        nn = nn->p;
        qq[top++] = nn;
    }
    while (top) {
        push(qq[--top]);
    }
    while (!isRoot(n)) {
        if (!isRoot(n->p)) {
            if (chnum(n) == chnum(n->p)) {
                rotate(n->p);
            } else {
                rotate(n);
            }
        }
        rotate(n);
    }
}

void expose(node *n) {
    splay(n);
    n->ch[1] = NULL;
    pull(n);
    while (n->p != NULL) {
        splay(n->p);
        attach(n, n->p, 1);
        pull(n->p);
        splay(n);
    }
}

void makeRoot(node *n) {
    expose(n);
    n->rev ^= 1;
}

node *nodes[MAXN];

int main() {
    int n;
    cin >> n;
    for (int i = 0; i <= n; i++) {
        nodes[i] = new node();
    }
    int q;
    cin >> q;
    while (q--) {
```

```
        string s;
        cin >> s;
        int u, v;
        cin >> u >> v;
        makeRoot(nodes[u]);
        makeRoot(nodes[v]);
        if (s == "get") {
            if (isRoot(nodes[u]) && u != v) {
                cout << "-1" << endl;
            } else {
                cout << getsz(nodes[v]) - 1 << endl;
            }
        } else if (s == "link") {
            nodes[v]->p = nodes[u];
        } else {
            push(nodes[v]);
            nodes[v]->ch[1] = NULL;
            nodes[u]->p = NULL;
        }
    }
}
```

Pushrelabel.cpp

Description: Maxflow
Time: $\mathcal{O}\left(n^2 m\right)$

1dbe57, 87 lines

```
#include <bits/stdc++.h>
using namespace std;

typedef long long ll;

struct MaxFlow {
    static const ll INF = 1e18 + 228; // maybe int?
    struct edge {
        int to, rev;
        ll cap; // maybe int?
    };

    int n;
    vector<vector<edge>> g;
    vector<ll> ex; // maybe int?
    vector<int> q;

    ll flow(int t) { // maybe int?
        while (true) {
            vector<int> dist(n, n);
            dist[t] = 0;
            int l = 0;
            int r = 1;
            q[0] = t;
            while (l != r) {
                int v = q[l++];
                for (auto e : g[v]) {
                    if (g[e.to][e.rev].cap > 0 && dist[e.to] > dist[v] + 1) {
                        dist[e.to] = dist[v] + 1;
                        q[r++] = e.to;
                    }
                }
            }
            ll was = ex[t];
            for (int ind = r - 1; ind >= 0; ind--) {
                int v = q[ind];
                if (ex[v] == 0)
                    continue;
                for (auto &e : g[v]) {
                    if (dist[e.to] + 1 == dist[v] && e.cap > 0) {
                        auto f = min(ex[v], e.cap);
                        e.cap -= f;
                        ex[e.to] += f;
                        ex[v] -= f;
                        g[e.to][e.rev].cap += f;
                    }
                }
            }
            if (was == ex[t]) {
                break;
            }
        }
        return ex[t];
    }
    MaxFlow(int n) : n(n) {
        g.resize(n);
        ex.resize(n);
        q.resize(n);
    }
    ll run(int s, int t) { // maybe int?
        ex[s] = INF;
        return flow(t);
    }
    void add_edge(int a, int b, int c, int cr = 0) {
        int sza = g[a].size();
        int szb = g[b].size();
        g[a].push_back({b, szb, c});
        g[b].push_back({a, sza, cr});
    }
};

int main() {
    int n;
    cin >> n;
    MaxFlow mf(n);
    int s = 0, t = n - 1;
    int m;
    cin >> m;
    for (int i = 0; i < m; i++) {
        int a, b, c;
        cin >> a >> b >> c;
        a--;
        b--;
        mf.add_edge(a, b, c);
    }
    cout << mf.run(s, t) << endl;
}
```

GlobalMincut.cpp

Description: Global min cut
Time: $\mathcal{O}(n^3)$

7b8a6b, 35 lines

```
const int MAXN = 500;
int n, g[MAXN][MAXN];
int best_cost = 1000000000;
vector<int> best_cut;
void mincut() {
    vector<int> v[MAXN];
    for (int i = 0; i < n; ++i)
        v[i].assign(1, i);
    int w[MAXN];
    bool exist[MAXN], in_a[MAXN];
    memset(exist, true, sizeof exist);
    for (int ph = 0; ph < n - 1; ++ph) {
        memset(in_a, false, sizeof in_a);
        memset(w, 0, sizeof w);
        for (int it = 0, prev; it < n - ph; ++it) {
            int sel = -1;
            for (int i = 0; i < n; ++i)
                if (exist[i] && !in_a[i] && (sel == -1 || w[i] > w[sel]))
                    sel = i;
            if (it == n - ph - 1) {
                if (w[sel] < best_cost)
                    best_cost = w[sel], best_cut = v[sel];
                v[prev].insert(v[prev].end(), v[sel].begin(), v[sel].end());
                for (int i = 0; i < n; ++i)
                    g[prev][i] = g[i][prev] += g[sel][i];
                exist[sel] = false;
            } else {
                in_a[sel] = true;
                for (int i = 0; i < n; ++i)
                    w[i] += g[sel][i];
                prev = sel;
            }
        }
    }
}
```

Geometry (5)

Point.cpp

Description: struct Point

cbfa4e, 37 lines

```
struct Point {
    ld x = 0, y = 0;
    Point() = default;
    Point(ld _x, ld _y) : x(_x), y(_y) {}
    Point ort() const { return Point(-y, x); }
    int half() const { return sign(y) == 1 || (sign(y) == 0 && sign(x) >= 0); }
    bool operator<(const Point &other) const {
        if (sign(y - other.y) != 0) {
            return y < other.y;
        } else if (sign(x - other.x) != 0) {
            return x < other.x;
        } else {
            return false;
        }
    }
    Point turn(ld sin, ld cos) const {
        return Point(x * cos - y * sin, x * sin + y * cos);
    }
    Point turn(ld phi) const { return turn(sin(phi), cos(phi)); }
};

#define Vec Point

ld getAngle(Vec &lhs, Vec &rhs) { return atan2(lhs ^ rhs, lhs * rhs); }

bool cmpHalf(const Vec &lhs, const Vec &rhs) {
    if (lhs.half() != rhs.half()) {
        return lhs.half();
    } else {
        int sgn = sign(lhs ^ rhs);
        if (!sgn) {
            return lhs.len2() < rhs.len2();
        } else {
            return sgn == 1;
        }
    }
}
```

Line.cpp

Description: struct Line

02e3a0, 24 lines

```
struct Line {
    ld a = 0, b = 0, c = 0;
    Line() = default;
    void norm() {
        // for half planes
        ld d = Vec(a, b).len();
        assert(sign(d) > 0);
        a /= d;
        b /= d;
        c /= d;
    }
    Line(ld _a, ld _b, ld _c) : a(_a), b(_b), c(_c) { norm(); }
```

```
    Line(Point x, Point y)
        : a(y.y - x.y), b(x.x - y.x), c(x.y * y.x - x.x * y.y) {
        norm();
    }
    ld eval(Point p) const { return a * p.x + b * p.y + c; }
    bool isIn(Point p) const { return sign(eval(p)) <= 0; }
    bool operator==(const Line &other) const {
        return sign(a * other.b - b * other.a) == 0 &&
               sign(a * other.c - c * other.a) == 0 &&
               sign(b * other.c - c * other.b) == 0;
    }
};
```

Intersections.cpp

Description: Geometry intersections

a7a42d, 84 lines

```
bool isCrossed(ld lx, ld rx, ld ly, ld ry) {
    if (lx > rx)
        swap(lx, rx);
    if (ly > ry)
        swap(ly, ry);
    return sign(min(rx, ry) - max(lx, ly)) >= 0;
}

// if two segments [a, b] and [c, d] has AT LEAST one common point -> true
bool isCrossed(Point &a, Point &b, Point &c, Point &d) {
    if (!isCrossed(a.x, b.x, c.x, d.x))
        return false;
    if (!isCrossed(a.y, b.y, c.y, d.y))
        return false;
    Vec v1, v2, v3;
    v1 = b - a;
    v2 = c - a;
    v3 = d - a;
    if (sign(v1 ^ v2) * sign(v1 ^ v3) == 1)
        return false;
    v1 = d - c;
    v2 = a - c;
    v3 = b - c;
    if (sign(v1 ^ v2) * sign(v1 ^ v3) == 1)
        return false;
    return true;
}

bool cross(Line &l, Line &m, Point &I) {
    ld d = l.b * m.a - m.b * l.a;
    if (sign(d) == 0) {
        return false;
    }
    ld dx = m.b * l.c - m.c * l.b;
    ld dy = m.c * l.a - l.c * m.a;
    I = Point(dx / d, dy / d);
    return true;
}

int cross(Point o1, ld r1, Point o2, ld r2, Point &I1, Point &I2) {
    if (r1 < r2) {
        swap(o1, o2);
        swap(r1, r2);
    }
    if (sign(r1 - r2) == 0 && o1 == o2) {
        return 3;
    }
    ld len = (o1 - o2).len();
    if (sign(len - r1 - r2) == 1 || sign(r1 - len - r2) == 1) {
        return 0;
    }
    ld d = (sq(r1) - sq(r2) + sq(len)) / 2 / len;
    Vec v = (o2 - o1).norm();
    Point a = o1 + v * d;
    if (sign(len - r1 - r2) == 0 || sign(len + r2 - r1) == 0) {
        I1 = a;
        return 1;
    }
    v = v.ort() * sqrt(sq(r1) - sq(d));
    I1 = a + v;
    I2 = a - v;
    return 2;
}

int cross(Point &o, ld r, Line &l, Point &I1, Point &I2) {
    ld len = dist(l, o);
    int sgn = sign(len - r);
    if (sgn == 1) {
        return 0;
    }
    Vec v = Vec(l.a, l.b).norm() * len;
    if (sign(l.eval(o + v)) != 0) {
        v = Point() - v;
    }
    Point a = o + v;
    if (sgn == 0) {
        I1 = a;
        return 1;
    }
    v = Vec(-l.b, l.a).norm() * sqrt(sq(r) - sq(len));
    I1 = a + v;
    I2 = a - v;
    return 2;
}
```

Tangents.cpp

Description: Tangents to circles.

649ac8, 42 lines

```
int tangents(Point &o, ld r, Point &p, Point &I1, Point &I2) {
    ld len = (o - p).len();
    int sgn = sign(len - r);
    if (sgn == -1) {
        return 0;
    } else if (sgn == 0) {
        I1 = p;
        return 1;
    } else {
        ld x = sq(r) / len;
        Vec v = (p - o).norm() * x;
        Point a = o + v;
```

```
        v = (p - o).norm().ort() * sqrt(sq(r) - sq(x));
        I1 = a + v;
        I2 = a - v;
        return 2;
    }
}

void tangents(Point c, ld r1, ld r2, vector<Line> &ans) {
    ld r = r2 - r1;
    ld z = sq(c.x) + sq(c.y);
    ld d = z - sq(r);
    if (sign(d) == -1)
        return;
    d = sqrt(abs(d));
    Line l;
    l.a = (c.x * r + c.y * d) / z;
    l.b = (c.y * r - c.x * d) / z;
    l.c = r1;
    ans.push_back(l);
}

vector<Line> tangents(Point o1, ld r1, Point o2, ld r2) {
    vector<Line> ans;
    for (int i = -1; i <= 1; i += 2)
        for (int j = -1; j <= 1; j += 2)
            tangents(o2 - o1, r1 * i, r2 * j, ans);
    for (int i = 0; i < (int)ans.size(); ++i)
        ans[i].c -= ans[i].a * o1.x + ans[i].b * o1.y;
    return ans;
}
```

Polygon.cpp

Description: Polygon functions

48483d, 71 lines

```
ld area(vector<Point> &p) {
    ld ans = 0;
    int n = p.size();
    for (int i = 0; i < n; ++i) {
        ans += p[i] ^ p[i + 1 < n ? i + 1 : 0];
    }
    return abs(ans) / 2;
}

ld perimeter(vector<Point> &p) {
    ld ans = 0;
    int n = p.size();
    for (int i = 0; i < n; ++i) {
        ans += (p[i] - p[i + 1 < n ? i + 1 : 0]).len();
    }
    return ans;
}

bool isCounterclockwise(vector<Point> &p) {
    int n = p.size();
    int pos = min_element(all(p)) - p.begin();
    return sign((p[pos + 1 < n ? pos + 1 : 0] - p[pos]) ^
                (p[pos - 1 >= 0 ? pos - 1 : n - 1] - p[pos])) == 1;
}

bool isConvex(vector<Point> &p) {
    int n = p.size();
    int sgn = 0;
    for (int i = 0; i < n; ++i) {
        int cur_sgn = sign((p[i - 1 >= 0 ? i - 1 : n - 1] - p[i]) ^
                           (p[i + 1 < n ? i + 1 : 0] - p[i]));
        if (sgn && sgn != cur_sgn) {
            return false;
        }
        sgn = cur_sgn;
    }
    return true;
}

vector<Point> convexHull(vector<Point> p) {
    if (p.empty()) {
        return {};
    }
    int n = p.size();
    int pos = min_element(all(p)) - p.begin();
    swap(p[0], p[pos]);
    for (int i = 1; i < n; ++i)
        p[i] = p[i] - p[0];
    sort(p.begin() + 1, p.end(), [&](Point &lhs, Point &rhs) -> bool {
        int sgn = sign(lhs ^ rhs);
        if (!sgn) {
            return lhs.len2() < rhs.len2();
        }
        return sgn == 1;
    });
    for (int i = 1; i < n; ++i)
        p[i] = p[i] + p[0];
    int top = 0;
    for (int i = 0; i < n; ++i) {
        while (top >= 2) {
            Vec v1 = p[top - 1] - p[top - 2];
            Vec v2 = p[i] - p[top - 1];
            if (sign(v1 ^ v2) == 1)
                break;
            --top;
        }
        p[top++] = p[i];
    }
    p.resize(top);
    return p;
}
```

IsInPolygon.cpp

Description: Is in polygon functions

c97da7, 66 lines

```
bool isOnSegment(Point &a, Point &b, Point &x) {
    if (a == b) {
        return a == x;
    }
    return sign((b - a) ^ (x - a)) == 0 && sign((b - a) * (x - a)) >= 0 &&
        sign((a - b) * (x - b)) >= 0;
    // optional (slower, but works better if there are some precision
```

```
    // problems) return sign((b - a).len() - (x - a).len() - (x - b).len())
    // == 0;
}

bool isIn(vector<Point> &p, Point &a) {
    int n = p.size();
    // depents on limitations
    Point b = a + Point(1e9, 1);
    int cnt = 0;
    for (int i = 0; i < n; ++i) {
        Point x = p[i];
        Point y = p[i + 1 < n ? i + 1 : 0];
        if (isOnSegment(x, y, a)) {
            // depends on the problem statement
            return true;
        }
        cnt += isCrossed(x, y, a, b);
    }
    return cnt % 2 == 1;
    /*optional (atan2 is VERY SLOW)!
    ld ans = 0;
    int n = p.size();
    for (int i = 0; i < n; ++i) {
      Point x = p[i];
      Point y = p[i + 1 < n ? i + 1 : 0];
      if (isOnSegment(x, y, a)) {
        // depends on the problem statement
        return true;
      }
      x = x - a;
      y = y - a;
      ans += atan2(x ^ y, x * y);
    }
    return abs(ans) > 1;*/
}

bool isInTriangle(Point &a, Point &b, Point &c, Point &x) {
    return sign((b - a) ^ (x - a)) >= 0 && sign((c - b) ^ (x - b)) >= 0 &&
           sign((a - c) ^ (x - c)) >= 0;
}

// points should be in the counterclockwise order
bool isInConvex(vector<Point> &p, Point &a) {
    int n = p.size();
    assert(n >= 3);
    // assert(isConvex(p));
    // assert(isCounterclockwise(p));
    if (sign((p[1] - p[0]) ^ (a - p[0])) < 0)
        return false;
    if (sign((p[n - 1] - p[0]) ^ (a - p[0])) > 0)
        return false;
    int pos = lower_bound(p.begin() + 2, p.end(), a,
                          [&](Point lhs, Point rhs) -> bool {
                              return sign((lhs - p[0]) ^ (rhs - p[0])) > 0;
                          }) -
              p.begin();
    assert(pos > 1 && pos < n);
    return isInTriangle(p[0], p[pos - 1], p[pos], a);
}
```

Diameter.cpp

Description: Rotating calipers.
Time: $\mathcal{O}(n)$

3a9573, 21 lines

```
ld diameter(vector<Point> p) {
    p = convexHull(p);
    int n = p.size();
    if (n <= 1) {
        return 0;
    }
    if (n == 2) {
        return (p[0] - p[1]).len();
    }
    ld ans = 0;
    int i = 0, j = 1;
    while (i < n) {
        while (sign((p[(i + 1) % n] - p[i]) ^ (p[(j + 1) % n] - p[j])) >= 0) {
            chkmax(ans, (p[i] - p[j]).len());
            j = (j + 1) % n;
        }
        chkmax(ans, (p[i] - p[j]).len());
        ++i;
    }
    return ans;
}
```

TangentsAlex.cpp

Description: Find both tangets to the convex polygon.
(Zakaldovany algos mozhet sgonyat za pivom tak zhe).
Time: $\mathcal{O}(\log(n))$

b2b424, 17 lines

```
pair<int, int> tangents_alex(vector<Point> &p, Point &a) {
    int n = p.size();
    int l = __lg(n);
    auto findWithSign = [&](int val) {
        int i = 0;
        for (int k = l; k >= 0; --k) {
            int i1 = (i - (1 << k) + n) % n;
            int i2 = (i + (1 << k)) % n;
            if (sign((p[i1] - a) ^ (p[i] - a)) == val)
                i = i1;
            if (sign((p[i2] - a) ^ (p[i] - a)) == val)
                i = i2;
        }
        return i;
    };
    return {findWithSign(1), findWithSign(-1)};
}
```

IsHpiEmpty.cpp

Description: Determines is half plane intersectinos.
Time: $\mathcal{O}(n)$ (expected)

4cb75f, 41 lines

```
bool isHpiEmpty(vector<Line> lines) {
    // return hpi(lines).empty();
    // overflow/precision problems?
    shuffle(all(lines), rnd);
```

```
    const ld C = 1e9;
    Point ans(C, C);
    vector<Point> box = {{-C, -C}, {C, -C}, {C, C}, {-C, C}};
    for (int i = 0; i < 4; ++i)
        lines.push_back({box[i], box[(i + 1) % 4]});
    int n = lines.size();
    for (int i = n - 4; i >= 0; --i) {
        if (lines[i].isIn(ans))
            continue;
        Point up(0, C + 1), down(0, -C - 1), pi = getPoint(lines[i]);
        for (int j = i + 1; j < n; ++j) {
            if (lines[i] == lines[j])
                continue;
            Point p, pj = getPoint(lines[j]);
            if (!cross(lines[i], lines[j], p)) {
                if (sign(pi * pj) != -1)
                    continue;
                if (sign(lines[i].c + lines[j].c) *
                        (!sign(pi.y) ? sign(pi.x) : -1) ==
                    -1)
                    return true;
            } else {
                if ((!sign(pi.y) ? sign(pi.x) : sign(pi.y)) * (sign(pi ^ pj)) ==
                    1)
                    chkmin(up, p);
                else
                    chkmax(down, p);
            }
        }
        if ((ans = up) < down)
            return true;
    }
    // for (int i = 0; i < n; ++i) {
    //     assert(lines[i].eval(ans) < EPS);
    // }
    return false;
}
```

HalfPlaneIntersection.cpp

Description: Find the intersection of the half planes.

Time: $\mathcal{O}(n \log(n))$

2a2340, 67 lines

```
Vec getPoint(Line l) { return Vec(-l.b, l.a); }

bool bad(Line a, Line b, Line c) {
    Point x;
    assert(cross(b, c, x) == 1);
    return a.eval(x) > 0;
}

// Do not forget about the bounding box
vector<Point> hpi(vector<Line> lines) {
    sort(all(lines), [](Line al, Line bl) -> bool {
        Point a = getPoint(al);
        Point b = getPoint(bl);
        if (a.y >= 0 && b.y < 0)
            return 1;
        if (a.y < 0 && b.y >= 0)
            return 0;
        if (a.y == 0 && b.y == 0)
            return a.x > 0 && b.x < 0;
        return (a ^ b) > 0;
    });

    vector<pair<Line, int>> st;
    for (int it = 0; it < 2; it++) {
        for (int i = 0; i < (int)lines.size(); i++) {
            bool flag = false;
            while (!st.empty()) {
                if ((getPoint(st.back().first) - getPoint(lines[i])).len() <
                    EPS) {
                    if (lines[i].c <= st.back().first.c) {
                        flag = true;
                        break;
                    } else {
                        st.pop_back();
                    }
                } else if ((getPoint(st.back().first) ^ getPoint(lines[i])) <
                           EPS / 2) {
                    return {};
                } else if (st.size() >= 2 &&
                           bad(st[st.size() - 2].first, st[st.size() - 1].first,
                               lines[i])) {
                    st.pop_back();
                } else {
                    break;
                }
            }
            if (!flag)
                st.push_back({lines[i], i});
        }
    }

    vector<int> en(lines.size(), -1);
    vector<Point> ans;
    for (int i = 0; i < (int)st.size(); i++) {
        if (en[st[i].second] == -1) {
            en[st[i].second] = i;
            continue;
        }
        for (int j = en[st[i].second]; j < i; j++) {
            Point I;
            assert(cross(st[j].first, st[j + 1].first, I) == 1);
            ans.push_back(I);
        }
        break;
    }
    return ans;
}
```

Math (6)

BerlekampMassey.cpp

Description: Find the shortest linear-feedback shift register
Time: $\mathcal{O}(n^2)$

505033, 36 lines

```
vector<int> berlekamp_massey(vector<int> x) {
    vector<int> ls, cur;
    int lf = 0, d = 0;
    for (int i = 0; i < x.size(); ++i) {
        ll t = 0;
        for (int j = 0; j < cur.size(); ++j) {
            t = (t + 1ll * x[i - j - 1] * cur[j]) % MOD;
        }
        if ((t - x[i]) % MOD == 0)
            continue;
        if (cur.empty()) {
            cur.resize(i + 1);
            lf = i;
            d = (t - x[i]) % MOD;
            continue;
        }
        ll k = -(x[i] - t) * pw(d, MOD - 2) % MOD;
        vector<int> c(i - lf - 1);
        c.push_back(k);
        for (auto &j : ls)
            c.push_back(-j * k % MOD);
        if (c.size() < cur.size())
            c.resize(cur.size());
        for (int j = 0; j < cur.size(); ++j) {
            c[j] = (c[j] + cur[j]) % MOD;
        }
        if (i - lf + (int)ls.size() >= (int)cur.size()) {
            tie(ls, lf, d) = make_tuple(cur, i, (t - x[i]) % MOD);
        }
        cur = c;
    }
    for (auto &i : cur)
        i = (i % MOD + MOD) % MOD;
    return cur;
}
// for a_i = 2 * a_i-1 + a_i-1 returns {2, 1}
```

GoncharFedor.cpp

Description: Calculating number of points $x, y \geq 0, Ax + By \leq C$
Time: $\mathcal{O}(\log(C))$

0ef10e, 11 lines

```
ll solve_triangle(ll A, ll B, ll C) { // x,y >=0, Ax+By<=C
    if (C < 0)
        return 0;
    if (A > B)
        swap(A, B);
    ll p = C / B;
    ll k = B / A;
    ll d = (C - p * B) / A;
    return solve_triangle(B - k * A, A, C - A * (k * p + d + 1)) +
          (p + 1) * (d + 1) + k * p * (p + 1) / 2;
}
```

PrimalityTest.cpp

Description: Checking primality of p
Time: $\mathcal{O}(\log(C))$

af473a, 32 lines

```
const int iters = 8; // can change
bool isprime(ll p) {
    if (p == 1 || p == 4)
        return 0;
    if (p == 2 || p == 3)
        return 1;
    for (int it = 0; it < iters; ++it) {
        ll a = rnd() % (p - 2) + 2;
        ll nw = p - 1;
        while (nw % 2 == 0)
            nw /= 2;
        ll x = binpow(a, nw, p); // int128
        if (x == 1)
            continue;
        ll last = x;
        nw *= 2;
        while (nw <= p - 1) {
            x = (__int128_t)x * x % mod;
            if (x == 1) {
                if (last != p - 1) {
                    return 0;
                }
                break;
            }
            last = x;
            nw *= 2;
        }
        if (x != 1)
            return 0;
    }
    return 1;
}
```

XorConvolution.cpp

Description: Calculating xor-convolution of 2 vectors modulo smth
Time: $\mathcal{O}(n\log(n))$

454afd, 23 lines

```
void fwht(vector<int> &a) {
    int n = a.size();
    for (int l = 1; l < n; l <<= 1) {
        for (int i = 0; i < n; i += 2 * l) {
            for (int j = 0; j < l; ++j) {
                int u = a[i + j], v = a[i + j + l];
                a[i + j] = add(u, v), a[i + j + l] = sub(u, v);
            }
        }
    }
} // https://judge.yosupo.jp/problem/bitwise_xor_convolution
vector<int> xorconvo(vector<int> a, vector<int> b) {
    int n = 1;
    while (n < max(a.size(), b.size()))
        n *= 2;
    a.resize(n), b.resize(n);
    fwht(a), fwht(b);
    int in = inv(n);
```

```
    for (int i = 0; i < n; ++i)
        a[i] = mul(a[i], mul(b[i], in));
    fwht(a);
    return a;
}
```

Factorization.cpp

Description: Factorizing a number real quick
Time: $\mathcal{O}\left(n^{\frac{1}{4}}\right)$

f0d7c6, 51 lines

```
ll gcd(ll a, ll b) {
    while (b)
        a %= b, swap(a, b);
    return a;
}

ll f(ll a, ll n) { return ((__int128_t)a * a % n + 1) % n; }

vector<ll> factorize(ll n) {
    if (n <= 1e6) { // can add primality check for speed?
        vector<ll> res;
        for (ll i = 2; i * i <= n; ++i) {
            while (n % i == 0) {
                res.pbc(i);
                n /= i;
            }
        }
        if (n != 1)
            res.pbc(n);
        return res;
    }
    ll x = rnd() % (n - 1) + 1;
    ll y = x;
    ll tries = 10 * sqrt(sqrt(n));
    const int C = 60;
    for (ll i = 0; i < tries; i += C) {
        ll xs = x;
        ll ys = y;
        ll m = 1;
        for (int k = 0; k < C; ++k) {
            x = f(x, n);
            y = f(f(y, n), n);
            m = (__int128_t)m * abs(x - y) % n;
        }
        if (gcd(n, m) == 1)
            continue;
        x = xs, y = ys;
        for (int k = 0; k < C; ++k) {
            x = f(x, n);
            y = f(f(y, n), n);
            ll res = gcd(n, abs(x - y));
            if (res != 1 && res != n) {
                vector<ll> v1 = factorize(res), v2 = factorize(n / res);
                for (auto j : v2)
                    v1.pbc(j);
                return v1;
            }
        }
```

```
    }
    return {n};
}
```

NTT.cpp

Description: Calculating FFT modulo MOD
Time: $\mathcal{O}(n \log(n))$

8b7830, 57 lines

```
// DONT FORGET TO CALL initNTT() AND CHECK MAXLOG
const int MOD = 998244353, MAXLOG = 20;
const int N = (1 << MAXLOG), MAXN = (1 << MAXLOG) + 228;
int rev[MAXN], w[MAXN], n, m, a[MAXN], b[MAXN], fans[MAXN];
void initNTT() {
    int g = 2;
    for (;; g++) {
        int y = g;
        for (int i = 0; i < MAXLOG - 1; ++i) {
            y = mul(y, y);
        }
        if (y == MOD - 1) {
            break;
        }
    }
    w[0] = 1;
    for (int i = 1; i < N; ++i) {
        w[i] = mul(w[i - 1], g);
    }
    rev[0] = 0;
    for (int i = 1; i < N; ++i) {
        rev[i] = (rev[i >> 1] >> 1) ^ ((i & 1) << (MAXLOG - 1));
    }
}
void NTT(int n, int LOG, int *a) {
    for (int i = 0; i < n; ++i) {
        if (i < (rev[i] >> (MAXLOG - LOG))) {
            swap(a[i], a[(rev[i] >> (MAXLOG - LOG))]);
        }
    }
    for (int lvl = 0; lvl < LOG; lvl++) {
        int len = 1 << lvl;
        for (int st = 0; st < n; st += len << 1) {
            for (int i = 0; i < len; ++i) {
                int x = a[st + i],
                    y = mul(a[st + len + i], w[i << (MAXLOG - 1 - lvl)]);
                a[st + i] = add(x, y);
                a[st + i + len] = sub(x, y);
            }
        }
    }
}
void mul() {
    int LOG = __lg(2 * max(n, m) - 1) + 1;
    int sz = 1 << LOG;
    fill(a + n, a + sz, 0);
    fill(b + m, b + sz, 0);
    NTT(sz, LOG, a), NTT(sz, LOG, b);
    for (int i = 0; i < sz; ++i)
        a[i] = mul(a[i], b[i]);
```

```
    NTT(sz, LOG, a);
    int inv_sz = inv(sz);
    for (int i = 0; i < sz; ++i)
        fans[i] = mul(a[i], inv_sz);
    reverse(fans + 1, fans + sz);
}
// DONT FORGET TO CALL initNTT() AND CHECK MAXLOG
```

FFT.cpp

Description: Calculating product of two polynomials
Time: $\mathcal{O}(n\log(n))$

ed80ec, 47 lines

```
// DONT FORGET TO INITFFT() AND CHECK MAXLOG
const ld PI = acos(-1);
using cd = complex<long double>;
const int MAXLOG = 20, N = (1 << MAXLOG), MAXN = (1 << MAXLOG) + 228;
int rev[MAXN], n, m, fans[MAXN];
cd w[MAXN], a[MAXN], b[MAXN];
void initFFT() {
    for (int i = 0; i < N; i++) {
        w[i] = cd(cos(2 * PI * i / N), sin(2 * PI * i / N));
    }
    rev[0] = 0;
    for (int i = 1; i < N; i++) {
        rev[i] = (rev[i >> 1] >> 1) ^ ((i & 1) << (MAXLOG - 1));
    }
}
void FFT(int n, int LOG, cd *a) {
    for (int i = 0; i < n; i++) {
        if (i < (rev[i] >> (MAXLOG - LOG))) {
            swap(a[i], a[(rev[i] >> (MAXLOG - LOG))]);
        }
    }
    for (int lvl = 0; lvl < LOG; lvl++) {
        int len = 1 << lvl;
        for (int st = 0; st < n; st += len << 1) {
            for (int i = 0; i < len; i++) {
                cd x = a[st + i],
                   y = a[st + len + i] * w[i << (MAXLOG - 1 - lvl)];
                a[st + i] = x + y;
                a[st + i + len] = x - y;
            }
        }
    }
}
void mul() {
    int LOG = __lg(2 * max(n, m) - 1) + 1;
    int sz = 1 << LOG;
    fill(a + n, a + sz, 0);
    fill(b + m, b + sz, 0);
    FFT(sz, LOG, a), FFT(sz, LOG, b);
    for (int i = 0; i < sz; i++)
        a[i] *= b[i];
    FFT(sz, LOG, a);
    for (int i = 0; i < sz; i++)
        fans[i] = (int)(a[i].real() / sz + 0.5);
    reverse(fans + 1, fans + sz);
}
// DONT FORGET TO INITFFT() AND CHECK MAXLOG
```

AndConvolution.cpp

Description: Calculating and-convolution modulo smth
Time: $\mathcal{O}(n\log(n))$

5dedf4, 24 lines

```
void conv(vector<int> &a, bool x) {
    int n = a.size();
    for (int j = 0; (1 << j) < n; ++j) {
        for (int i = 0; i < n; ++i) {
            if (!(i & (1 << j))) {
                if (x)
                    a[i] = add(a[i], a[i | (1 << j)]);
                else
                    a[i] = sub(a[i], a[i | (1 << j)]);
            }
        }
    }
} // https://judge.yosupo.jp/problem/bitwise_and_convolution
vector<int> andcon(vector<int> a, vector<int> b) {
    int n = 1;
    while (n < max(a.size(), b.size()))
        n *= 2;
    a.resize(n), b.resize(n);
    conv(a, 1), conv(b, 1);
    for (int i = 0; i < n; ++i)
        a[i] = mul(a[i], b[i]);
    conv(a, 0);
    return a;
}
```

6.1 Fun things

$ClassesCount = \dfrac{1}{|G|} \sum_{\pi \in G} I(\pi)$
$ClassesCount = \dfrac{1}{|G|} \sum_{\pi \in G} k^{C(\pi)}$
Stirling 2kind - count of partitions of n objects into k nonempty sets:
$S(n,k) = S(n-1,k-1) + kS(n-1,k)$
$S(n,k) = \sum_{j=0}^{n-1} \binom{n-1}{j} S(j,k-1)$
$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k+j} \binom{k}{j} j^n$
$n! \approx \sqrt{2n\pi} (\frac{n}{e})^n$
$\binom{n}{k} \equiv \prod_i \binom{n_i}{k_i}$, n_i, k_i - digits of n, k in p-adic system
$\int_a^b f(x)dx \approx \dfrac{b-a}{6}(f(a) + 4f(\dfrac{a+b}{2}) + f(b))$
$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$, $O(loglog)$
$G(n) = n \oplus (n >> 1)$
$g(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} g(d)\mu(\frac{n}{d})$
$\sum_{d|n} \mu(d) = [n = 1], \mu(1) = 1, \mu(p) = -1, \mu(p^k) = 0$
$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$
$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$
$\mathrm{tg}(a \pm b) = \dfrac{\mathrm{tg}\, a \pm \mathrm{tg}\, b}{1 \mp \mathrm{tg}\, a\, \mathrm{tg}\, b}$
$\mathrm{ctg}(a \pm b) = \dfrac{\mathrm{ctg}\, a\, \mathrm{ctg}\, b \mp 1}{\mathrm{ctg}\, b \pm \mathrm{ctg}\, a}$
$\sin \dfrac{a}{2} = \pm\sqrt{\dfrac{1 - \cos a}{2}}$
$\cos \dfrac{a}{2} = \pm\sqrt{\dfrac{1 + \cos a}{2}}$
$\mathrm{tg}\, \dfrac{a}{2} = \dfrac{\sin a}{1 - \cos a} = \dfrac{1 - cosa}{sina}$
$\sin a \sin b = \dfrac{\cos(a-b) - \cos(a+b)}{2}$
$\sin a \cos b = \dfrac{\sin(a-b) + \sin(a+b)}{2}$
$\cos a \cos b = \dfrac{\cos(a-b) + \cos(a+b)}{2}$
1 jan 2000 - saturday, 1 jan 1900 - monday, 14 apr 1961 - friday
Bell numbers: 0:1, 1:1, 2:2, 3:5, 4:15, 5:52, 6:203, 7:877, 8:4140, 9:21147, 10:115975, 11:678570, 12:4213597, 13:27644437, 14:190899322, 15:1382958545, 16:10480142147, 17:82864869804, 18:682076806159, 19:5832742205057, 20:51724158235372, 21:474869816156751, 22:4506715738447323, 23:44152005855084346
Fibonacci: 45:1134903170. 46:1836311903(max int), 91: 4660046610375530309
Highly composite numbers:
$\le 1000 : d(840) = 32, \le 10^4 : d(9240) = 64, \le 10^5 : d(83160) = 128, \le 10^6 : d(720720) = 240, \le 10^7 : d(8648640) = 448, \le 10^8 : d(91891800) = 768, \le 10^9 : d(931170240) = 1344, \le 10^{11} : d(97772875200) = 4032, \le 10^{15} : d(866421317361600) = 26880, \le 10^{18} : d(897612484786617600) = 103680$

Table of Basic Integrals (7)

Basic Forms

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1 \tag{7.1}$$

$$\int \frac{1}{x} dx = \ln|x| \tag{7.2}$$

$$\int u dv = uv - \int v du \tag{7.3}$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a}\ln|ax+b| \tag{7.4}$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} \tag{7.5}$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1 \tag{7.6}$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \tag{7.7}$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x \tag{7.8}$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} \tag{7.9}$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2}\ln|a^2+x^2| \tag{7.10}$$

$$\int \frac{x^2}{a^2+x^2} dx = x - a\tan^{-1}\frac{x}{a} \tag{7.11}$$

$$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln|a^2+x^2| \tag{7.12}$$

$$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{7.13}$$

$$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a}\ln\frac{a+x}{b+x},\ a \neq b \tag{7.14}$$

$$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln|a+x| \tag{7.15}$$

$$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{7.16}$$

Integrals with Roots

$$\int \sqrt{x-a}\ dx = \frac{2}{3}(x-a)^{3/2} \tag{7.17}$$

$$\int \frac{1}{\sqrt{x\pm a}}\ dx = 2\sqrt{x\pm a} \tag{7.18}$$

$$\int \frac{1}{\sqrt{a-x}}\ dx = -2\sqrt{a-x} \tag{7.19}$$

$$\int x\sqrt{x-a}\ dx = \begin{cases} \frac{2a}{3}(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2}, \text{ or} \\ \frac{2}{3}x(x-a)^{3/2} - \frac{4}{15}(x-a)^{5/2}, \text{ or} \\ \frac{2}{15}(2a+3x)(x-a)^{3/2} \end{cases} \tag{7.20}$$

$$\int \sqrt{ax+b}\ dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b} \tag{7.21}$$

$$\int (ax+b)^{3/2}\ dx = \frac{2}{5a}(ax+b)^{5/2} \tag{7.22}$$

$$\int \frac{x}{\sqrt{x\pm a}}\ dx = \frac{2}{3}(x\mp 2a)\sqrt{x\pm a} \tag{7.23}$$

$$\int \sqrt{\frac{x}{a-x}}\ dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a} \tag{7.24}$$

$$\int \sqrt{\frac{x}{a+x}}\ dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x}+\sqrt{x+a}\right] \tag{7.25}$$

$$\int x\sqrt{ax+b}\ dx = \frac{2}{15a^2}(-2b^2+abx+3a^2x^2)\sqrt{ax+b} \tag{7.26}$$

$$\int \sqrt{x(ax+b)}\ dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2\ln\left|a\sqrt{x}+\sqrt{a(ax+b)}\right|\right] \tag{7.27}$$

$$\int \sqrt{x^3(ax+b)}\ dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}}\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right| \tag{7.28}$$

$$\int \sqrt{x^2 \pm a^2}\ dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2 \ln\left|x + \sqrt{x^2 \pm a^2}\right| \tag{7.29}$$

$$\int \sqrt{a^2 - x^2}\ dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2 \tan^{-1}\frac{x}{\sqrt{a^2 - x^2}} \tag{7.30}$$

$$\int x\sqrt{x^2 \pm a^2}\ dx = \frac{1}{3}\left(x^2 \pm a^2\right)^{3/2} \tag{7.31}$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}}\ dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| \tag{7.32}$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\ dx = \sin^{-1}\frac{x}{a} \tag{7.33}$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}}\ dx = \sqrt{x^2 \pm a^2} \tag{7.34}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}}\ dx = -\sqrt{a^2 - x^2} \tag{7.35}$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}}\ dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2 \ln\left|x + \sqrt{x^2 \pm a^2}\right| \tag{7.36}$$

$$\int \sqrt{ax^2 + bx + c}\ dx = \frac{b + 2ax}{4a}\sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a^{3/2}}\ln\left|2ax + b + 2\sqrt{a(ax^2 + bx^+ c)}\right| \tag{7.37}$$

$$\begin{aligned}\int x\sqrt{ax^2 + bx + c}\ dx = \frac{1}{48a^{5/2}}&\left(2\sqrt{a}\sqrt{ax^2 + bx + c}\left(-3b^2 + 2abx + 8a(c + ax^2)\right)\right.\\ &\left.+3(b^3 - 4abc)\ln\left|b + 2ax + 2\sqrt{a}\sqrt{ax^2 + bx + c}\right|\right)\end{aligned} \tag{7.38}$$

$$\int \frac{1}{\sqrt{ax^2 + bx + c}}\ dx = \frac{1}{\sqrt{a}}\ln\left|2ax + b + 2\sqrt{a(ax^2 + bx + c)}\right| \tag{7.39}$$

$$\int \frac{x}{\sqrt{ax^2 + bx + c}}\ dx = \frac{1}{a}\sqrt{ax^2 + bx + c} - \frac{b}{2a^{3/2}}\ln\left|2ax + b + 2\sqrt{a(ax^2 + bx + c)}\right| \tag{7.40}$$

$$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 + x^2}} \tag{7.41}$$

Integrals with Logarithms

$$\int \ln ax\ dx = x\ln ax - x \tag{7.42}$$

$$\int x\ln x\ dx = \frac{1}{2}x^2\ln x - \frac{x^2}{4} \tag{7.43}$$

$$\int x^2\ln x\ dx = \frac{1}{3}x^3\ln x - \frac{x^3}{9} \tag{7.44}$$

$$\int x^n\ln x\ dx = x^{n+1}\left(\frac{\ln x}{n+1} - \frac{1}{(n+1)^2}\right),\quad n \neq -1 \tag{7.45}$$

$$\int \frac{\ln ax}{x}\ dx = \frac{1}{2}(\ln ax)^2 \tag{7.46}$$

$$\int \frac{\ln x}{x^2}\ dx = -\frac{1}{x} - \frac{\ln x}{x} \tag{7.47}$$

$$\int \ln(ax + b)\ dx = \left(x + \frac{b}{a}\right)\ln(ax + b) - x, a \neq 0 \tag{7.48}$$

$$\int \ln(x^2 + a^2)\,dx = x\ln(x^2 + a^2) + 2a\tan^{-1}\frac{x}{a} - 2x \tag{7.49}$$

$$\int \ln(x^2 - a^2)\,dx = x\ln(x^2 - a^2) + a\ln\frac{x + a}{x - a} - 2x \tag{7.50}$$

$$\int \ln\left(ax^2 + bx + c\right)\ dx = \frac{1}{a}\sqrt{4ac - b^2}\tan^{-1}\frac{2ax + b}{\sqrt{4ac - b^2}} - 2x + \left(\frac{b}{2a} + x\right)\ln\left(ax^2 + bx + c\right) \tag{7.51}$$

$$\int x\ln(ax + b)\ dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax + b) \tag{7.52}$$

$$\int x\ln\left(a^2 - b^2x^2\right)\ dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 - \frac{a^2}{b^2}\right)\ln\left(a^2 - b^2x^2\right) \tag{7.53}$$

$$\int (\ln x)^2 \; dx = 2x - 2x\ln x + x(\ln x)^2 \tag{7.54}$$

$$\int (\ln x)^3 \; dx = -6x + x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x \tag{7.55}$$

$$\int x(\ln x)^2 \; dx = \frac{x^2}{4} + \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x \tag{7.56}$$

$$\int x^2(\ln x)^2 \; dx = \frac{2x^3}{27} + \frac{1}{3}x^3(\ln x)^2 - \frac{2}{9}x^3\ln x \tag{7.57}$$

Integrals with Exponentials

$$\int e^{ax} \; dx = \frac{1}{a}e^{ax} \tag{7.58}$$

$$\int \sqrt{x}e^{ax} \; dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\mathrm{erf}\left(i\sqrt{ax}\right), \text{ where } \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \tag{7.59}$$

$$\int xe^x \; dx = (x-1)e^x \tag{7.60}$$

$$\int xe^{ax} \; dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax} \tag{7.61}$$

$$\int x^2e^x \; dx = \left(x^2 - 2x + 2\right)e^x \tag{7.62}$$

$$\int x^2e^{ax} \; dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax} \tag{7.63}$$

$$\int x^3e^x \; dx = \left(x^3 - 3x^2 + 6x - 6\right)e^x \tag{7.64}$$

$$\int x^ne^{ax} \; dx = \frac{x^ne^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,\mathrm{d}x \tag{7.65}$$

$$\int x^ne^{ax} \; dx = \frac{(-1)^n}{a^{n+1}}\Gamma[1+n, -ax], \text{ where } \Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}\,\mathrm{d}t \tag{7.66}$$

$$\int e^{ax^2} \; dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(ix\sqrt{a}\right) \tag{7.67}$$

$$\int e^{-ax^2} \; dx = \frac{\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(x\sqrt{a}\right) \tag{7.68}$$

$$\int xe^{-ax^2} \; dx = -\frac{1}{2a}e^{-ax^2} \tag{7.69}$$

$$\int x^2e^{-ax^2} \; dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}\mathrm{erf}(x\sqrt{a}) - \frac{x}{2a}e^{-ax^2} \tag{7.70}$$

Integrals with Trigonometric Functions

$$\int \sin ax \; dx = -\frac{1}{a}\cos ax \tag{7.71}$$

$$\int \sin^2 ax \; dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \tag{7.72}$$

$$\int \sin^3 ax \; dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a} \tag{7.73}$$

$$\int \sin^n ax \; dx = -\frac{1}{a}\cos ax \;\; {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right] \tag{7.74}$$

$$\int \cos ax \; dx = \frac{1}{a}\sin ax \tag{7.75}$$

$$\int \cos^2 ax \; dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \tag{7.76}$$

$$\int \cos^3 ax dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a} \tag{7.77}$$

$$\int \cos^p ax dx = -\frac{1}{a(1+p)}\cos^{1+p} ax \times {}_2F_1\left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right] \tag{7.78}$$

$$\int \cos x\sin x \; dx = \frac{1}{2}\sin^2 x + c_1 = -\frac{1}{2}\cos^2 x + c_2 = -\frac{1}{4}\cos 2x + c_3 \tag{7.79}$$

$$\int \cos ax\sin bx \; dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b \tag{7.80}$$

$$\int \sin^2 ax\cos bx \; dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)} \tag{7.81}$$

$$\int \sin^2 x\cos x \; dx = \frac{1}{3}\sin^3 x \tag{7.82}$$

$$\int \cos^2 ax \sin bx \; dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)} \tag{7.83}$$

$$\int \cos^2 ax \sin ax \; dx = -\frac{1}{3a}\cos^3 ax \tag{7.84}$$

$$\int \sin^2 ax \cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} \tag{7.85}$$

$$\int \sin^2 ax \cos^2 ax \; dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \tag{7.86}$$

$$\int \tan ax \; dx = -\frac{1}{a}\ln\cos ax \tag{7.87}$$

$$\int \tan^2 ax \; dx = -x + \frac{1}{a}\tan ax \tag{7.88}$$

$$\int \tan^n ax \; dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right) \tag{7.89}$$

$$\int \tan^3 ax dx = \frac{1}{a}\ln\cos ax + \frac{1}{2a}\sec^2 ax \tag{7.90}$$

$$\int \sec x \; dx = \ln|\sec x + \tan x| = 2\tanh^{-1}\left(\tan\frac{x}{2}\right) \tag{7.91}$$

$$\int \sec^2 ax \; dx = \frac{1}{a}\tan ax \tag{7.92}$$

$$\int \sec^3 x \; dx = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x + \tan x| \tag{7.93}$$

$$\int \sec x \tan x \; dx = \sec x \tag{7.94}$$

$$\int \sec^2 x \tan x \; dx = \frac{1}{2}\sec^2 x \tag{7.95}$$

$$\int \sec^n x \tan x \; dx = \frac{1}{n}\sec^n x, n \neq 0 \tag{7.96}$$

$$\int \csc x \; dx = \ln\left|\tan\frac{x}{2}\right| = \ln|\csc x - \cot x| + C \tag{7.97}$$

$$\int \csc^2 ax \; dx = -\frac{1}{a}\cot ax \tag{7.98}$$

$$\int \csc^3 x \; dx = -\frac{1}{2}\cot x\csc x + \frac{1}{2}\ln|\csc x - \cot x| \tag{7.99}$$

$$\int \csc^n x \cot x \; dx = -\frac{1}{n}\csc^n x, n \neq 0 \tag{7.100}$$

$$\int \sec x \csc x \; dx = \ln|\tan x| \tag{7.101}$$

Products of Trigonometric Functions and Monomials

$$\int x\cos x \; dx = \cos x + x\sin x \tag{7.102}$$

$$\int x\cos ax \; dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax \tag{7.103}$$

$$\int x^2\cos x \; dx = 2x\cos x + \left(x^2 - 2\right)\sin x \tag{7.104}$$

$$\int x^2\cos ax \; dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2 - 2}{a^3}\sin ax \tag{7.105}$$

$$\int x^n\cos x dx = -\frac{1}{2}(i)^{n+1}\left[\Gamma(n+1, -ix) + (-1)^n\Gamma(n+1, ix)\right] \tag{7.106}$$

$$\int x^n\cos ax \; dx = \frac{1}{2}(ia)^{1-n}\left[(-1)^n\Gamma(n+1, -iax) - \Gamma(n+1, ixa)\right] \tag{7.107}$$

$$\int x\sin x \; dx = -x\cos x + \sin x \tag{7.108}$$

$$\int x\sin ax \; dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2} \tag{7.109}$$

$$\int x^2\sin x \; dx = \left(2 - x^2\right)\cos x + 2x\sin x \tag{7.110}$$

$$\int x^2 \sin ax\ dx = \frac{2-a^2x^2}{a^3}\cos ax + \frac{2x\sin ax}{a^2} \tag{7.111}$$

$$\int x^n \sin x\ dx = -\frac{1}{2}(i)^n\left[\Gamma(n+1,-ix)-(-1)^n\Gamma(n+1,-ix)\right] \tag{7.112}$$

$$\int x\cos^2 x\ dx = \frac{x^2}{4}+\frac{1}{8}\cos 2x+\frac{1}{4}x\sin 2x \tag{7.113}$$

$$\int x\sin^2 x\ dx = \frac{x^2}{4}-\frac{1}{8}\cos 2x-\frac{1}{4}x\sin 2x \tag{7.114}$$

$$\int x\tan^2 x\ dx = -\frac{x^2}{2}+\ln\cos x + x\tan x \tag{7.115}$$

$$\int x\sec^2 x\ dx = \ln\cos x + x\tan x \tag{7.116}$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x\ dx = \frac{1}{2}e^x(\sin x - \cos x) \tag{7.117}$$

$$\int e^{bx}\sin ax\ dx = \frac{1}{a^2+b^2}e^{bx}(b\sin ax - a\cos ax) \tag{7.118}$$

$$\int e^x \cos x\ dx = \frac{1}{2}e^x(\sin x + \cos x) \tag{7.119}$$

$$\int e^{bx}\cos ax\ dx = \frac{1}{a^2+b^2}e^{bx}(a\sin ax + b\cos ax) \tag{7.120}$$

$$\int xe^x \sin x\ dx = \frac{1}{2}e^x(\cos x - x\cos x + x\sin x) \tag{7.121}$$

$$\int xe^x \cos x\ dx = \frac{1}{2}e^x(x\cos x - \sin x + x\sin x) \tag{7.122}$$

Integrals of Hyperbolic Functions

$$\int \cosh ax\ dx = \frac{1}{a}\sinh ax \tag{7.123}$$

$$\int e^{ax}\cosh bx\ dx = \begin{cases} \dfrac{e^{ax}}{a^2-b^2}[a\cosh bx - b\sinh bx] & a\neq b \\ \dfrac{e^{2ax}}{4a}+\dfrac{x}{2} & a=b \end{cases} \tag{7.124}$$

$$\int \sinh ax\ dx = \frac{1}{a}\cosh ax \tag{7.125}$$

$$\int e^{ax}\sinh bx\ dx = \begin{cases} \dfrac{e^{ax}}{a^2-b^2}[-b\cosh bx + a\sinh bx] & a\neq b \\ \dfrac{e^{2ax}}{4a}-\dfrac{x}{2} & a=b \end{cases} \tag{7.126}$$

$$\int \tanh ax\, dx = \frac{1}{a}\ln\cosh ax \tag{7.127}$$

$$\int e^{ax}\tanh bx\ dx = \begin{cases} \dfrac{e^{(a+2b)x}}{(a+2b)}{}_2F_1\left[1+\dfrac{a}{2b},1,2+\dfrac{a}{2b},-e^{2bx}\right] & \\ \qquad -\dfrac{1}{a}e^{ax}{}_2F_1\left[1,\dfrac{a}{2b},1+\dfrac{a}{2b},-e^{2bx}\right] & a\neq b \\ \dfrac{e^{ax}-2\tan^{-1}[e^{ax}]}{a} & a=b \end{cases} \tag{7.128}$$

$$\int \cos ax\cosh bx\ dx = \frac{1}{a^2+b^2}\left[a\sin ax\cosh bx + b\cos ax\sinh bx\right] \tag{7.129}$$

$$\int \cos ax\sinh bx\ dx = \frac{1}{a^2+b^2}\left[b\cos ax\cosh bx + a\sin ax\sinh bx\right] \tag{7.130}$$

$$\int \sin ax\cosh bx\ dx = \frac{1}{a^2+b^2}\left[-a\cos ax\cosh bx + b\sin ax\sinh bx\right] \tag{7.131}$$

$$\int \sin ax\sinh bx\ dx = \frac{1}{a^2+b^2}\left[b\cosh bx\sin ax - a\cos ax\sinh bx\right] \tag{7.132}$$

$$\int \sinh ax\cosh ax dx = \frac{1}{4a}\left[-2ax+\sinh 2ax\right] \tag{7.133}$$

$$\int \sinh ax\cosh bx\ dx = \frac{1}{b^2-a^2}\left[b\cosh bx\sinh ax - a\cosh ax\sinh bx\right] \tag{7.134}$$

Problem	Status	Comment	Iurii	Alex	Leha
A - 1					
B - 2					
C - 3					
D - 4					
E - 5					
F - 6					
G - 7					
H - 8					
I - 9					
J - 10					
K - 11					
L - 12					
M - 13					
N - 14					
O - 15					