1 $\mathbb{R}/\mathbb{Z}$ is compact

Suppose $\{U_\lambda \mid \lambda \in \Lambda\}$ is an open cover of $\frac{\mathbb{R}}{\mathbb{Z}}$. Define $V_\lambda = \pi_\sim^{-1}(U_\lambda)$. Note that $\{V_\lambda \cap [0,1] \mid \lambda \in \Lambda\}$ is an open cover of $[0,1]$ with subspace topology. Hence, there exists an open cover $\{V_1 \cap [0,1], \ldots, V_n \cap [0,1]\}$ of $[0,1]$. Note that saturation of $[0,1]$ is $\mathbb{R}$ and V_λ are $\sim$ - saturated. Hence, $\{V_1, \ldots, V_n\}$ is an open cover of $\mathbb{R}$. Lastly, $\bigcup_{i=1}^n \pi_\sim(V_{\lambda_i}) = \bigcup_{i=1}^n U_{\lambda_i} = \pi_\sim(\bigcup_{i=1}^n V_{\lambda_i}) = \pi_\sim(\mathbb{R}) = \frac{\mathbb{R}}{\mathbb{Z}}$.

Hence, we found a finite subcover.