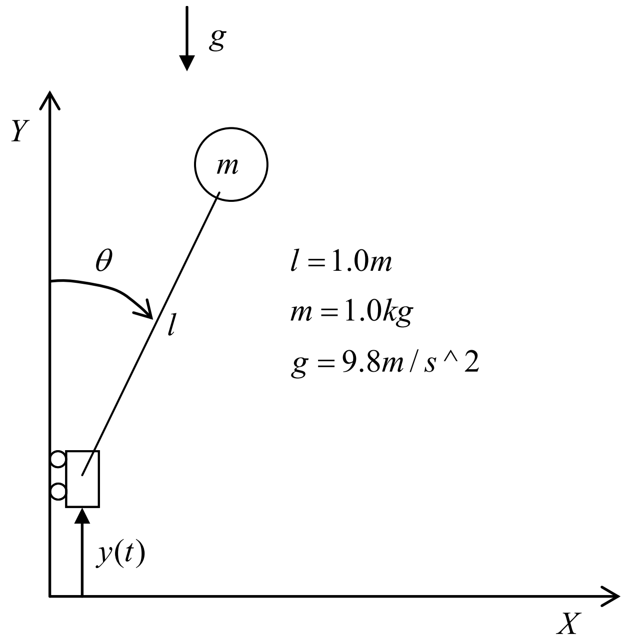
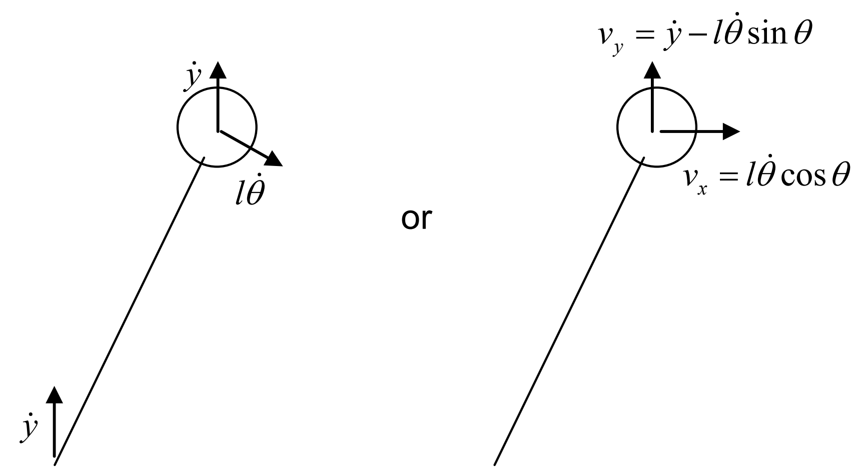
EME 271 Simulation Number 2

Professor Margolis, Winter Quarter, 2026



This inverted pendulum with vertical base excitation is pretty interesting. An analysis can be done which shows that if $y(t)$ is oscillated harmonically at some frequency and amplitude, the pendulum will be stable and will return to a vertical up position after being displaced by some angle, θ. Your job is to test this hard to believe behavior using simulation.

Simulate the equations of motion for a base input, $y(t) = Y \sin \omega t$, for various values of Y and ω .See if you can find a combination that stabilizes the pendulum for initial angles of $\theta_{ini} = 10^0, 20^0, 30^0$. Do you think it will work for $\theta_{ini} = 90^0$? Present results for θ vs t. When the system is stable it will go back and forth about the vertical position. When the system is unstable it will fall over and might go around 360 plus degrees.

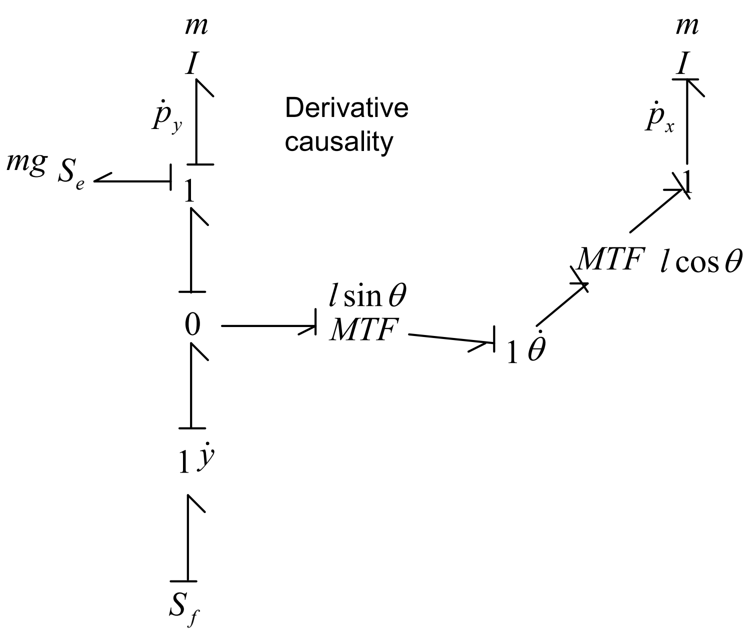


Velocity diagam

The figure shows a velocity diagram using the variables from the schematic.

Bond Graph Model

A bond graph of the system is shown below.



Bond graph of the system

Using procedures from the book that you will learn in the near future, we use causality and write,

$$p_y = m[\dot{y} - \frac{\sin\theta}{\cos\theta}\frac{p_x}{m}] \qquad (1)$$

We will need the derivative of this quantity, thus,

$$\dot{p}_y = m[\ddot{y} - \frac{\sin\theta}{\cos\theta}\frac{\dot{p}_x}{m} - \frac{p_x}{m}\frac{\dot{\theta}}{\cos^2\theta}] \qquad (2)$$

We now derive the state equations in the normal way yielding,

$$\dot{p}_x = \frac{\sin\theta}{\cos\theta}[mg + \dot{p}_y] \qquad (3)$$

Substituting from Eq. (2) and carrying out some algebra yields,

$$\dot{p}_x = mg\sin\theta\cos\theta + m\ddot{y}\sin\theta\cos\theta - \frac{\sin\theta}{\cos\theta}\dot{\theta}p_x \qquad (4)$$

and we need to expand the state space to allow calculation of θ, thus

$$\dot{\theta} = \frac{1}{l\cos\theta}\frac{p_x}{m} \qquad (5)$$

Eqs. (4) and (5) are the first order equations you will use for the simulation.

It turns out that the governing nonlinear equations have an approximate solution that was developed by a very famous applied mechanics professor, J P DenHartog. In his analysis he developed a range of amplitudes and frequencies that are stabilizing for this system. Here are an amplitude and frequency that does stabilize the system and this is a good starting point for your simulation.

$$Y = 0.1\ m$$
$$f = 20\ Hz, \quad \omega = 2\pi f$$

Try and determine a Y *and* f that yield the largest initial angle that is stable.