



# Applications of #SAT Solvers on Feature Models

Chico Sundermann, Michael Nieke, Paul Maximilian Bittner, Tobias Heß, Thomas Thüm, Ina Schaefer  
VaMoS'21 | February 10, 2020

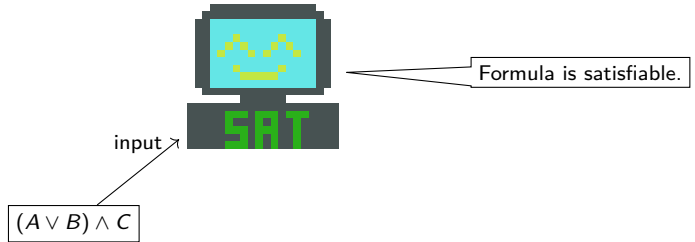


Technische  
Universität  
Braunschweig

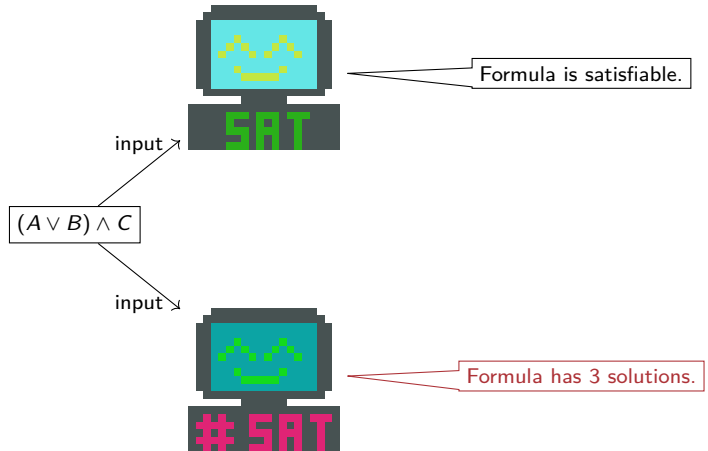


ulm university universität  
**uulm**

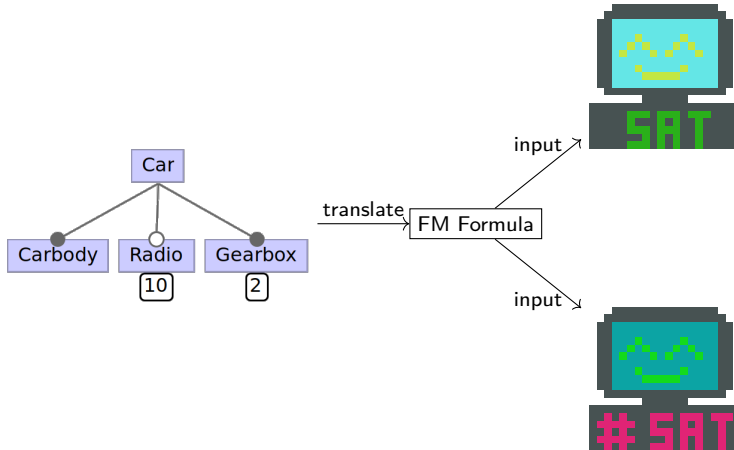
# What is SAT?



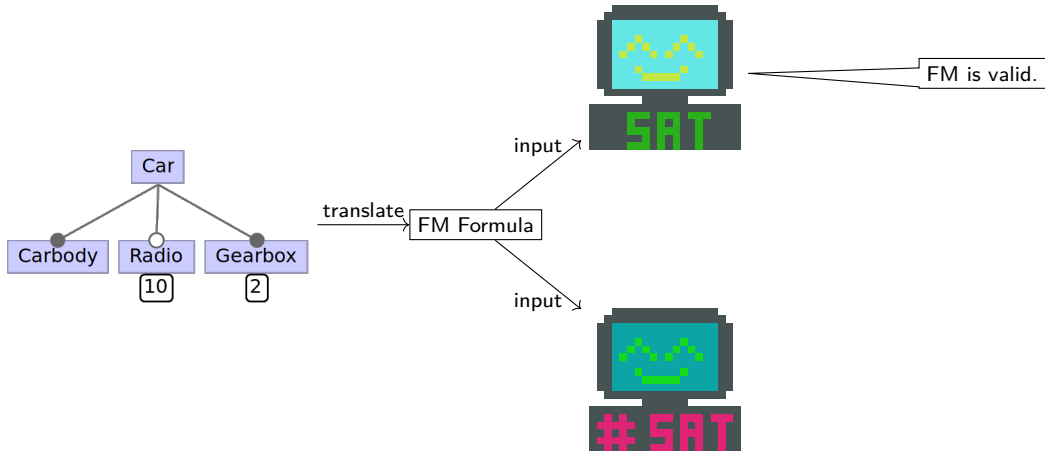
# What is #SAT?



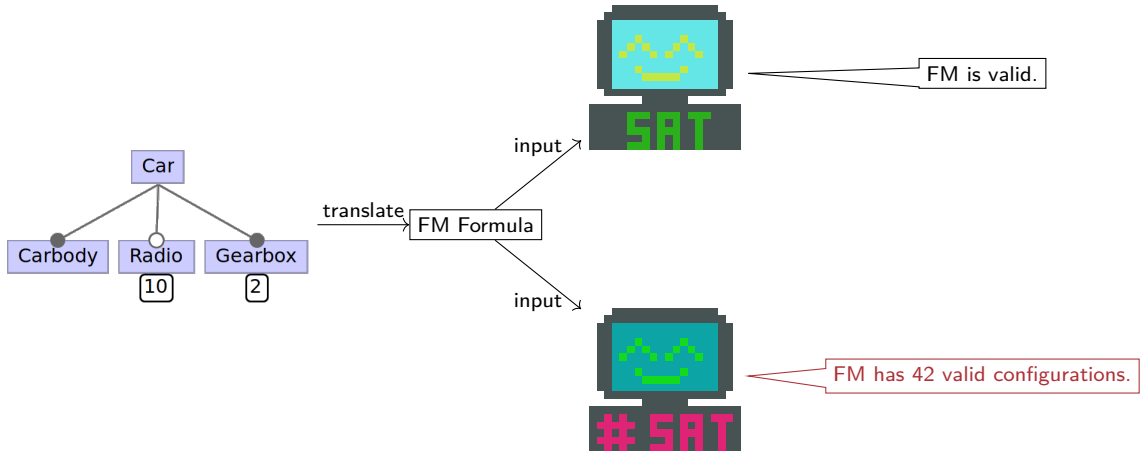
# What is #SAT?

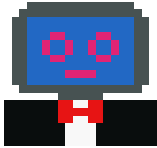


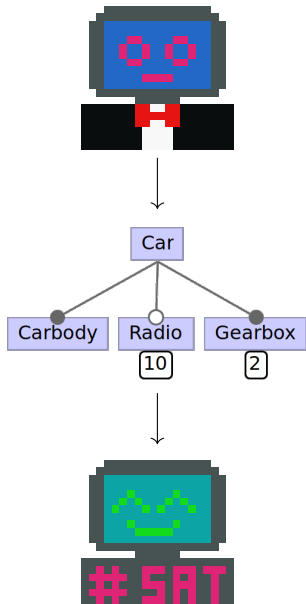
# What is #SAT?



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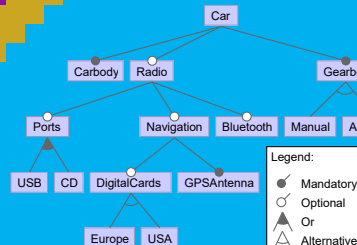
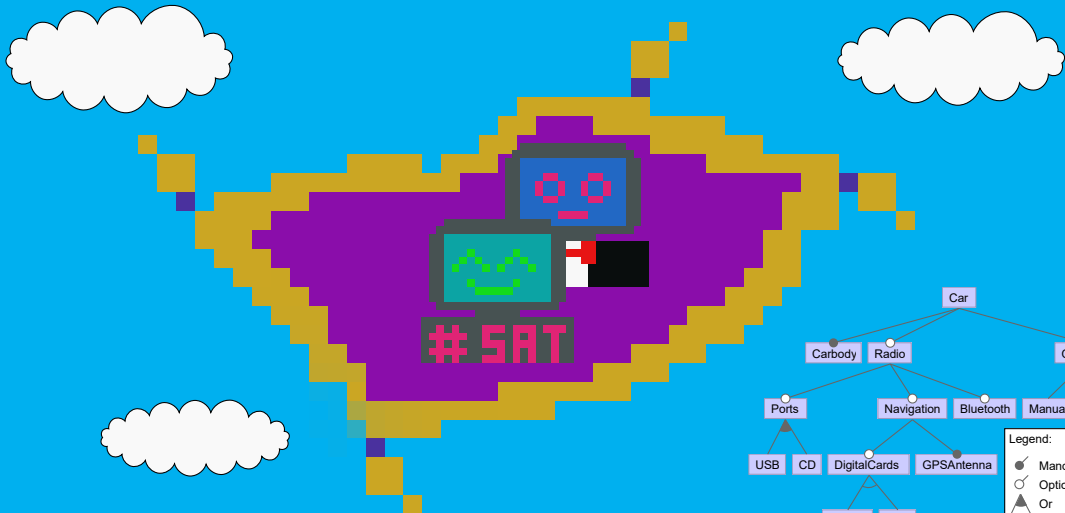








# A Whole New World!



Variability Factor

Void Feature Model

Degree of Orthogonality

Cost Savings

Maintainability Prediction

Homogeneity

Payoff Threshold

Degree of Reuse

Uniform Random Sampling

Literature  
Adapted  
New

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Rating Errors

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Atomic Set Candidates

Feature Prioritization

CTC Restrictiveness

Optimize Configuring

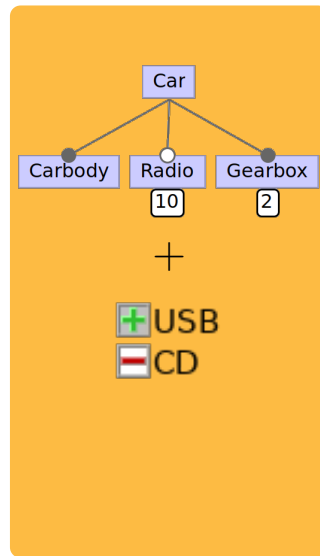
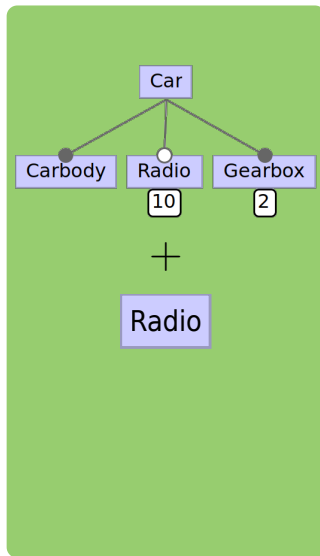
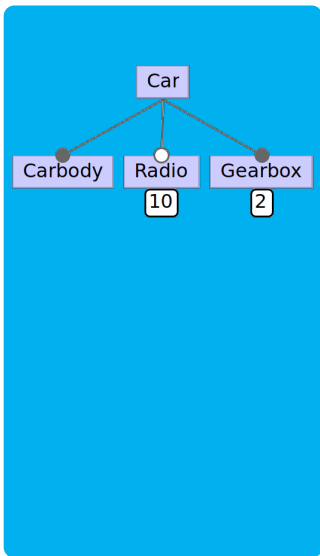
Uniform Random Sampling

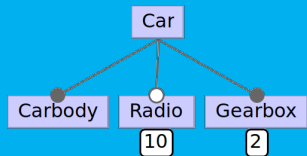
Atomic Sets

Rate Interactions

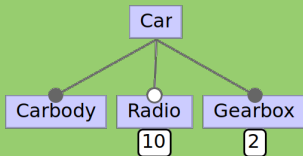
Configuration Derivation

Literature  
Adapted  
New

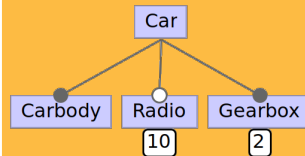




Cardinality of  
**Feature Models**



Cardinality of  
**Features**



Cardinality of  
**Partial Configurations**



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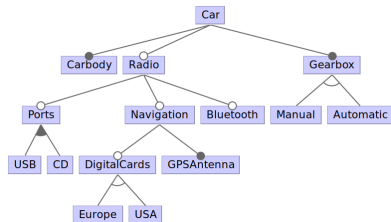




Variability Reduction

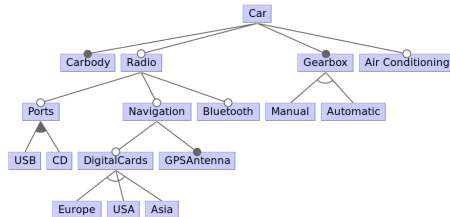
Variability Reduction

New

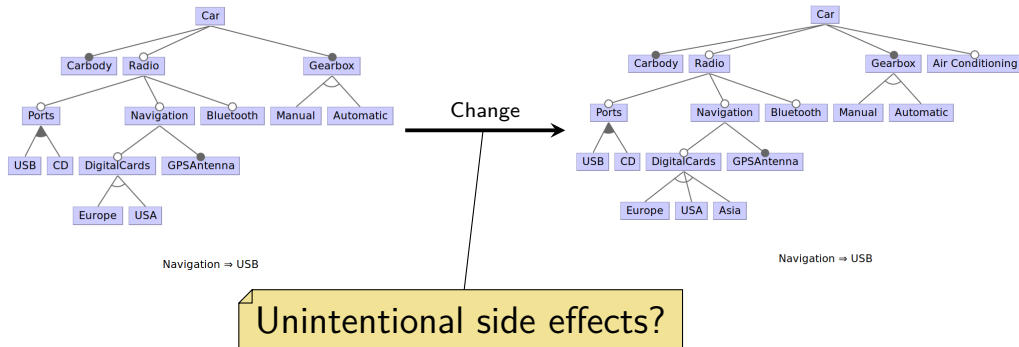


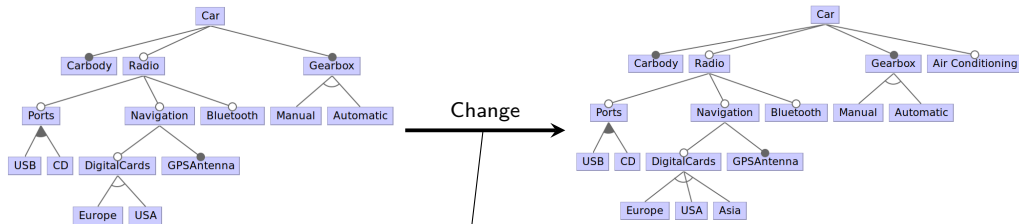
Navigation  $\Rightarrow$  USB

Change



Navigation  $\Rightarrow$  USB

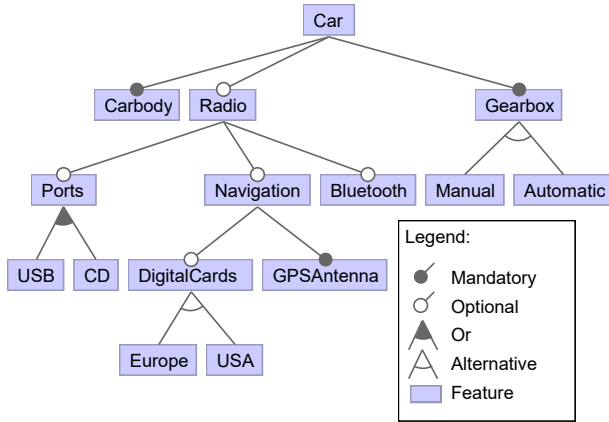




Unintentional side effects?

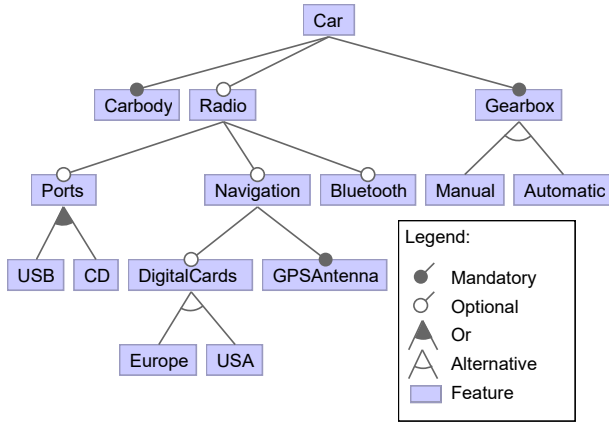
Difference in cardinality

## Variability Reduction



Navigation  $\Rightarrow$  USB

## Variability Reduction

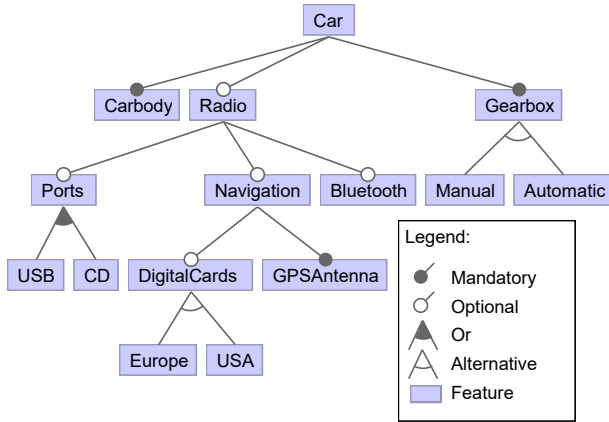


Navigation  $\Rightarrow$  USB

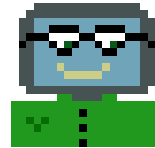
42 valid configurations



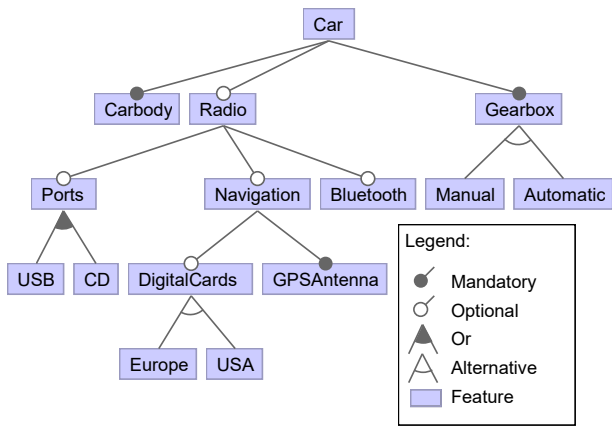
## Variability Reduction



Navigation  $\Rightarrow$  USB



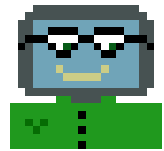




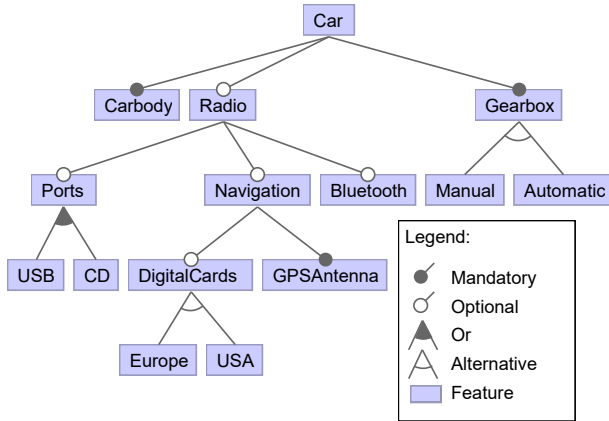
Navigation  $\Rightarrow$  USB

Variability Reduction

Automatic  $\Rightarrow$  USA

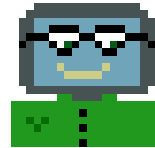


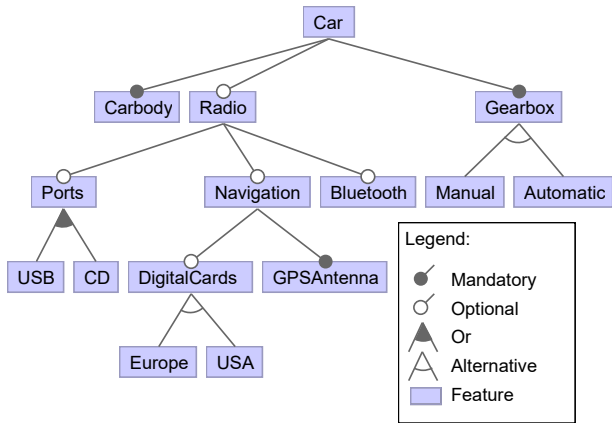
## Variability Reduction



Navigation  $\Rightarrow$  USB

Automatic  $\Rightarrow$  USA

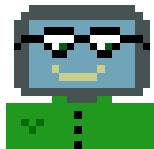




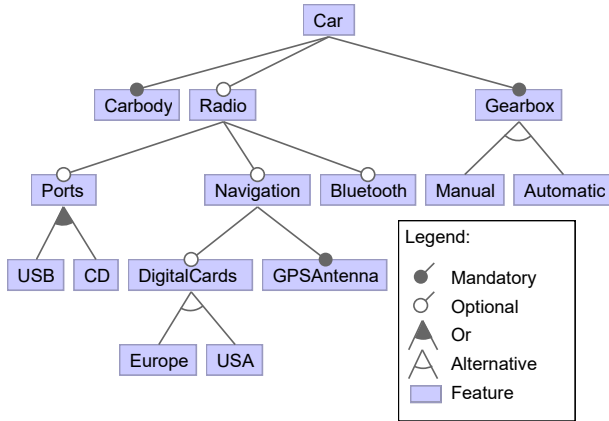
Navigation  $\Rightarrow$  USB  
 Automatic  $\Rightarrow$  USA

Variability Reduction

No anomalies! ✓

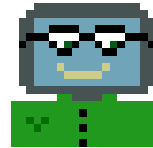


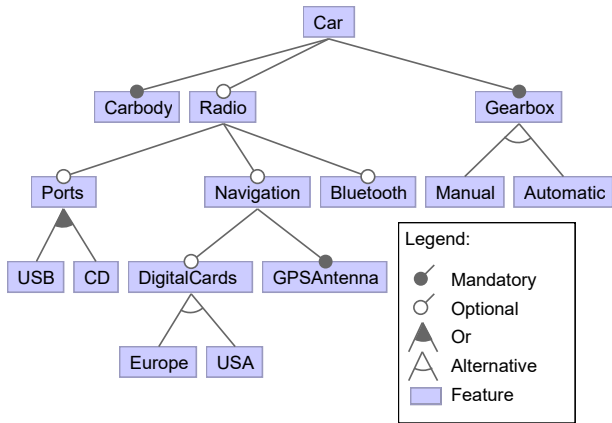
## Variability Reduction



Navigation  $\Rightarrow$  USB

Automatic  $\Rightarrow$  USA

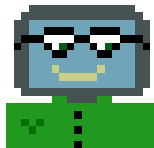




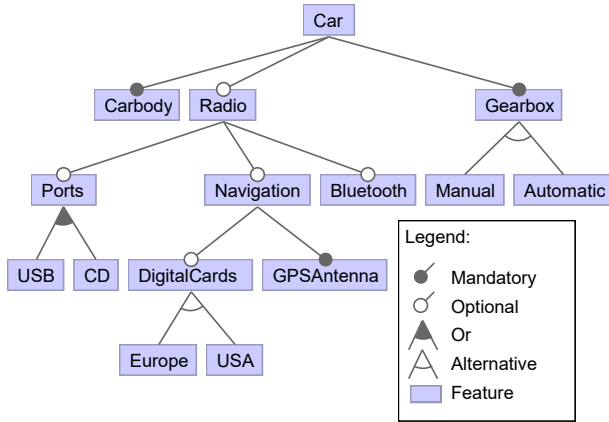
Navigation  $\Rightarrow$  USB  
 Automatic  $\Rightarrow$  USA

## Variability Reduction

Only 25 valid configurations!

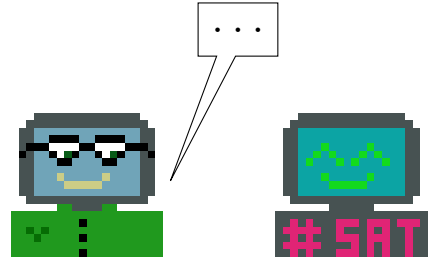


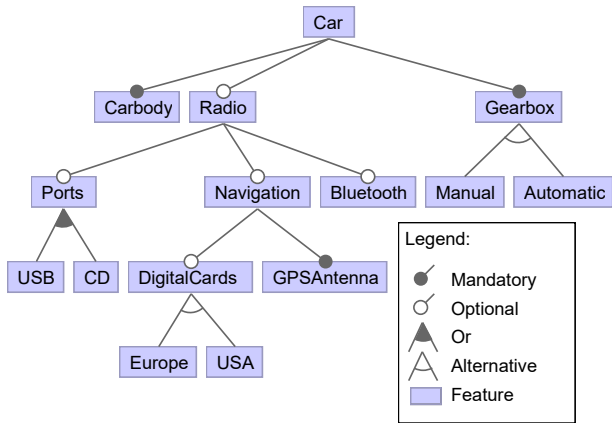
## Variability Reduction



Navigation  $\Rightarrow$  USB

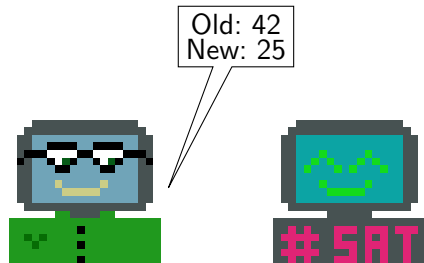
Automatic  $\Rightarrow$  USA

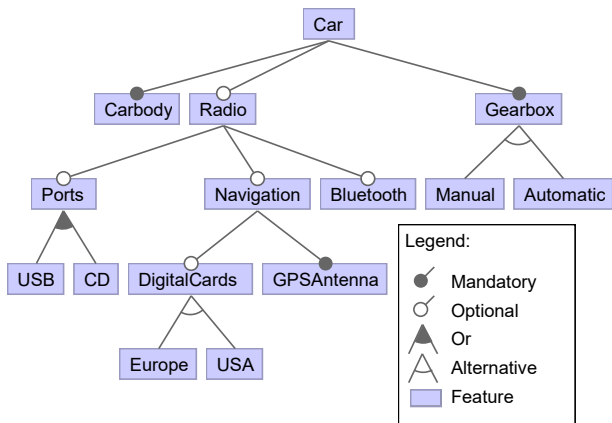




Navigation  $\Rightarrow$  USB  
Automatic  $\Rightarrow$  USA

## Variability Reduction

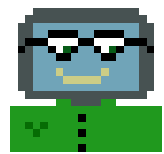




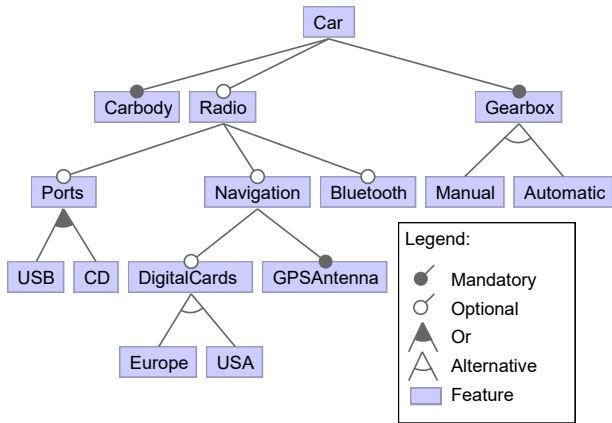
Navigation ⇒ USB  
 Automatic ⇒ USA

## Variability Reduction

Old: 21 manual, 21 automatic  
 New: 21 manual, 4 automatic



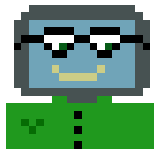




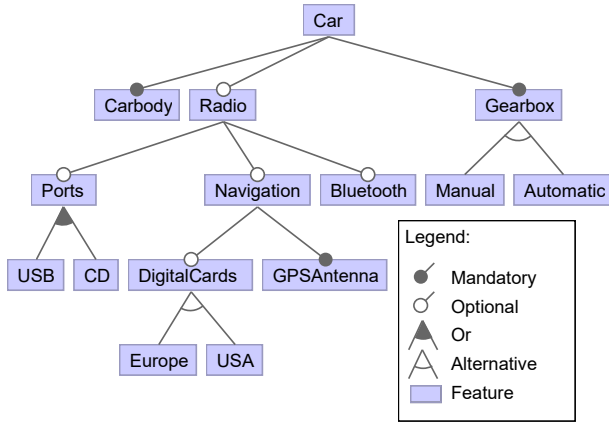
Navigation  $\Rightarrow$  USB  
 Automatic  $\Rightarrow$  USA

## Variability Reduction

Automatic  $\wedge$  DigitalCards  $\Rightarrow$  USA

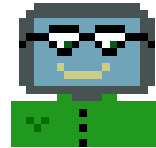


## Variability Reduction



Navigation  $\Rightarrow$  USB

Automatic  $\wedge$  DigitalCards  $\Rightarrow$  USA



# How to use #SAT on Feature Models



Variability Reduction

# How to use #SAT on Feature Models

Variability Factor

Void Feature Model

Degree of Orthogonality

Cost Savings

Maintainability Prediction

Configuration Relevance

Rating Errors

Variability Reduction

Subset Variability

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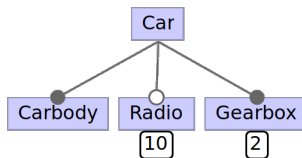
Maintainability Prediction

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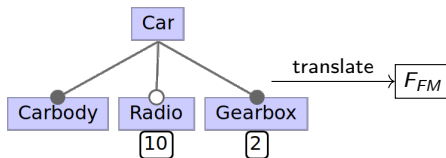
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Cost Savings

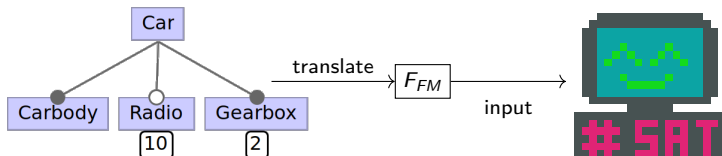
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Cost Savings

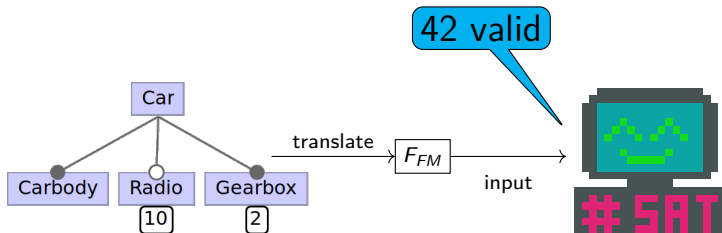
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Subset Variability

Homogeneity

Payoff Threshold

Degree of Reuse

Core, Dead & False-Optional

Atomic Set Candidates

Feature Prioritization

CTC Restrictiveness

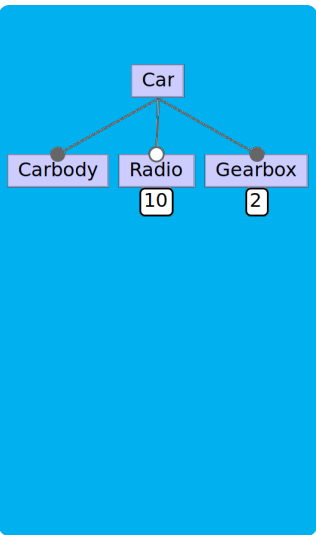
Optimize Configuring

Uniform Random Sampling

Atomic Sets

Rate Interactions

Configuration Derivation



Homogeneity

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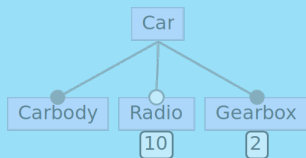
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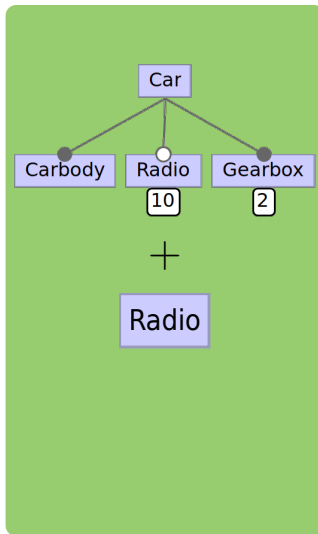
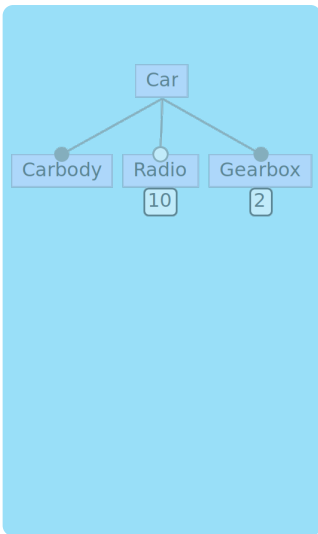
Optimize Configuring

Uniform Random Sampling

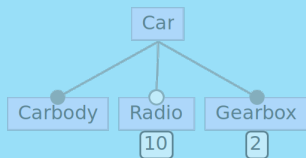
Atomic Sets

Rate Interactions

Configuration Derivation



- Uniform Random Sampling
- Atomic Sets
- Rate Interactions
- Configuration Derivation



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## Feature Prioritization

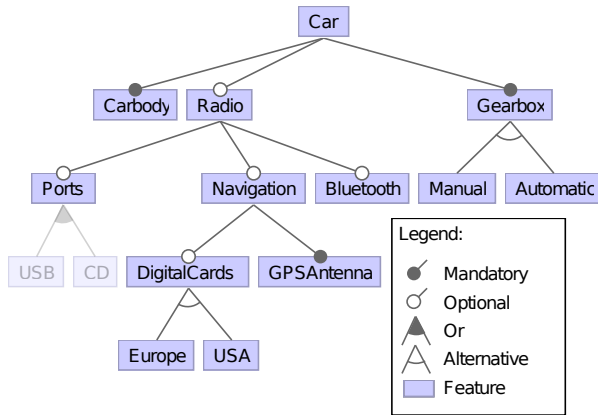


Feature Prioritization

New

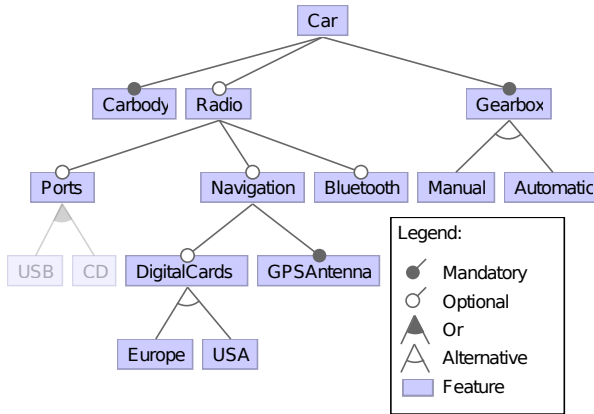


## Feature Prioritization



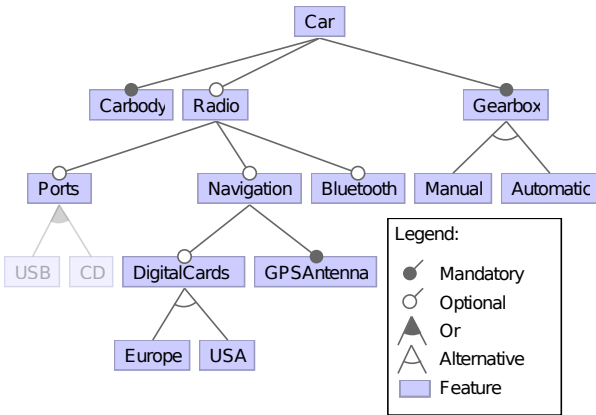
Navigation  $\Rightarrow$  USB

## Feature Prioritization



Navigation  $\Rightarrow$  USB

Which feature to develop first?



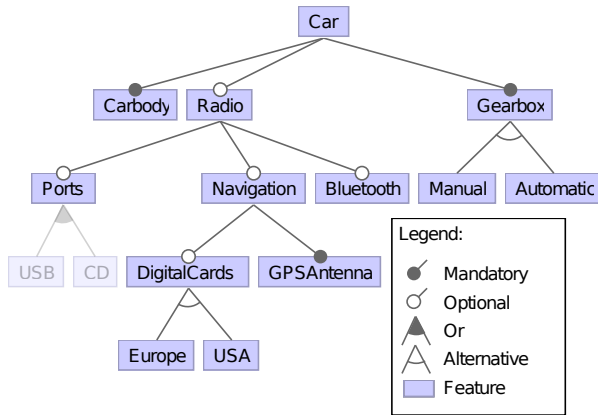
Navigation  $\Rightarrow$  USB

## Feature Prioritization

Which feature to develop first?

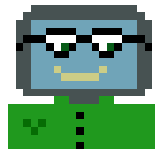
Prioritize based on cardinality

## Feature Prioritization



Navigation  $\Rightarrow$  USB

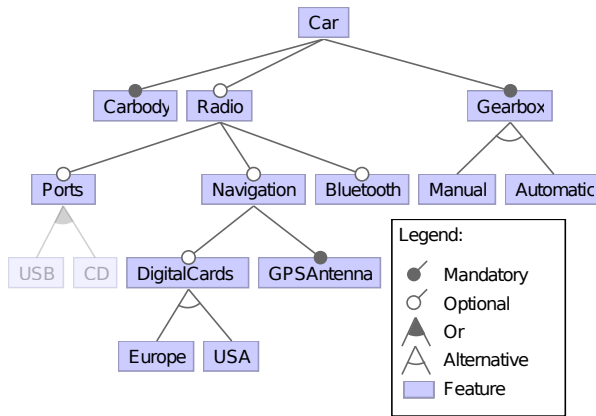
USB or CD?



## Feature Prioritization

**Prioritize USB**

$\#USB > \#CD$

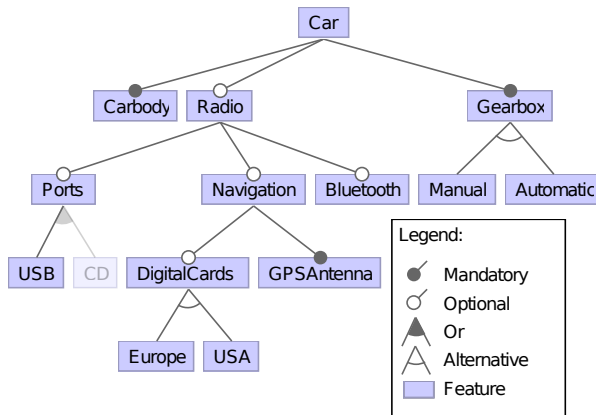


Navigation  $\Rightarrow$  USB

## Feature Prioritization

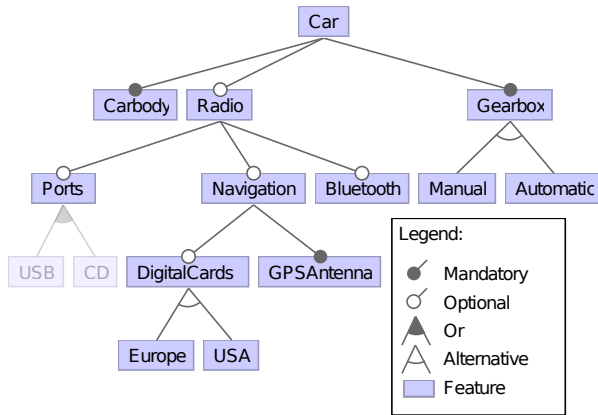
**Prioritize USB**

$\#USB > \#CD$



Navigation  $\Rightarrow$  USB

## Feature Prioritization

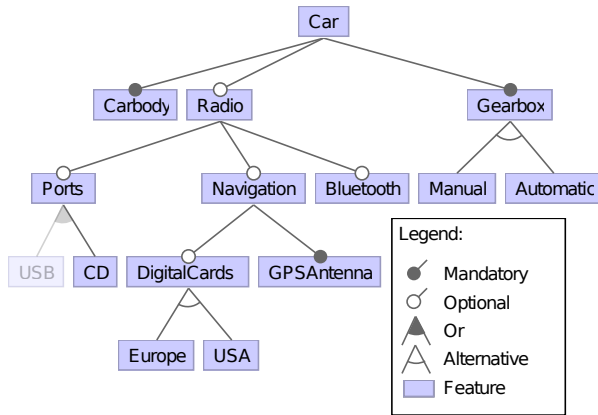


Navigation  $\Rightarrow$  USB

**Prioritize CD**  
 $\frac{\#USB}{Cost(USB)} < \frac{\#CD}{Cost(CD)}$



## Feature Prioritization



Navigation  $\Rightarrow$  USB

**Prioritize CD**  

$$\frac{\#USB}{Cost(USB)} < \frac{\#CD}{Cost(CD)}$$





# How to use #SAT on Features



Feature Prioritization

# How to use #SAT on Features

Homogeneity

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# How to use #SAT on Features

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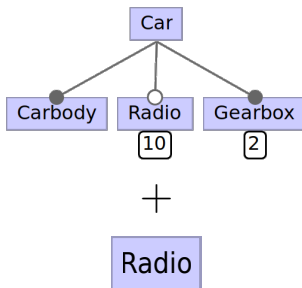
Core, Dead & False-Optional

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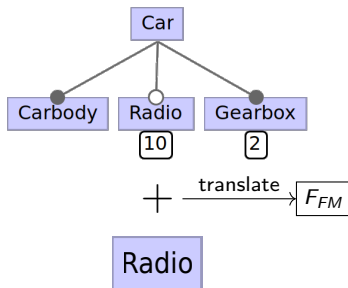
Core, Dead & False-Optional

Atomic Set Candidates

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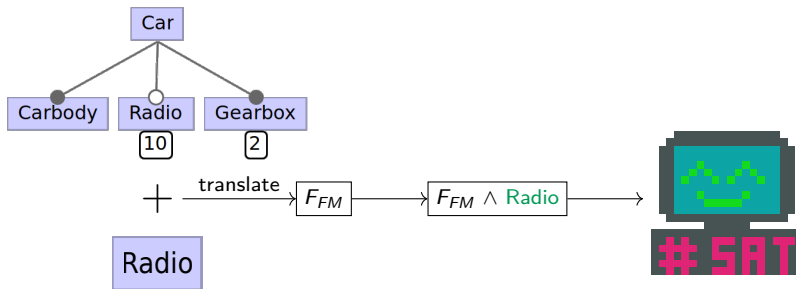
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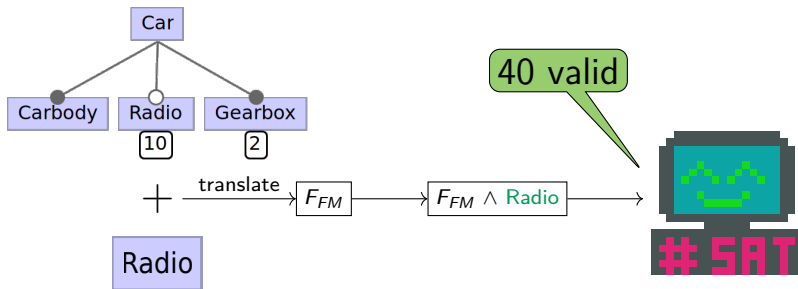
Core, Dead & False-Optional

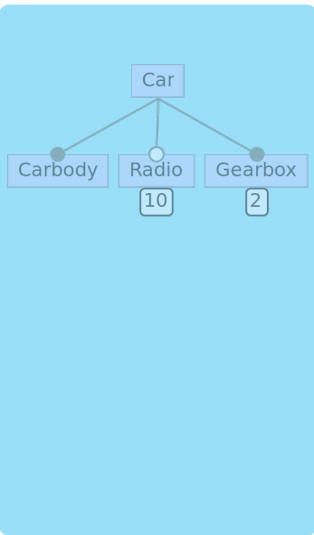
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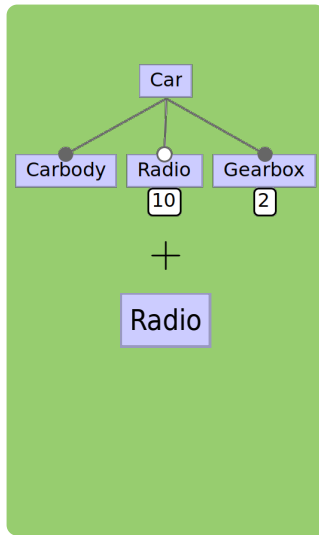
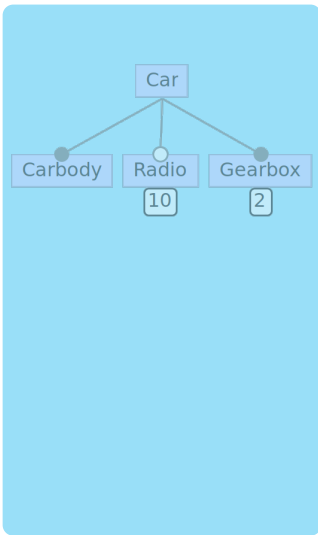
Uniform Random Sampling

Atomic Sets

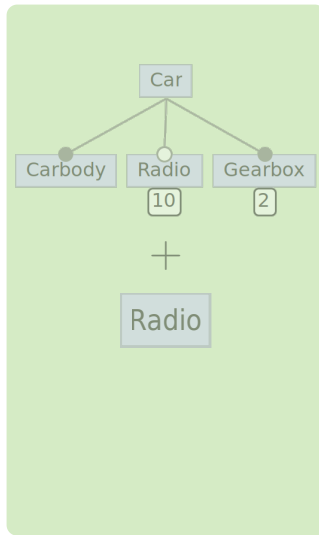
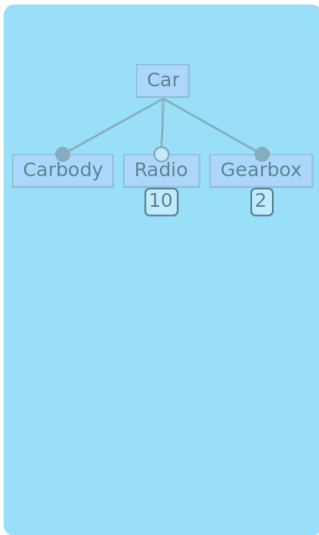
Rate Interactions

Configuration Derivation

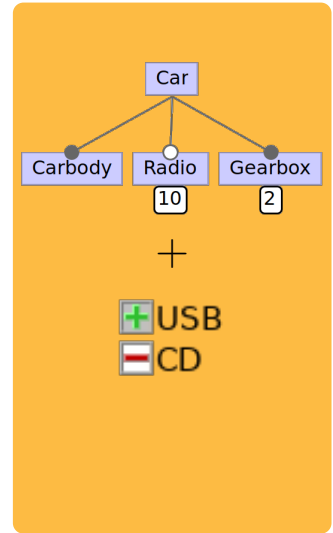
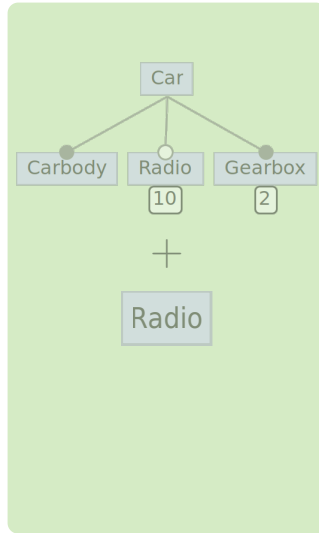
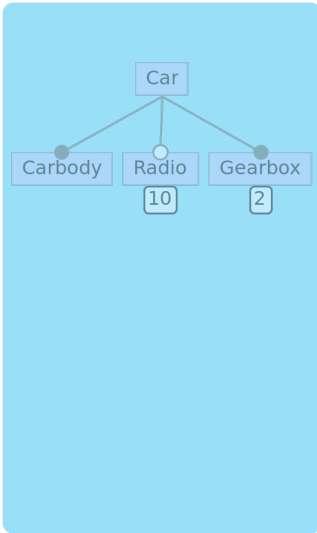


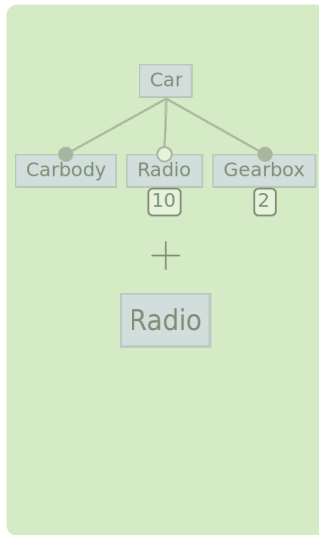
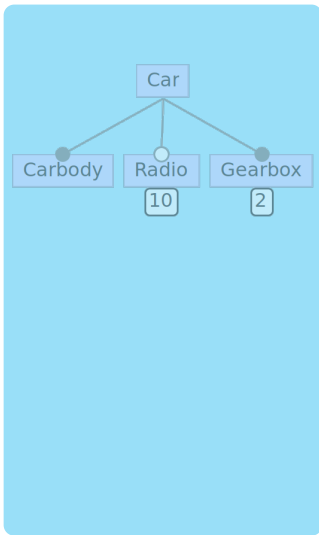


- Uniform Random Sampling
- Atomic Sets
- Rate Interactions
- Configuration Derivation



- Uniform Random Sampling
- Atomic Sets
- Rate Interactions
- Configuration Derivation





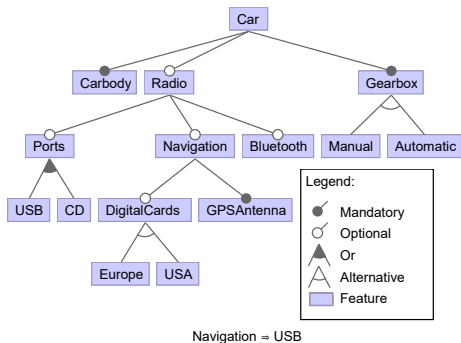
- Uniform Random Sampling
- Atomic Sets
- Rate Interactions
- Configuration Derivation

## Uniform Random Sampling

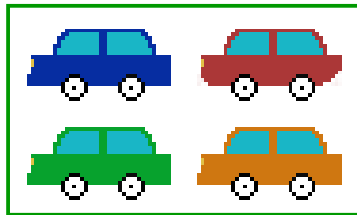
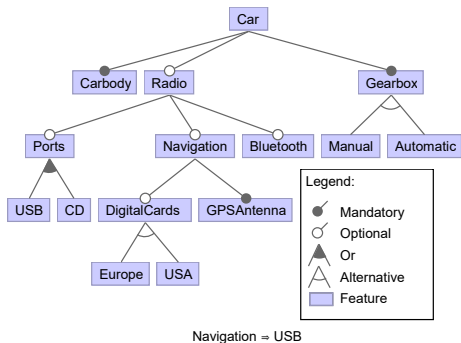
Old

## Uniform Random Sampling

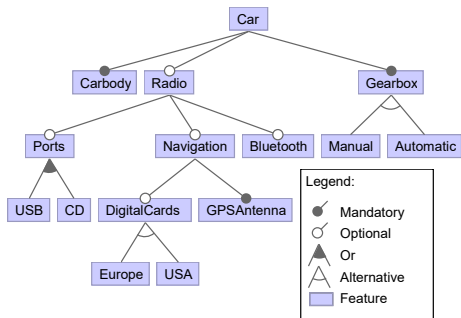
## Uniform Random Sampling



## Uniform Random Sampling

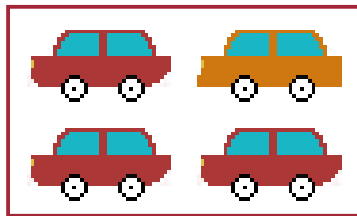




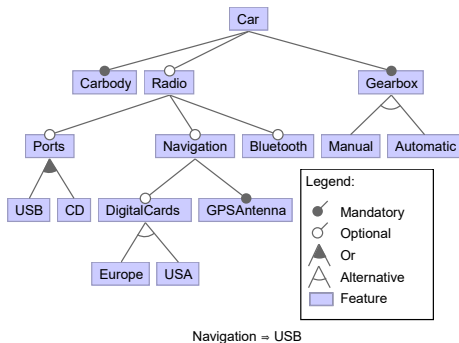


Navigation ⇒ USB

Uniform Random Sampling

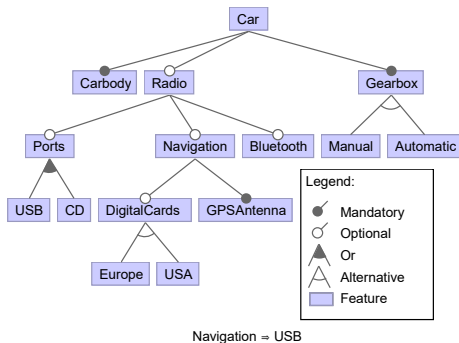


Bias



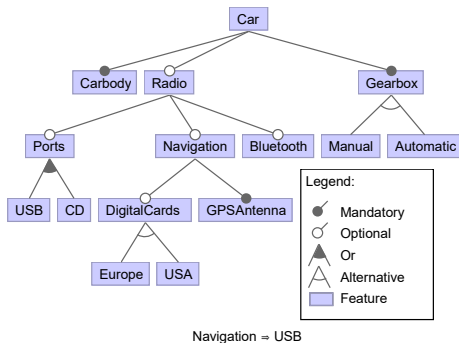
## Uniform Random Sampling

- Same chance for each configuration



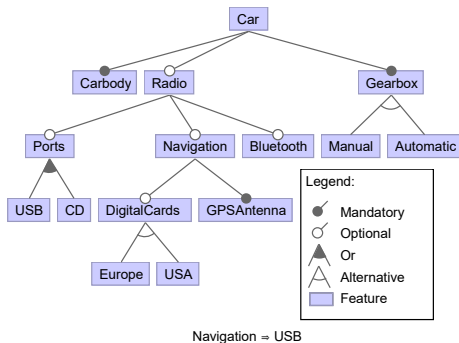
## Uniform Random Sampling

- Same chance for each configuration
- Representative sample



## Uniform Random Sampling

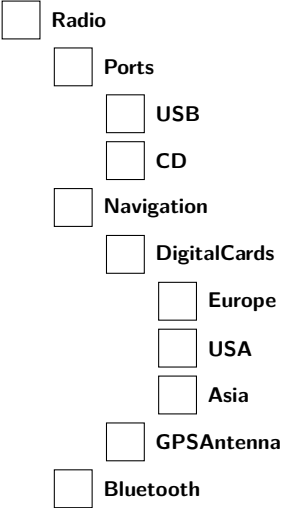
- Same chance for each configuration
- Representative sample
- Why is this difficult?

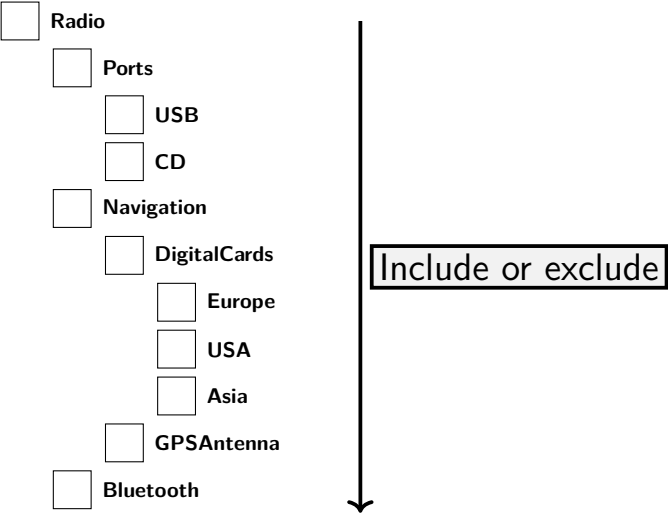


## Uniform Random Sampling

- Same chance for each configuration
- Representative sample
- Why is this difficult?
- Why do we need #SAT?

Uniform Random Sampling





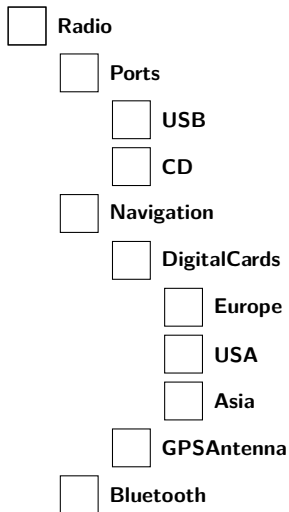
Uniform Random Sampling

- ☐ Radio
  - ☐ Ports
    - ☐ USB
    - ☐ CD
  - ☐ Navigation
    - ☐ DigitalCards
      - ☐ Europe
      - ☐ USA
      - ☐ Asia
    - ☐ GPSAntenna
  - ☐ Bluetooth

Include by  $X\%$



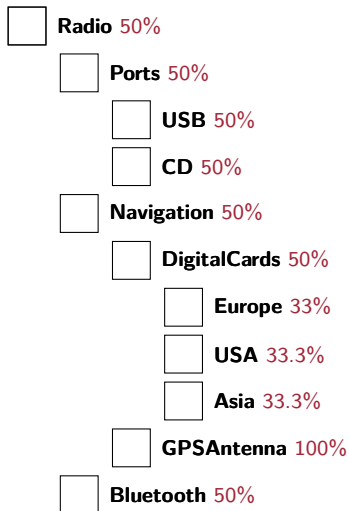




Uniform Random Sampling

Naive (SAT)  
SharpSAT

## Uniform Random Sampling



Naive (SAT)  
SharpSAT

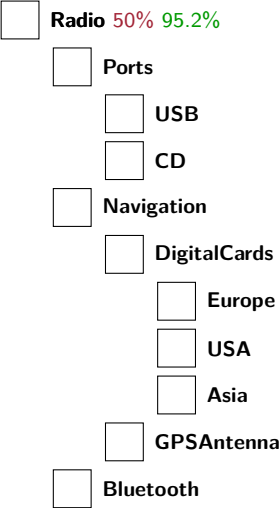
Uniform Random Sampling

- ☐ **Radio** 50% 95.2%
- ☐ **Ports** 50% 90%
- ☐ **USB** 50% 88.9%
- ☐ **CD** 50% 55.6%
- ☐ **Navigation** 50% 60%
- ☐ **DigitalCards** 50% 66.67%
- ☐ **Europe** 33.3% 33.3%
- ☐ **USA** 33.3% 33.3%
- ☐ **Asia** 33.3% 33.3%
- ☐ **GPSAntenna** 100% 100%
- ☐ **Bluetooth** 50% 50%



Naive (SAT)  
SharpSAT

Uniform Random Sampling



Naive (SAT)  
SharpSAT



**Radio**

☐

**Ports** 50% 90%

☐

**USB**

☐

**CD**

☐

**Navigation**

☐

**DigitalCards**

☐

**Europe**

☐

**USA**

☐

**Asia**

☐

**GPSAntenna**

☐

**Bluetooth**

Uniform Random Sampling

Naive (SAT)  
SharpSAT

Uniform Random Sampling

- ☒ Radio
  - ☒ Ports
    - ☐ USB 50% 88.9%
    - ☐ CD
  - ☐ Navigation
    - ☐ DigitalCards
      - ☐ Europe
      - ☐ USA
      - ☐ Asia
    - ☐ GPSAntenna
  - ☐ Bluetooth

Naive (SAT)  
SharpSAT

## Uniform Random Sampling

- ☒ **Radio**
  - ☒ **Ports**
    - ☒ **USB**
    - ☐ **CD** 50% 55.6%
  - ☐ **Navigation**
    - ☐ **DigitalCards**
      - ☐ **Europe**
      - ☐ **USA**
      - ☐ **Asia**
    - ☐ **GPSAntenna**
  - ☐ **Bluetooth**

Naive (SAT)  
SharpSAT

Uniform Random Sampling

- ☒ Radio
  - ☒ Ports
    - ☒ USB
    - ☒ CD
  - ☐ Navigation 50% 75%
  - ☐ DigitalCards
    - ☐ Europe
    - ☐ USA
    - ☐ Asia
  - ☐ GPSAntenna
- ☐ Bluetooth

Naive (SAT)  
SharpSAT



Uniform Random Sampling

- ☒ Radio
  - ☒ Ports
    - ☒ USB
    - ☒ CD
  - ☒ Navigation
    - ☐ DigitalCards 50% 66.67%
    - ☐ Europe
    - ☐ USA
    - ☐ Asia
    - ☐ GPSAntenna
  - ☐ Bluetooth

Naive (SAT)  
SharpSAT

Uniform Random Sampling

- ☒ Radio
  - ☒ Ports
    - ☒ USB
    - ☒ CD
  - ☒ Navigation
    - ☒ DigitalCards
    - ☐ Europe
    - ☐ USA
    - ☐ Asia
  - ☐ GPSAntenna
- ☐ Bluetooth

Naive (SAT)  
SharpSAT

## Uniform Random Sampling

- ☒ Radio
  - ☒ Ports
    - ☒ USB
    - ☒ CD
  - ☒ Navigation
    - ☒ DigitalCards
    - ☒ Europe
    - ☒ USA
    - ☒ Asia
  - ☐ GPSAntenna
- ☐ Bluetooth

Naive (SAT)  
SharpSAT

Uniform Random Sampling

- ☒ Radio
  - ☒ Ports
    - ☒ USB
    - ☒ CD
  - ☒ Navigation
    - ☒ DigitalCards
    - ☒ Europe
    - ☒ USA
    - ☒ Asia
  - ☐ GPSAntenna 100% 100%
- ☐ Bluetooth

Naive (SAT)  
SharpSAT

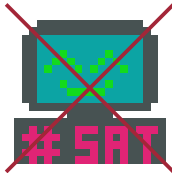
## Uniform Random Sampling

- ☒ **Radio**
  - ☒ **Ports**
    - ☒ **USB**
    - ☒ **CD**
  - ☒ **Navigation**
    - ☒ **DigitalCards**
    - ☒ **Europe**
    - ☒ **USA**
    - ☒ **Asia**
  - ☒ **GPSAntenna**
- ☐ **Bluetooth** 50% 50%

Naive (SAT)  
SharpSAT

Uniform Random Sampling

- ☒ Radio
  - ☒ Ports
    - ☒ USB
    - ☒ CD
  - ☒ Navigation
    - ☒ DigitalCards
    - ☒ Europe
    - ☒ USA
    - ☒ Asia
  - ☒ GPSAntenna
- ☒ Bluetooth

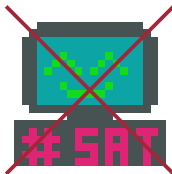


: Bias 6/11

Naive (SAT)  
SharpSAT

Uniform Random Sampling

- ☒ Radio
  - ☒ Ports
    - ☒ USB
    - ☒ CD
  - ☒ Navigation
    - ☒ DigitalCards
    - ☒ Europe
    - ☒ USA
    - ☒ Asia
  - ☒ GPSAntenna
- ☒ Bluetooth



: Bias 6/11



: No bias

Naive (SAT)  
SharpSAT

# How to use #SAT on Partial Configurations

Uniform Random Sampling



# How to use #SAT on Partial Configurations

Uniform Random Sampling

Atomic Sets

Rate Interactions

Configuration Derivation

# How to use #SAT on Partial Configurations

Uniform Random Sampling

Atomic Sets

Rate Interactions

Configuration Derivation



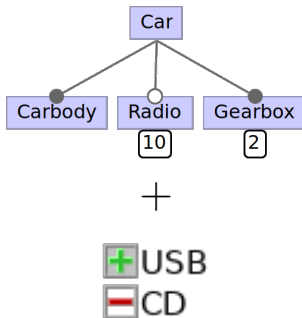
# How to use #SAT on Partial Configurations

Uniform Random Sampling

Atomic Sets

Rate Interactions

Configuration Derivation



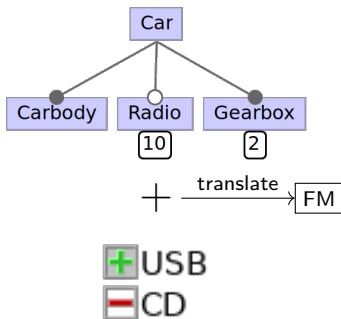
# How to use #SAT on Partial Configurations

Uniform Random Sampling

Atomic Sets

Rate Interactions

Configuration Derivation



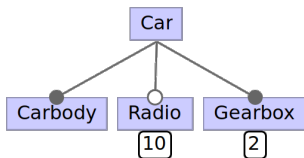
# How to use #SAT on Partial Configurations

Uniform Random Sampling

Atomic Sets

Rate Interactions

Configuration Derivation



$+$  translate

FM

$\text{FM} \wedge \text{USB} \wedge \neg \text{CD}$

 USB  
 CD



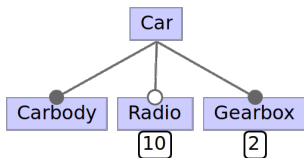
# How to use #SAT on Partial Configurations

Uniform Random Sampling

Atomic Sets

Rate Interactions

Configuration Derivation



+    translate

FM

$FM \wedge \text{USB} \wedge \neg \text{CD}$

 USB  
 CD

16 valid



Variability Factor

Void Feature Model

Degree of Orthogonality

Cost Savings

Maintainability Prediction

Configuration Relevance

Rating Errors

Variability Reduction

Subset Variability

Homogeneity

Payoff Threshold

Degree of Reuse

Core, Dead & False-Optional

Atomic Set Candidates

Feature Prioritization

CTC Restrictiveness

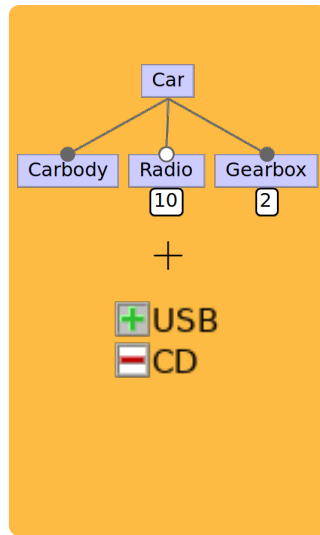
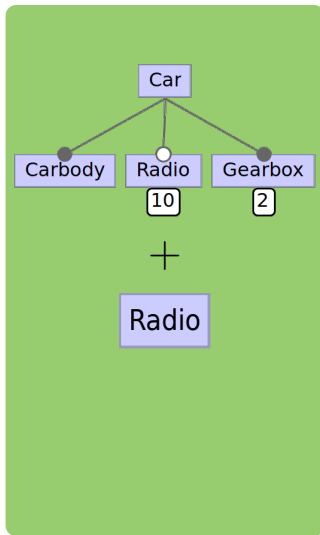
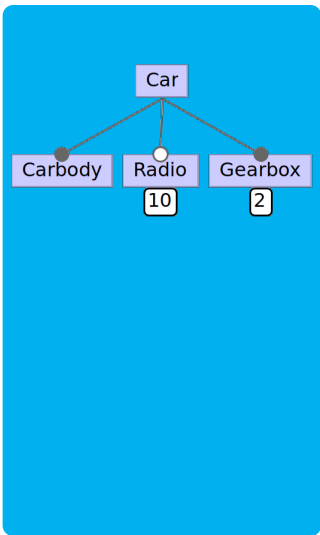
Optimize Configuring

Uniform Random Sampling

Atomic Sets

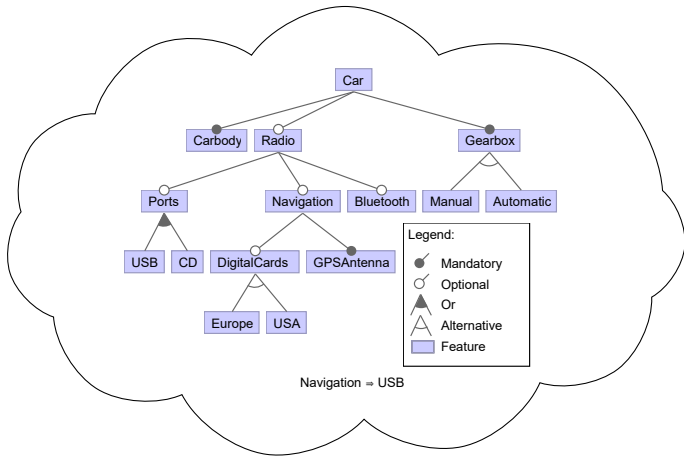
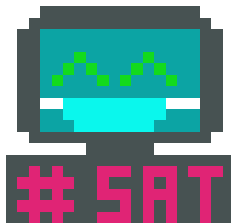
Rate Interactions

Configuration Derivation

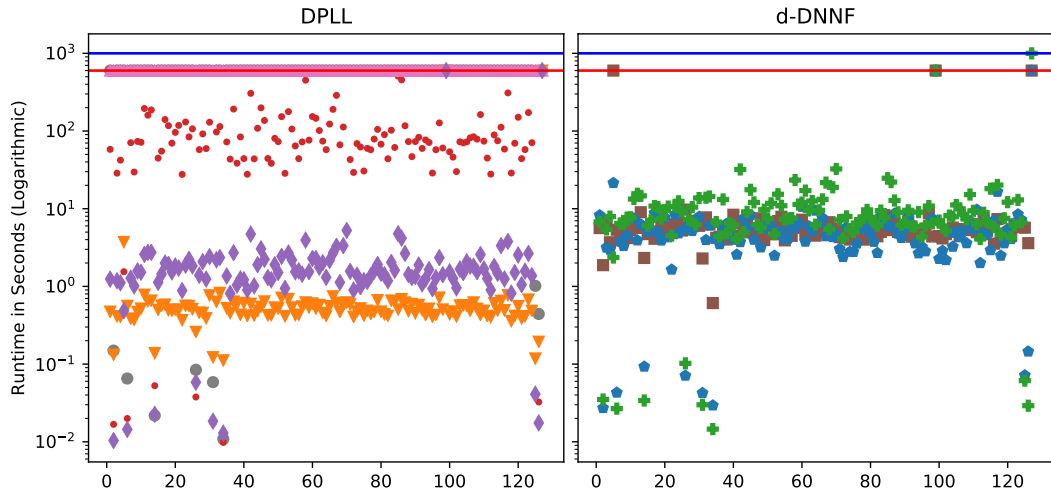




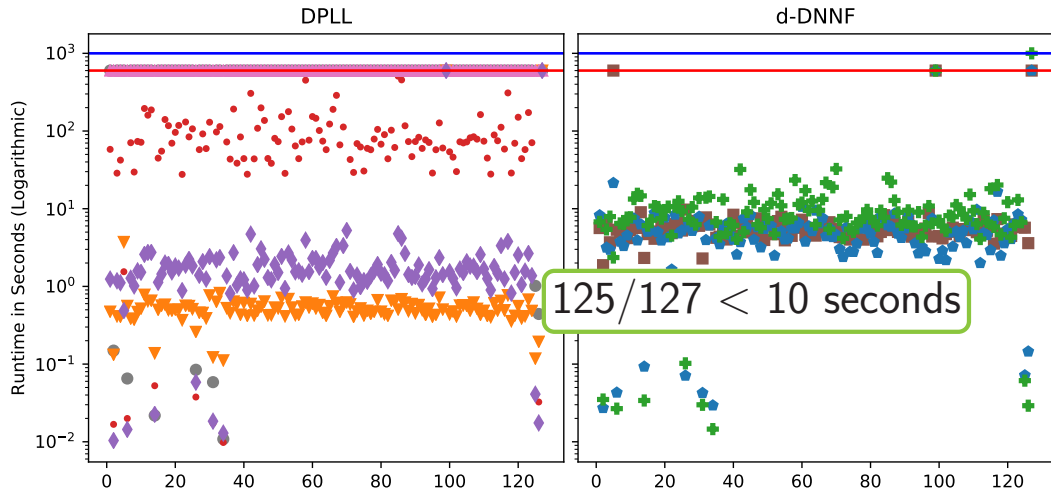
# Applications of #SAT Solvers on Feature Models



# Scalability of #SAT



# Scalability of #SAT



# Scalability of #SAT

