Master Thesis

Modelling the Yield Curve - Factoring in the Macroeconomy

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under the supervision of

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To Prof. Dr. Helmut Kramer, who not only taught me how to be a considerate economist, but, more importantly, what it means to be a loving father.

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1 Introduction

Over the past decade, people have become increasingly concerned about personal health. Ones own health is constantly monitored and assessed using different metrics, while potential risks are forecasted and mitigated as good as possible. For the masses, simplicity is key in this context as easy to understand metrics offer a good indication of one's current health status. In a similar fashion, economists, finance professionals and laypeople alike have some key metrics to assess the health of the economy. Especially in times of turmoil, simple albeit highly relevant metrics often enter the center stage. One of the most well-known and most studied among those key metrics is the yield curve. Its various shapes, slopes and spreads are analysed, forecasted and interpreted in order to assess the health of the overall economy as well as market expectations of the future. Beware an inverted yield curve, which the market usually interprets as the onset of a recession, similar to a doctor facing elevated inflammation markers and inferring the onset of illness. But analogous to a doctor, who is faced with an individual with various idiosyncratic characteristics, economic researchers are faced with a tremendous amount of information from which they try to infer a signal in order to predict future economic activity.

Though the yield curve - mapping interest rates to specific maturities - is a key indicator for economic agents, it is often not well understood, either in a general sense and especially in regards to which factors determine the shapes and movements of the yield curve. Ultimately, correctly assessing and predicting the impact of movements in the underlying factors on the yield curve enables central bank policymakers to calibrate their policies to current market expectations as well as giving asset managers the opportunity to position their portfolios in order to maximize returns.

In previous research, the yield curve has been studied in a rather dichotomous fashion, either from the lens of a finance professional, or from the perspective of an economist. However, yields are known to contain a tremendous amount of information useful in both realms, be it on the current stance of monetary or fiscal policy as well as expectations of future economic activity and inflation (Evans and Marshall, 2007). What is more, ever since the introduction of the expectation theory by Hicks (1946), monetary policy (expectations) seem to be one of the main drivers of yield curve movements (Evans and Marshall, 1998). Thus, correctly assessing future monetary policy is invaluable for both portfolio managers and economists alike. But how does one combine those two worlds? How exactly is the yield curve related to the economy? Is the yield curve a leading indicator for economic activity, or are economic variables the reason for shifts in the yield curve? Could there even be a bidirectional link, where the causality runs in both directions? This thesis is aimed at answering these questions.

Largely following Diebold, Rudebusch, and Aruoba (2006), a decomposition of the United States and Euro Area yield curves using the well-known model introduced by Nelson and Siegel (1987) is conducted, yielding factors representing the level, slope and curvature. In a second step, a Bayesian sVAR is estimated using said factors while also incorporating various economic variables in order to disentagle this messy relationship using distinct shocks and analysing the responses from each variable deducing potential interpretations regarding their relationship.

The thesis is structured in the following way. Chapter 2 offers a brief overview of the literature, highlighting past findings regarding the yield curve's relevance and connection to macroeconomic factors. Chapter 3 introduces the methodological approach as well as the data. The next section presents an empirical analysis of the US (Section 4.1) & Euro Area yield curves (Section 4.2), where the relevance of macroeconomic variables as drivers of yield curve movements is assessed, while simultaneously examining the impacts shifts in the yield curve have on economic variables. Thereafter, a conclusion summarizing the main findings is provided.

2 Literature Review

There has historically been an abundance of literature about the relationship between the term structure of interest rates and economic activity. This section aims at narrowing down this wealth of research, providing an overview of the hitherto available and relevant literature, focusing on identifying the potential one- or bidirectional link between the yield curve and the economy. Thus, it follows the distinction provided by Diebold, Rudebusch, and Aruoba (2006), who divide this research into yields-to-macro versus and macro-to-yields studies.

Some of the central contributions regarding the yields-to-macro relationship have come from Harvey (1988), who shows that the expected real term structure contains information about future consumption growth. Among other researchers, Estrella and Hardouvelis (1991), Estrella and Mishkin (1995) and Estrella and Mishkin (1996), use a probit model approach, modelling the probability of a recession as a function of the term spread[1], through which the authors have provided some remarkable results corroborating the hypothesis in regard to the relevance of the yield curve (slope) for macroeconomic activity, at least for the US and other major European economies such as Germany, the UK and France. In a similar fashion, Bernard and Gerlach (1998) extend the analysis to other major economies such as Canada, Japan, Belgium and the Netherlands, confirming the finding that the term spread offers sig-

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[1]The term spread is mainly known as the difference between a long-term and a short-term interest rate, e.g. the difference between the 10-year and 3-month government bond yields

nificant information regarding future recessions. Solely for Japan the significance is limited, which can possibly be attributed to differences in financial regulation and the fact that interest rates in Japan did not fully reflect market participants expectations of the future path of the economy (Bernard and Gerlach, 1998). Evgenidis, Papadamou, and Siriopoulos (2018) offer a meta-analysis of the yields-to-macro view, again focusing on the yield spread's ability to forecast economic activity. Although they again corroborate the findings of the previous 30 years, they note that the modelling strategy should take non-linearities into account as well as underlining the importance of incorporating monetary policy variables into the models, as some predictive power of the yield spread is attributed to expectations about the future path of monetary policy. The authors also note that there still is no widely accepted theory explaining the usefulness of the yield curve. Apart from these rather promising studies affirming one's confidence in the yields-to-macro view, authors such as Dotsey (1998) and Stock and W.Watson (2003) note that yields (spreads) have somewhat lost their predictive ability ever since the 1980s.

On the spectrum of authors leaning towards a macro-to-yields approach, central research has come from Evans and Marshall (1998), Ang and Piazzesi (2003) and Evans and Marshall (2007), all using some variation of a VAR framework.

Using three different identification strategies within a VAR setting, Evans and Marshall (1998) find that monetary policy shocks primarily have a significant effect on short-term interest rates while also reducing expected inflation, implying a rise in the real interest rate. The authors interpret this as monetary policy having a significant effect on the yield curve through a liquidity, rather than an expected inflation effect. Though delivering promising results of a potential macro-to-yields relationship, one key weakness is the sole focus on monetary policy shocks alone, whilst excluding other macro variables. Counteracting these shortcomings and extending their previous research, Evans and Marshall (2007) include other macro variables into their model, such as technology, fiscal policy and preferences for current consumption, an identification set predominantly derived from previous research on DSGE models. As before, they conclude that, apart from fiscal policy shocks, macroeconomic factors such as technology are able to explain a large part of yields movements along the short- to medium- end of the curve.

By including multiple macro variables into a multi-factor model of the term structure, using the assumption of no-arbitrage as identifying restrictions, Ang and Piazzesi (2003) find that macro variables explain a tremendous amount of the variation in bond yields, especially at the short-end of the curve, where the main drivers are shocks to inflation. This is consistent with the previous findings by Evans and Marshall (1998). The authors also conclude that incorporating macro factors in latent factor term structure models enhances out-of-sample forecasts.

Taking the stance of the opposite direction of causality, i.e. yields-to-macro, Ang, Piazzesi, and Wei (2006) offer a comparison to previous research of Estrella and Hardouvelis (1991) etc. Through estimating a relatively simple VAR only including a set of yields and GDP growth, they conclude that the nominal short rate dominates the slope of the yield curve in predicting future economic activity.

Some other notable contributions in the realm of linking the macroeconomy and the yield curve have come from Dewachter and Lyrio (2006) and Rudebusch and T. Wu (2008). Both sets of authors extend the latent factor model of the term structure with macroeconomic factors. The former specifically reference to Ang and Piazzesi (2003) in the context of a possible model misspecification, since previous research has apparently failed to correctly model the long end of the yield curve. Through their slightly modified approach, the authors introduce long-run inflation expectations in their model and show that not only the short-end of the yield curve, but also longer maturities are indeed driven by macroeconomic variables. They also suggest interesting and seemingly plausible interpretations to the latent factors, chiefly among them finding that the level factor is closely related to the aforementioned long-run inflation expectations. Rudebusch and T. Wu (2008) combine a small scale macro model, assuming that the short-term rate is represented as a monetary policy reaction functions, such as the well known function developed by Taylor (1993), with a standard no-arbitrage latent factor model of the term structure, where the short-term interest rate is modelled as a function of latent factors often interpreted as a level and/or slope factor, the authors are able to synthesize both the finance and the economists approach to modelling interest rates. Obtaining promising results, they offer insightful and intuitive interpretations, e.g. how the level factor and the central banks inflation target are linked, underlining the significance of a holistic approach.

Among the major contributions regarding the application of the Nelson and Siegel (1987) method in a macroeconometric framework are Diebold and Li (2006). Since other prominent theoretical models of the yield curve, chiefly among them are the no-arbitrage as well as equilibrium models, fail to provide the tools necessary for accurate predictions, the authors try to fill this gap using the aforementioned and well-known Nelson-Siegel decomposition. More precisely, the authors compare the term structure forecasting performance of said strategy with that of various benchmark forecasts (e.g. random walk) in order to test their hypothesis that a Nelson-Siegel approach yields superior out-of-sample forecasting results. Through decomposing a set of yields, representing the term structure, into three factors, which are then assumed to be evolving as an AR(1) process over time, Diebold and Li (2006) obtain forecasts of the yield curve based on the forecasts of said factors. The authors find that this approach appears to yield superior forecasts, especially at longer horizons of one year and beyond. Their promising results are one of the main reasons, why a Nelson-Siegel factor model approach has been chosen for this thesis. Another

central contribution of the authors is their theoretical as well as empirical argumentation regarding the interpretation of the Nelson-Siegel factors as representing the level, slope and curvature of the decpomposed yield curve. This interpretation is conveniently used in Section 4 to get a first glimpse how well the estimated factors approximate their theoretical counterparts.

One of the central papers serving as an inspiration and template alike for this thesis is the seminal work by Diebold, Rudebusch, and Aruoba (2006). Extending upon the bipartite available literature at that time, the authors are among the most significant to investigate the joint dynamics of both macro and financial variables (i.e., level, slope and curvature factors representing the yield curve). They build upon the approach used by Diebold and Li (2006), using a state-space representation of the Nelson-Siegel model, estimated via maximum-likelihood using the Kalman filter, which simultaneously models the dynamics of the yield curve for each point in time as well as the dynamics of other variables. Through this one-step approach the authors investigate the relationship between the macroeconomy and yields, laying forth the research question of this thesis, how bond yields and the macroeconomy are linked and, more interestingly, if there even is a bi-directional relationship. Diebold, Rudebusch, and Aruoba (2006) conclude that there is strong evidence for a macro-to yield effect and a somewhat weaker evidence for the yield curve to affect (future) macroeconomic dynamics.

Using the same two-step approach as in this thesis, some interesting contributions for emerging economies such as India and Chile, have come from Kanjilal (2011) and Morales (2010), respectively. Both authors find evidence of a potential bi-directional link between macroeconomic variables and the yield curve. Interestingly, Kanjilal (2011) finds a stronger evidence for the yields-to-macro direction, which supports the literature by authors such as Estrella and Hardouvelis (1991) and somewhat contrasts the findings by Diebold, Rudebusch, and Aruoba (2006). Suitably for this thesis, Morales (2010) compares the two-step estimation introduced by Diebold and Li (2006) with the more complicated state space representation (one-step estimation process) by Diebold, Rudebusch, and Aruoba (2006) and concludes that the simplified estimation methodology does not contradict the basic intuition of the results. Furthermore, the author finds evidence for a two-way relationship between yield curve factors and macroeconomic variables for the Chilean economy.

Notable theoretical contributions have come from Hicks (1946), who is among the economists credited with developing the well-known expectations theory, which is also tested in Diebold, Rudebusch, and Aruoba (2006). Kessel (1971) offers a broad overview of common (past) theories explaining the term structure of interest rates. Some of the latest contributions in the corresponding literature have mainly focused on how and how-well a Nelson-Siegel approach can be implemented in a machine learning framework, where, among others, Pedersen and Swanson (2019) offer a

comprehensive examination.

3   Methodology and Data

This section is aimed at outlining the methodology applied in this thesis. After giving a comprehensive primer on the aforementioned model of the yield curve established by Nelson and Siegel (1987), the identification strategy for the VAR model containing macroeconomic and yield curve variables is introduced. Lastly, an overview of the data and its sources is offered.

The first step of the two-step methodological approach involves modelling the yield curve using the Nelson-Siegel three factor model. The genius of the Nelson-Siegel decomposition lies within its flexibility as well as its parsimony. Based on a set of observable yields, the model is able to capture a wide range of yield curve shapes. One of its core strength is its ability to enable analysts to inter- and extrapolate between yields within the sample, thus providing yields for all maturities along the curve. As mentioned in section 2, the Nelson-Siegel model performs relatively well in out-of sample forecasting. What is more, though it does not explicitly ensure the absence of arbitrage, Coroneo, Nyholm, and Vidova-Koleva (2011) show that the Nelson-Siegel model aligns with the assumption of no-arbitrage. Given these highly promising attributes, the Nelson-Siegel approach appears to be well suited for the task at hand.

Following the representation of Diebold, Rudebusch, and Aruoba (2006), the yield curve at any time t is thus represented as the Nelson and Siegel (1987) model:

$$y_t(\tau) = \beta_{1t} + \beta_{2t} \left(\frac{1 - \mathrm{e}^{-\lambda\tau}}{\lambda\tau} \right) + \beta_{3t} \left(\frac{1 - \mathrm{e}^{-\lambda\tau}}{\lambda\tau} - \mathrm{e}^{-\lambda\tau} \right), \qquad (1)^2$$

where $t = 1, \ldots, T$, $y_t(\tau)$ is a set of N yields, each with a distinct maturity τ, used for the decomposition, and β_{1t}, β_{2t}, β_{3t} and λ are the parameters to be estimated. The regressors are also known as the factor loadings of the β coefficients, where λ shows their respective rate of decay. These factor loadings are central for ensuring the flexibility to represent various yield curve shapes. The loading on β_{1t} is 1 for all maturities and is thus, interpreted as the long-term factor, while the loading on the second factor, β_{2t} starts at 1 and decays monotonically towards 0, and thus

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[2]Mathematically, this representation can be thought of as a constant plus a Laguerre function (Nelson and Siegel, 1987)

can be viewed as a short-term factor. Finally, the loading on the third factor, β_{3t}, starts at 0, increases, but then decays back to 0, hence, it may be thought of as the medium-term factor (Diebold and Li, 2006). The contribution of each factor loading to the overall shape of the estimated yield curve is then given by each respective factor, e.g. β_{1t} shows the contribution of the long-term factor, β_{2t} that of the short-term component, while β_{3t} illustrates the contribution of the medium-term component(Nelson and Siegel, 1987).

As described in Nelson and Siegel (1987), the estimation can be conducted in the following way. For a provisional value λ, the sample values for the factor loadings are calculated. Based thereupon, the best-fitting values of the β coefficients are estimated. This procedure is repeated over a grid of values for λ, yielding the overall best-fitting values for λ, β_{1t}, β_{2t}, and β_{3t}.

Conveniently, as demonstrated by Diebold and Li (2006), the factors β_{1t}, β_{2t}, and β_{3t} can have economically meaningful interpretations, i.e. they are the time-varying level, slope[3] and curvature factors, L_t, S_t, C_t, respectively.

Thus, the estimated[4] model of the yield curve used in this thesis is represented by:

$$y_t(\tau) = L_t + S_t \left(\frac{1 - \mathrm{e}^{-\lambda\tau}}{\lambda\tau} \right) + C_t \left(\frac{1 - \mathrm{e}^{-\lambda\tau}}{\lambda\tau} - \mathrm{e}^{-\lambda\tau} \right), \tag{2}$$

In a more comprehensive matrix representation, the estimation of the factors is done using the following system of linear equations, where each row corresponds to equation 2 with a specific yield y_t and a corresponding maturity τ_N as well as an error term $\varepsilon_t(\tau_N)$:

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-\mathrm{e}^{-\tau_1\lambda}}{\tau_1\lambda} & \frac{1-\mathrm{e}^{-\tau_1\lambda}}{\tau_1\lambda} - \mathrm{e}^{-\tau_1\lambda} \\ 1 & \frac{1-\mathrm{e}^{-\tau_2\lambda}}{\tau_2\lambda} & \frac{1-\mathrm{e}^{-\tau_2\lambda}}{\tau_2\lambda} - \mathrm{e}^{-\tau_2\lambda} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-\mathrm{e}^{-\tau_N\lambda}}{\tau_N\lambda} & \frac{1-\mathrm{e}^{-\tau_N\lambda}}{\tau_N\lambda} - \mathrm{e}^{-\tau_N\lambda} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_N) \end{pmatrix} \tag{3}^5$$

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[3]As shown by Diebold and Li (2006), β_{2t}, and hence S_t, equals the negative slope, that is, the difference between short-term and long-term yields

[4]The estimation of the yield curve factors is conducted using the Nelson.Siegel method from the R package YieldCurve

[5]This corresponds to the measurement equation in Diebold, Rudebusch, and Aruoba (2006)

Thus, for each time period t, a level, slope and curvature factor is estimated based on the prevailing yields contained in vector y_t with distinct maturities τ_N. Consequently, the L_t, S_t and C_t factors are assumed to be an approximate representation of the yield curve at any given time t and, together with the macroeconomic variables, are included in a VAR(p) model aimed at studying the link between the economy and the yield curve.

Aforesaid VAR(p) model forms the second step of the analysis and is represented in the following way:

$$\mathbf{Y}_t = \mathbf{c} + \mathbf{A}_p \mathbf{Y}_{t-p} + \varepsilon_{\mathbf{t}}, \ \varepsilon_{\mathbf{t}} \sim \mathcal{N}\left(0, \Sigma_\varepsilon\right), \tag{4}$$

where Y_t denotes the $(K \times 1)$ matrix containing K endogenous variables, c is a $(K \times 1)$ vector of intercept terms, p denotes the maximum lag length, A_p is a $(K \times K)$ matrix of the autoregressive coefficients for lag length p and ε_t is a $(K \times 1)$ matrix of the reduced-form error terms.

The identification strategy is based on a structural VAR approach using contemporary recursive restrictions via a Cholesky decomposition of the variance-covariance matrix of the reduced-form errors Σ_ε. Largely following the notation of Kilian and Lütkepohl (2017), the representation of the structural VAR, ignoring the intercept vector c, as well as the relationship between the reduced-form and structural form VAR can be seen by:

$$\mathbf{B_0 Y}_t = \mathbf{B}_p \mathbf{Y}_{t-p} + \omega_{\mathbf{t}}, \ \omega_{\mathbf{t}} \sim \mathcal{N}\left(0, \mathbf{I}\right),$$

$$\underbrace{\mathbf{B_0^{-1} B_0}}_{\mathbf{I}} \mathbf{Y}_t = \underbrace{\mathbf{B_0^{-1} B}_p}_{\mathbf{A_p}} \mathbf{Y}_{t-p} + \underbrace{\mathbf{B_0^{-1} \omega_t}}_{\varepsilon_{\mathbf{t}}}, \tag{5}$$

where ω_t is the serially uncorrelated vector of the structural shocks. Since these structural shocks permit to deduce causal conclusions based on the correlations within the data, the central aim of this thesis is the identification of the structural shocks, whereupon the relationship between the model variables are examined. As can be seen in equation 5, identification depends upon identifying the inverse of matrix B_0, which governs the contemporaneous relationship between the model variables. In this thesis, it is obtained applying the aforementioned Cholesky decomposition of the variance-covariance matrix of the reduced-form errors. B_0^{-1} shows how each (correlated) reduced-form shock is a linear combination of specific (uncorrelated) structural shocks. To be precise, B_0^{-1} is a lower-triangular matrix, where each element above the main diagonal is zero, and it depicts how a structural shock to each variable affects each of the models variables on impact, thus enabling one

to infer how the variables in the model might be related. Since the identification of B_0^{-1} is not unique w.r.t. the ordering of the variables, assessing the economic adequacy of the chosen ordering is crucial. In particular, as all elements above the main diagonal are zero, implying that one assumes that the variables ordered first tend to be affect by the other variables with a delay of at least one period, it would generally be reasonable to order the variables from the slowest to the fastest reacting.

In summary, following the estimation of a VAR(p) model containing specific macroeconomic and yield curve variables and applying the identification strategy described above, the orthogonal shocks, based on a certain variable ordering, are used to identify the relationship between the macroeconomy and the yield curve.

Based on the standard identification set proposed by Furlanetto, Ravazzolo, and Sarferaz (2017), the empirical analysis presented in section 4 is conducted using $K = 8$ variables, where the relevant macroeconomic variables are industrial production (IP_t), inflation (π_t), a short-term interest rate (i_t), while the financial variables are an indicator for financial stress (FS_t) and a variable representing the stock market (M_t). The variables representing the yield curve are the three factors obtained from the first step via the Nelson-Siegel decomposition (L_t, S_t, C_t). Drawing from Martins and Afonso (2010), who argue that financial variables may more prone to be affected instantaneously by shocks to the macroeconomy while the latter may react more slowly to shocks to the former, the variables are ordered from the most exogenous to the least exogenous. Thus, the ordering of vector Y_t^{T} is given by $Y_t^{\mathrm{T}} = [IP_t, \ \pi_t, \ i_t, \ FS_t, \ L_t, \ S_t, \ C_t, \ M_t]$. While the macroeconomic variables being ordered first appears plausible given the fact that they often tend to react in a lagged manner, the stock market being ordered last broadly follows Kilian and Park (2009), assuming that the stock market reacts to shocks to the other variables contemporaneously, which seems reasonable as it is generally assumed that stocks instantaneously incorporate any new information available[6], while, following the previously mentioned assumption that macro variables tend to react inertly, it does affect all other variables only with a delay of at least one period.

Though the selected methodology applied in the thesis has, in various forms and alterations, been applied quite often in the literature, a tremendous amount of alternative approaches is available in the economist's toolbox. A comprehensive overview of these numerous potential methodological approaches modelling the yield curve and studying it's relationship with the macroeconomy is offered in Diebold and Rudebusch (2013).

As to the concrete data used, the sample for the United States contains monthly

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[6]See, for example, Pearce and Roley (1984), Beaudry and Portier (2006), and Ormos and Vázsonyi (2011) for a discussion about the response of stock prices to economic news

data ranging from January 1975 to December 2022. Except for data on US yields, the excess bond premium, which is selected as the indicator for financial stress, and the S&P 500 stock market index, all data has been obtained from the FRED. A more detailed description of the data and its sources is provided in the Appendix A.

Of the highest significance for any analysis involving a Nelson-Siegel decomposition are the yields data. So far, the literature, heavily focused on the US, has primarily used unsmoothed Fama and Bliss (1987) Treasury forward rates obtained via the CRSP[7], which are then converted to unsmoothed Fama-Bliss zero yields. This thesis uses zero-coupon US Treasury yields obtained via a novel and improved approach kindly provided by Liu and J. C. Wu (2021). Following Diebold, Rudebusch, and Aruoba (2006), the maturities used are 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 months.

The data used for the analysis of the Euro Area consists of monthly data, spanning from October 2004 to December 2022. Again, the bulk of the dataset containing macro variables has been obtained via the FRED. Solely the Eurostoxx 50 index, representing the European stock market, as well as the VSTOXX index, again serving as the indicator for financial stress in the European economy, have been obtained via Yahoo Finance and the Bloomberg Terminal, respectively. The Euro Area yield curve data consists of spot rates derived from bonds with finite maturity denominated in EUR and issued by a euro area central government with an (issuer) rating of triple A. The data has been obtained via the ECB and, due to somewhat limited data availability, includes yields with maturities of 3, 6, 9, 12, 24, 36, 48, 60, 72, 84, 96 and 120 months.

4 Analysis

This last section lays out the main corpus of the thesis, presenting the empirical analysis, which tries to answer the research question how the macroeconomy and the yield curve are related in the US and the Euro Area based on the methodology outlined in section 3. Subsection 4.1 is concerned with the US, while subsection 4.2 focuses on the Euro Area. Each subsection starts in a descriptive manner, focusing on the respective time series of interest, especially the estimated yield curve factors and their characteristics, after which interpretations of the potential link between the macroeconomy and the yield curve are presented based on the results obtained via a structural vector autoregression model.
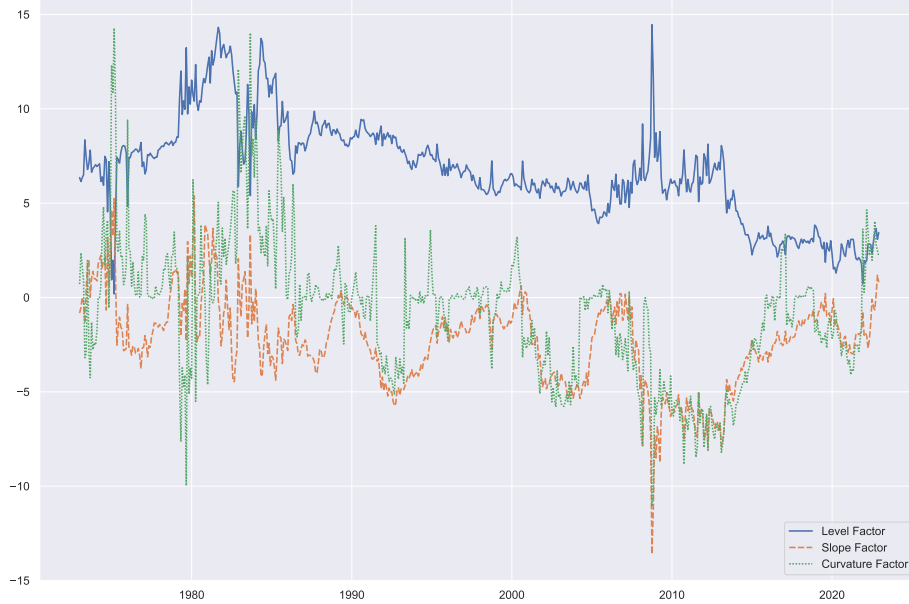
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[7]https://www.crsp.org/

Figure 1: Estimated level, slope and curvature factor, USA (in %)

4.1 The Yield Curve and the Macroeconomy in the USA

In order to get an initial sense of comparability, Figure 1 depicts all three estimated yield curve factors obtained via the Nelson-Siegel model for the US from January 1973 until December 2022. Among other things, one can see that the level factor, $\hat{L}_t^{US}$, has consistently been positive and well above the level of 5% for most of the time. Being above 10%, the estimated level factor is fairly high during the second oil price crisis induced by the Iranian Revolution in 1979, which caused world oil prices and thus, also US inflation to spike - an interesting observation that is further discussed when looking at Figure 2. The spike in October 2008 could potentially be attributed to the onset of the financial crisis, where the bursting of the real estate bubble in the US led to the biggest financial crisis since the Great Depression - an idiosyncratic shock that might reveal possible shortcomings of the Nelson-Siegel model during times of turmoil. Though a decreasing trend in the estimated level can be observed since the 1990s, the pace of the decrease has been the highest with the level being the lowest during the 2010s - a time marked by QE and deflationary risks. As for the estimated slope ($\hat{S}_t^{US}$) and the curvature ($\hat{C}_t^{US}$) factors, both factors assume positive and negative values during the sample period. It is apparent that the curvature factor is the most volatile, especially during the 1970s and 1980s,
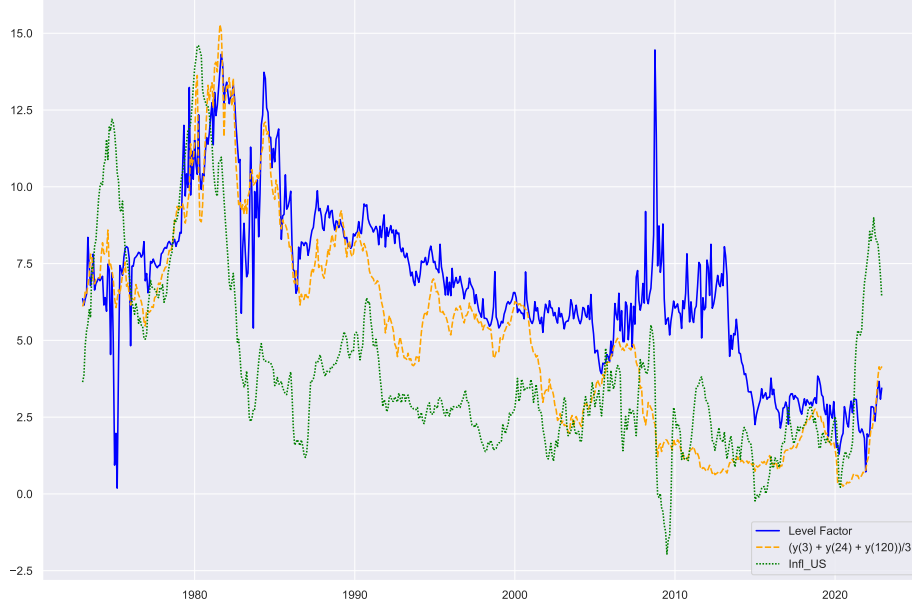
Figure 2: Level factor, empirical proxy and inflation, USA (in %)

while from the 2000s onwards, the correlation between the slope and the curvature factors is fairly high[8]. As a mirror image to the likely erroneous spike of the level factor during the financial crisis of 2008, the (negative) slope factor displays a similar spike, only downwards, which, since one would expect an inverted yield curve and thus, an increase of the negative slope at times of a looming a recession, contradicts economic theory as well as countless empirical analyses outlined in the literature review of section 2, again underlining the possible inaptitude of the Nelson-Siegel approach when faced with an idiosyncratic event.

Figures 2-4 show the estimated factors together with their empirical proxies. These proxies are insofar relevant as they offer a first indication of the gap between the estimated coefficients and the actually realized yields (Kanjilal, 2011). Figure 2 depicts the estimated level factor, $\hat{L}_t^{US}$ and the proxy variable proposed in the literature being the arithmetic average of the 10-year, 2-year and 3-month yield. It is apparent that the empirical proxy is fairly close to the estimated level factor, especially in the beginning of the sample, while a stronger divergence can be observed from the financial crisis onwards, though the level proxy has been close to the estimate in

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[8]The correlation between estimates of the slope and curvature factor is about 32% from 01-1973 until 12-1999, while it is around 77% from 01-2000 until the end of the sample
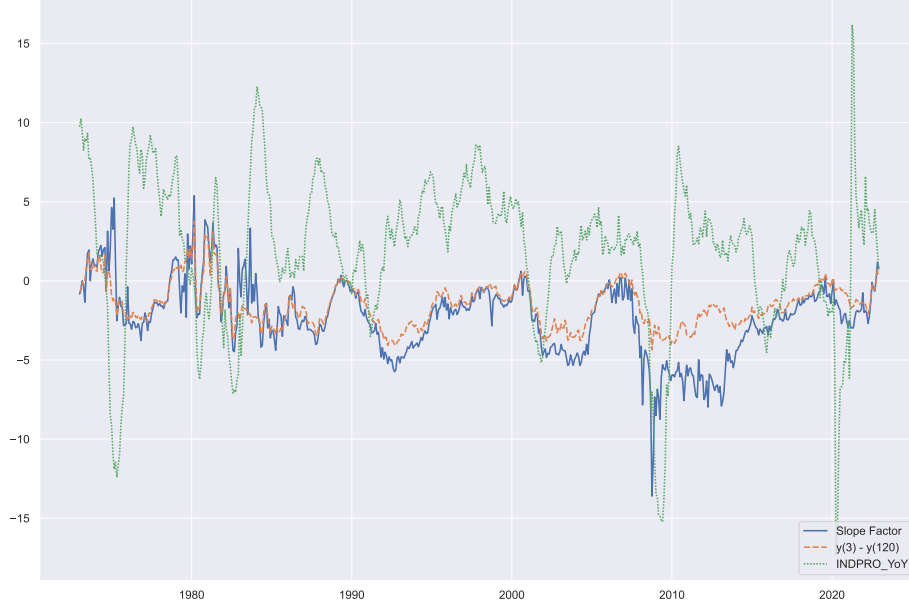
Figure 3: Slope factor and empirical proxy, USA (in %)

recent years. Over the whole sample period, the correlation between the estimated level factor and the respective proxy is 82%, being in line with the findings of Rude-busch and T. Wu (2008), and validating the estimate as being a good representation for level factor, i.e. the long-run component of the yield curve. Since it is the first indication of a link between the macroeconomy and the yield curve, a highly inter-esting observation for the task at hand can also be drawn when comparing the level factor with the inflation rate over time. From the Fisher equation one can derive how nominal interest rates and inflation expectations could be linked. Based on Fisher (1930), one would expect that nominal interest rates move one-to-one with a change in expected inflation, with the real interest rate being unaffected. In order to study this proposition Figure 2 also includes the year-on-year change in the CPI via the dotted green line. Though this inflation measure is concerned with actual and not expected inflation, one can see the spikes in the inflation rate during the second oil crisis beginning in 1979, where both inflation and the level factor have increased tremendously - a first indication that the inflation rate is indeed linked with the long-run factor of the yield curve. In a similar fashion, $\hat{L}_t^{US}$ first spikes in October 2008 and then decreases strongly, a pattern that is almost perfectly coin-ciding with the inflation rate during that time. When looking at the recent period of inflationary pressures induced by supply bottlenecks during the COVID-19 pan-demic in 2021 as well as the Russian aggression in Ukraine in 2022, one can see
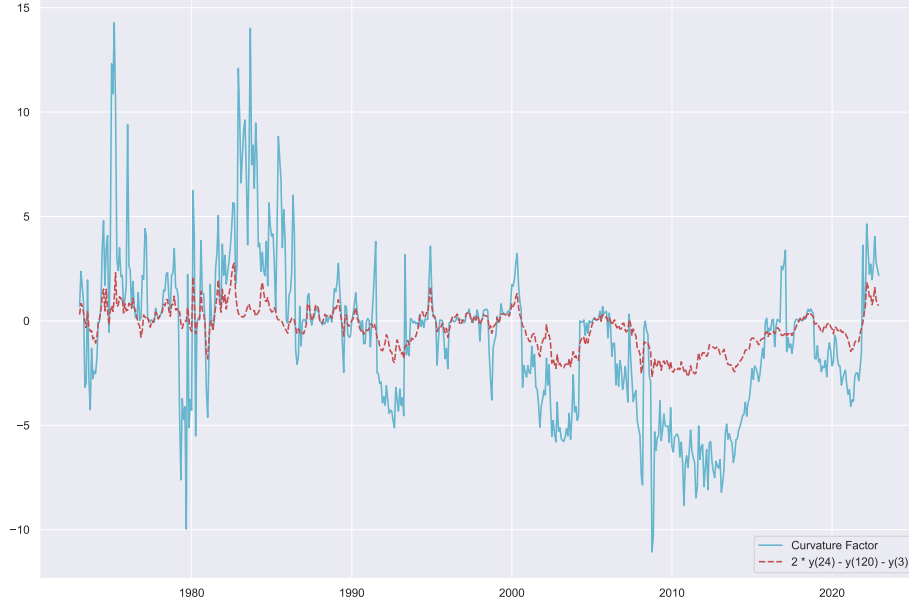
Figure 4: Curvature factor and empirical proxy, USA (in %)

that again the level factor, which has of course been flanked by increasing interest rates through monetary tightening by the FED beginning in 2022, has moved in accordance with actual inflation, though not rising as sharply as the inflation rate. Overall, the estimated level factor and inflation exhibit a rather strong co-movement in the US. In fact, the correlation between $\hat{L}_t^{US}$ and inflation over the sample period is 44% and significant[9], a finding that is consistent with the interpretation of the level factor as being related to inflation expectations, as described in, among others, Dewachter and Lyrio (2006), Rudebusch and T. Wu (2008), and Diebold, Rudebusch, and Aruoba (2006).

Looking at Figure 3, one can see that the estimated slope factor, $\hat{S}_t^{US}$ is extremely close to its empirical proxy being the (negative) slope of the yield curve, i.e. the difference between the 3-month and 10-year yield. In fact, the correlation is strikingly high and significant with 81%. As often done in the literature, the slope factor is said to be correlated, in fact, even able to predict economic activity. Thus, to again get a glimpse of a possible link between the macroeconomy and the yield curve represented by the slope factor, Figure 3 also depicts economic activity in the form

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[9]From hereon, a significance level of 5% is postulated with regards to statistical hypothesis testing

	t-Statistic	Critical value	p-value
IP_t^{US}	-5.6906	-2.8665	0.0000
π_t^{US}	-3.0096	-2.8665	0.0340
i_t^{US}	-2.7681	-2.8665	0.0630
FS_t^{US}	-5.4727	-2.8664	0.0000
L_t^{US}	-1.7930	-2.8665	0.3839
S_t^{US}	-3.0791	-2.8665	0.0281
C_t^{US}	-2.9236	-2.8665	0.0427
M_t^{US}	-4.8305	-2.8665	0.0000

Table 1: Augmented Dickey-Fuller (ADF) unit root test, US

of the year-on-year growth rate industrial of production over time. Evidently, there is some co-movement between the slope and industrial production, where most economic downturns are preceded by an upward spike in the term spread, corroborating the findings related to the yields-to macro literature outlined in section 2 and again strengthening the case of a link between the yield curve and the macroeconomy. Similarly in Figure 4, with a correlation of 79%, the curvature factor tends to be somewhat close to its proxy, where they generally have similar trends over time, though it unambiguously has a very high volatility relative to it's proxy variable.

In summary, on the one hand the observations described above tend to validate the indication, that the estimated Nelson-Siegel factors accurately represents the yield curve and are thus, fitted for studying the relationship between the macroeconomy and the term structure. On the other hand, Figures 2 and 3 have also offered preemptive evidence confirming that there indeed seems to be a relationship consistent with economic theory as well as the literature, namely that the level factor is the bonds market long-run inflation gauge, as well as an increasing slope factor indicating an upcoming recession.

In order to further understand the relationship between the yield curve and the macroeconomy, the next step involves analysing macroeconomic and yield curve variables together in a comprehensive structural VAR methodology outlined in section 3.

Before fitting the VAR model, it is first important to understand the nature of the variables involved. Thus, Table 1 shows the results of an Augmented Dickey-Fuller test of each variable testing whether it is stationary, i.e. it possesses a unit root at the 5% significance level. Apart from the Federal Funds rate (i_t^{US}) and the level factor (L_t^{US}), one can reject the null hypothesis of the existence of a unit root at the 5% level for all remaining model variables. This is insofar not surprising given the fact that the year-on-year growth rates for industrial production, inflation and

stock prices are used, which likely results in detrended time-series. Note that taking the first differences of both the Federal Funds rate as well as the level factor would result in stationary time series, though due to consistency considerations as well as methodologies used in the literature, both time series are included in levels.

Note that the VAR model is estimated via Ordinary Least Squares (OLS).

To get an initial gauge - apart from impulse response functions - how the yield curve factors and the macro variables are related, Table xxx depicts block Granger causality tests, where one can test if a block of variables Granger cause another block of variables. Conveniently, this method enables to determine whether there is a one or bi-directional relationship present, e.g. testing whether solely a set macro variables Granger cause the yield curve factors or if both the macro variables do Granger cause and are Granger caused by the yield curve factors.

Granger Causality Tests?

4.2   The Yield Curve and the Macroeconomy in the Euro Area

Though, starting in January 2004, the sample period is markedly smaller compared to that of the United States, the analysis of the Euro Area nevertheless reveals some interesting insights into the relationship between the yield curve and the economy. Again Figure 6 offers a comparison between all three estimated Nelson-Siegel factors over time. Just like in the US, the level factor has been positive during the whole sample period, while also exhibiting a decreasing trend for most of the time, especially since the mid 2010s, a period, as mentioned before, marked by QE, negative interest rates, and in the case of the Euro Area, the European debt crisis. Since the start of 2022, where residual inflationary pressures due to the COVID-19 pandemic as well inflation induced by soaring energy prices due to the conflict in Ukraine also severely affected the European economy, the level factor has increased quite strongly, surpassing the 2% mark - an interesting observation that is not only in line with the above mentioned Fisher effect regarding the relationship between the level effect and inflation, but this co-movement is also best seen in Figure 7. With a correlation of 75%, the slope factor displays a strong co-movement with the curvature factor. Evidently, the curvature factor has the highest volatility, especially due to the spike in May 2011, which could again be due to an idiosyncratic shock in the form of the Euro crisis, but could also be attributed to the fact that the Euro Area yields are based on a rather diverse pool of issuers with possible unobserved heterogeneity, which could potentially have confounding effects when using the Nelson-Siegel model.

	IP_t^{US}	π_t^{US}	i_t^{US}	ebp	L_t^{US}	S_t^{US}	C_t^{US}	M_t^{US}
const	-0.165084	0.202589	-0.050101	-0.034507	0.500211	-0.558095	-0.717369	1.531798
L1.IP	0.881565	0.013160	0.008638	0.000489	0.004619	0.002506	-0.013431	-0.430458
L1.Infl_US	-0.047693	0.979509	0.012629	0.005795	0.010239	0.022340	0.003853	-0.407215
L1.FFR	-0.273028	-0.002447	0.539283	0.021977	-0.080132	-0.010575	0.322698	-0.150216
L1.ebp	-0.660866	-0.146757	0.052072	0.896605	0.068537	-0.147965	-0.217483	-4.401908
L1.L	0.360447	-0.001100	0.515452	-0.023670	1.021382	0.049261	-0.252230	0.534527
L1.S	0.257082	0.043083	0.540740	-0.030537	0.120604	0.916111	-0.292822	0.404042
L1.C	0.030263	-0.016314	-0.015762	0.003469	0.042556	-0.015642	0.814918	-0.021891
L1.S&P_500	0.014820	-0.002988	-0.000949	0.000075	0.000059	-0.001273	0.004854	0.908624
Log-Likelihood	-5132.3844							
AIC	-4.8391							
BIC	-3.3678							
HQIC	-4.2662							

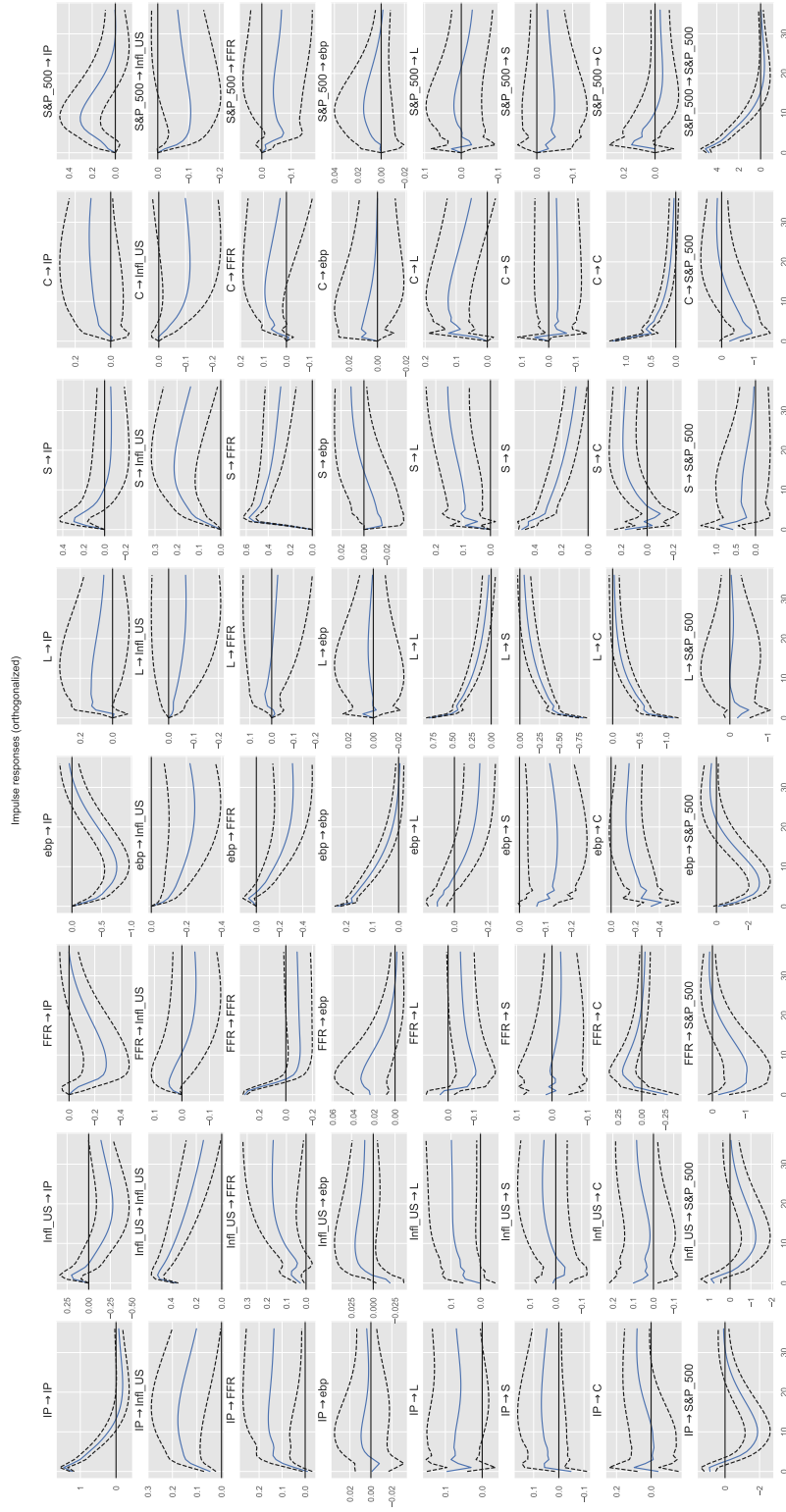Table 2: Vector Autoregression estimation results, US
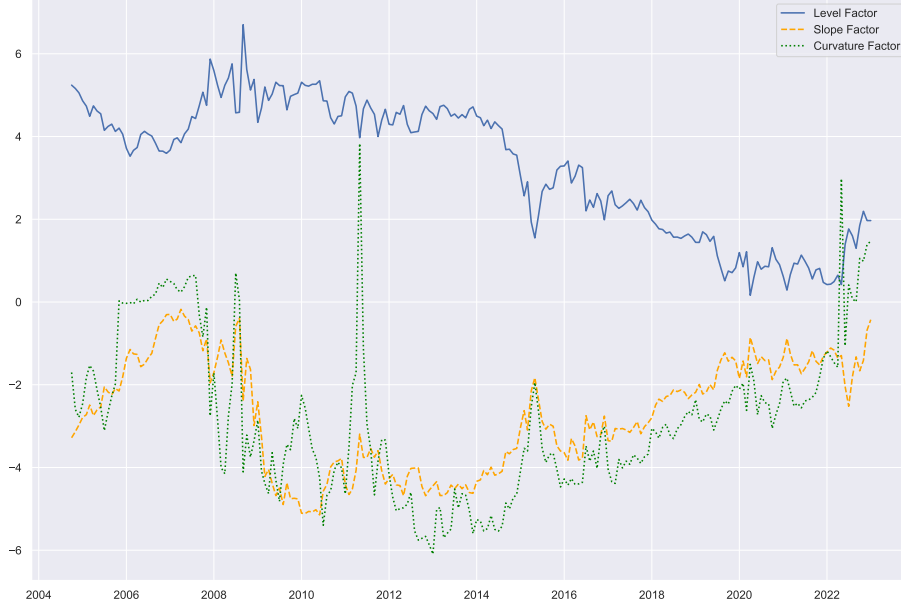
Figure 5: Impulse Responses, US

Figure 6: Estimated level, slope and curvature factor, EA (in %)

Once more, in order to get an indication of how well the factor estimates represent the yield curve, Figures 7-9 display the estimated factors, $\hat{L}_t^{EA}$, $\hat{S}_t^{EA}$, $\hat{C}_t^{EA}$ with their respective empirical proxies. All three estimates exhibit a very strong and significant correlation with their proxies ranging from 74% ($\hat{L}_t^{EA}$) to 86% ($\hat{C}_t^{EA}$), which underlines the suitability of the estimates as accurately representing the level, slope and curvature of the Euro Area yield curve. Most interestingly, the correlation between inflation and the level factor, though not being significant, is negative during the sample period, which of course contradicts the Fisher effect and the interpretation, that the level factor is an approximation for the expected inflation rate. Notably, not only is the sample period rather short, four highly idiosyncratic shocks have occurred during that short period of time, i.e. the financial crisis of 2007/08, the European debt crisis starting in 2010, the COVID-19 pandemic starting in 2020 and the ongoing war in Ukraine since February 2022, which likely stretches the rather simplistic approach used in this thesis to its limits. Adding to that the comparatively high heterogeneity of the Euro Area, it is by no means surprising to obtain counterintuitive results.

Apart from the estimated slope factor and it's proxy variable, Figure 8 also includes Euro Area industrial production representing economic activity. The first thing that stands out is the very high variability of industrial production, especially during the
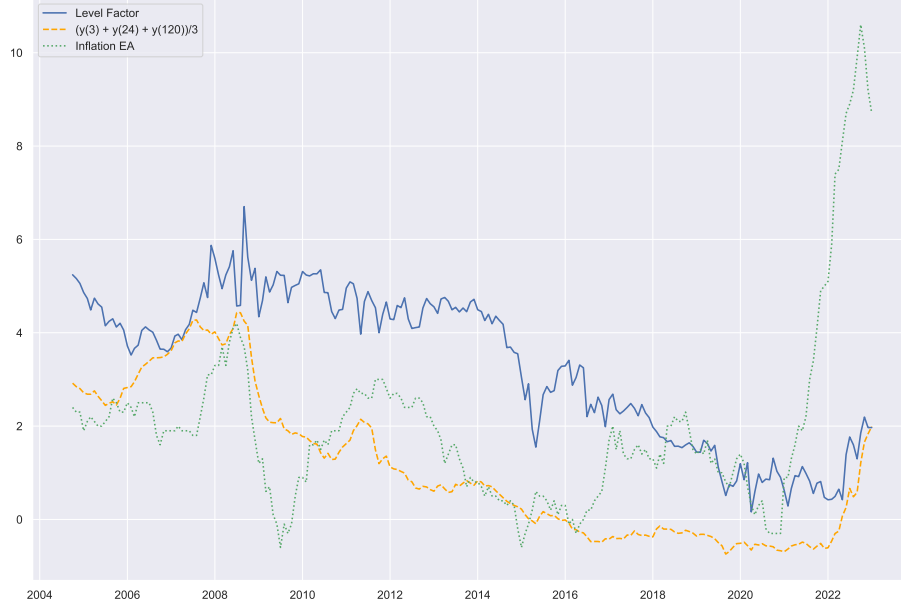
Figure 7: Level factor, empirical proxy and inflation, EA (in %)

onset of the COVID-19 pandemic in early 2020, ranging from -19,91% to 23,90%. Though the recession following the financial crisis of 2007/08 was preceded by an upward trend of the estimated slope factor - being consistent with economic theory - the number of economic downturns simply is too small during the short sample to infer any substantial conclusions regarding the validity of the relationship between the term spread and economic activity in the Euro Area.

In short, Figures 6-9 show that the obtained estimates are very close to their respective proxy variables, and are thus well suited to represent the level, slope and curvature of the Euro Area yield curve. Nevertheless, they have also indicated that the limited sample period of 235 months marked by various distinct exogenous shocks impede to draw viable conclusions regarding the relationship between the yield curve and the macroeconomy in the Euro Area.

Again, the VAR model is estimated using OLS.

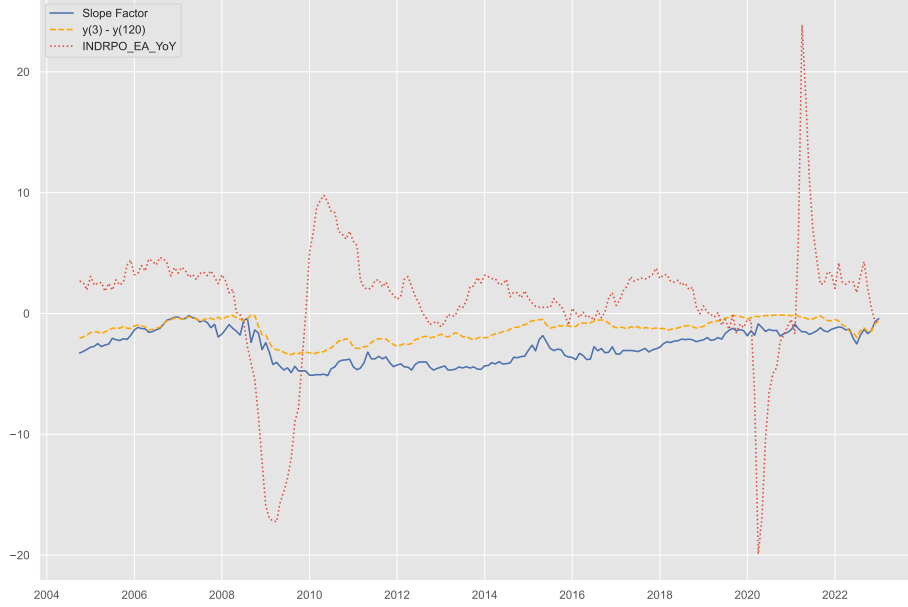**Granger Causality Tests?**

Figure 8: Slope factor and empirical proxy, EA (in %)

5   Conclusion

In conclusion, this thesis has investigated how the yield curve and the macroeconomy are related in the United States as well as the Euro Area, especially focusing on the question, if there is a one or bi-directional link. By using a two step approach similar to Diebold, Rudebusch, and Aruoba (2006), first modelling the yield curve using the Nelson-Siegel model to obtain a level, slope and curvature factor and subsequently including said factors in a VAR model containing various other macroeconomic as well as financial variables, the resulting structural shocks have given an indication of said relationship.

A possible extension of this thesis would be to actually use the estimated slope factors in a probabilistic setting like the one proposed by Estrella and Hardouvelis (1991) and see if said factors, being an approximation of the term spread, are able to correctly predict the onset of past recessions - a finding that would be consistent with the literature. Furthermore, testing the performance in yield (curve) forecasting of a general Nelson-Siegel model versus the extended model including macro variables used in this thesis could be further enlightening in regards to the usefulness of a yields-macro model.
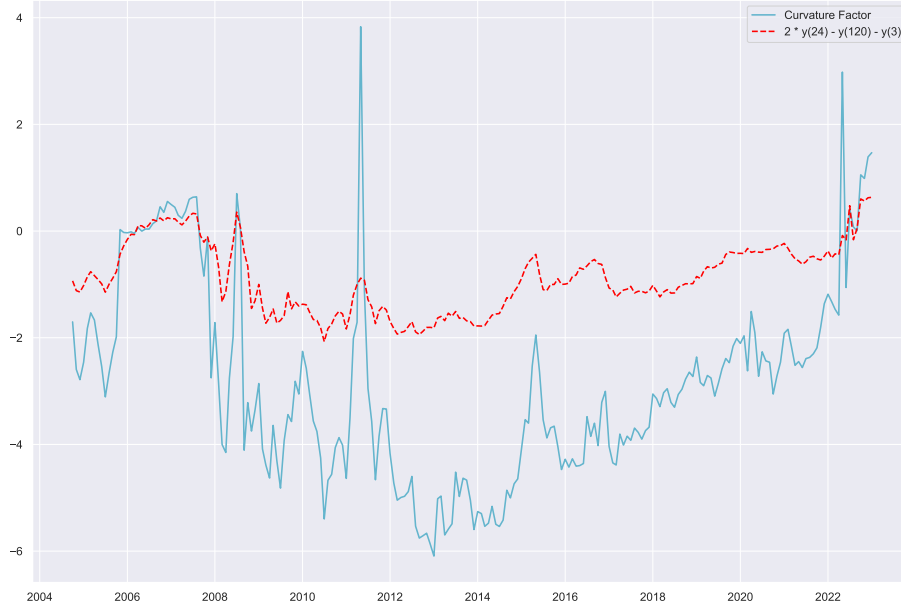
Figure 9: Curvature factor and empirical proxy, EA (in %)

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A Appendix

Data US

- Industrial Production (FRED: INDPRO)

- CPI (FRED: CPIAUCSL)

- Federal Funds Effective Rate (FRED: DFF)

- S&P 500 (Yahho Finance: GSPC)

- Excess Bond Premium (EBP)[10]

- β factors based on yields data provided by Liu and J. C. Wu (2021)[11]

Data EA

- Industrial Production (FRED: INDPRO EA)

- Inflation (FRED: Inflation EA)

- 3M Interbank Rate (FRED: 3M Rate EA)

- Eurostoxx 50 (Yahho Finance: GDAXI)

- VSTOXX Index (source: Bloomberg Terminal)

- β factors based on yields data provided by the ECB[12]

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[10]source: Fed Note Data

[11]source: Liu-Wu Yield Data

[12]source: dataset "All years - AAA"