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In 2008 all ECB publications feature a motif taken from the €10 banknote.

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Abstract

We test whether the Nelson and Siegel (1987) yield curve model is arbitrage-free in a statistical sense. Theoretically, the Nelson-Siegel model does not ensure the absence of arbitrage opportunities, as shown by Bjork and Christensen (1999). Still, central banks and public wealth managers rely heavily on it. Using a non-parametric resampling technique and zero-coupon yield curve data from the US market, we find that the no-arbitrage parameters are not statistically different from those obtained from the NS model, at a 95 percent confidence level. We therefore conclude that the Nelson and Siegel yield curve model is compatible with arbitrage-freeness. To corroborate this result, we show that the Nelson-Siegel model performs as well as its no-arbitrage counterpart in an out-of-sample forecasting experiment.

JEL classification codes: C14, C15, G12
Keywords *Nelson-Siegel model; No-arbitrage restrictions; affine term structure models; non-parametric test*

Non-technical summary

Academic literature and practitioner oriented publications show that the parametric yield curve model suggested by Nelson and Siegel (1987) fits data well. Due to its intuitive appeal and implementational easiness the Nelson-Siegel model has grown to be very popular among practitioners, in particular in central banks. A dynamic formulation of the model, suggested by Diebold and Li (2006), has added significantly to the models popularity.

A concern that might be raised against the Nelson-Siegel model is that it does not ensure consistency between the dynamic evolution of yields over time, and the shape of the yield curve at a given point in time. Such consistency is hard-coded into the so-called arbitrage-free yield curve models, and is generally a desirable property. For example, arbitrage-free yield curve models ensure that the expected future path of the yields is appropriately accounted for in the curve that is estimated today. Although a similar consistency is not guaranteed by the Nelson-Siegel model per se, it may be that the Nelson-Siegel model fulfills the arbitrage constraints in a statistical sense. In particular, if the Nelson-Siegel model is sufficiently flexible and if it is applied to data that is generated in a competitive trading environment, it is likely that most of the yield curves generated by the model fulfill the no-arbitrage constraints.

In the current paper we test the hypothesis that the Nelson-Siegel model fulfils the no-arbitrage constraints. The test is performed on resampled data, i.e. re-generated from the original data. Our procedure can be summarised as follows: (1) we generate a yield curve data sample with statistical properties similar to the original data sample; (2) we estimate the Nelson-Siegel yield curve factors; (3) we use the estimated Nelson-Siegel yield curve factors as exogenous input to a no-arbitrage model, and we estimate the parameters of the no-arbitrage model. These steps are repeated many times and eventually produce distributions for the no-arbitrage model parameters. On the basis of these distributions we test whether the no-arbitrage and the Nelson-Siegel factor loadings are statistically different.

We apply the framework to US yield curve data from January 1970 to December 2000 and we cannot reject the null hypothesis that the loading structures of the Nelson-Siegel and the no-arbitrage model are equal at a 95 percent level of confidence. Thus we conclude that the Nelson-Siegel model is compatible with arbitrage-freeness.

I Introduction

Fixed-income wealth managers in public organizations, investment banks and central banks rely heavily on Nelson and Siegel (1987) type models to fit and forecast yield curves. According to BIS (2005), the central banks of Belgium, Finland, France, Germany, Italy, Norway, Spain, and Switzerland, use these models for estimating zero-coupon yield curves. The European Central Bank (ECB) publishes daily Eurosystem-wide yield curves on the basis of the Soderlind and Svensson (1997) model, which is an extension of the Nelson-Siegel model.[1] In its foreign reserve management framework the ECB uses a regime-switching extension of the Nelson-Siegel model, see Bernadell, Coche and Nyholm (2005).

There are at least four reasons for the popularity of the Nelson-Siegel model. First, it is easy to estimate. In fact, if the so-called time-decay-parameter is fixed, then Nelson-Siegel curves are obtained by linear regression techniques. If this parameter is not fixed, one has to resort to non-linear regression techniques. In addition, the Nelson-Siegel model can be adapted in a time-series context, as shown by Diebold and Li (2006). In this case the Nelson-Siegel yield-curve model can be seen as the observation equation in a state-space model, and the dynamic evolution of yield curve factors constitutes the transition equation. As a state-space model, estimation can be carried out via the Kalman filter. Second, by construction, the model provides yields for all maturities, i.e. also maturities that are not covered by the data sample. As such it lends itself as an interpolation and extrapolation

[1] For Eurosystem-wide yield curves see http://www.ecb.int/stats/money/yc/html/index.en.html.

tool for the analyst who often is interested in yields at maturities that are not directly observable.[2] Third, estimated yield curve factors obtained from the Nelson and Siegel model have intuitive interpretations, as level, slope (the difference between the long and the short end of the yield curve), and curvature of the yield curve. This interpretation is akin to that obtained by a principal component analysis (see, e.g. Litterman and Scheinkman (1991) and Diebold and Li (2006)). Due to the intuitive appeal of the Nelson-Siegel parameters, estimates and conclusions drawn on the basis of the model are easy to communicate. Fourth, empirically the Nelson-Siegel model fits data well and performs well in out-of-sample forecasting exercises, as shown by e.g. Diebold and Li (2006) and De Pooter, Ravazzolo and van Dijk (2007).

Despite its empirical merits and wide-spread use in the finance community, two theoretical concerns can be raised against the Nelson-Siegel model. First, it is not theoretically arbitrage-free, as shown by Bjork and Christensen (1999). Second, as demonstrated by Diebold, Ji and Li (2004), it falls outside the class of affine yield curve models defined by Duffie and Kan (1996) and Dai and Singleton (2000).

The Nelson-Siegel yield curve model operates at the level of yields, as they are observed, i.e. under the so-called empirical measure. In contrast, affine arbitrage-free yield curve models specify the dynamic evolution of yields under a risk-neutral measure and then map this dynamic evolution back to the physical measure via a functional form for the market price of risk. The advantage of the no-arbitrage approach is that it automatically ensures a certain

[2] This is relevant e.g. in a situation where fixed-income returns are calculated to take into account the roll-down/maturity shortening effect.

consistency between the parameters that describe the dynamic evolution of the yield curve factors under the risk-neutral measure, and the translation of yield curve factors into yields under the physical measure. An arbitrage-free setup will, by construction, ensure internal consistency as it cross-sectionally restricts, in an appropriate manner, the estimated parameters of the model. It is this consistency that guarantees arbitrage freeness. Since a similar consistency is not hard-coded into the Nelson-Siegel model, this model is not necessarily arbitrage-free.[3]

The main contribution of the current paper is to conduct a statistical test for the equality between the factor loadings of Nelson-Siegel model and the implied arbitrage-free loadings. In the context of a Monte Carlo study, the Nelson and Siegel factors are estimated and used as exogenous factors in an essentially-affine term structure model to estimate the implied arbitrage-free factor loadings. The no-arbitrage model with time-varying term premia is estimated using the two-step approach of Ang, Piazzesi and Wei (2006), while we use the re-parametrization suggested by Diebold and Li (2006) as our specification of the Nelson-Siegel model.

In a recent study Christensen, Diebold and Rudebusch (2007) reconcile the Nelson and Siegel modelling setup with the absence of arbitrage by deriving a class of dynamic Nelson-Siegel models that fulfill the no-arbitrage constraints. They maintain the original Nelson-Siegel factor-loading structure and derive mathematically, a correction term that, when added to the dynamic Nelson-Siegel model, ensures the fulfillments of the no-arbitrage

[3] An illustrative example of this issue for a two-factor Nelson-Siegel model is presented by Diebold, Piazzesi and Rudebusch (2005).

constraints. The correction term is shown to impact mainly very long maturities, in particular maturities above the ten-year segment.

While being different in setup and analysis method, our paper confirms the findings of Christensen et al. (2007). In particular, we find that the Nelson-Siegel model is not significantly different from a three-factor no-arbitrage model when it is applied to US zero-coupon yield-curve data. In addition, we outline a general method for empirically testing for the fulfillment of the no-arbitrage constraints in yield curve models that are not necessarily arbitrage-free. Our results furthermore indicate that non-compliance with the no-arbitrage constraints is most likely to stem from "mis-specification" in the Nelson-Siegel factor loading structure pertaining to the third factor, i.e. the one often referred to as the curvature factor.

Our test is conducted on U.S. Treasury zero-coupon yield data covering the period from January 1970 to December 2000 and spanning 18 maturities from 1 month to 10 years. We rely on a non-parametric resampling procedure to generate multiple realizations of the original data. Our approach to regenerate yield curve samples can be seen as a simplified version of the yield-curve bootstrapping approach suggested by Rebonato, Mahal, Joshi, Bucholz and Nyholm (2005).

In summary, we (1) generate a realization from the original yield curve data using a block-bootstrapping technique; (2) estimate the Nelson-Siegel model on the regenerated yield curve sample; (3) use the obtained Nelson-Siegel yield curve factors as input for the essentially affine no-arbitrage model; (4) estimate the implied no-arbitrage yield curve factor loadings on the regenerated data sample. Steps (1) to (4) are repeated 1000 times in order to obtain bootstrapped distributions for the no-arbitrage parameters. These distributions are then used to test whether the implied no-arbitrage factor loadings are significantly different from the Nelson-Siegel loadings.

Our results show that the Nelson Siegel factor loadings are not statistically different from the implied no-arbitrage factor loadings at a 95 percent level of confidence. In an out-of-sample forecasting experiment, we show that the performance of the Nelson-Siegel model is as good as the no-arbitrage counterpart. We therefore conclude that the Nelson and Siegel model is compatible with arbitrage-freeness at this level of confidence.

II Modeling framework

Term-structure factor models describe the relationship between observed yields, yield curve factors and loadings as given by

$$y_t = a + bX_t + \epsilon_t, \tag{1}$$

where y_t denotes a vector of yields observed at time t for N different maturities; y_t is then of dimension $(N \times 1)$. X_t denotes a $(K \times 1)$ vector of yield curve factors, where K counts the number of factors included in the model. The variable a is a $(N \times 1)$ vector of constants, b is of dimension $(N \times K)$ and contains the yield curve factor loadings. ϵ_t is a zero-mean $(N \times 1)$ vector of measurement errors.

The reason for the popularity of factor models in the area of yield curve modeling is the empirical observation that yields at different maturities generally are highly correlated. So, when the yield for one maturity changes, it is very likely that yields at other maturities also change. As a consequence, a parsimonious representation of the yield curve can be obtained by modeling fewer factors than observed maturities.

This empirical feature of yields was first exploited in the continuous-time one factor models, where, in terms of equation (1), $X_t = r_t$, r_t being the short rate, see e.g. Merton (1973), Vasicek (1977), Cox, Ingersoll and Ross (1985), Black, Derman and Toy (1990), and Black and Karasinski (1993).[4] A richer structure for the dynamic evolution of yield curves can be obtained by adding more yield curve factors to the model. Accordingly, X_t becomes a column-vector with a dimension equal to the number of included factors.[5] The multifactor representation of the yield curve is also supported empirically by principal component analysis, see e.g. Litterman and Scheinkman (1991).

Multifactor yield curve models can be specified in different ways: the yield curve factors can be observable or unobserved, in which case their values have to be estimated alongside the other parameters of the model; the structure of the factor loadings can be specified in a way such that a particular interpretation is given to the unobserved yield curve factors, as e.g. Nelson and Siegel (1987) and Soderlind and Svensson (1997); or the factor loadings can be derived from no-arbitrage constraints, as in, among many others, Duffee (2002), Ang and Piazzesi (2003) and Ang, Bekaert and Wei (2007).

Yield curve models that are linear functions of the underlying factors can

[4]The merit of these models mainly lies in the area of derivatives pricing.

[5]Yield curve factor models are categorized by Duffie and Kan (1996) and Dai and Singleton (2000).

be written as special cases of equation (1).[6] In this context, the two models used in the current paper are presented below.

A The Nelson-Siegel model

The Nelson and Siegel (1987) model, as re-parameterized by Diebold and Li (2006), can be seen as a restricted version of equation (1) by imposing the following constraints:

$$a^{NS} = 0 \tag{2}$$

$$b^{NS} = \left[1 \quad \frac{1-\exp(-\lambda\tau)}{\lambda\tau} \quad \frac{1-\exp(-\lambda\tau)}{\lambda\tau} - \exp(-\lambda\tau) \right], \tag{3}$$

where λ is the exponential decay rate of the loadings for different maturities, and τ is time to maturity. This particular loading structure implies that the first factor is responsible for parallel yield curve shifts, since the effect of this factor is identical for all maturities; the second factor represents minus the yield curve slope, because it has a maximal impact on short maturities and minimal effect on the longer maturity yields; and, the third factor can be interpreted as the curvature of the yield curve, because its loading has a hump in the middle part of the maturity spectrum, and little effect on both short and long maturities. In summary, the three factors have the interpretation of a yield curve level, slope and curvature.

[FIGURE 1 AROUND HERE]

[6]Excluded from this list are naturally the quadratic term structure models as proposed by Ahn, Dittmar and Gallant (2002).

A visual representation of the Nelson and Siegel factor loading structure is given in Figure 1. By imposing the restrictions (2) to (3) on equation (1) we obtain

$$y_t = b^{NS} X_t^{NS} + \epsilon_t^{NS}, \tag{4}$$

where $X_t^{NS} = [L_t \quad S_t \quad C_t]$ represents the Nelson-Siegel yield curve factors: Level, Slope and Curvature, at time t.

Empirically the Nelson-Siegel model fits data well, as shown by Nelson and Siegel (1987), and performs relatively well in out-of-sample forecasting exercises (see among others, Diebold and Li (2006) and De Pooter et al. (2007)). However, as mentioned in the introduction, from a theoretical viewpoint the Nelson-Siegel yield curve model is not necessarily arbitrage-free (e.g. see Bjork and Christensen (1999)) and does not belong to the class of affine yield curve models (e.g. see Diebold et al. (2004)).

B Gaussian arbitrage-free models

The Gaussian discrete-time arbitrage-free affine term structure model can also be seen as a particular case of equation (1), where the factor loadings are cross-sectionally restricted to ensure the absence of arbitrage opportunities. This class of no-arbitrage (NA) models can be represented by

$$y_t = a^{NA} + b^{NA} X_t^{NA} + \epsilon_t^{NA}, \tag{5}$$

where the underlying factors are assumed to follow a Gaussian VAR(1) process

$$X_t^{NA} = \mu + \Phi X_{t-1}^{NA} + u_t,$$

with $u_t \sim N(0, \Sigma\Sigma')$ being a $(K \times 1)$ vector of errors, μ is a $(K \times 1)$ vector of means, and Φ is a $(K \times K)$ matrix collecting the autocorrelation coefficients. The elements of a^{NA} and b^{NA} in equation (5) are defined by

$$a_\tau^{NA} = -\frac{A_\tau}{\tau}, \quad b_\tau^{NA} = -\frac{B_\tau}{\tau}, \tag{6}$$

where, as shown by e.g. Ang and Piazzesi (2003), A_τ and B_τ satisfy the following recursive formulas to preclude arbitrage opportunities

$$A_{\tau+1} = A_\tau + B_\tau' \left(\mu - \Sigma\, \lambda_0\right) + \frac{1}{2} B_\tau' \Sigma\Sigma' B_\tau - A_1, \tag{7}$$

$$B_{\tau+1}' = B_\tau' \left(\Phi - \Sigma\, \lambda_1\right) - B_1', \tag{8}$$

with boundary conditions $A_0 = 0$ and $B_0 = 0$. The parameters λ_0 and λ_1 govern the time-varying market price of risk, specified as an affine function of the yield curve factors

$$\Lambda_t = \lambda_0 + \lambda_1 X_t^{NA}.$$

The coefficients $A_1 = -a_1^{NA}$ and $B_1 = -b_1^{NA}$ in equations (7) to (8) refer to the short rate equation

$$r_t = a_1^{NA} + b_1^{NA} X_t^{NA} + v_t,$$

where usually r_t is approximated by the one-month yield.

If the factors X_t^{NA} driving the dynamics of the yield curve are assumed to be unobservable, the estimation of affine term structure models requires a joint procedure to extract the factors and to estimate the parameters of the model. This is a difficult task, given the non-linearity of the model and that the number of parameters grows with the number of included factors. As the factors are latent, identifying restrictions have to be imposed. Moreover, as mentioned by Ang and Piazzesi (2003), the likelihood function is flat in the market-price-of-risk parameters and this further complicates the numerical estimation process.

The most common procedure to estimate affine term structure models is described by Chen and Scott (1993). It relies on the assumption that as many yields, as factors, are observed without measurement error. Hence, it allows for recovering the latent factors from the observed yields by inverting the yield curve equation. Unfortunately, the estimation results will depend on which yields are assumed to be measured without error and will vary according to the choice made. Alternatively, to reduce the degree of arbitrariness, observable factor can be used. For example, Ang et al. (2006) use the short rate, the spread and the quarterly GDP growth rate as yield curve factors. It is also possible to rely on pure statistical techniques in the determination of yield curve factors, as e.g. De Pooter et al. (2007) who use extracted principal components as yield curve factors.

C Motivation

The affine no-arbitrage term structure models impose a structure on the loadings a^{NA} and b^{NA}, presented in equations (6) to (8), such that the resulting yield curves, in the maturity dimension, are compatible with the estimated time-series dynamics for the yield curve factors. This hard-coded internal consistency between the dynamic evolution of the yield curve factors, and hence the yields at different maturity segments of the curve, is what ensures the absence of arbitrage opportunities. A similar constraint is not integrated in the setup of the Nelson-Siegel model (see, Bjork and Christensen (1999)).

However, in practice, when the Nelson-Siegel model is estimated, it is possible that the no-arbitrage constraints are approximately fulfilled, i.e. fulfilled in a statistical sense, while not being explicitly imposed on the model. It cannot be excluded that the functional form of the yield curve, as it is imposed by the Nelson and Siegel factor loading structure in equations (2) and (3), fulfils the no-arbitrage constraints most of the times.

As a preliminary check for the comparability of the Nelson-Siegel model and the no-arbitrage model, Figure 2 compares extracted yield curve factors i.e. $\widehat{X}_t^{NA}$ and $\widehat{X}_t^{NS}$ for US data from 1970 to 2000 (the data is presented in Section III). We estimate the Nelson-Siegel factors as in Diebold and Li (2006), and the no-arbitrage model as in Ang and Piazzesi (2003) using the Chen and Scott (1993) method, and assuming that yields at maturities $3, 24, 120$ months are observed without error.

[FIGURE 2 AROUND HERE]

Although the two models have different theoretical backgrounds and use different estimation procedures, the extracted factors are highly correlated. Indeed, the estimated correlation between the Nelson-Siegel level factor and the first latent factor from the no-arbitrage model is 0.95. The correlation between the slope and the second latent factor is 0.96 and between the curvature and the third latent factor is 0.65.[7]

On the basis of these results and in order to properly investigate whether the Nelson-Siegel model is compatible with arbitrage-freeness, we conduct a test for the equality of the Nelson-Siegel factor loadings to the implied no-arbitrage ones obtained from an arbitrage-free model. To ensure correspondence between the Nelson-Siegel model and its arbitrage-free counterpart, we use extracted Nelson-Siegel factors as exogenous factors in the no-arbitrage setup. The model that we estimate is the following

$$y_t = a^{NA} + b^{NA}\widehat{X}_t^{NS} + \epsilon_t^{NA}, \quad \epsilon_t^{NA} \sim (0, \Omega), \tag{9}$$

where $\widehat{X}_t^{NS}$ are the estimated Nelson-Siegel factors from equations (2) to (4), the observation errors ϵ_t^{NA} are not assumed to be normally distributed and a^{NA} and b^{NA} satisfy the no-arbitrage restrictions presented in equations (6) to (8). In order to impose these no-arbitrage restrictions we have to fit a VAR(1) on the estimated Nelson-Siegel factors

$$\hat{X}_t^{NS} = \mu + \Phi\hat{X}_{t-1}^{NS} + u_t, \tag{10}$$

with $u_t \sim N(0, \Sigma\Sigma')$, to specify the market price of risk as an affine function

[7]Correlations are reported in absolute value.

of the estimated Nelson-Siegel factors

$$\Lambda_t = \lambda_0 + \lambda_1 \hat{X}_t^{NS}, \tag{11}$$

and the short rate equation as

$$r_t = a_1^{NA} + b_1^{NA} \hat{X}_t^{NS} + v_t. \tag{12}$$

In this way, we estimate the no-arbitrage factor loading structure that emerges when the underlying yield curve factors are identical to the Nelson-Siegel yield curve factors. The test is then formulated in terms of the equality between the interceps of the two models, a^{NS} and a^{NA}, and the relative loadings, b^{NA} and b^{NS}.

III Data

We use U.S. Treasury zero-coupon yield curve data covering the period from January 1970 to December 2000 constructed by Diebold and Li (2006), based on end-of-month CRSP government bond files.[8] The data is sampled at a monthly frequency providing a total of 372 observations for each of the maturities observed at the $(1, 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, 120)$ month segments.

[FIGURE 3 AROUND HERE]

[8]The data can be downloaded from http://www.ssc.upenn.edu/ fdiebold/papers/ paper49/FBFITTED.txt and Diebold and Li (2006, pp. 344-345) give a detailed description of the data treatment methodology applied.

The data is presented in Figure 3. The surface plot illustrates how the yield curve evolves over time. Table 1 reports the mean, standard deviation and autocorrelations to further illustrate the properties of the data.

[TABLE 1 AROUND HERE]

The estimated autocorrelation coefficients are significantly different from zero at a 95 percent level of confidence for lag one through twelve, across all maturities.[9] Such high autocorrelations could suggest that the underlying yield series are integrated of order one. If this is the case, we would need to take first-differences to make the variables stationary before valid statistical inference could be drawn, or we would have to resort to co-integration analysis. However, economic theory tells us that nominal yield series cannot be integrated, since they have a lower bound support at zero and an upper bound support lower than infinity. Consequently, and in accordance with the yield-curve literature, we model yields in levels and thus disregard that their in-sample properties could indicate otherwise.[10]

IV Estimation Procedure

To estimate the Nelson-Siegel factors $\widehat{X}_t^{NS}$ in equation (4), we follow Diebold and Li (2006) by fixing the decay parameter $\lambda = 0.0609$ in equation (3) and

[9]A similar degree of persistence in yield curve data is also noted by Diebold and Li (2006).

[10]It is often the case in yield-curve modeling that yields are in levels. See, among others, Nelson and Siegel (1987), Diebold and Li (2006), Diebold, Rudebusch and Aruoba (2006), Diebold, Ji and Yue (2007), Duffee (2006), Ang and Piazzesi (2003), Bansal and Zhou (2002), and Dai and Singleton (2000).

by using OLS.[11] We treat the obtained Nelson-Siegel factors as observable in the estimation of the no-arbitrage model presented in equations (6) to (12). To estimate the parameters of the arbitrage-free model we standardize the Nelson and Siegel factors and use the two-step procedure proposed by Ang et al. (2006). In the first step, we fit a VAR(1) for the standardized Nelson-Siegel factors to estimate $\widehat{\mu}$, $\widehat{\Phi}$ and $\widehat{\Sigma}$ from equation (10). And, we project the short rate (one-month yield) on the standardized Nelson-Siegel yield curve factors, to estimate the parameters in the short rate equation (12). In the second step, we minimize the sum of squared residuals between observed yields and fitted yields to estimate the market-price-of-risk parameters $\widehat{\lambda}_0$ and $\widehat{\lambda}_1$ of equation (11). Finally, we un-standardize the Nelson-Siegel factors and compute $\widehat{a}^{NA}$ and $\widehat{b}^{NA}$.

Our goal is to test whether the Nelson-Siegel model in equations (2) to (4) is statistically different from the no-arbitrage model in equations (6) to (12). Since the estimated factors, $\widehat{X}_t^{NS}$ are the same for both models we can formulate our hypotheses is the following way:

$$
\begin{aligned}
H_0^1 &: a_\tau^{NA} = a_\tau^{NS} \equiv 0, \\
H_0^2 &: b_\tau^{NA}(1) = b_\tau^{NS}(1), \\
H_0^3 &: b_\tau^{NA}(2) = b_\tau^{NS}(2), \\
H_0^4 &: b_\tau^{NA}(3) = b_\tau^{NS}(3),
\end{aligned}
$$

where $b_\tau^{NA}(k)$ denotes the loadings on the k-th factor in the no-arbitrage

[11] This value of λ maximizes the loading on the curvature at 30 months maturity as shown by Diebold and Li (2006).

model at maturity τ, and $b_\tau^{NS}(k)$ denotes the corresponding variable from the Nelson-Siegel model.

We claim that the Nelson-Siegel model is compatible with arbitrage-freeness if H_0^1 to H_0^4 are not rejected at traditional levels of confidence. Notice that to test for H_0^1 to H_0^4 we only need to estimate a^{NA} and b^{NA}, since the Nelson-Siegel loading structure is fixed from the model. To account for the two-step estimation procedure of the no-arbitrage model and for the generated regressor problem, we construct confidence intervals around $\hat{a}^{NA}$ and $\hat{b}^{NA}$ using the resampling procedure described in the next section.

A Resampling procedure

To recover the empirical distributions of the estimated parameters we conduct block resampling and reconstruct multiple yield curve data samples from the original yield curve data in the following way. We denote with G the matrix of observed yield ratios with elements $y_{t,\tau}/y_{t-1,\tau}$ where $t = (2, \ldots, T)$ and $\tau = (1, \ldots, N)$.

We first randomly select a starting yield curve y_k, where the index k is an integer drawn randomly from a discrete uniform distribution $[1, \ldots, T]$. The resulting k marks the random index value at which the starting yield curve is taken.

In a second step, blocks of length w are sampled from the matrix of yield ratios G. The generic i-th block can be denoted by $\widetilde{g}_{z,i}$ where z is a random number from $[2, \ldots, T - w + 1]$ denoting the first observation of the block and I is the maximum number of blocks drawn, $i = 1 \ldots I$. [12] A full

[12] We use $\sim$ to indicate the re-sampled variables.

data-sample of regenerated yield curve ratios $\widetilde{G}$ can then be constructed by vertical concatenation of the drawn data blocks $\widetilde{g}_{z,i}$ for $i = 1 \dots I$.

Finally, a new data set of resampled yields can be constructed via:

$$\begin{cases} \widetilde{y}_1 &= y_k \\ \widetilde{y}_s &= \widetilde{y}_{s-1} \odot \{\widetilde{G}\}_s, \qquad s = 2, \dots, S, \end{cases} \tag{13}$$

where $\{\widetilde{G}\}_s$ denotes the s^{th} row of the matrix of resampled ratios $\widetilde{G}$, and $\odot$ denotes element by element multiplication.

We choose to resample from yield ratios for two reasons. First, it ensures positiveness of the resampled yields. Second, as reported in Table 1, yields are highly autocorrelated and close to $I(1)$. Therefore, one could resample from first differences, but as reported in Table 2, first differences of yields are highly autocorrelated and not variance-stationary. Yield ratios display better statistical properties regarding variance-stationarity, as can be seen by comparing the correlation coefficients for squared differences and ratios in Table 2. Block-bootstrapping is used to account for serial correlation in the yield curve ratios.

[TABLE 2 AROUND HERE]

A similar resampling technique has been proposed by Rebonato et al. (2005). They provide a detailed account for the desirable statistical features of this approach. In the present context we recall that the method ensures: (i) the exact asymptotic recovery of all the eigenvalues and eigenvectors of yields; (ii) the correct reproduction of the distribution of curvatures of the yield curve across maturities; (iii) the correct qualitative recovery of the transition from super- to sub-linearity as the yield maturity is increased in the variance of n-day changes, and (iv) satisfactory accounting of the empirically-observed positive serial correlations in the yields.

To test hypotheses H_0^1 to H_0^4 we employ the following scheme:

(1) Construct a yield curve sample $\widetilde{y}$ following equation (13);

(2) Estimate the Nelson-Siegel yield curve factors $\widetilde{X}_t^{NS}$ on $\widetilde{y}$;

(3) Use $\widetilde{X}_t^{NS}$ to estimate the parameters $\widetilde{a}^{NA}$ and $\widetilde{b}^{NA}$ from the arbitrage-free model given in equations (6) - (12);

(4) Repeat steps 1 to 3, 1000 times to build a distribution for the parameter estimates $\hat{a}^{NA}$ and $\hat{b}^{NA}$;

(5) Construct confidence intervals for $\hat{a}^{NA}$ and $\hat{b}^{NA}$ using the sample quantiles of the empirical distribution of the estimated parameters.

Note that by fixing λ in step 2, the Nelson-Siegel factor loading structure remains unchanged from repetition to repetition. We set the block length equal to 50 observations, i.e. $w = 50$, and generate a total of 370 yield curve observations for each replication, i.e. $S = 370$.[13]

[13]The last block is drawn to contain 20 observations as to obtain a total number of observations for each regenerated sample close to the number of observations of the original sample, 372.

V Results

This section presents three sets of results to help assess whether the Nelson-Siegel model is compatible with arbitrage-freeness when applied to US zero-coupon data. Our main result is a test of equality of the factor loadings on the basis of the resampling technique outlined in section IV. In addition we compare the in-sample and out-of-sample performance of the Nelson-Siegel model, equations (2) - (4), to the no-arbitrage model based on exogenous Nelson-Siegel yield curve factors, equations (6) - (12).

A Testing results

Using the resampling methodology outlined in section IV, we generate empirical distributions for each factor loading of the no-arbitrage yield curve model in equation (9). Results are presented for each maturity covered by the original data sample. The Nelson-Siegel factor loading structure, in equations (2) and (3), is constant across all bootstrapped data sampled because λ is treated as a known parameter.[14] Hence, only the extracted Nelson-Siegel factors vary across the bootstrap samples.

Parameter estimates and corresponding empirical confidence intervals for the no-arbitrage model, equations (6) - (12), are shown in Table 3. The diagonal elements of the matrices holding the estimated autoregressive coefficients $\widehat{\Phi}$ and the covariance matrix of the VAR residuals $\widehat{\Sigma}$, in equation (10), are significantly different from zero at a 95 percent level of confidence.

[14] The results presented in the paper are robust to changes in λ. We have performed the calculations for other values of λ, namely $\lambda = 0.08$, $\lambda = 0.045$, and $\lambda = 0.0996$, and the results for these values of λ are qualitatively the same as the ones presented in the paper.

In addition, the estimates of a_1^{NA}, and the two first elements of the (3×1) vector b_1^{NA} in equation (12), are also different from zero, judged at the same level of confidence.

[TABLE 3 AROUND HERE]

The estimated intercepts of the no-arbitrage model $\hat{a}^{NA}$, computed as in equations (6)- (7), are presented in Table 4, for each maturity covered by the original data. This table reports also the 95 percent confidence intervals, obtained from the resampling, and the Nelson-Siegel intercepts, a^{NS}. Therefore, results in Table 4 allow for testing H_0^1 for the equality between the intercepts in the yield curve equations for the no-arbitrage and the Nelson-Siegel models. Tables 5 to 7 present the corresponding results that allow us to test H_0^2, H_0^3, and H_0^4, i.e. whether the corresponding yield curve factor loadings are identical, in a statistical sense.

[TABLE 4 to 7 AROUND HERE]

Figure 4 gives a visual representation of the results contained in Tables 4 to 7. The figure shows the estimated no-arbitrage loadings, $\hat{a}^{NA}$ and $\hat{b}^{NA}$, with the relative 50 percent and 95 percent empirical confidence intervals obtained from resampling, as well as the parameter values for the Nelson-Siegel model, b^{NS}, for comparison.

It is clear from Figure 4 that the empirical distributions are highly skewed for most of the maturities. Consider, for example, the plot for the intercept estimates (the top left plot in Figure 4) at maturity 120. It is evident that the distribution of the no-arbitrage coefficient is highly right skewed.

[FIGURE 4 AROUND HERE]

This non-normality of the distributions for the estimated no-arbitrage parameters, is further analyzed in Table 8. This table shows that all distributions display skewness, excess kurtosis, or both. Selected maturities are shown in Table 8, however, this result holds for all maturities included in the sample. We also perform the Jarque-Bera test for normality, and reject normality at a 95 percent confidence level for all maturities.

[TABLE 8 AROUND HERE]

Visual confirmation of the documented non-normality is provided by Figures 5 to 6. For a representative selection of maturities, these figures show the empirical distribution of the estimated no-arbitrage loadings, and a normal distribution approximation. In addition, the figures show the 95 percent confidence intervals derived from the empirical distribution and the normal approximation.

[FIGURE 5 to 6 AROUND HERE]

The non-normality of the empirical distributions for the bootstrapped intercepts $\hat{a}^{NA}$, and factor loadings $\hat{b}^{NA}$, indicates that the confidence intervals should be constructed using the sample quantiles of the empirical distribution. The empirical 95 percent confidence intervals are included in Tables 4, 5, 6 and 7. The lower bound of the confidence intervals is denoted by a subscript L, and the upper bound by a U.

By inspecting the tables, we reach the following conclusions for the tested hypotheses:

$H_0^1 : a_\tau^{NA} = a_\tau^{NS} \equiv 0$	not rejected at a 95% level of confidence,
$H_0^2 : b_\tau^{NA}(1) = b_\tau^{NS}(1)$	not rejected at a 95% level of confidence,
$H_0^3 : b_\tau^{NA}(2) = b_\tau^{NS}(2)$	not rejected at a 95% level of confidence,
$H_0^4 : b_\tau^{NA}(3) = b_\tau^{NS}(3)$	not rejected at a 95% level of confidence.

For the test of the curvature parameter in H_0^4 an additional comment is warranted. As can be seen from Figure 4, the curvature parameter, at middle maturities, is the closest to violating the 95 percent confidence band, and this parameter thus constitutes the "weak point" of the Nelson-Siegel model in relation to the no-arbitrage constraints. This finding is in line with Bjork and Christensen (1999) who prove that a Nelson-Siegel type model with two additional curvature factors, each with its own λ, theoretically would be arbitrage-free. However, when acknowledging that Litterman and Scheinkman (1991) find that the curvature factor only accounts for approximately 2 percent of the variation of yields, and in the light of our results, one can question the significance of imposing constraints on parameters that have an explanatory power in the range of 2 percent. Our empirical finding is also supported by the theoretical results in Christensen et al. (2007) who show that adding an additional term at very long maturities reconciles the dynamic Nelson-Siegel model with the affine arbitrage-free term structure models.

Using yield curve modeling for purposes other than relative pricing, as for example central bankers and fixed-income strategists do, one might be tempted to use the Nelson-Siegel model on the basis of its compatibility with arbitrage-freeness.

The hypothesis H_0^1 through H_0^4 test the equality between each no-arbitrage factor loading and the corresponding Nelson-Siegel factor loading separately. The results reported above are confirmed by a joint F test. To perform the test we use the empirical variance-covariance matrix of the estimates. The test statistic is 0.22 and the 95 percent critical F-value with 72 and 300 degrees of freedom is 1.34. Therefore, we also cannot reject the hypothesis that the loading structures of the two models are equal in a statistical sense.

B In-sample comparison

To conduct an in-sample comparison of the two models, we estimate the Nelson-Siegel model in equations (2) - (4) and the no-arbitrage model in equations (6) - (12), where the latter model uses the yield curve factors extracted from the former. Measures of fit are displayed in Table 9.

A general observation is that both models fit data well: the means of the residuals for all maturities are close to zero and show low standard deviations. The root mean squared error, RMSE, and the mean absolute deviation, MAD, are also low and similar for both models.

More specifically, Table 9 shows that the averages of the residuals from the fitted Nelson-Siegel model, $\hat{\epsilon}^{NS}$, for the included maturities, are all lower than 16 basis points, in absolute value. In fact, the mean of the absolute residuals across maturities is 5 basis points, while the corresponding number for $\hat{\epsilon}^{NA}$

is 3 basis points. The 3 months maturity is the worst fitted maturity for the no-arbitrage model with a mean of the residuals of 8 basis points. For the Nelson-Siegel model the worst fitted maturity is the 1 month segment with a mean of the residuals close to -16 bp. Furthermore, the two models have the same amount of autocorrelation in the residuals. A similar observation is made for the Nelson-Siegel model alone by Diebold and Li (2006).

[TABLE 9 AROUND HERE]

Drawing a comparison on the basis of RMSE and MAD figures gives the conclusion that both models fit data equally well.

C Out-of-sample comparison

As a last comparison-check of the equivalence of the Nelson-Siegel model and the no-arbitrage counterpart, we perform an out-of-sample forecast experiment. In particular, we generate h-steps ahead iterative forecasts in the following way. First, the yield curve factors are projected forward using the estimated VAR parameters from equation (10)

$$\hat{X}^{NS}_{t+h|t} = \sum_{s=0}^{h-1} \hat{\Phi}^s \hat{\mu} + \hat{\Phi}^h \hat{X}^{NS}_t,$$

where $h \in \{1, 6, 12\}$ is the forecasting horizon in months. Second, out-of-sample forecasts are calculated for the two models, given the projected factors,

$$\hat{y}^{NS}_{t+h|t} = b^{NS} \hat{X}^{NS}_{t+h|t},$$
$$\hat{y}^{NA}_{t+h|t} = \widehat{a}^{NA}_t + \widehat{b}^{NA}_t \hat{X}^{NS}_{t+h|t},$$

where subscripts t on $\widehat{a}^{NA}_t$ and $\widehat{a}^{NA}_t$ indicate that parameters are estimated using data until time t. To evaluate the prediction accuracy at a given forecasting horizon, we use the mean squared forecast error, MSFE, the average squared error over the evaluation period, between t_0 and t_1, for the h-months ahead forecast of the yield with maturity τ

$$MSFE(\tau, h, m) = \frac{1}{t_1 - t_0 + 1} \sum_{t=t_0}^{t_1} \left(\hat{y}^m_{t+h,\tau|t} - y_{t+h,\tau} \right)^2, \tag{14}$$

where $m \in \{NA, NS\}$ denotes the model.

The results presented are expressed as ratios of the MSFEs of the two models against the MSFE of a random walk. The random walk represents a naïve forecasting model that historically has proven very difficult to outperform. The success of the random walk model in the area of yield curve forecasting is due to the high degree of persistence exhibited by observed yields. The random walk h-step ahead prediction, at time t, of the yield with maturity τ is

$$\hat{y}_{t+h,\tau|t} = y_{t,\tau}.$$

To produce the first set of forecasts, the model parameters are estimated on a sample defined from 1970:01 to 1993:01, and yields are forecasted for the chosen horizons, h. The data sample is then increased by one month and the parameters are re-estimated on the new data covering 1970:01 to 1993:02. Again, forecasts are produced for the forecasting horizons. This procedure is repeated for the full sample, generating forecasts on successively increasing data samples. The forecasting performances are then evaluated over the period 1994:01 to 2000:12 using the mean squared forecast error, as shown in equation (14).

Table 10 reports on the out-of-sample forecast performance of the Nelson-Siegel and the implied no-arbitrage model evaluated against the random walk forecasts.

[FIGURE 10 AROUND HERE]

The well-known phenomenon of the good forecasting performance of the random walk model is observed for the 1 month forecasting horizon. For the 6 and 12 month forecasting horizons, the Nelson-Siegel model and the no-arbitrage counterpart generally perform better than the random walk model, as shown by ratios being less than one.

Turning now to the relative comparison of the no-arbitrage model against the Nelson-Siegel model, it can be concluded that they exhibit very similar forecasting performances. If we consider every maturity for each forecasting horizon as an individual observation, then there are in total 54 observations. In 18 of these cases the Nelson-Siegel model is better, in 24 cases the no-arbitrage model is better, and in the remaining 12 cases the models perform equally well. Even when one model is judged to be better than its competitor, the differences in the performance ratios are very small. Typically, a difference is only seen at the second decimal with a magnitude of 1 to 3 basis points.

In summary, it can be concluded that there is no systematic pattern across maturities and forecasting horizons showing when one model is better than its competitor. Indeed, to formally compare the forecasting performance of the two models we calculate the Diebold-Mariano statistic for each maturity and forecasting horizon. At a 5 percent level we do not reject the hypothesis that the no-arbitrage model and the Nelson-Siegel model forecast equally well, see Table 11.

[TABLE 11 AROUND HERE]

VI Conclusion

In this paper we show that the model proposed by Nelson and Siegel (1987) is compatible with arbitrage-freeness, in the sense that the factor loadings from the model are not statistically different from those derived from an arbitrage-free model which uses the Nelson-Siegel factors as exogenous factors, at a 95 percent level of confidence.

In theory, the Nelson-Siegel model is not arbitrage-free as shown by Bjork and Christensen (1999). However, using US zero-coupon data from 1970 to 2000, a yield curve bootstrapping approach and the implied arbitrage-free factor loadings, we cannot reject the hypothesis that Nelson-Siegel factor loadings fulfill the no-arbitrage constraints, at a 95 percent confidence level. Furthermore, we show that the Nelson-Siegel model performs as well as the no-arbitrage counterpart in an out-of-sample forecasting experiment. Based on these empirical observations, we conclude that the Nelson-Siegel model is compatible with arbitrage-freeness.

This conclusion is of relevance to fixed-income money managers and central banks in particular, since such organizations traditionally rely heavily on the Nelson-Siegel model for policy and strategic investment decisions.

Tables and Graphs

Table 1: Summary statistics of the US zero-coupon data

τ	mean	std dev	min	max	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(12)$
1	6.44	2.58	2.69	16.16	0.97*	0.93*	0.89*	0.69*
3	6.75	2.66	2.73	16.02	0.97*	0.94*	0.91*	0.71*
6	6.98	2.66	2.89	16.48	0.97*	0.94*	0.91*	0.73*
9	7.10	2.64	2.98	16.39	0.97*	0.94*	0.91*	0.73*
12	7.20	2.57	3.11	15.82	0.97*	0.94*	0.91*	0.74*
15	7.31	2.52	3.29	16.04	0.97*	0.94*	0.91*	0.75*
18	7.38	2.50	3.48	16.23	0.98*	0.94*	0.92*	0.75*
21	7.44	2.49	3.64	16.18	0.98*	0.95*	0.92*	0.76*
24	7.46	2.44	3.78	15.65	0.98*	0.94*	0.92*	0.75*
30	7.55	2.36	4.04	15.40	0.98*	0.95*	0.92*	0.76*
36	7.63	2.34	4.20	15.77	0.98*	0.95*	0.93*	0.77*
48	7.77	2.28	4.31	15.82	0.98*	0.95*	0.93*	0.78*
60	7.84	2.25	4.35	15.01	0.98*	0.96*	0.94*	0.79*
72	7.96	2.22	4.38	14.98	0.98*	0.96*	0.94*	0.80*
84	7.99	2.18	4.35	14.98	0.98*	0.96*	0.94*	0.78*
96	8.05	2.17	4.43	14.94	0.98*	0.96*	0.95*	0.81*
108	8.08	2.18	4.43	15.02	0.98*	0.96*	0.95*	0.81*
120	8.05	2.14	4.44	14.93	0.98*	0.96*	0.94*	0.78*

Descriptive statistics of monthly yields at different maturities, τ, for the sample from January 1970 to December 2000. $\rho(p)$ refers to the sample autocorrelation of the series at lag p and * denotes significance at 95 percent confidence level. Confidence intervals are computed according to Box and Jenkins (1976).

Table 2: Autocorrelations

Yield differences

τ	$\rho(1)$	$\rho(3)$	$\rho(12)$	$\rho^2(1)$	$\rho^2(3)$	$\rho^2(12)$
1	0.06	-0.07	-0.06	0.23*	0.08	0.08
3	0.12*	-0.05	-0.13*	0.34*	0.07	0.22*
6	0.16*	-0.09	-0.08	0.32*	0.09	0.20*
12	0.15*	-0.10	-0.05	0.16*	0.11*	0.13*
24	0.18*	-0.11*	0.00	0.21*	0.13*	0.13*
36	0.14*	-0.11*	0.03	0.12*	0.14*	0.14*
60	0.13*	-0.07	0.03	0.09	0.13*	0.13*
84	0.10	-0.09	-0.03	0.17*	0.22*	0.18*
120	0.10	-0.05	-0.03	0.15*	0.19*	0.23*

Yield ratios

τ	$\rho(1)$	$\rho(3)$	$\rho(12)$	$\rho^2(1)$	$\rho^2(3)$	$\rho^2(12)$
1	0.07	-0.05	0.10	0.23*	0.12*	0.02
3	0.11*	0.00	0.01	0.34*	0.10	0.16*
6	0.16*	0.00	0.04	0.25*	0.13*	0.13*
12	0.16*	-0.04	0.04	0.10	0.13*	0.07
24	0.16*	-0.07	0.03	0.06	0.12*	0.03
36	0.13*	-0.09	0.06	0.01	0.06	0.05
60	0.12*	-0.04	0.05	0.01	0.01	0.01
84	0.11*	-0.04	0.00	0.04	0.07	0.03
120	0.08	-0.03	0.00	0.03	0.06	0.06

Sample autocorrelations of first yield differences $\triangle y$, squared first yield differences $\triangle y^2$, yield ratios $\frac{y_t}{y_{t-1}}$ and squared demeaned yield ratios $\left(\frac{y_t}{y_{t-1}} - \bar{\mu}\right)^2$, for selected maturities τ, at lags 1, 3 and 12. $*$ denotes significance at 95 percent confidence level. Confidence intervals are computed according to Box and Jenkins (1976). $\rho(p)$ and $\rho^2(p)$ denote, respectively, the correlation of the variables and their squares, at lag p.

Table 3: Parameter estimates

Parameter	Estimated value	$Q_{2.5}$	$Q_{97.5}$
$\hat{\mu}_1$	-0.247	-1.170	0.911
$\hat{\mu}_2$	-0.006	-0.992	1.158
$\hat{\mu}_3$	-0.408	-1.164	0.895
$\hat{\Phi}_{11}$	0.991*	0.926	1.021
$\hat{\Phi}_{21}$	-0.031	-0.094	0.032
$\hat{\Phi}_{31}$	0.070	-0.102	0.154
$\hat{\Phi}_{12}$	0.024	-0.037	0.068
$\hat{\Phi}_{22}$	0.933*	0.888	1.013
$\hat{\Phi}_{32}$	0.036	-0.140	0.185
$\hat{\Phi}_{13}$	0.000	-0.035	0.062
$\hat{\Phi}_{23}$	0.038	-0.015	0.082
$\hat{\Phi}_{33}$	0.771*	0.755	0.975
$\hat{\Sigma}_{11}$	0.162*	0.086	0.306
$\hat{\Sigma}_{21}$	-0.051	-0.192	0.042
$\hat{\Sigma}_{31}$	-0.110	-0.302	0.014
$\hat{\Sigma}_{22}$	0.324*	0.067	0.305
$\hat{\Sigma}_{32}$	0.009	-0.170	0.071
$\hat{\Sigma}_{33}$	0.596*	0.150	0.532

Parameter estimates (continued)

Parameter	Estimated value	$Q_{2.5}$	$Q_{97.5}$
$\hat{\lambda}_{0,1}$	-0.215	-3.672	1.967
$\hat{\lambda}_{0,2}$	-0.354	-3.043	1.995
$\hat{\lambda}_{0,3}$	0.297	-2.390	3.053
$\hat{\lambda}_{1,11}$	-0.062	-0.470	1.262
$\hat{\lambda}_{1,21}$	-0.123	-0.799	0.523
$\hat{\lambda}_{1,31}$	0.124	-1.098	0.728
$\hat{\lambda}_{1,12}$	0.117	-2.734	1.051
$\hat{\lambda}_{1,22}$	-0.049	-0.633	1.343
$\hat{\lambda}_{1,32}$	0.150	-1.080	1.378
$\hat{\lambda}_{1,13}$	-0.187	-4.208	0.209
$\hat{\lambda}_{1,23}$	-0.169	-2.238	-0.019
$\hat{\lambda}_{1,33}$	-0.024	-0.399	3.209
$\hat{a}_1^{NA}$	0.537*	0.115	1.202
$\hat{b}_1^{NA}(1)$	0.168*	0.064	0.390
$\hat{b}_1^{NA}(2)$	0.146*	0.061	0.623
$\hat{b}_1^{NA}(3)$	0.000	-0.039	0.023

Estimated parameters from the no-arbitrage model in equations (6) to (12) with the 95 percent confidence intervals obtained by resampling. The confidence intervals [$Q_{2.5}$ $Q_{97.5}$] refer to the empirical 2.5 percent and 97.5 percent quantiles of the distributions of the parameters. A star * is used to indicate when a parameter estimate is significantly different from zero at a 95 percent level of confidence.

Table 4: Estimation results for a^{NA}

τ	a^{NS}	$\hat{a}^{NA}$	$\widetilde{a}_L^{NA}$	$\widetilde{a}_U^{NA}$
1	0.00	0.00	-0.10	0.05
3	0.00	0.00	-0.04	0.05
6	0.00	0.00	-0.02	0.06
9	0.00	0.01	-0.02	0.05
12	0.00	0.01	-0.02	0.05
15	0.00	0.00	-0.02	0.04
18	0.00	0.00	-0.02	0.03
21	0.00	0.00	-0.03	0.02
24	0.00	0.00	-0.04	0.01
30	0.00	0.00	-0.05	0.01
36	0.00	-0.01	-0.06	0.02
48	0.00	-0.01	-0.07	0.03
60	0.00	-0.01	-0.06	0.03
72	0.00	0.00	-0.04	0.03
84	0.00	0.00	-0.02	0.02
96	0.00	0.00	-0.01	0.04
108	0.00	0.01	-0.02	0.07
120	0.00	0.01	-0.04	0.10

Estimated intercepts from the no-arbitrage model $\hat{a}^{NA}$ with the 95 percent confidence intervals obtained from the resampling $[\widetilde{a}_L^{NA}\ \ \widetilde{a}_U^{NA}]$. The confidence intervals refer to the empirical 2.5 percent and 97.5 percent quantiles of the distribution of the parameters. The second column of the Table reports the Nelson-Siegel loadings.

Table 5: Estimation results for $b^{NA}(1)$

τ	$b^{NS}(1)$	$\hat{b}^{NA}(1)$	$\widetilde{b}^{NA}_L(1)$	$\widetilde{b}^{NA}_U(1)$
1	1.00	0.98	0.87	1.16
3	1.00	0.99	0.90	1.06
6	1.00	0.99	0.89	1.04
9	1.00	1.00	0.92	1.04
12	1.00	1.00	0.93	1.04
15	1.00	1.00	0.94	1.04
18	1.00	1.00	0.96	1.05
21	1.00	1.00	0.97	1.06
24	1.00	1.00	0.98	1.06
30	1.00	1.01	0.98	1.08
36	1.00	1.01	0.96	1.10
48	1.00	1.00	0.95	1.10
60	1.00	1.00	0.95	1.09
72	1.00	1.00	0.95	1.06
84	1.00	1.00	0.96	1.03
96	1.00	1.00	0.92	1.01
108	1.00	0.99	0.88	1.04
120	1.00	0.99	0.82	1.08

Estimated loadings of the level factor from the no-arbitrage model $\hat{b}^{NA}(1)$ with the 95 percent confidence intervals obtained from the resampling $[\widetilde{b}^{NA}_L(1)\ \widetilde{b}^{NA}_U(1)]$. The confidence intervals refer to the empirical 2.5 percent and 97.5 percent quantiles of the distribution of the parameters. The second column of the Table reports the Nelson-Siegel loadings on the level.

Table 6: Estimation results for $b^{NA}(2)$

τ	$b^{NS}(2)$	$\hat{b}^{NA}(2)$	$\widetilde{b}_L^{NA}(2)$	$\widetilde{b}_U^{NA}(2)$
1	0.97	0.93	0.83	1.08
3	0.91	0.89	0.83	0.98
6	0.84	0.83	0.77	0.92
9	0.77	0.77	0.71	0.84
12	0.71	0.72	0.66	0.76
15	0.66	0.66	0.62	0.70
18	0.61	0.62	0.57	0.64
21	0.56	0.57	0.52	0.59
24	0.53	0.53	0.48	0.56
30	0.46	0.46	0.40	0.50
36	0.41	0.41	0.35	0.45
48	0.32	0.32	0.27	0.38
60	0.27	0.26	0.23	0.32
72	0.23	0.22	0.20	0.26
84	0.19	0.19	0.18	0.22
96	0.17	0.17	0.15	0.21
108	0.15	0.15	0.11	0.20
120	0.14	0.13	0.07	0.19

Estimated loadings of the slope factor from the no-arbitrage model $\hat{b}^{NA}(2)$ with the 95 percent confidence intervals obtained from the resampling $[\widetilde{b}_L^{NA}(2)\ \widetilde{b}_U^{NA}(2)]$. The confidence intervals refer to the empirical 2.5 percent and 97.5 percent quantiles of the distribution of the parameters. The second column of the Table reports the Nelson-Siegel loadings on the slope.

Table 7: Estimation results for $b^{NA}(3)$

τ	$b^{NS}(3)$	$\hat{b}^{NA}(3)$	$\widetilde{b}_L^{NA}(3)$	$\widetilde{b}_U^{NA}(3)$
1	0.03	0.00	-0.10	0.06
3	0.08	0.10	0.05	0.18
6	0.14	0.19	0.13	0.26
9	0.19	0.24	0.17	0.27
12	0.23	0.26	0.21	0.28
15	0.25	0.27	0.23	0.29
18	0.27	0.28	0.24	0.30
21	0.29	0.28	0.23	0.30
24	0.29	0.27	0.24	0.30
30	0.30	0.26	0.23	0.31
36	0.29	0.25	0.23	0.31
48	0.27	0.23	0.22	0.29
60	0.24	0.21	0.20	0.27
72	0.21	0.20	0.19	0.23
84	0.19	0.19	0.18	0.22
96	0.17	0.19	0.16	0.21
108	0.15	0.18	0.13	0.21
120	0.14	0.18	0.11	0.21

Estimated loadings of the curvature factor from the no-arbitrage model $\hat{b}^{NA}(3)$ with the 95 percent confidence intervals obtained from the resampling $[\widetilde{b}_L^{NA}(3)\ \widetilde{b}_U^{NA}(3)]$. The confidence intervals refer to the empirical 2.5 percent and 97.5 percent quantiles of the distribution of the parameters. The second column of the Table reports the Nelson-Siegel loadings on the curvature.

Table 8: Summary statistics for the resampled parameters

Intercept $\widetilde{a}^{NA}$

τ	mean	st.dev.	skewness	kurtosis
3	0.00	0.02	0.11	9.66
12	0.01	0.02	-0.24	8.91
24	0.00	0.01	-3.11	18.77
60	-0.01	0.02	0.34	9.25
84	0.00	0.01	5.49	57.71
120	0.02	0.04	1.06	7.71

Loading of the level $\widetilde{b}^{NA}(1)$

τ	mean	st.dev.	skewness	kurtosis
3	0.99	0.04	0.28	9.39
12	0.99	0.03	0.76	9.02
24	1.01	0.02	2.85	17.25
60	1.01	0.04	-0.88	10.97
84	1.00	0.02	-5.66	60.42
120	0.97	0.06	-1.03	8.17

Loading of the slope $\widetilde{b}^{NA}(2)$

τ	mean	st.dev.	skewness	kurtosis
3	0.91	0.03	0.47	5.56
12	0.71	0.02	-0.08	3.45
24	0.53	0.02	-0.99	6.67
60	0.27	0.02	0.52	5.01
84	0.20	0.01	3.00	34.43
120	0.14	0.03	-0.10	3.97

Loading of the curvature $\widetilde{b}^{NA}(3)$

τ	mean	st.dev.	skewness	kurtosis
3	0.10	0.03	0.93	3.39
12	0.25	0.02	-0.52	4.59
24	0.28	0.02	-0.73	2.71
60	0.22	0.02	1.72	8.99
84	0.19	0.01	1.05	5.42
120	0.16	0.02	-0.85	6.80

Summary statistics of the empirical distributions of the estimated parameters obtained using resampled data.

Table 9: Measures of Fit

Residuals from the Nelson-Siegel model

τ	mean	st dev	min	max	RMSE	MAD	$\rho(1)$	$\rho(6)$	$\rho(12)$
1	-0.159	0.200	-1.046	0.387	0.200	0.040	0.513	0.332	0.443
3	0.027	0.114	-0.496	0.584	0.114	0.013	0.274	0.159	0.326
6	0.091	0.135	-0.412	0.680	0.135	0.018	0.543	0.346	0.471
12	0.046	0.122	-0.279	0.483	0.122	0.015	0.586	0.127	0.289
24	-0.040	0.073	-0.398	0.261	0.073	0.005	0.493	0.044	0.153
36	-0.066	0.090	-0.432	0.339	0.089	0.008	0.417	0.256	0.183
60	-0.053	0.096	-0.520	0.292	0.096	0.009	0.655	0.312	-0.037
84	0.006	0.097	-0.446	0.337	0.096	0.009	0.518	0.159	-0.083
120	0.002	0.140	-0.763	0.436	0.140	0.020	0.699	0.345	0.091

Residuals from no-arbitrage model

τ	Mean	st dev	min	max	RMSE	MAD	$\rho(1)$	$\rho(6)$	$\rho(12)$
1	0.000	0.168	-0.730	0.752	0.168	0.028	0.361	0.197	0.363
3	0.080	0.132	-0.508	0.817	0.132	0.018	0.448	0.219	0.312
6	0.060	0.135	-0.295	0.795	0.134	0.018	0.579	0.361	0.432
12	-0.019	0.109	-0.355	0.439	0.109	0.012	0.514	0.147	0.306
24	-0.041	0.071	-0.323	0.217	0.071	0.005	0.491	0.134	0.096
36	-0.018	0.088	-0.286	0.405	0.088	0.008	0.474	0.320	0.263
60	0.004	0.100	-0.332	0.379	0.100	0.010	0.688	0.350	0.101
84	0.019	0.097	-0.479	0.343	0.097	0.009	0.527	0.157	-0.070
120	-0.060	0.144	-0.801	0.375	0.144	0.021	0.705	0.464	0.249

Summary statistics of residuals of the Nelson-Siegel and the no-arbitrage models. The Nelson-Siegel model is estimated according to equations (2) - (4). The no-arbitrage yield curve model is estimated according to equations (6) - (12). Statistics are shown for selected maturities, τ. RMSE is the root mean squared error and MAD is the mean absolute deviation. Autocorrelations are denoted by $\rho(p)$, where p is the lag.

Table 10: Out-of-sample performance

	1-m ahead		6-m ahead		12-m ahead	
τ	NS	NA	NS	NA	NS	NA
1	0.82	**0.67**	0.67	**0.56**	0.66	**0.59**
3	0.91	**0.89**	0.72	**0.70**	0.64	**0.63**
6	1.08	**1.03**	**0.81**	0.82	**0.65**	0.67
9	**1.06**	1.21	**0.80**	0.83	**0.64**	0.66
12	1.01	**1.00**	**0.80**	0.81	**0.64**	0.65
15	1.06	**0.98**	0.79	0.79	**0.64**	0.65
18	1.04	**1.03**	0.80	0.80	0.65	0.65
21	**1.06**	1.07	0.80	0.80	0.66	0.66
24	**1.09**	1.11	0.80	0.80	0.67	0.67
30	1.04	1.04	0.80	**0.78**	0.68	**0.67**
36	0.99	**0.98**	0.80	**0.78**	0.70	**0.69**
48	0.98	0.98	0.84	**0.81**	0.76	**0.73**
60	1.10	**1.04**	0.88	**0.85**	0.81	**0.79**
72	1.02	**1.01**	0.90	**0.88**	0.85	**0.84**
84	1.08	1.08	0.91	0.91	0.87	**0.86**
96	1.03	1.03	**0.93**	0.94	**0.91**	0.92
108	**1.04**	1.08	**0.95**	0.98	**0.93**	0.96
120	**1.08**	1.32	**1.02**	1.08	**1.00**	1.05

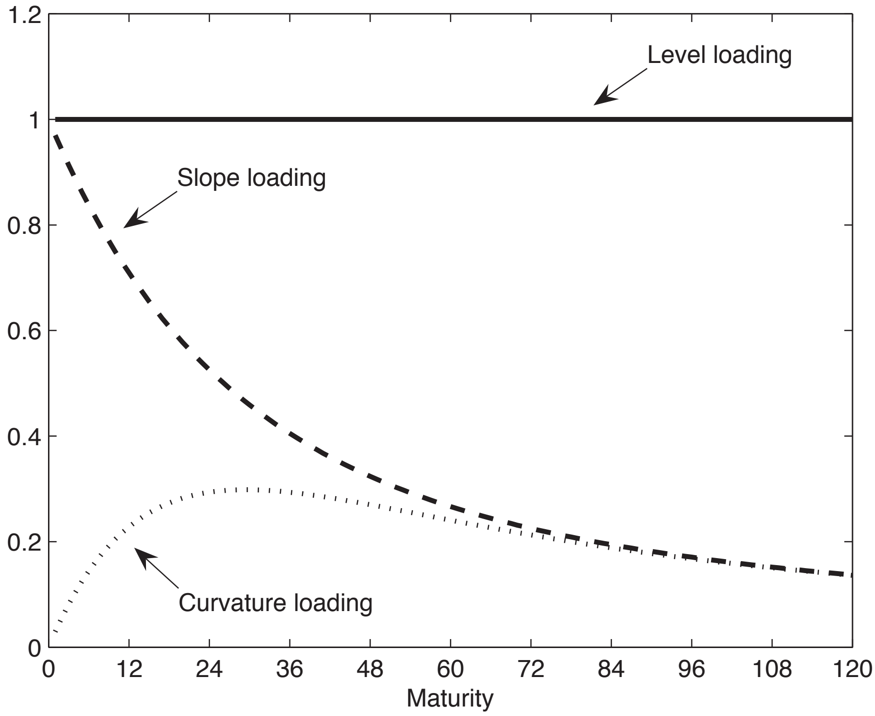
Ratios of the Mean Squared Forecast Error (MSFE) of the no-arbitrage model (NA) and the Nelson-Siegel model (NS) both measured against the performance of the random walk model. A ratio lower than 1 means that the MSFE for the respective model is lower than the forecast error generated by the random walk, and hence that the model performs better than the random walk model. The models are estimated on successively increasing data samples starting 1970:1 until the time the forecast is made, and expanded by one month each time a new set of forecasts are generated. Forecasts for horizons of 1, 6 and 12 months ahead are evaluated on the sample from 1994:1 to 2000:12. Bold entries in the table indicate superior performance of one model (NA or NS) against the other model.

Table 11: Diebold-Mariano test statistics

τ	1-m ahead	6-m ahead	12-m ahead
1	-0.080	-0.214	-0.250
3	-0.037	-0.129	-0.146
6	-0.051	0.132	0.262
9	0.147	0.159	0.222
12	-0.015	0.085	0.154
15	-0.117	0.021	0.098
18	-0.040	0.017	0.086
21	0.048	-0.025	0.046
24	0.070	-0.318	- 0.165
30	-0.003	-0.174	-0.290
36	-0.022	-0.149	-0.239
48	0.002	-0.128	- 0.215
60	-0.082	-0.153	-0.233
72	-0.025	-0.121	-0.215
84	-0.007	-0.047	- 0.166
96	-0.016	0.315	0.447
108	0.069	0.231	0.322
120	0.266	0.290	0.366

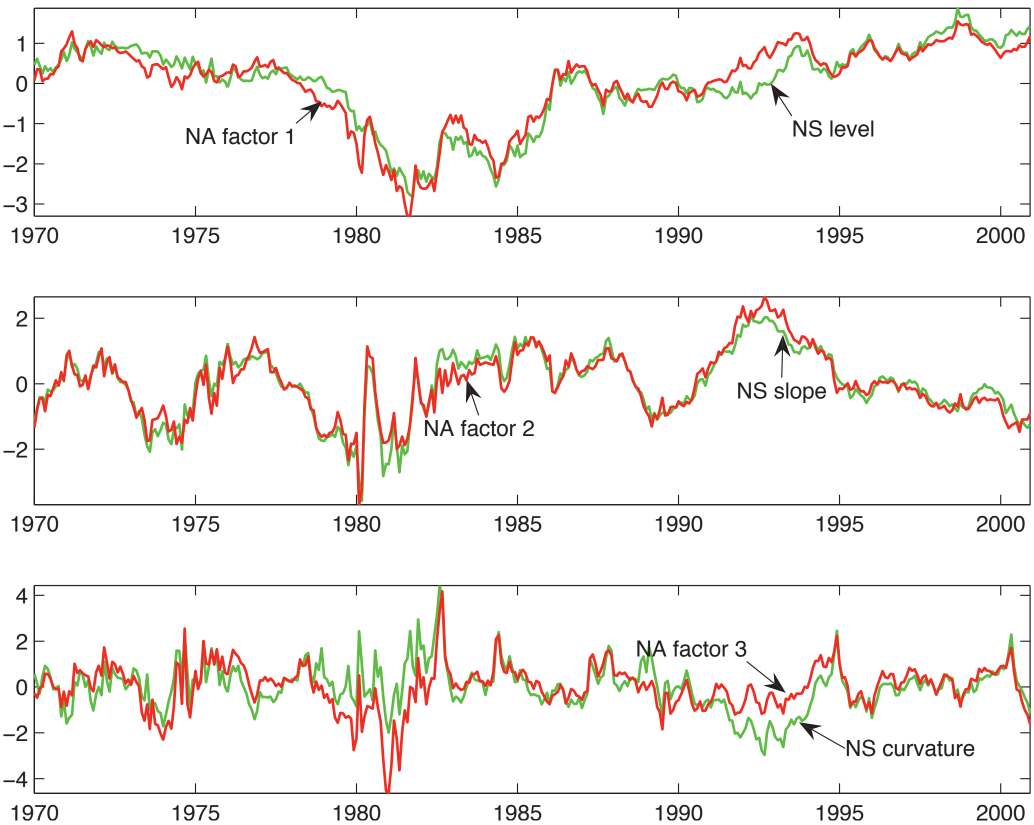
Diebold-Mariano test statistic to compare forecast accuracy of two models. We compare the no-arbitrage model against the Nelson-Siegel model. Negative numbers reflect superiority of the no-arbitrage model, and positive numbers indicate that the Nelson-Siegel model performs better. The null hypothesis is that the mean squared forecast error of the two models is identical. A number larger than 1.96 in absolute terms indicates that the forecasts produced by the models are significantly different at a 5 percent level.

Figure 1: Nelson-Siegel factor loadings



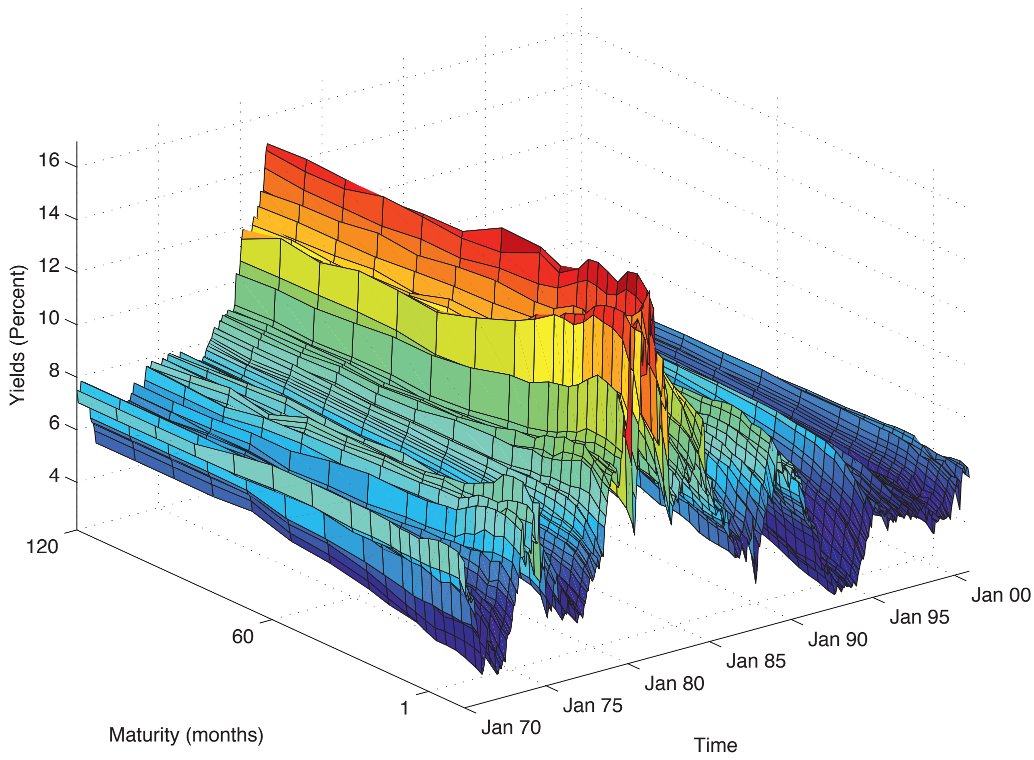
Nelson and Siegel (1987) factor loadings using the re-parameterized version of the model as presented by Diebold and Li (2006). The factor loadings b^{NS} are computed using $\lambda = 0.0609$ and equation (3).

Figure 2: No-Arbitrage Latent factors and Nelson and Siegel factors



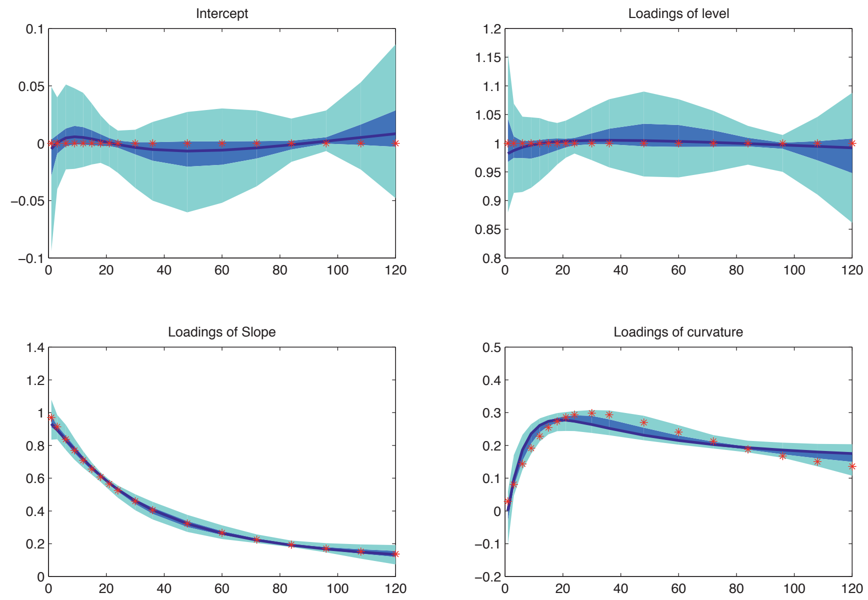
Extracted yield curve factors using US zero-coupon data observed at a monthly frequency and covering the period from 1970:1 to 2000:12. Factors are extracted from the Nelson-Siegel model and from the no-arbitrage model. "NS level" and "NA factor 1" refer to the first extracted factor from each model. The second and third extracted factors are correspondingly labeled "NS slope", "NA factor 2" and "NS curvature", "NA factor 3".

Figure 3: Zero-coupon yields data



U.S. zero-coupon yield curve data observed at monthly frequency from 1970:1 to 2000:12 at maturities $1, 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108$ and 120 months.

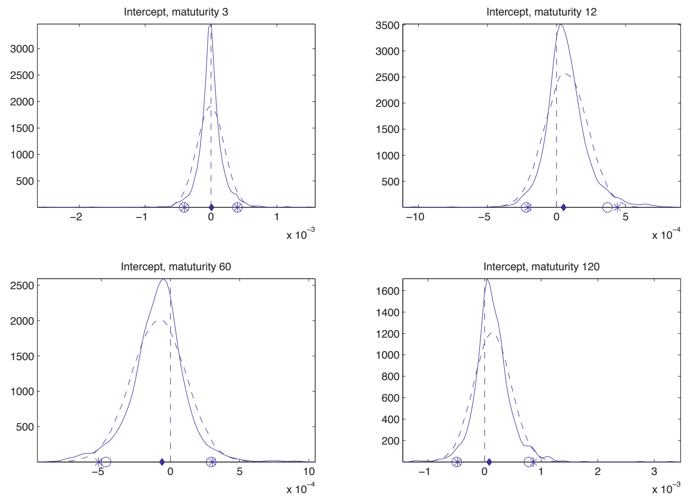
Figure 4: No-Arbitrage loadings of the Nelson and Siegel factors



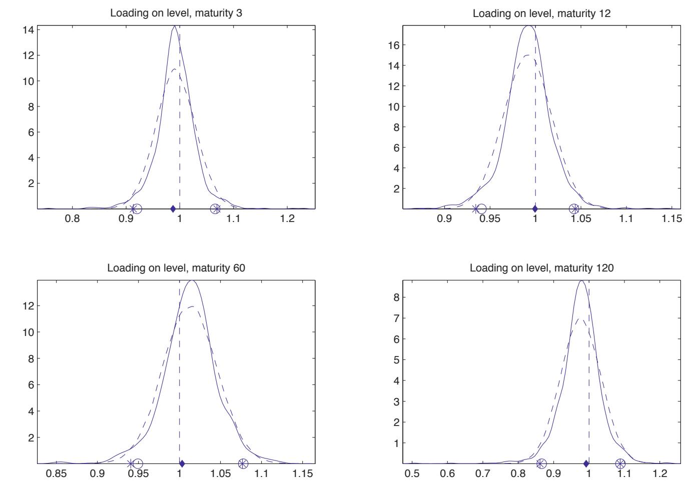
Estimated factor loadings and empirical 50 and 95 percent confidence intervals. Star * indicate the factor loadings from the Nelson-Siegel model, i.e. a^{NS} and b^{NS} in equations (2) and (3), while the continuous lines indicate the corresponding factor loadings estimated from the no-arbitrage model, i.e. $\widehat{a}^{NA}$ and $\widehat{b}^{NA}$ in equations (6) to (8). The distributions of the latter are obtained through resampling. The dark-shaded areas are the 50 percent confidence intervals, while the light-shaded areas show the 95 percent confidence intervals. These are computed as empirical quantiles.

Figure 5: Distribution of the estimated no-arbitrage loadings

Estimated intercepts a^{NA}



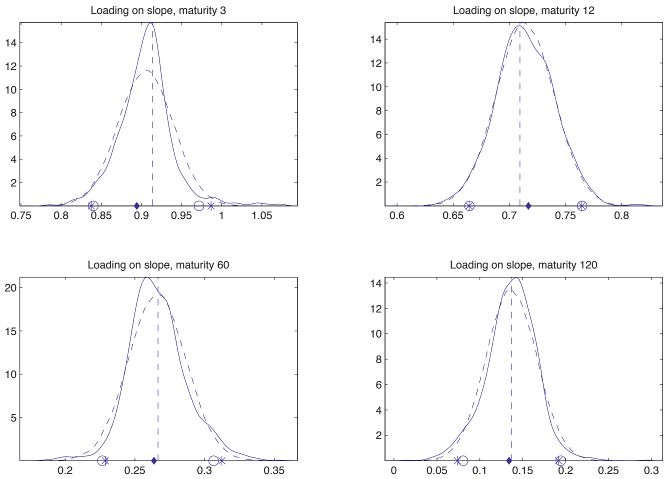
Estimated loadings of the level $b^{NA}(1)$



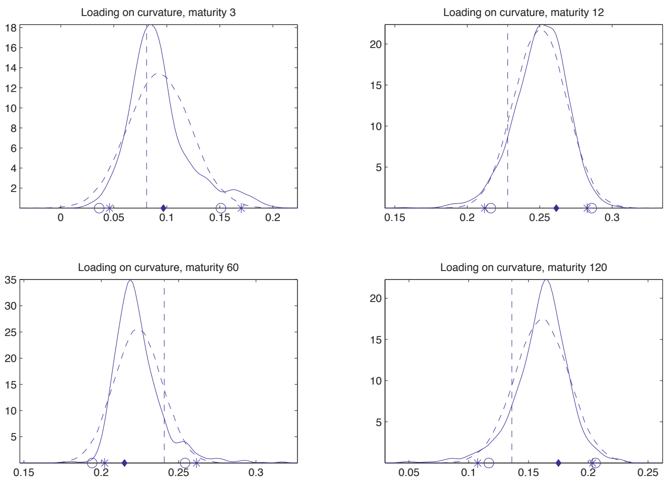
Empirical distributions, for selected maturities, of the no-arbitrage intercepts (first panel) and loadings of the level (second panel)obtained from the resampling (continuous line), with the relative 95 percent confidence interval (asterisks). The dashed line is the Gaussian approximation with the relative 95 percent confidence intervals (circles). The diamonds are the estimated no-arbitrage parameters and the dashed vertical line indicates the Nelson and Siegel ones.

Figure 6: Distribution of the estimated no-arbitrage loadings

Estimated loadings of the slope $b^{NA}(2)$



Estimated loadings of the curvature $b^{NA}(3)$



Empirical distributions, for selected maturities, of the no-arbitrage loadings of the slope (first panel) and of the curvature (second panel) obtained from the resampling (continuous line), with the relative 95 percent confidence interval (asterisks). The dashed line is the Gaussian approximation with the relative 95 percent confidence intervals (circles). The diamonds are the estimated no-arbitrage parameters and the dashed vertical line indicates the Nelson and Siegel ones.

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