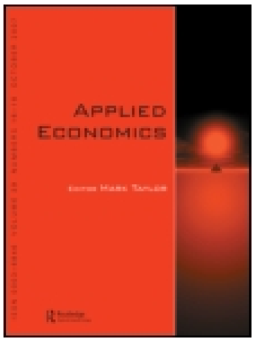

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This article estimates a dynamic model for the yield curve incorporating latent and macro factors to represent the term structure of the real interest rates. The representation of the yield curve is based on the popular latent factor model of Nelson and Siegel (1987), but under a dynamic interpretation due to Diebold and Li (2006). After assuming the data generating process for the latent and macro factors can be represented by a VAR process, the yields-macro model can be regarded as a state-space representation and estimated by a Kalman Filter approach or by using a simplified two-step procedure proposed by Diebold and Li (2006). This article follows the simple two-step method and makes a comparison check with the Kalman Filter estimation, concluding that the basic intuition of the results is not significantly affected by the use of the simplified approach. Estimation results give support to the dynamic interaction between yield curve latent factors and macroeconomic variables. In particular, monetary policy implemented by the Central Bank seems to be influenced by the market players given the significant response of the monetary policy rate to the yield curve factors as shown by impulse-response functions. In addition, the level and slope of the yield curve seems to be responsive to real activity and monetary policy shocks, issues that should be considered by monetary authorities given the dependency of monetary policy effectiveness on the shape of the yield curve.

I. Introduction

Even though the relationship between the yield curve and macroeconomic variables has been a longer considered fact by policy makers, its formal modelling just started recently. Among the practitioners it is widely accepted that economic news are quickly assimilated by bond markets. Moreover, under the Expectation Hypothesis long-term rates depend on the expected future short-term rates, which will be related to monetary policy. So, expected future policy actions should affect the shape of the yield curve. On the other hand, the use of the short-term interest rate as policy target gives a fundamental role to the term structure in determining the real effects from a monetary shock, provided the consumption-investment decisions are based on long-term rates more than on the policy rate.

In particular, Campbell (1995) and Taylor (1992) – beyond providing a good introduction to the term

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structure literature – present some empirical evidence (for US and UK, respectively) on the validity of popular theories for the yield curve, pointing out that the government debt policy can affect and be affected by the form of the curve. In fact, for US during the 90's Campbell (1995) suggests that the financial position of the Treasury was improved by shortening the maturity of the government debt, given a steep yield curve. On the other hand, Taylor (1992) concludes that the policy of repurchasing government debt in UK, during the second half of the 80's, produced the inversion of the yield curve.

While the finance and macroeconomic literature contains several works studying the unidirectional effects, from macro to yields and vice versa, the analysis of bidirectional feedback is presented in just a few number of recent articles.

For the Chilean case, the bidirectional feedback between the term structure and the macroeconomy has not been considered yet. There is only a few number of studies related to the econometric fitting of the yield curve or to the analysis of inflation expectations implied by the associated forward curve (see Fernández, 1999, Herrera and Magendzo, 1997 and Parisi, 1998).

The Chilean economy offers an interesting case to analyse the interaction between macro variables and the real term structure, given that bond indexation is based on the last month inflation rate, which could be considered as almost fully indexed when compared – for example – to the UK or Israeli bonds. That is, in the Chilean case to obtain the real term structure directly from the prices of indexed bonds implies small biases compared to the case for economies where the holders of the debt are exposed to a larger inflation risk.

Theoretically, the framework to analyse the dynamic interaction is far from being a settled issue. On the one hand, there is no consensus on the way to estimate the term structure of the interest rates, in order to consider its dynamic relationship with macroeconomic variables. In financial theory, there are two main traditions in estimating yield curves. One is the no-arbitrage approach, fitting the curve at each point in time to rule out any arbitrage opportunity. The other approach corresponds to the equilibrium models, assuming the short-term rate driven by an specific stochastic process, and then the rest of the term structure defined by the particular assumptions made on the term premium behaviour. While the no-arbitrage tradition is not directly concerned with dynamics, the equilibrium tradition has shown a poor performance in out of sample forecasting of the complete term structure. In order to establish the links between yields and macro factors a good dynamic fit is essential to the analysis. On the other hand, the way the macroeconomic variables are incorporated makes fundamental differences in the conclusions obtained from the empirical analysis. One approach is to provide a structural representation of the macroeconomy, usually by means of a small (linear) model or consistently modelling long run expectations about future inflation. The other possibility is to use a nonstructural representation of the macroeconomy, considering observed macro variables or latent macro factors. While the conclusions from the first approach depends a lot on the structural model assumptions, the implications from the second one, despite its statistic robustness, are always subject to the noneconomic content critique. In addition, the conclusions from either of both approaches depend on the definition of macro variables used in the study. In some cases, the macroeconomic meaning given to some latent factors are not easily accepted, weakening the implications derived from the empirical analysis.

Most studies in the literature relates the term structure to macroeconomic variables by using impulse-responses and variance-decompositions from an estimated VAR system (see Ang and Piazzesi, 2003 and Evans and Marshall, 2002). However, any inferred dynamic from the VAR is not valid for maturities not included in the estimation of the yield curve. This way, a latent factor approach is required in order to analyse the interaction between the term structure and the macroeconomy.

Diebold and Li (2006), and Diebold *et al*. (2006), interpreting the Nelson and Siegel (1987) yield curve as a dynamic latent factor model of the term structure, allows for the use of an state-space representation to investigate the bidirectional feedback between yields and macro factors. By assuming the dynamic latent factors follow a vector autoregressive process, the corresponding state-space model can be estimated either by means of a two-step procedure (Diebold and Li, 2006) or by following a Kalman Filter approach (Diebold *et al*., 2006). This way, the estimation method is able to produce a good dynamic forecast for the complete term structure, in contrast with the mentioned traditions in finance theory failing in simultaneously matching cross-section and dynamic fit of the yield curve required for the study of bidirectional feedback.

The purpose of this article is to apply the same basic methodology described above for the Chilean case. The analysis will be based on indexed bonds – dominating the domestic market – and observable macroeconomic aggregates representing monetary policy, inflation and economic activity. The conclusions from this study are assumed of great interest for the Central Bank in implementing monetary policy, as well as for the market players in forming their expectations about long-term rates and future policy shocks.

II. Yield Curve Representation

The most popular representation of the yield curve, specially among Central Banks, is the parsimonious model proposed by Nelson and Siegel (1987). The main advantage of this model is related to its parsimony and flexibility to accommodate the usual shapes exhibited by the term structure (e.g. upward-sloping, inverted and hump-shaped). Following Diebold and Li (2006), a modified version of the Nelson–Siegel model is given by:

$$R(\tau)=\beta_1+\beta_2\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right)+\beta_3\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}-e^{-\lambda_t\tau}\right)+\varepsilon_t(\tau) \tag{1}$$

where β_1, β_2, β_3 and λ_t are parameters to be estimated for each cross-section, at any point in time.

However, as pointed out by Nelson and Siegel (1987) the parameter λ_t could be considered as a fix value (λ), provided the sum of squares residuals is not very sensitive to changes on it. If β is a latent time varying vector, and follows a vector autoregressive process, the model above can be regarded as a state-space representation of the yield curve. Looking at the latent variables, β_{1t} could be considered as a long-term factor, because it has a loading equal to 1, which is constant and not decaying to zero in the limit. On the other hand, the loading for β_{2t} is a function starting at 1 but decaying monotonically and quickly to 0, allowing us to consider this as a short-term factor. Finally, β_{3t} is considered as a medium-term factor, given that its loading starts and ends at 0 but it is increasing in the transition.

Diebold and Li (2006) give to the coefficients in the latent β vector the interpretation of level (L_t), slope (S_t) and curvature (C_t) of the yield curve. The reasons are related to the aspect of the curve that each factor governs. An increase in the long-term factor, β_{1t}, produces the same change in all the yields given the constancy of its loading factor, and hence it affects the level of the curve. An increase in β_{2t} has a stronger effect in short yields compared to long yields, affecting the slope of the curve (actually the slope is usually defined as long term minus instantaneous yield, which is exactly the same as $-\beta_{2t}$). The medium term factor, β_{3t}, on the other hand, is not affecting the instantaneous or the long-term yields, but it has a direct effect on the medium-term rates, modifying the curvature of the term structure.

By assuming the latent vector is generated by a VAR(1) process, then the state-space representation of the Nelson–Siegel yield curve is given by

Signal Equation:

$$\begin{pmatrix} R_t(\tau_1) \\ \vdots \\ R_t(\tau_n) \end{pmatrix} = \begin{pmatrix} 1 & \dfrac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \dfrac{1-e^{-\lambda\tau_1}}{\lambda\tau_1}-e^{-\lambda\tau_1} \\ \vdots & \vdots & \vdots \\ 1 & \dfrac{1-e^{-\lambda\tau_n}}{\lambda\tau_n} & \dfrac{1-e^{-\lambda\tau_n}}{\lambda\tau_n}-e^{-\lambda\tau_n} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \vdots \\ \varepsilon_t(\tau_n) \end{pmatrix} \tag{2}$$

Transition equation:

$$\begin{pmatrix} L_t-\mu_L \\ S_t-\mu_S \\ C_t-\mu_C \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} \begin{pmatrix} L_{t-1}-\mu_L \\ S_{t-1}-\mu_S \\ C_{t-1}-\mu_C \end{pmatrix} + \begin{pmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{pmatrix} \tag{3}$$

and white noise disturbances:

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim iid\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} H & 0 \\ 0 & Q \end{pmatrix}\right] \tag{4}$$

with H diagonal, and Q nondiagonal matrices. The assumption of H being a diagonal matrix, implies uncorrelated deviations of yields of different maturities from the yield curve. On the other hand, a nondiagonal Q matrix allows for correlation between the shocks to the three latent factors.

It is important to note that the proposed model is not imposing a no-arbitrage restriction as usually done in finance literature. The reason to not impose such a restriction on the model is the evidence of poor dynamic fit of no-arbitrage models. As mentioned in the introduction, this class of specifications do a good job in fitting the yield curve at each cross-section, but less well in a dynamic framework. On the other hand, the possible loss of efficiency by not imposing the restriction, must be compared to the potential missespecification that could arise if arbitrage opportunities are not eliminated in the market. This last situation could be not unusual in the Chilean bond market, so the case for imposing the restriction is not clear for the present study.

III. Yield Curve and Macroeconomic Factors

In order to analyse the dynamic interaction between the latent factors – determining the shape of the yield curve – and the macroeconomic variables, the state-space representation above can be extended by including macro factors. To maintain the parsimony and nonstructural nature of the yield curve model, the macroeconomy is represented through the minimum set of variables required to asses the macro dynamics. Then, the variables expanding the state-space representation are: real activity – measured as deviations from its potential – (Y_t), Central Bank target interest rate (MPR_t), and 12-months inflation rate (INF_t).

This way, the state-space representation is given by:

Signal equation:

$$R_t(\tau) = \begin{pmatrix} 1 & \dfrac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \dfrac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \dfrac{1-e^{-\lambda\tau_n}}{\lambda\tau_n} & \dfrac{1-e^{-\lambda\tau_n}}{\lambda\tau_n} - e^{-\lambda\tau_n} & 0 & 0 & 0 \end{pmatrix} \times f_t + \varepsilon_t(\tau) \tag{5}$$

Transition equation:

$$(f_t - \mu) = A(f_{t-1} - \mu) + \eta_t \tag{6}$$

and white noise disturbances:

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim iid\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} H & 0 \\ 0 & Q \end{pmatrix}\right] \tag{7}$$

where $f_t = (L_t, S_t, C_t, Y_t, \mathrm{MPR}_t, \mathrm{INF}_t)$ and Q, μ and η_t are increased as appropriate. It is worth noting that the signal equation implies no change from the previous version of the model, recognizing the fact that the yield curve is fully described by the three latent factors: level, slope and curvature.

IV. Estimation Methods

To estimate the yields-macro model above, there are two basic approaches to follow. First, as in Diebold *et al.* (2006), we could implement a simultaneous estimation of the signal and transition equations by using the Kalman Filter method. Second, it is possible to use a simply two-step recursive method as proposed by Diebold and Li (2006).

The one-step estimation of the dynamic model by a Kalman Filter approach gives maximum likelihood estimates of the coefficients, and optimal smoothed estimates for the latent factors determining the shape of the yield curve. However, the sensitivity of the final results on initial conditions, could make the conclusions made out of the model estimates and forecasts less robust. On the other hand, the two-step method followed by Diebold and Li (2006), even though robust and intuitively appealing, is suboptimal in the sense that the potential estimation errors from the first step are not considered in the final step.

Based on the considerations above, to emphasize intuition the yields-macro model is initially estimated by means of the two-step approach. Then, to evaluate the potential suboptimality of the two-step method some comparisons are made for the basic model (yields only) in terms of the conclusions obtained from the two alternative approaches.

Two-step approach

Consider the estimation of the yield curve:

$$R(\tau) = \beta_1 + \beta_2\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_3\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + \varepsilon_t(\tau)$$

This estimation requires the use of a nonlinear method, for each cross-section at any date period. However, if we impose a fixed value for λ, then it is possible to obtain estimates of the β vector for each time period by simply using an Ordinary Least Square (OLS) estimator. Collecting the corresponding $\hat{\beta}_t$ vectors, a time series for each of its components is readily available.

If we identify $\hat{\beta}_{1t}$, $\hat{\beta}_{2t}$, $\hat{\beta}_{3t}$ as the estimated latent factors $\hat{L}_t$, $\hat{S}_t$, $\hat{C}_t$, then a vector autoregression estimator can be implemented to asses the dynamic interaction between the latent yield curve factors and macroeconomic variables. That is, Equation 6 of the yields-macro model could be estimated by replacing actual by estimated latent factors.

If λ_t were estimated, as remarked by Diebold and Li (2006), the required nonlinear estimation at each cross-section can make the $\hat{\beta}_t$ to vary a lot over time. This variation in latent factors due to the estimation of λ_t could induce spurious dynamics for the estimated level, slope and curvature invalidating the conclusions from the VAR process of Equation 6.

Kalman Filter

The Kalman Filter is an updating algorithm for the linear projection of the state-vector (latent variables) based on observable variables, that allows you to write down – under normality – the likelihood function of the model based on the prediction error decomposition. Once the likelihood function is obtained, the coefficients are estimated by means of numerical optimization methods. In addition, a smoothed state-vector estimate – for the full sample – can be obtained if the value of the latent variables are of interest and they could be given a structural interpretation.

The implementation of the filter requires meaningful start up conditions (parameters values, first observation for the state-vector, and its mean squared error matrix) to achieve convergence. This is not a minor issue given the potentially high nonlinearity in the likelihood function for the state-space representation of the yields-macro model.

A possible set of initial conditions can be extracted from the two-step estimation above, as proposed by Diebold *et al*. (2006).

Impulse-response functions

The most intuitive tool to analyse the interaction among variables in the system is the impulse-response function for each of the series. To see this, by using recursive substitution we can write the unrestricted VAR(1) in its Vector Moving Average (VMA) representation:

$$(f_t - \mu) = \sum_{i=0}^{\infty} A^i \eta_{t-i} \tag{8}$$

However, to trace the impact of an 'impulse-' to one of the variables on itself and on the rest of the variables in the system, what is required is the VMA representation based on the orthogonal structural shocks instead of using the reduced form residuals, which are correlated with each other.

Given that the structural errors are not available, a popular way to identify impulse–response functions is by using a lower triangular matrix coming from a Cholesky decomposition of the variance–covariance matrix of the residuals. That is, instead of using the true structural shocks, the VMA representation of the VAR model is given by:

$$(f_t - \mu) = \sum_{i=0}^{\infty} A^i P v_{t-i} = \sum_{i=0}^{\infty} \Phi_i \upsilon_{t-i} \tag{9}$$

where, the variance–covariance matrix of residuals is decomposed as $\Omega = PP'$.

By updating this equation we get the response of $(f_{t+i} - \mu)$ to a one-SD impulse- at time t. If we graph each element of Φ_i against i periods, we have the response of each variable in the system from the impulse- to the different orthogonalized shocks.

This way to identify impulse- responses, however, is not free of strong implicit assumptions about the contemporaneous relationship among the variables in the system. By using this kind of triangular decomposition there is a forced asymmetry in the system, since the first variable is assumed as not contemporaneously affected by any other endogenous variable, while the rest of them are affected just by the preceding ones. This way, the Cholesky decomposition implies a specific order of the variables. Changing the order produces different impulse-response functions for each variable.

The usual rule of thumb to deal with the ordering problem described above is to try all the possible orders in the system, and then if the implications from impulse- response functions are not too different you can analyse the dynamics of the model based on the estimated impulse- responses. However, remember that all the possible combinations are given by $n!$, which can be a huge number even for small dimension VAR models.

To solve the ordering problem affecting the Cholesky decomposition, Pesaran and Shin (1998) propose a 'generalized' impulse-response function for unrestricted VARs, which is invariant to the ordering of the variables in the system. The main idea is to understand the impulse-response as the difference between the expected value of the variable at time $t+i$ after a hypothetical shock at time t, and the expected value of the same variable at time $t+i$ given the observed history of the system. That is, the generalized impulse–response function is given by:

$$\text{GIR} = E\big((f_{t+i} - \mu)|(\eta_j \Theta_{t-1})\big) - E((f_{t+i} - \mu)|(\Theta_{t-1})) \tag{10}$$

where, η_j corresponds to a $n \times 1$ vector with unity at the jth element and zeros elsewhere. The matrix Θ_{t-1} represents the information set at time $t-1$.

Assuming the residuals from the VAR model are multivariate normally distributed, we have that the generalized impulse-response from a shock (one SD) to the jth residual is given by,

$$\text{GIR}_j = \frac{1}{\sqrt{\sigma_j^2}} A^i \Omega \eta_j \tag{11}$$

As mentioned before, the generalized impulse-responses are invariant to the ordering of the

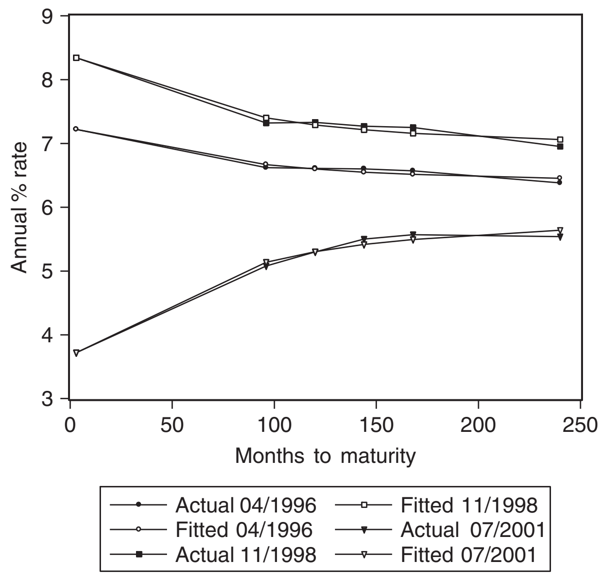

Fig. 1. Actual and fitted yield curves

variables in the system. Even though, this feature is an improvement with respect to the Cholesky decomposition based impulse- responses, the generalized version suffers from the same lack of economic content as in the other case. Indeed, for the first variable in the ordering of the system, both impulse-response functions are exactly the same.

V. Fitting the Models

The data

The data used corresponds to the Internal Rate of Return (IRR) for the Central Bank bonds, between April 1996 and July 2001. The data-set only includes bonds sold by the Central Bank, not secondary market transactions. All the rates are expressed in annual terms and indexed by the variation of the 'Unidad de Fomento' (index that follows last month inflation rate). The restriction of not using secondary market traded bonds, implies a reduced number of rates for the estimation of the yield curve. The maturities included are: 3, 96, 120, 144, 168 and 240 months. Actually, the sample period was selected in order to maximize the number of indexed bonds with different maturities.

All the bonds, except by the 3 months maturity, are coupon bonds. However, given that the coupon profile is almost the same for all the bonds, the 'coupon effect' is not supposed to be relevant for our analysis. This way, following a common practice in studying movements for the term structure, the yield curve is estimated directly from the IRR of Central Bank bonds.

Finally, since the maturities are not given at fixed intervals, the estimation of the yield curve implicitly is driven by the most active zone of the curve. That is, maturities between 96 and 168 months have a larger weight in fitting the model.

Estimation results

To implement the two-step method a fixed value for λ is required. The selected value is taken from Herrera and Magendzo (1997), where they estimated the inverse of λ_t at four different time period (the last week from March to June of 1996). Given that they considered maturities in quarters, instead of months, the inverse of their average estimate is divided by 3 to obtain $\lambda = 0.083$. It is worth noting that this fixed value is not far from the one used by Diebold and Li (2006), of $\lambda = 0.0609$, for the US case.

With this fixed value for λ, factor loadings are calculated and used as independent variables in the OLS cross-section regressions estimating the latent factors: $\hat{L}_t, \hat{S}_t$ and $\hat{C}_t$.

In Fig. 1 we can see the fit for three selected dates (start, middle and end of the sample period). As is evident, the Nelson–Siegel model is replicating almost exactly the shape of the actual yield curves.

In Figs 2–4 we plot the estimated level, slope and curvature along with the corresponding empirical proxies for them (called here as 'actual' level, slope and curvature). For the level we use the rate with the longest maturity ($R(240)$). The actual slope is calculated as the difference between short- and

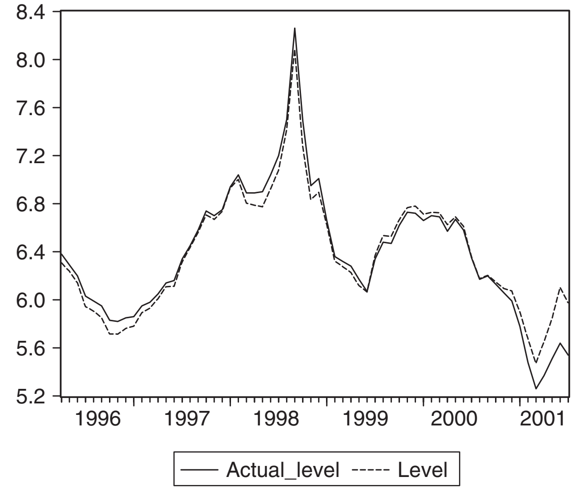

Fig. 2. Empirical and estimated level factor

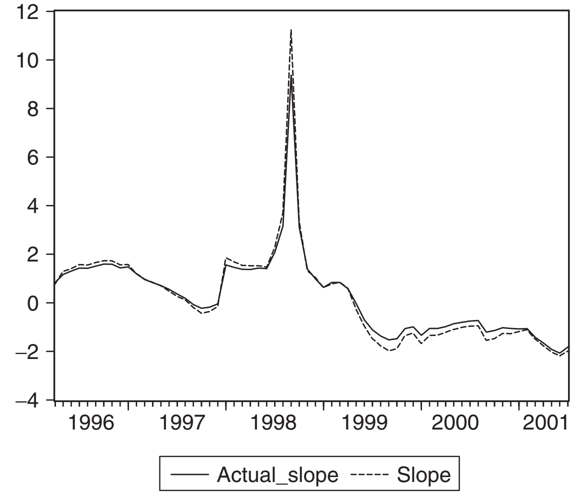

Fig. 3. Empirical and estimated slope factor

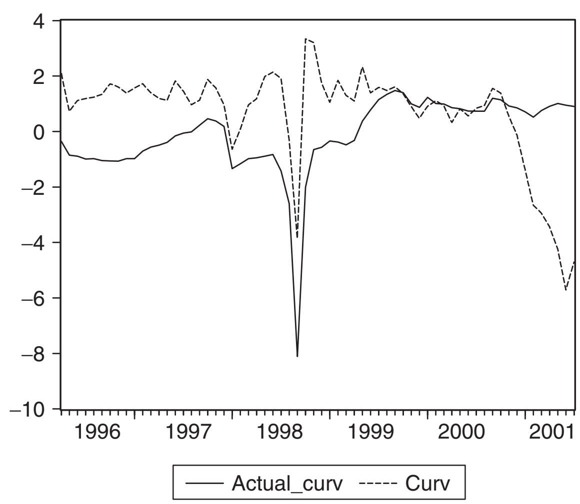

Fig. 4. Empirical and estimated curvature factor

	Level	Slope	Curv	Y	MPR	INF
Level(–1)	0.937765	0.586927	0.168473	0.263828	0.693594	–0.298581
	(0.07711)	(0.55175)	(0.41144)	(0.42092)	(0.13804)	(0.15217)
	[12.1615]	[1.06375]	[0.40947]	[0.62679]	[5.02471]	[–1.96220]
Slope(–1)	–0.008709	0.446802	0.638396	–0.144009	0.443399	0.018492
	(0.03062)	(0.21908)	(0.16337)	(0.16713)	(0.05481)	(0.06042)
	[–0.28446]	[2.03946]	[3.90779]	[–0.86166]	[8.08993]	[0.30606]
Curv(–1)	0.037581	0.042025	0.963465	0.038935	0.096912	0.090894
	(0.02163)	(0.15478)	(0.11542)	(0.11808)	(0.03872)	(0.04269)
	[1.73732]	[0.27151]	[8.34746]	[0.32973]	[2.50268]	[2.12931]
Y(–1)	0.034300	0.093386	–0.061403	0.835033	0.046522	0.071943
	(0.01451)	(0.10382)	(0.07741)	(0.07920)	(0.02597)	(0.02863)
	[2.36408]	[0.89953]	[–0.79317]	[10.5435]	[1.79121]	[2.51274]
MPR(–1)	–0.031197	0.153383	–0.412698	–0.096911	0.309422	0.022954
	(0.03758)	(0.26889)	(0.20051)	(0.20513)	(0.06727)	(0.07416)
	[–0.83021]	[0.57044]	[–2.05827]	[–0.47244]	[4.59971]	[0.30953]
INF(–1)	0.006244	0.338904	–0.150276	0.187525	0.169679	0.853486
	(0.02424)	(0.17348)	(0.12936)	(0.13235)	(0.04340)	(0.04784)
	[0.25754]	[1.95355]	[–1.16165]	[1.41694]	[3.90953]	[17.8389]
C	0.542536	–6.346397	2.124920	–1.935712	–0.962658	2.320728
	(0.57945)	(4.14620)	(3.09179)	(3.16305)	(1.03729)	(1.14347)
	[0.93630]	[–1.53066]	[0.68728]	[–0.61198]	[–0.92805]	[2.02954]
R-squared	0.872177	0.621819	0.736024	0.762440	0.968355	0.944349
Adj. *R*-squared	0.858481	0.581299	0.707741	0.736987	0.964964	0.938386
Sum sq. resids	1.899481	97.25426	54.07912	56.60070	6.087111	7.397108
S.E. equation	0.184172	1.317833	0.982700	1.005349	0.329694	0.363444
F-statistic	63.68405	15.34620	26.02338	29.95492	285.6025	158.3783
Log-likelihood	20.90582	–103.0702	–84.58350	–86.01905	–15.77885	–21.91869
Akaike AIC	–0.441455	3.494293	2.907413	2.952986	0.723138	0.918054
Schwarz SC	–0.203328	3.732419	3.145539	3.191112	0.961264	1.156180
Mean dependent	6.386915	0.240130	0.634341	0.019451	6.545714	4.825397
SD dependent	0.489572	2.036613	1.817760	1.960325	1.761388	1.464193

Fig. 5. Estimation results for yields-macro transition equation

long-term yields $(R(3) - (R(240))$.[1] Finally, the proxy for curvature is given by $2 * R(96) - (R(3) + R(240))$. The match between estimated and actual factors is remarkably good for level and slope (correlation coefficients are 0.98 and 0.998, respectively). For the case of curvature the case is less clear, even though the correlation between actual and estimated curvature is significant until the middle of 2000 (0.52), but low for the full sample (0.02).

Figure 5 presents the estimated coefficients from the VAR(1) of Equation 6 above. The order of the vector autoregressive process is supported by Schwarz (BIC) and Hannan–Quinn (HQ) information criteria both selecting 1 lag, while Final Prediction Error (FPE) and Akaike (AIC) information criteria selected 3 and 5, respectively. The first order model is preferred based on parsimony considerations. Figure 6 shows the correlograms and cross-correlograms, indicating the residuals from the VAR(1) can be considered as well behaved.

Figure 7 plot generalized impulse-response functions along with Monte Carlo-based confidence intervals. The responses are consistent with a yields only models, for latent factors, as well as with a small macro model considering the same basic macro variables included here. Figures 8 and 9 present

[1] This is the negative of the usual slope definition, which is the difference between the long and the short yields.

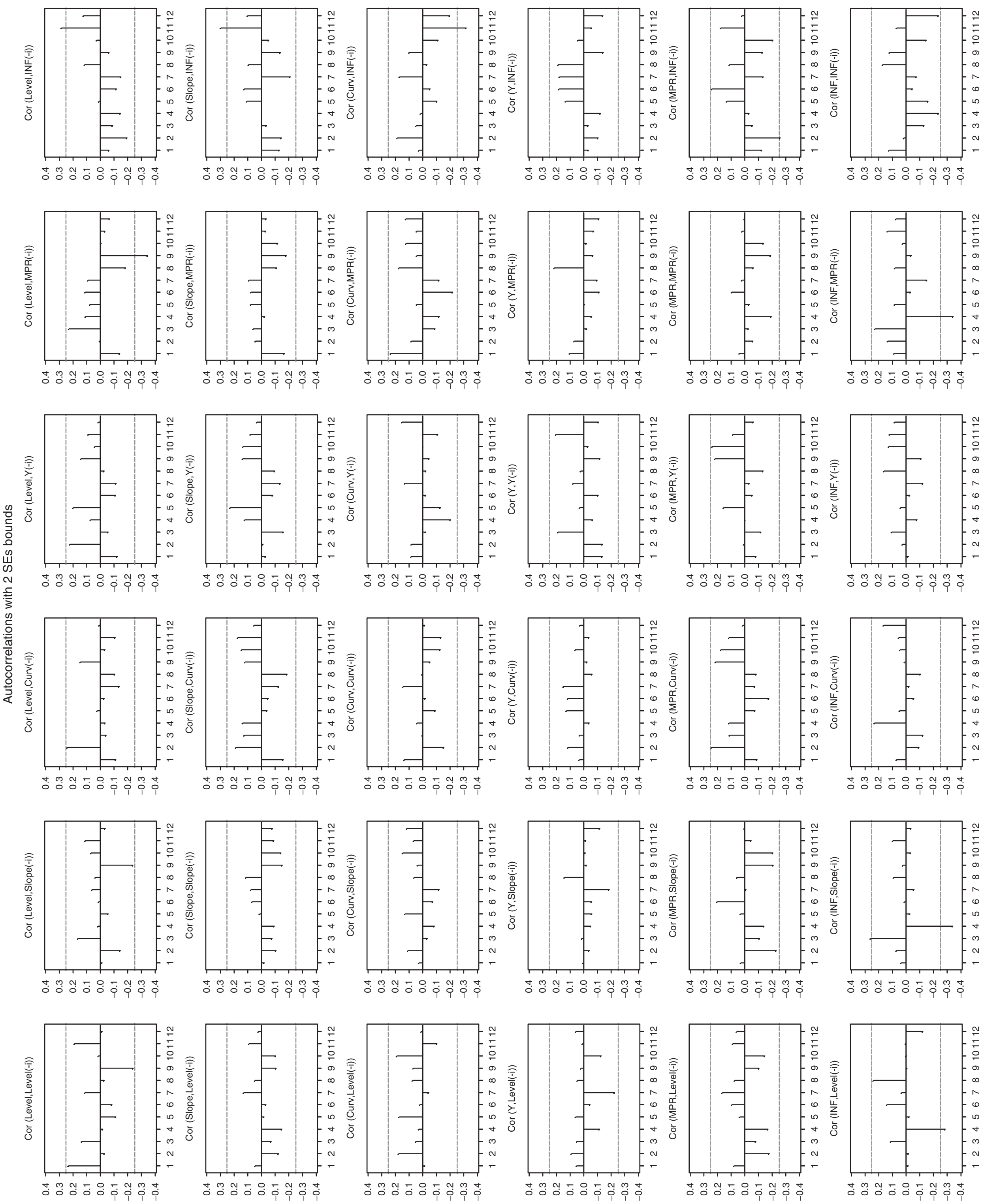

Fig. 6. Correlograms and cross-correlograms for VAR(1)

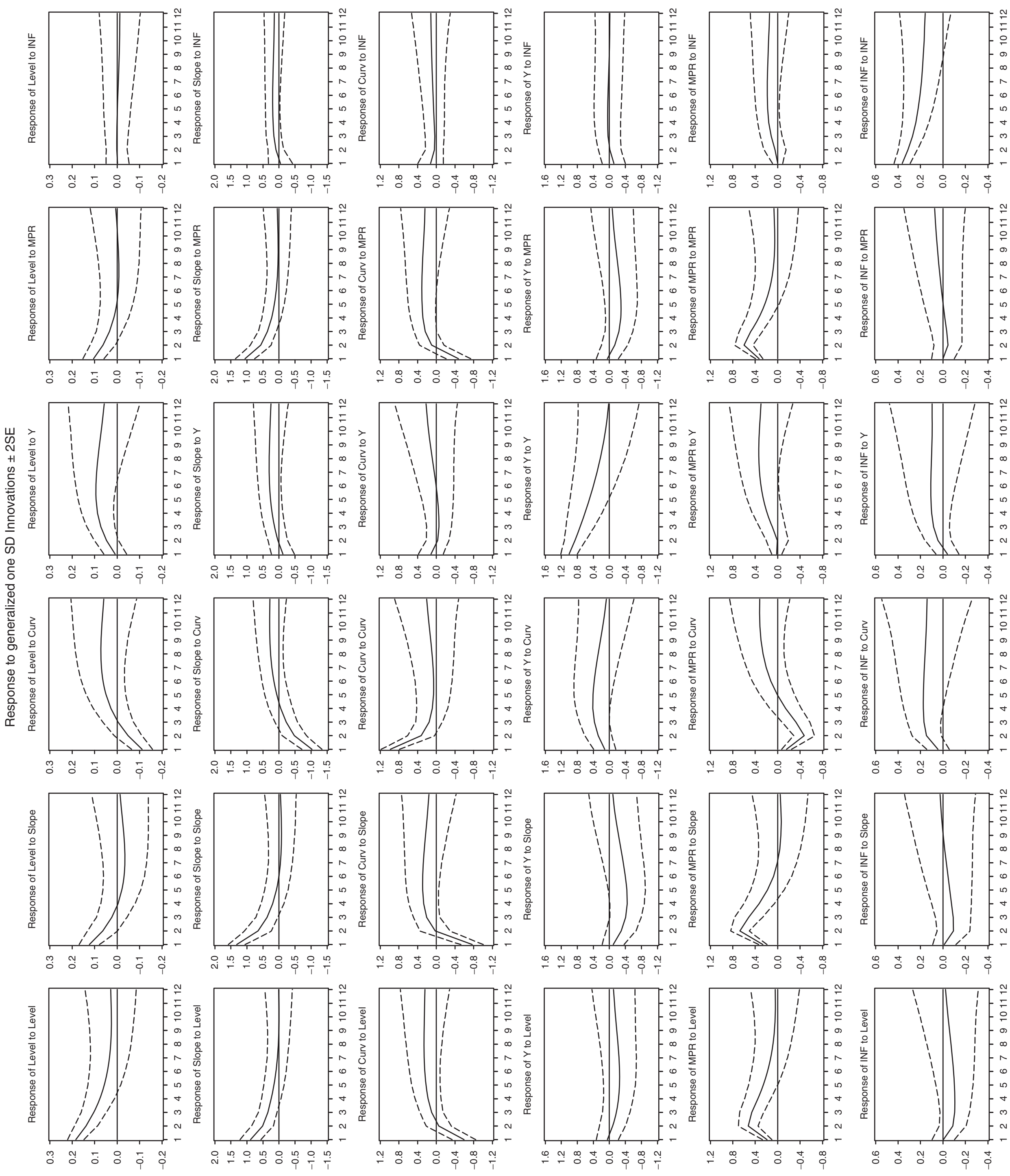

Fig. 7. Impulse-response functions for yields-macro model

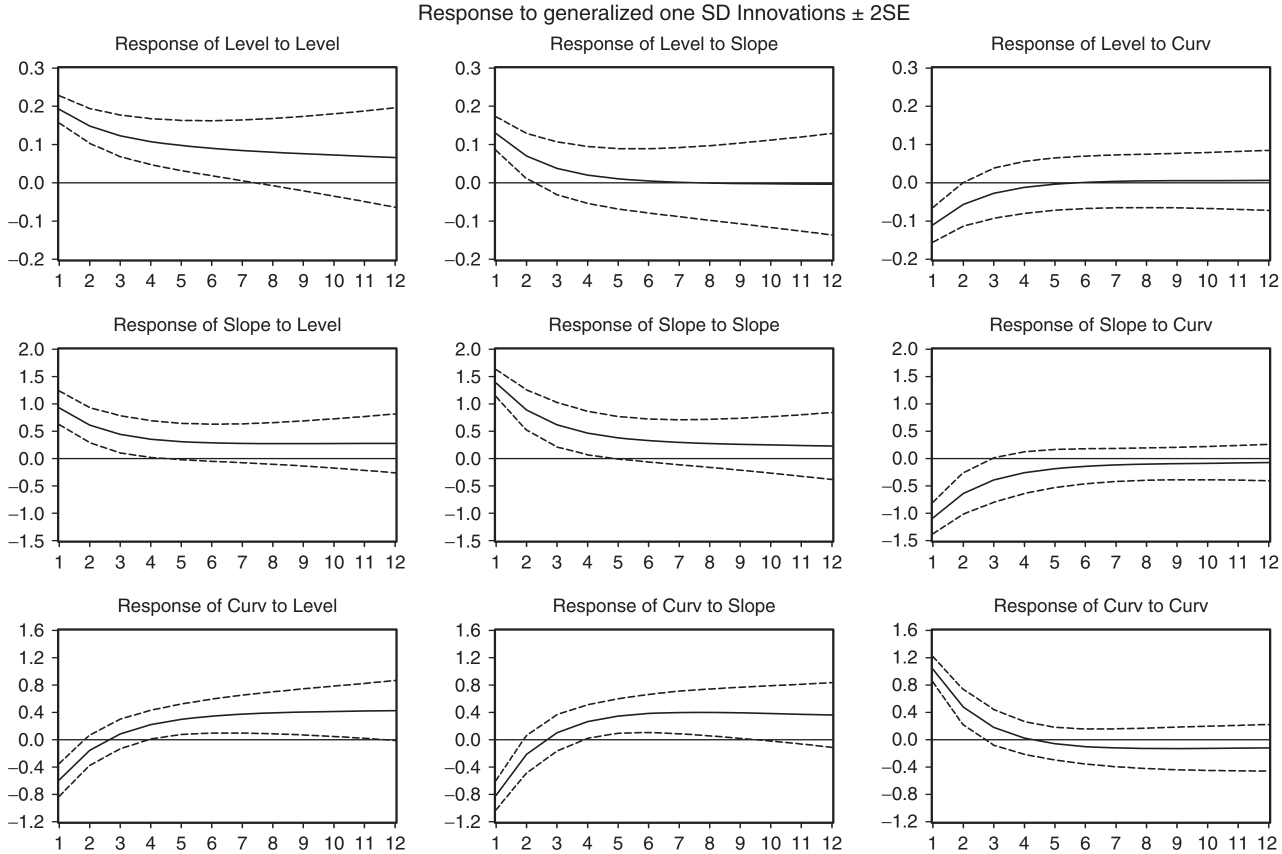

Fig. 8. Impulse-response functions for yields-only model

impulse–response functions for the yields-only and macro-only models, in order to check consistency with our aggregate yields-macro model.

In terms of macro responses to latent factors, the real activity shows a transitory negative response to an increase in the slope of the yield curve. This reaction to the slope impulse- is consistent with an increase in the short rates relative to long rates, producing a hump-shaped response on output as observed after a monetary policy shock. The close link between slope and the monetary policy rate is confirmed by the direct response of MPR_t to the slope impulse-.

As mentioned before, the monetary policy rate responds to the slope, as well as to the level and curvature factors. This could be evidence in favour of a Central Bank reaction function taking into account the bond market reactions to monetary policy shocks, or just the anticipation of the bond market players to macroeconomic news.

In terms of yield curve responses to macro shocks, level is transitory affected by output capturing a possible future reaction of the Central Bank increasing the monetary policy rate. On the other hand, slope presents a positive but declining response to a MPR_t impulse-. This is consistent with the close link between these two variables mentioned before. The curvature factor exhibits a transitory negative effect from the monetary policy rate, which is consistent with the adjustment of medium and long rates to the initial increase in the short rate. Finally, it is good pointing out that the nonsignificant effect of inflation on the yield curve factors is not surprising, given that we are modelling the yield curve in real terms, so that any inflation news must be affecting the latent factors only through the other macro variables.

Comparing to Kalman Filter estimation

In order to have a first glance of the potential loss in not using the optimal simultaneous estimation approach, based on the Kalman Filter method, we compare the smoothed estimates for the latent factors

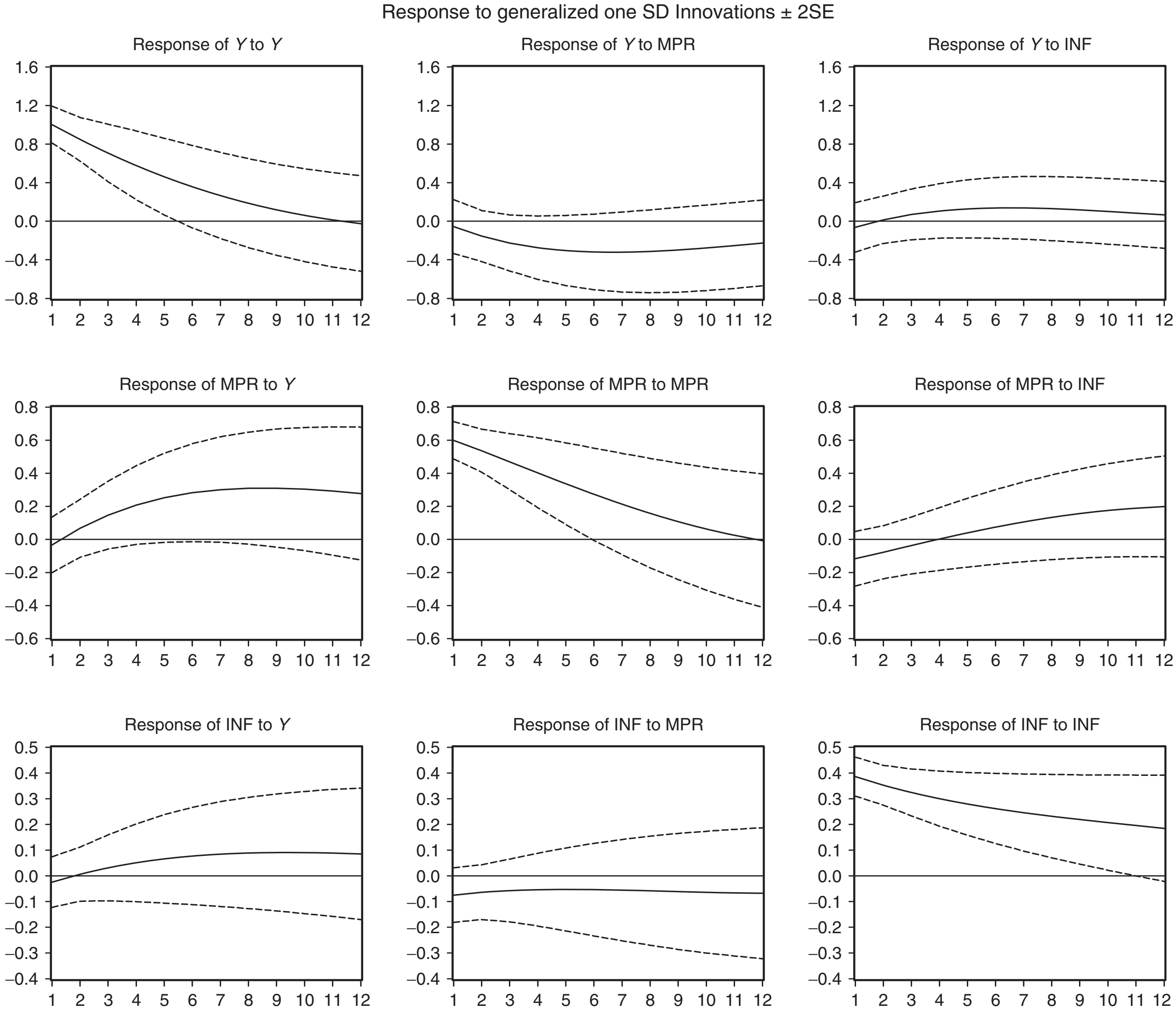

Fig. 9. Impulse-response functions for macro-only model

with the corresponding factors estimated by using the two-step approach, for the simplest case of the yields-only model.

As initial coefficients for the filter we use the parameters from the two-step method, as well as the initial values for the state vector and the corresponding Mean Squared Error matrix.

Figures 10–12 plot the estimated latent factors from the two alternative approaches (the smoothed factors from the Kalman Filter estimation are called level_s, slope_s and curv_s). The graphs give clear evidence that the optimality loss by using the two-step method is not at all significant. However, in terms of the curvature factor it seems possible to do a better job by implementing the estimation based on the Kalman Filter approach.

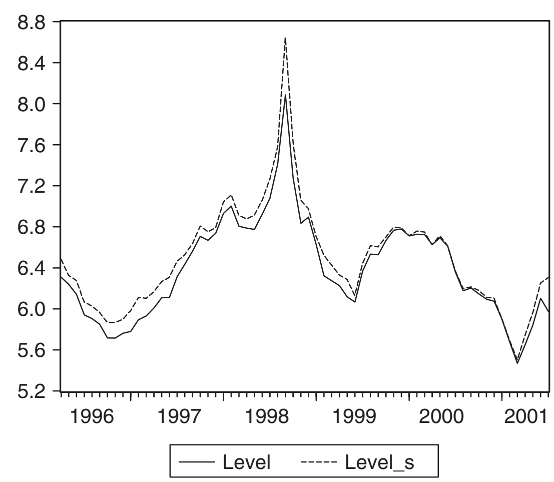

Fig. 10. Two-step level versus smoothed level

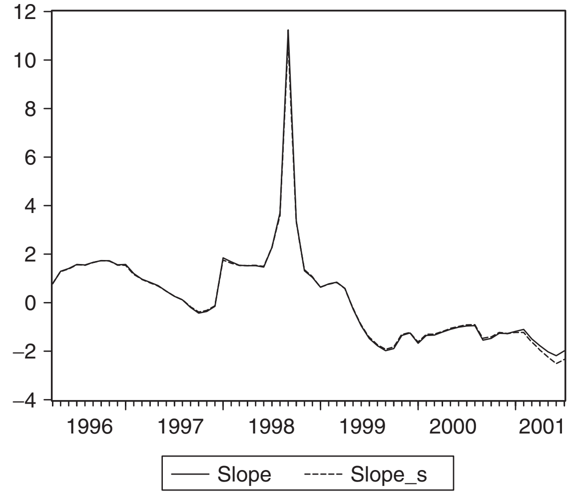

Fig. 11. Two-step slope versus smoothed slope

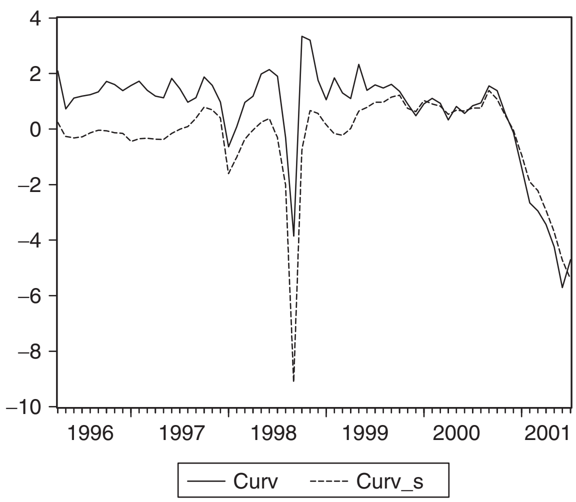

Fig. 12. Two-step curvature versus smoothed curvature

VI. Conclusions

This article estimates a dynamic model for the yield curve incorporating latent and macro factors to represent the term structure of the real interest rates. The representation of the yield curve is based on the popular latent factor model of Nelson and Siegel (1987), but under a dynamic interpretation due to Diebold and Li (2006). After assuming the data generating process for the latent and macro factors can be represented by a VAR process, the yields-macro model can be regarded as a state-space representation and estimated by a Kalman Filter approach or by using a simplified two-step procedure proposed by Diebold and Li (2006). This article follows the simple two-step method and makes a comparison check with the Kalman Filter estimation, concluding that the basic intuition of the results is not significantly affected by the use of the simplified approach.

Estimation results give support to the dynamic interaction between yield curve latent factors and macroeconomic variables. In particular, monetary policy implemented by the Central Bank seems to be influenced by the market players given the significant response of the monetary policy rate to the yield curve factors as shown by impulse–response functions. In addition, the level and slope of the yield curve seems to be responsive to real activity and monetary policy shocks, issues that should be considered by monetary authorities given the dependency of monetary policy effectiveness on the shape of the yield curve.

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