The Euro Area Yield Curve: an Analysis with Nelson-Siegel type models

Alessandro Carboni and Andrea Carboni*

Master in Economics and Banking, University of Siena, Piazza San Francesco, 8, 53100, Siena, Italy

Abstract

We present Nelson-Siegel-type yield curves for the Euro Area, by studying the in-sample fit and the out-of sample forecasting properties. Moreover, we add macro variables (output gap, EONIA and HICP or alternatively Eurocoin) to estimate the interactions with yield curve factors (level, slope and curvature), following the rationale of Diebold *et al* (2006). We use a two step procedure as in Diebold and Li (2006) and we find three interesting results. The Svensson model has better in-sample fit properties with respect to the Nelson-Siegel model. There exist interactions among both the level and the curvature with macroeconomic variables, while on the other hand, the slope interacts with all macroeconomic variables. The addiction of a business cycle indicator like Eurocoin gives better results with respect to both yields-only and alterative yields-macro models, confirming the success of parsimonious models for out-of sample forecasting.

JEL Classification: E43**,** E44, E47

Keywords: Nelson-Siegel curves, Eurocoin, macrofinancial models

* Corresponding authors. Postal address: Master in Economics and Banking, University of Siena, Piazza San Francesco, 8, 53100, Siena, Italy. E-mail: andreacarbo123@gmail.com. All errors and omissions remain our own.

1. Introduction

Investment banks and Central banks rely heavily on the Nelson and Siegel (1987) model to fit and forecast yield curves, as shown in BIS (2005). The European Central Bank (ECB) publishes its daily yield curves based on Söderlind and Svensson (1997) model, which is an extension of the seminal Nelson-Siegel work. Diebold and Li (2006) present a dynamic version of the original framework which has become the usual workhorse in modelling and forecasting yield curves. Moreover, Diebold *et al.* (2006) among others (see Ang *et al.* (2003), Hördal *et al.* (2004) and Rudebusch and Wu (2003) for example) show and estimate a yield curve model that integrates macroeconomic and financial factors. Following these contributions, we present an analysis on European data: on the one hand, we estimate Nelson-Siegel type models by studying the in sample-fit properties of the yield curve, while on the other, we analyse the interactions between macroeconomic variables and financial factors through a vector autoregression (VAR). Finally, we present out-of sample forecasts, by using both yields-only and yields-macro models. The rest of the Letter is organized as follows: Section 2 describes the model, Section 3 data and estimation methodology, while Section 4 discusses results. We offer conclusions in Section 5.

2. Model

We study the Euro Area yield curve by using the classic Nelson and Siegel (1987) model, as re-parametrized by Diebold and Li (2006) in a dynamic version. We then add the Svensson (1994) model, which is the well-known four factors extension:

(1) $$y_t(\tau) = L_t + S_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + C_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right),$$

(2) $$y_t(\tau) = L_t + S_t\left(\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau}\right) + C_t^1\left(\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau} - e^{-\lambda_1\tau}\right) + C_t^2\left(\frac{1-e^{-\lambda_2\tau}}{\lambda_2\tau} - e^{-\lambda_2\tau}\right).$$

Equation (1) is the Diebold and Li (2006) version: $y_t(\tau)$ is the zero-coupon yield with τ the time to maturity, L_t, S_t, C_t and λ are model parameters, that can be interpreted respectively as level, slope and curvature (given their factor loadings in parenthesis), while λ is the time decay parameter, which governs the exponential decay rate.

Equation (2) is the modified version of the Svensson model, with two curvature factors (C_t^1 and C_t^2) and two time decay parameters (λ_1 and λ_2), as in Christensen *et al* (2009). The second curvature factor is introduced to obtain a better fitting of the yield curve, as to avoid that the factor loadings for the slope and the curvature decay rapidly to zero.

Moreover we construct a VAR(1) model, which is an extension of the yields-only model developed in (1), expressed by the following relation:

$$(3) \qquad f_t = \mu + A_t f_{t-1} + \varepsilon_t,$$

where f_t is a vector of our set of yield curve and macro variables, f_{t-1} contains one lagged values, A_t is an appropriate matrix of coefficients, while ε_t is a vector of residuals. This framework allows us to evaluate the interactions among yield factors and macro variables, and vice-versa.

3. The data and estimation procedure

We use zero-coupon bond monthly data for the Euro Area Yield curve, from March 2001 to August 2009, gathered from Reuters. The maturities considered are 1, 3, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 months. Our macro dataset includes Eurocoin, the monthly indicator of the Euro Area business cycle published by Bank of Italy and CEPR, the EONIA, the inflation rate, as measured by the year-on-year growth rate of the Harmonized index of consumer prices (HICP) and a measure of output gap.[1,2]

We develop a two-step procedure estimation, as in Diebold and Li (2006) to obtain unobserved factors in Equation (1): we first use OLS, and secondly we perform a VAR(1) for the factors, considered either alone or with macro variables, following the idea of Diebold *et al* (2006). Then we use Equation (2) only for the in-sample fit, estimating unobserved factors by OLS. We do not impose no-arbitrage restrictions following Diebold and Li (2006).[3]

Time decay parameters are chosen in this way: λ_1 is chosen to maximize the first curvature factor at 3 months, λ_2 to maximize the second curvature factor at 120 months, while λ is chosen as the average of the values that maximize the curvature at 24 and 36 months.

We can define the vector f_t in equation (3) in three ways, according to interactions we want to test: $f_t = (L_t\ S_t\ C_t)'$, $f_t = (L_t\ S_t\ C_t\ Output\ gap_t\ Inflation_t\ EONIA_t)'$ or alternatively $f_t = (L_t\ S_t\ C_t\ Eurocoin_t)'$. We then perform impulse response functions for the factors together with the macro variables, and finally we run out-of-sample forecasts.

4. Results

We present our results dividing them in goodness of fit, interactions between yield factors and macro variables and finally in out-of sample forecasting performance.

4.1 Fitting the yield curve

Figure 1, 2 and 3 present respectively the factor loadings and the raw yield curves versus the three and four factors fitted yield curves for some selected dates. We obtain $\lambda = 0.0623$, $\lambda_1 = 0.5960$ and $\lambda_2 = 0.0160$. Our estimation method for the time decay parameters is different from the Söderlind and Svensson (1997) model, which is used daily by the ECB to publish the Euro Area Yield curve.[4]

We see that the addiction of a second curvature factor gives a better in-sample fit, which is particularly evident for the short end of the yield curve. In Table 1 we present statistics on raw yields, while in Table 2 residuals from Equation (2). The average yield curve is upward sloping and short term rates are more volatile and less persistent than long term ones. Moreover, the residual sample autocorrelations indicate that pricing errors are persistent and presumably arise from persistent tax and/or liquidity effects, as suggested in Diebold and Li (2006).

4.2 Macroeconomic and yield curve interactions

In Table 3 we present results for the VAR(1). Our idea is that, although the Svensson model produces a better in-sample fit, it is well known in the literature that parsimonious models are often more successful for out-of-sample forecasts.

In the yields-only VAR(1) all the Nelson-Siegel own lagged factors are statistically significant at 99 percent. Moreover, the lagged curvature is significant for the level (at 90 percent), while the lagged level for the slope (at 95 percent).

In the yields-macro VAR(1) we note that: Eurocoin helps explain both slope and curvature factors, while it is not significant for the level. Moreover both output gap and EONIA are significantly different from zero for the slope, and only EONIA for the curvature. Vice-versa, level, slope and curvature together explain both the output gap and EONIA, even if with an unexpected sign: if level is a proxy for future inflation, EONIA should increase. On the contrary we have the right response from the output gap. This is the same for the relation among slope and both EONIA and output gap. Finally, lagged values of output gap and EONIA together are able to explain their contemporaneous values, while HICP is explained only by its first lagged value, together with the same lag for output gap.

To complete our analysis, we present impulse response functions in Figure 3.[5] We define four groups of responses: macro responses to macro shocks, macro responses to yield curve shocks, yield curve responses to macro shocks and yield curve responses to yield curve shocks.

For the first one, an increase in the EONIA has a marginal impact on the output gap and vice-versa the EONIA rises with output gap. On the other hand, the EONIA does not react in a properly manner, as a consequence of an increase in inflation. The evidence shows that the Taylor principle is not respected, although the response of EONIA to output gap shocks is in line with Taylor-type reaction functions. Inflation exhibits the so-called “price puzzle”, with an initial upward response of inflation to a shock in the EONIA. Moreover, inflation shows an aggregate supply response to increased output gap.

For the second one, yield curve components add some interesting elements to the macro responses. An increase in the level produces an increase in the EONIA and in the output gap. Given the relation between the level and perceived future inflation, as suggested by Rudebusch and Wu (2004), a shock in the level generates an initial increase in the output gap, which is completely absorbed. On the other hand, the slope factor produces a marginal effect on macro variables dynamics. Finally, a curvature shock generates an initial rise in all macroeconomic variables, even if it is completely absorbed after 10 and 15 months for the output gap and the EONIA, respectively.

For the third one, shocks in the EONIA and in the HICP produce marginal effects on level and curvature factors. The slope factor responds directly to positive shocks in the EONIA, while indirectly to positive shocks in the inflation. It seems interesting that the response of the slope to these two kinds of shocks is similar to the same encountered by the output gap: this confirms that the slope factor is linked strongly to economic activity. If the slope factor is empirically represented by the difference between the short term and long term interest rates, a positive shock on output gap reflects the reaction of the Central bank which increases the policy rate.[6]

For the fourth one, there is a negligible impact of the slope on all yield factors. On the other hand, if we consider a level surprise as a surprise in expected inflation, we can see that this is associated with a loosening of policy interest rates, as measured by a more pronounced upward sloping of the yield curve, up to seven months.

4.3 Out-of sample forecasting performance

We produce yield out-of sample forecasts based on our yields-only model and on our yields-macro model, for the period June 2007 - August 2009. In Table 4 we compare h-month-ahead forecasting results, for all maturities: our chosen measure of forecasting performance is the ratio of the root mean squared error (RMSE) of each model to the RMSE of a random walk. We decide for this period because it has unprecedented aspects of uncertainty: in this framework, the random walk should produce better forecasts, so this could be a good test for our models.

Results show that our models have better results only for the short-end of the yield curve: the macro-finance model with Eurocoin beats the random walk for 3 and 6 months yields for all h periods ahead considered, and also for the 1 month yield 6 and 12 months ahead. Empirically, our results suggest that the addiction of macroeconomic variables produces better results: we use the business cycle indicator Eurocoin which is a composite indicator as a substitute of our three macro variables. Hence, this confirms the idea that parsimonious models have better out-of sample forecasting performance.

5. Conclusions

We present Nelson-Siegel-type yield curves for the Euro Area, by studying the in-sample fit and the out-of sample forecasting properties. Moreover, we add macro variables (output gap, EONIA and HICP or alternatively Eurocoin) to estimate the interactions with yield curve factors (level, slope and curvature), following Diebold *et al* (2006). We found three important results: first, the Svensson model has a better goodness of fit relative to the Nelson and Siegel model, as confirmed by the choice of the ECB for the Euro Area yield curve. Second, there exist interactions among both the level and the curvature with macroeconomic variables, while on the other hand, the slope interacts with all macroeconomic variables. Finally, we find that the addiction of Eurocoin performs better out-of sample forecasts with respect to both yields-only model and to the other yields-macro model, confirming that parsimonious models are often more successful for out-of sample forecasting.

References.

Altissimo, F., Cristadoro, R., Forni, M., Lippi, M. and Veronese, G. F. (2007) New Eurocoin: Tracking economic growth in real time, *Banca d'Italia Temi di discussione*, **631**.

Ang, A., M. Piazzesi and M. Wei (2006) What does the yield curve tell us about GDP growth?, *Journal of Econometrics,* **131**, 359--403.

Bank for International Settlements (2005) Zero-coupon yield curves: technical documentation, *BIS Papers*, **25**.

Christensen, J. H. E., F. X. Diebold and G. D. Rudebusch (2009) An arbitrage-free generalized Nelson-Siegel term structure model, *Econometrics Journal*, **12**, 33--64.

Diebold, F. X. and C. Li (2006) Forecasting the term structure of government bond yields, *Journal of Econometrics*, **130**, 337--64.

Diebold, F. X., G. D. Rudebusch and S. B. Arouba (2006) The macroeconomy and the yield curve: a dynamic latent factor approach, *Journal of Econometrics*, **131**, 309--38.

Nelson, C. R. and A. F. Siegel (1987) Parsimonious modeling of yield Curves, *Journal of Business*, **60** (4), 473--89.

Hördal, P., O. Tristani and D. Vestin (2004) A joint econometric model of macroeconomic and term structure dynamics, *ECB Working paper*, 405.

Nyholm, K. (2008) Strategic asset allocation in fixed-income markets: a MATLAB-based user's guide, John Wiley & Sons, Ltd, Wiley Finance Series.

Ravn, M. e H. Uhlig (2002) On adjusting the HP-filter for the frequency of observations, *Review of Economics and Statistics*, **84**, 371--80.

Rudebusch, G.D and T. Wu (2003), A macro-finance model of the term structure, monetary policy, and the economy, *Federal Reserve Bank of San Francisco Working Paper Series*, **17**.

Sims, C. A. and T. Zha (1999) Error bands for impulse responses, *Econometrica*, **67**, 1113--56.

Söderlind, P., and L. E. O. Svensson (1997) New techniques to extract market expectations from financial instruments, *Journal of Monetary Economics*, **40**, 383--429.

Svensson L. E. O. (1994) Estimating and interpreting forward interest rates: Sweden 1992-1994, *CEPR Discussion Paper*, **1051**.

Tables and Graphs.

Maturity (Months)	Mean	Std. Dev.	Min.	Max.	$\rho(1)$	ρ (12)	ρ (30)
1	2.996	1.117	0.329	5.109	0.977	0.098	-0.529
3	4.343	0.923	0.801	5.376	0.975	0.068	-0.007
6	4.306	0.859	1.060	5.180	0.970	0.045	-0.006
12	4.203	0.846	1.168	5.307	0.973	0.104	-0.031
24	4.305	0.778	1.714	5.328	0.972	0.248	-0.063
36	4.430	0.697	2.140	5.265	0.973	0.339	-0.069
48	4.555	0.646	2.435	5.184	0.975	0.430	-0.080
60	4.677	0.618	2.683	5.105	0.976	0.507	-0.092
72	4.802	0.607	2.891	5.196	0.979	0.569	-0.102
84	4.923	0.604	3.071	5.340	0.980	0.615	-0.095
96	5.028	0.604	3.217	5.456	0.981	0.641	-0.080
108	5.124	0.606	3.343	5.552	0.981	0.658	-0.057
120	5.193	0.598	3.464	5.638	0.981	0.668	-0.046

Table 1. Descriptive statistics, raw yield curve.

Maturity (Months)	Mean	Std. Dev.	Min.	Max.	$\rho(1)$	ρ (12)	ρ (30)
1	-0.009	0.023	-0.040	0.051	0.925	0.545	-0.302
3	0.044	0.095	-0.206	0.169	0.927	0.555	-0.296
6	-0.079	0.108	-0.219	0.222	0.934	0.601	-0.259
12	0.070	0.030	-0.033	0.169	0.665	0.373	-0.207
24	0.081	0.058	-0.092	0.148	0.928	0.590	-0.375
36	-0.024	0.033	-0.125	0.047	0.742	0.316	-0.221
48	-0.099	0.036	-0.168	0.034	0.808	0.238	-0.280
60	-0.129	0.038	-0.178	0.006	0.817	0.277	-0.242
72	-0.109	0.030	-0.144	-0.008	0.848	0.243	-0.227
84	-0.053	0.015	-0.073	-0.009	0.821	0.219	-0.343
96	0.017	0.014	-0.016	0.044	0.853	0.228	-0.378
108	0.105	0.033	-0.011	0.159	0.788	0.146	-0.068
120	0.186	0.053	-0.003	0.296	0.812	0.056	0.020

Table 2. Descriptive statistics, yield curve residuals from Equation (2).

Exp. variables		**Constant**	**NS1**	**NS2**	**NS3**	**€-coin**	**Output gap**	**EONIA**	**HICP infl.**
Dep. variables	DW								
NS1 (level)	1.947***	0.349	0.933	-0.001	0.029	-	-	-	-
		(0.223)	(0.044)***	(0.015)	(0.016)*				
	1.947***	0.339	0.936	-0.001	0.030	-0.004	-	-	-
		(0.187)*	(0.039)***	(0.017)	(0.018)	(0.050)			
	1.906***	0.199	0.869	-0.078	0.050	-	-0.013	0.105	0.033
		(0.151)	(0.104)***	(0.113)	(0.030)*		(0.013)	(0.096)	(0.043)
NS2 (slope)	1.711***	-0.884	0.169	1.022	0.022	-	-	-	-
		(0.411)**	(0.071)**	(0.033)***	(0.014)				
	1.945**	-0.176	0.009	0.983	-0.009	0.241	-	-	-
		(0.258)	(0.047)	(0.031)***	(0.016)	(0.080)***			
	2.133***	-0.005	-0.365	0.425	0.058	-	0.052	0.323	0.051
		(0.261)	(0.086)***	(0.123)***	(0.023)**		(0.014)***	(0.073)***	(0.064)
NS3 (curvature)	1.734***	-1.145	0.178	-0.105	0.881	-	-	-	-
		(1.056)	(0.169)	(0.136)	(0.043)***				
	1.732***	-0.974	0.139	-0.115	0.874	0.058	-	-	-
		(1.067)	(0.206)	(0.157)	(0.068)***	(0.268)***			
	1.675***	-0.903	0.686	0.498	0.798	-	0.024	-0.529	-0.224
		(1.051)	(0.406)*	(0.476)	(0.071)***		(0.049)	(0.286)*	(0.138)
€-coin	0.346	0.087	-0.040	-0.072	-0.000	1.017	-	-	-
		(0.118)	(0.025)	(0.012)***	(0.008)	(0.033)***			
Output Gap	2.207***	0.488	-1.223	-1.819	0.333	-	1.079	1.218	-0.153
		(1.806)	(0.432)***	(0.560)***	(0.103)***		(0.056)***	(0.365)***	0.199)
EONIA	1.776***	0.117	-0.204	-0.338	0.046	-	0.045	1.129	0.023
		(0.160)	(0.093)**	(0.113)***	(0.023)***		(0.007)***	(0.088)***	(0.025)
HICP infl.	1.645	0.455	-0.122	-0.108	0.088	-	0.027	0.182	0.776
		(0.410)	(0.181)	(0.202)	(0.040)**		(0.015)*	(0.174)	(0.058)***

Table 3. VAR(1) estimates 2001m3 – 2009m8. */**/*** means 90%, 95% and 99% significance level. Standard errors are in parenthesis. €-coin stands for Eurocoin.

	Yields only				Yields with Eurocoin				Yields with Macro variables			
Maturities\ h	1	3	6	12	1	3	6	12	1	3	6	12
1	1,392	1,222	1,101	**0,946**	1,307	1,091	**0,970**	**0,873**	1,373	1,332	1,469	1,771
3	1,043	1,036	**0,960**	**0,933**	**0,916**	**0,915**	**0,871**	**0,865**	**0,977**	1,153	1,349	1,764
6	**0,899**	**0,915**	**0,897**	1,052	**0,854**	**0,872**	**0,878**	**0,940**	**0,929**	1,110	1,328	1,866
12	1,045	**0,991**	1,026	1,391	1,044	**0,998**	1,004	1,107	1,073	1,161	1,349	2,000
24	1,096	1,157	1,312	1,908	1,083	1,113	1,184	1,345	1,065	1,186	1,425	2,325
36	1,060	1,218	1,445	2,190	1,030	1,142	1,264	1,479	**0,995**	1,182	1,471	2,535
48	1,045	1,226	1,486	2,318	1,016	1,149	1,300	1,549	**0,981**	1,182	1,491	2,601
60	1,040	1,203	1,470	2,352	1,023	1,141	1,302	1,581	1,005	1,184	1,494	2,592
72	1,038	1,168	1,427	2,315	1,037	1,128	1,289	1,575	1,042	1,194	1,496	2,531
84	1,028	1,124	1,370	2,240	1,036	1,105	1,265	1,546	1,059	1,200	1,498	2,461
96	1,013	1,076	1,296	2,122	1,021	1,074	1,224	1,491	1,055	1,198	1,489	2,366
108	1,014	1,034	1,223	1,988	1,013	1,040	1,178	1,421	1,047	1,190	1,479	2,271
120	1,034	1,006	1,156	1,856	1,020	1,010	1,129	1,348	1,043	1,176	1,464	2,192

Table 4. Out-of sample h-month ahead forecasting results. H reflects 1, 3, 6 and 12 months ahead. Maturities are in months. Ratio of the RMSE of our models to the RMSE of the random walk forecast. In bold font we show values under one.

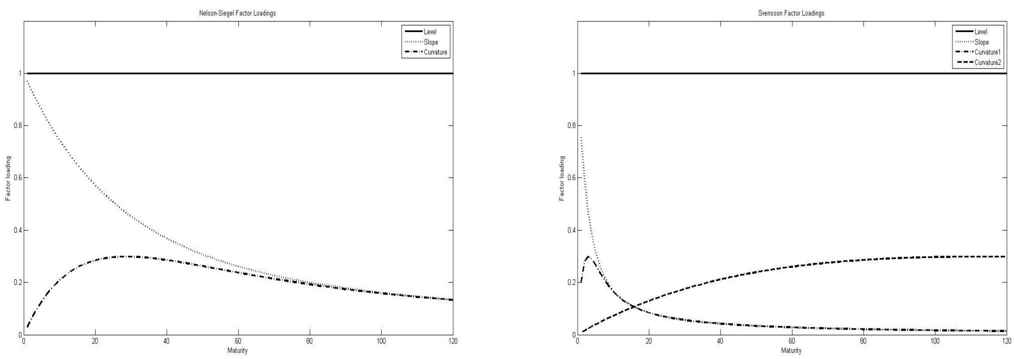

Figure 1. Factor loadings in the yield functions: $\lambda = 0.0623$ for Equation (1); $\lambda_1 = 0.5960$ and $\lambda_2 = 0.0160$ for Equation (2).

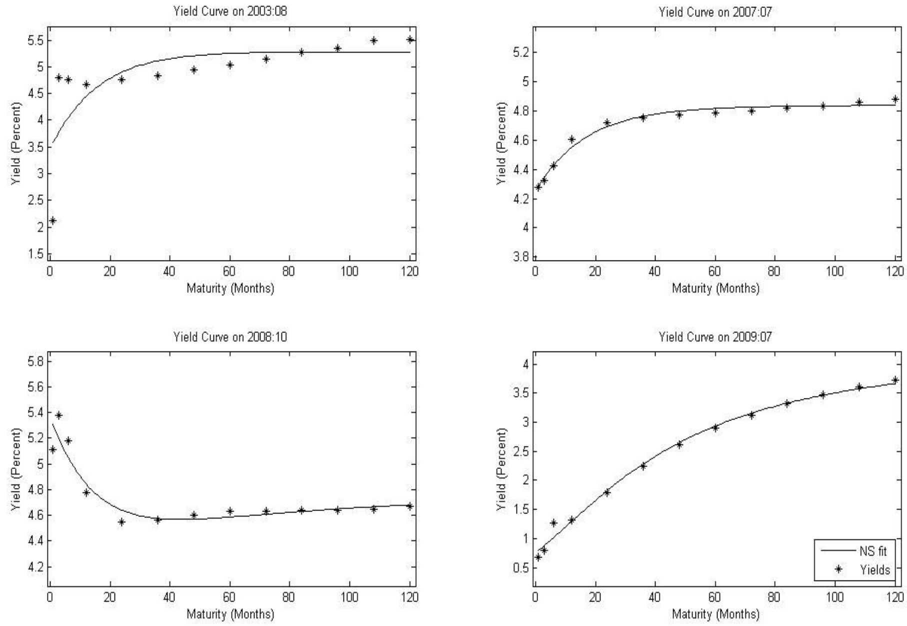

Figure 2. Fitted Nelson-Siegel yield curves on four specific dates.

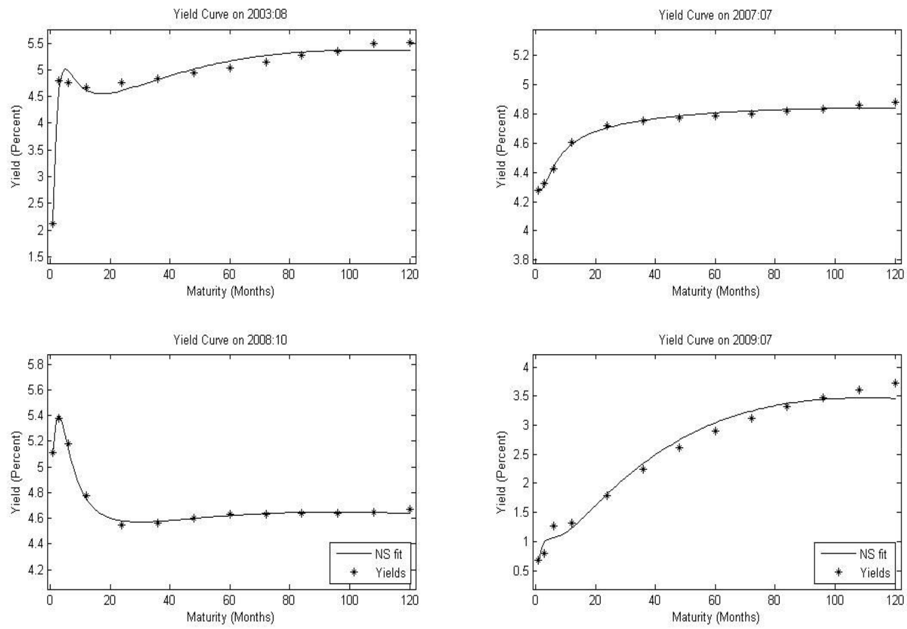

Figure 3. Fitted Svensson yield curves on four specific dates.

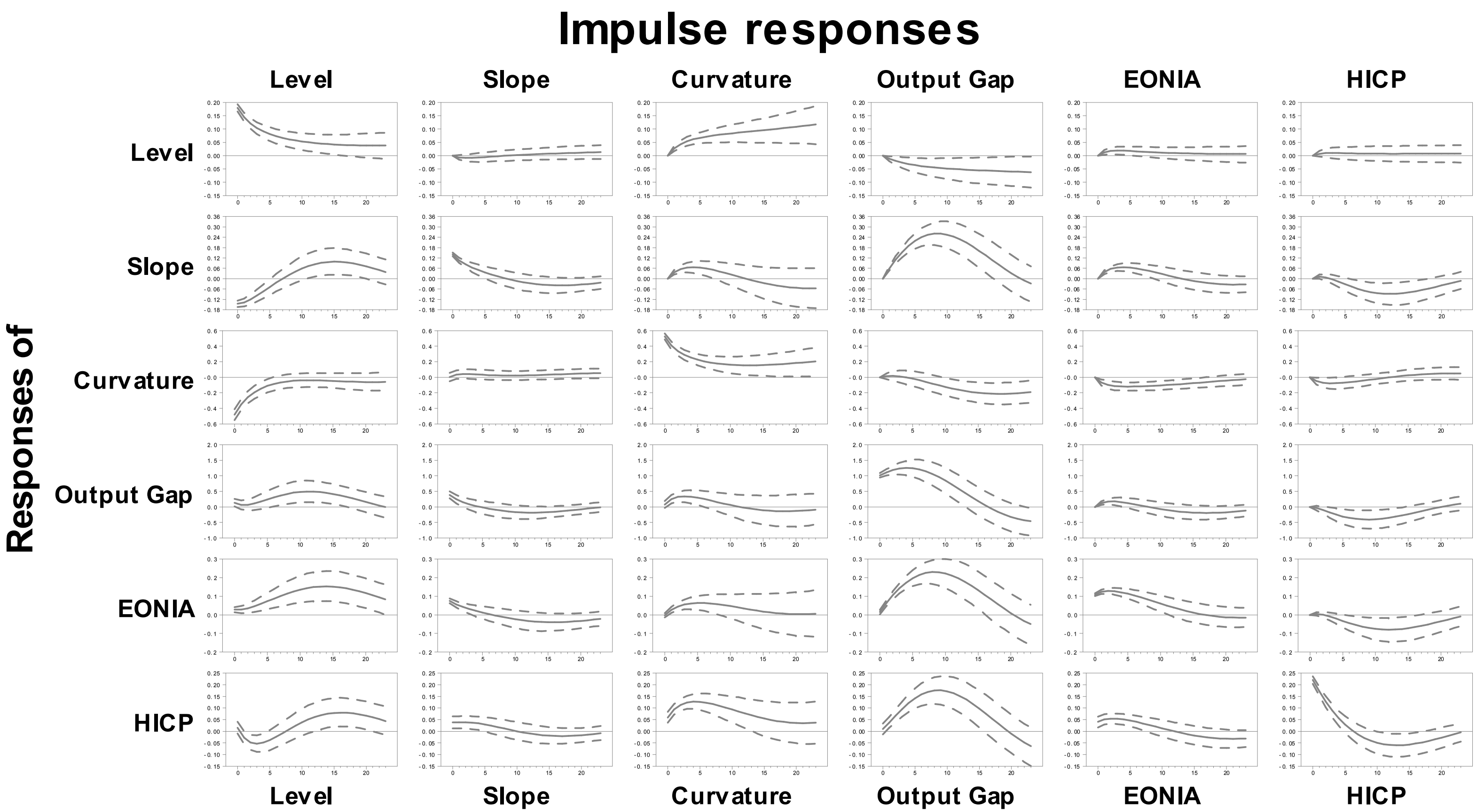

Figure 4. Impulse responses of yields-macro model, with the Output gap, the EONIA rate and the HICP.

Notes.

[1] We define output gap as the distance between the seasonally adjusted index of industrial production and its potential value, estimated by using the Hodrick-Prescott filter with smoothing parameter $\lambda = 129{,}600$ as in Ravn and Uhlig (2002). Eurocoin is taken from the website http://eurocoin.bancaditalia.it/, the HICP and the EONIA from Eurostat, while the index of industrial production from the OECD Main Economic Indicators Database. We use these variables because they are able to capture macroeconomic dynamics.

[2] For a reference on Eurocoin see Altissimo *et al* (2007).

[3] Moreover, in Christensen et al (2009) it is shown that the difference between arbitrage free and models with no arbitrage restrictions is negligible in fitting the curves, for yields up to ten years: see Figure 5 in the paper.

[4] For the Eurosystem-wide yield curves see http://www.ecb.int/stats/money/yc/html/index.en.html.

[5] See Sims and Zha (1999) for technical details.

[6] For a proxy for the slope factor see Diebold and Li (2006) and Diebold *et al* (2006).