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Abstract

Macroeconomic shocks account for most of the variability of nominal Treasury yields, inducing parallel shifts in the level of the yield curve. We develop a new approach to identifying macroeconomic shocks that exploits model-based empirical shock measures. Technology shocks shift yields through their effect on expected inflation and the term premium. Shocks to preferences for current consumption affect yields through their impact on real rates and expected inflation. For both shocks, the systematic reaction of monetary policy is an important transmission pathway. We find little evidence that fiscal policy shocks are an important source of interest rate variability.

Keywords: Term Structure, Monetary Policy, Vector Autoregression, identification

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1. Introduction

Treasury yields assimilate vast amounts of information about economic activity. The Treasury yield curve is often cited as providing information on the current stance of monetary and fiscal policy, as well as expectations of future economic activity, real interest rates, and inflation.¹ For these reasons one would expect to find links between movements in the nominal Treasury yields and observed macroeconomic shocks.

While a major theme of finance research is to understand the factors that move the term structure, little work to date has focused on observable macroeconomic factors. Rather, most recent work on the term structure assumes that interest rate changes are driven by unobserved factors. Notable examples include Litterman and Scheinkman (1991), Knez, Litterman and Scheinkman (1994), Backus, Foresi, and Telmer (1998), and the empirical affine term structure literature.² An important exception is Ang and Piazzesi (2003). They introduce two observable macroeconomic factors into a Dai and Singleton (2000)-type affine model of the yield curve. The first factor is the first principal component extracted from several measures of real economic activity; the second factor is similarly extracted from several price level indices. They find that macro factors explain up to 85% of the long-horizon variance of shorter-term yields, but have a much smaller effect on long yields.

In this paper, we ask how different macroeconomic impulses affect the nominal yield curve. Our first exercise confirms Ang and Piazzesi's (2003) result that most of the variability of short- and medium-term yields is driven by macroeconomic factors. However, our results for the long-term yield are rather different from these authors. In particular, we find that macro impulses account for almost 85% of the 5-year yield variance. The key source of this difference is that, unlike Ang and Piazzesi (2003), our model incorporates *interest rate smoothing*, in that interest rates depend on their own lagged values. We show that when interest rate smoothing is omitted, the importance of macroeconomic shocks for interest rates is severely attenuated.

Our second set of results provides evidence on how specific types of shocks affect the yield curve. To identify economic shocks, we develop an approach that is new to the VAR literature. Instead of imposing *a priori* covariance restrictions on the relation between the VAR innovations and shocks, we infer these relationships from empirical measures of economic shocks that economists have proposed, often based on dynamic general equilibrium models. Our model-based measures include: Basu, Fernald, and Shapiro's (2001a,b) measure of technology shocks; Blanchard and Perotti's (2000) measure of fiscal policy shocks; and a measure of marginal-rate-of-substitution (MRS) shocks similar to that studied by Hall (1997). We show how this information is easily incorporated into the analysis of impulse responses.

We find that our MRS shock moves output and inflation in the same direction. Many empirical macroeconomists refer to this sort of impulse as an aggregate de-

¹For example, see Bernanke and Blinder (1992), Estrella and Hardouvelis (1991), Blanchard (1985), Mishkin (1990).

²See Duffie and Kan (1996), Dai and Singleton (2000) and Backus, Foresi, Mozumdar, and Wu (2001).

mand shock (e.g., see Blanchard (1989)). An expansionary MRS shock increases both expected inflation and real interest rates, inducing a large, significant, and persistent response in all nominal rates and shifting the yield curve level. In contrast, the technology shock moves output and inflation in opposite directions. An expansionary technology shock drives real interest rates up and expected inflation down, so its effect on nominal interest rates is, in principle, ambiguous. However, we find that the expected inflation response dominates, so the expansionary technology shock tends to reduce interest rates of all maturities. Our model-based measure of fiscal shocks does not have a significant impact on interest rates.

Our third set of results relates to the transmission mechanisms by which these shocks move the yield curve. We find that the systematic response of monetary policy is an important pathway whereby macroeconomic shocks affect interest rates. Monetary policy generally reacts to these shocks in the manner predicted by the Taylor (1993) principle: shocks that increase expected inflation or the gap between actual and potential output tend to increase the Federal funds rate. Longer-term interest rates are affected by expectations of changes in the funds rate. In addition, macroeconomic shocks can directly affect term premiums.

The remainder of this paper is structured as follows: In section 2 we describe our basic statistical framework. In section 3 we conduct a preliminary empirical exploration on the effect of macroeconomic factors on the yield curve. This section uses an eigenvalue decomposition of impulse responses to examine the implications of interest rate smoothing for the transmission of macroeconomic shocks. Section 4 develops our identification methodology that uses model-based shock measures, and section 5 explains how we implement this methodology empirically. Section 6 presents our empirical findings. Section 7 concludes the paper.

2. Basic statistical framework

We use the following vector autoregression (VAR) framework throughout our empirical analysis. Let Z_t be an $(n - 1) \times 1$ vector of nonfinancial macroeconomic variables at time t ; let FF_t denote the federal funds rate (included as the instrument of monetary policy); let $Y_t \equiv (Z_t', FF_t)'$; and let R_t denote an $m \times 1$ vector of zero-coupon Treasury yields of different maturities. We estimate versions of the following structural VAR:

$$\begin{bmatrix} A & \mathbf{0} \\ G & H \end{bmatrix} \begin{bmatrix} Y_t \\ R_t \end{bmatrix} = \begin{bmatrix} \tilde{A}(L) & \mathbf{0} \\ \tilde{C}(L) & \tilde{D}(L) \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ R_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \gamma_t \end{bmatrix} \quad (1)$$

where A is an $n \times n$ nonsingular matrix; H is an $m \times m$ nonsingular matrix; G is a rectangular matrix, $\mathbf{0}$ is the zero matrix with appropriate dimensions; and $\tilde{A}(L)$, $\tilde{C}(L)$, and $\tilde{D}(L)$ are matrix polynomials in the lag operator L . The process $[\varepsilon_t, \gamma_t]'$ is an i.i.d. vector of mutually and serially uncorrelated shocks whose variance is the identity matrix. It is useful to partition $\varepsilon_t \equiv (\varepsilon_t^Z, \varepsilon_t^{FF})'$, where ε_t^Z is a $(n - 1) \times 1$ vector of macroeconomic shocks and ε_t^{FF} is a monetary policy shock. One can think of γ_t as yield shocks analogous to Ang and Piazzesi's (2003) vector of latent financial variables. The zero restrictions on the upper right-hand blocks

of the coefficient matrices in (1) imply that neither current nor lagged yields nor the yield shocks γ_t enter the law of motion for Y_t . Thus, current and lagged Y_t are a sufficient state vector for spanning the space of all macroeconomic driving shocks.

We estimate system (1) via ordinary least squares using the following reduced form:

$$\begin{bmatrix} Y_t \\ R_t \end{bmatrix} = \begin{bmatrix} 0 \\ \alpha Y_t \end{bmatrix} + \begin{bmatrix} a(L) & \mathbf{0} \\ c(L) & d(L) \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ R_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ v_t \end{bmatrix} \quad (2)$$

where $\alpha \equiv -H^{-1}G$ and $[u_t' v_t']'$ is the vector of OLS residuals. If A is identified, then the structural shocks ε_t can be recovered from the OLS residuals via the relation

$$Au_t = \varepsilon_t \quad (3)$$

Once ε_t is identified, variance decompositions and impulse responses can be computed.

3. Macroeconomic shocks as drivers of interest rates

Our first exercise using framework (1) is our “baseline model”. It explores the fraction of interest rate variability that can be attributed to macroeconomic shocks. The data vector is given by $Z \equiv (IP, P, PCOM)'$ ³, where IP denotes the logarithm of industrial production, P denotes the logarithm of the personal consumption expenditure chain-weight price index, and $PCOM$ denotes the smoothed change in an updated version of the index of sensitive materials prices originally published in the index of leading indicators. The yields we use here, and throughout the paper, are the 1-month, 12-month, and 60-month zero coupon bond yields from the Fama-Bliss files of the CRSP data base. The data are monthly, from January 1959 through December 2000. The VAR incorporates 12 lags.

For this exercise, we posit a lower-triangular structure for matrix A in system (1). This is equivalent to a simple recursive orthogonalization of the VAR residual vector u_t . We give no structural interpretation to the elements of ε_t thus constructed, except that its first three elements, ε_t^Z , span the impulses driving the nonfinancial macro variables Z . Panel A of Table 1 displays the fraction of the 5-year ahead conditional variance of each yield attributable to each of the orthogonalized macro residuals. (Numbers in parentheses are lower and upper 90% error bands, computed using 500 Monte Carlo draws from the Bayesian posterior distribution of the model parameters.) As can be seen from the Table, 75% to 84% of the 5-year ahead variance of the yields is due to nonfinancial macro factors. That is, macroeconomic impulses account for the vast preponderance of interest rate variability.

Our estimates for the shorter yields are similar to those reported by Ang and Piazzesi (2003). However, our estimate for the 60-month yield is much higher than that reported by those authors. In particular, they find that only 48% of the 5-year ahead variance of this long bond yield is explained by macroeconomic factors.

³We thank John Fernald for providing us with this time series on technology shocks.

What accounts for this discrepancy? One important reason is that our model incorporates *interest rate smoothing*. This term often refers to models in which monetary policy is a function of lagged interest rates. For example, Christiano, Eichenbaum and Evans (1999) study monetary policy rules in VARs that include substantial lags of interest rates. Similarly, the literature on Taylor rules tends to find that specifications with partial-adjustment match the data better (Orphanides, 1999; Evans, 1998).⁴ From a theoretical perspective, Woodford (2003) argues that interest-rate smoothing is more effective at imparting persistence in long-term interest rates and influencing the interest-sensitive sectors of the economy with smaller policy movements. Building on the empirical Taylor rule literature, we generalize the term “interest rate smoothing” to refer to any model in which lagged interest rates enter the interest rate equations. The model of Ang and Piazzesi (2003) excludes interest rate smoothing, since they model interest rates as functions exclusively of current and lagged macro factors, along with current and lagged latent exogenous factors that are orthogonal to the macro factors.

We construct a “no-smoothing” version of model (1) by setting all elements of $\tilde{D}(L)$ as well as the fourth row of $\tilde{A}(L)$ equal to zero. By doing so, the regression equations for FF and R exclude lagged values of these interest rates. Note that lagged values of the federal funds rate still *indirectly* affect interest rates because Z_t is still be a function of FF_{t-1} , FF_{t-2} , etc. In this no-smoothing version of equation (1) the interest rate residuals ε_t^{FF} and γ_t display pronounced serial correlation (a point noted by Rudebusch, 2002, and Ang and Piazzesi, 2003). To account for this serial persistence, our no-smoothing model assumes that these residuals are autoregressive processes.⁵

The implications of the no-smoothing model are given in Panel B of Table 1. Compared to the baseline model, the no-smoothing model has a much smaller fraction of interest rate variance accounted for by nonfinancial macro shocks. Specifically, only 47% - 53% of the 5-year ahead conditional interest rate variance is attributable to macro shocks when interest rate smoothing is excluded.

There appear to be two reasons for the dramatic differences between these two models. First, in the baseline model the macro shocks ε_t^Z are propagated to future interest rates R_{t+h} both through their direct affect on future macro variables $Z_t, Z_{t+1}, \dots, Z_{t+h}$, and through their effect on the interest rates (FF_t, R_t) , (FF_{t+1}, R_{t+1}) , \dots , (FF_{t+h-1}, R_{t+h-1}) . These interest rate channels are missing in the no-smoothing case. Rather, when interest rate smoothing is ruled out, the dynamic propagation of macro shocks ε_t^Z to R_{t+1} , R_{t+2} etc. proceeds exclusively through its effect on Z_t, Z_{t+1}, Z_{t+2} , etc. One might conjecture that, by restricting the propagation of macro shocks to this single pathway, the magnitude of the interest rate responses (and therefore the fraction of interest rate variation explained by macro shocks) would be attenuated.

Second, as we noted above, the no-smoothing model implies a yield residual γ_t that is highly serially correlated. This implies that a good deal of the dynamics of

⁴In contrast, Rudebusch (2002) argues that serial persistence in Taylor rules comes from serially correlated shocks and not interest-rate smoothing.

⁵In particular, ε_t^{FF} and each of the elements of γ_t are modelled as AR(8) processes.

R_t in the no-smoothing model is accounted for by the dynamic properties of γ_t which, *by construction*, is orthogonal to the macro shocks and the current and lagged macro variables. As a result, the estimator of the no-smoothing model relies less on the propagation of macro shocks in explaining interest rate persistence, and places a greater reliance on exogenous persistence that is orthogonal to the macroeconomy.

These two explanations should be reflected in an eigenvalue decomposition of impulse responses. In particular, in any linear dynamical system the h -step response of variable k to shock j can be written as $\sum_{i=1}^q \left[w_i^{(k,j)} \right] \lambda_i^h$, where q is the number of eigenvalues characterizing the dynamical system, λ_i is the i^{th} such eigenvalue, and $w_i^{(k,j)}$ denotes the weight put on the i^{th} eigenvalue λ_i in this particular impulse response. (See the Appendix for a detailed derivation of $\left\{ w_i^{(k,j)} \right\}$.) Note that the weights $w_i^{(k,j)}$ do not depend on h , the number of steps ahead in the impulse response. Note also that the eigenvalues can be complex, in which case the corresponding weight will also be complex. (Complex eigenvalues characterize oscillatory impulse responses.)

The 5-year ahead variance decompositions, displayed in Table 1, are proportional to the squared sum of the impulse responses for the first 60 steps, so they are largely captured by studying the largest eigenvalues. In the baseline model with interest rate smoothing, the medium- to long-run variation in interest rates is well-explained by focusing on only three large eigenvalues:

1. A complex conjugate pair with modulus = .9903:

$$\begin{aligned} \bar{\lambda} &= .9900 + .0242i \\ conj(\bar{\lambda}) &= .9900 - .0242i \end{aligned} \tag{4}$$

This eigenvalue pair receives substantial weight in the responses of both macro variables and yields to macro shocks. It has the potential to impart long-term dynamics in macro and interest rate responses. The periodicity of this complex pair is about 21 years, so we tentatively associate it with the long swings in productivity growth that characterize the post-war data.

2. A real eigenvalue with modulus .9606. This eigenvalue has a half-life of approximately 17.2 months. It seems to be associated with the medium-term dynamics induced by having lagged FF_{t-h} in the federal funds equation. It has small but non-zero weight for the Z_t responses, and a large weight for the responses of both FF_t and R_t .
3. A real eigenvalue with modulus .9590. This eigenvalue has a half-life of approximately 16.6 months. It appears to be associated with the additional medium-term interest rate dynamics induced by having lagged R_{t-h} in the R_t equations, since it receives a large weight for the R_t responses, but it receives a weight of zero for both Z_t and FF_t .

In contrast, in the no-smoothing model the medium- to long-run variation in interest rates is well-explained using only a single complex eigenvalue pair that is almost identical to $\bar{\lambda}$ in equation (4) and its conjugate.⁶ The subsidiary eigenvalues 0.96 and 0.959 that we found to be important in the baseline system are completely absent in the no-smoothing system. This is not surprising, since these subsidiary eigenvalues capture dynamics associated with the lagged fed funds rate and lagged yields, dynamics which have been excluded from the no-smoothing case. Thus, the absence of these subsidiary eigenvalues from the no-smoothing case is a formalization of the first explanation, discussed above. In particular, the no-smoothing model omits propagation paths for the macro shocks that are responsible for the medium-term dynamics of interest rates. In contrast, this medium-term propagation mechanism is present in the baseline model.

The eigenvalue decomposition of impulse responses also sheds light on the second explanation. As mentioned above, the complex eigenvalue pair $\{\bar{\lambda}, \text{conj}(\bar{\lambda})\}$ given in equation (4) is important for both the baseline and no-smoothing systems. However, the weights $w_i^{(k,j)}$ associated with this eigenvalue pair tend to be substantially larger for the baseline smoothing model than for the no-smoothing model. This is especially important for the interest rate responses to the *PCOM* shock, which is the most important macro shock for interest rate variability in Table 1. For example, the weight associated with $\bar{\lambda}$ for the response of the one-month rate to this shock has a modulus of 0.1849 in the baseline case, but only 0.1548 in the no-smoothing case. This increased weight on $\bar{\lambda}$ is the main reason why, in Table 1, the *PCOM* shock accounts for substantially more interest rate variation in the baseline model, as compared to the no-smoothing model.

What is the economics underlying this larger weight on $\bar{\lambda}$ in the baseline case? Recall that $\bar{\lambda}$ captures the low-frequency component of interest rate movements. The smoothing model ascribes essentially all of this component of interest rate variation to macro movements at the same frequency. In contrast, the no-smoothing model ascribes a portion of this component to low-frequency movements in the serially-correlated residual γ_t . In fact, there appears to be a pronounced peak in the spectral density of γ_t at a periodicity between 20 and 30 years (although estimates of spectra at very low frequencies are necessarily imprecise, for obvious reasons). With a process γ_t that is orthogonal to the macro shocks explaining a good deal of the low-frequency variation in interest rates, there is less remaining interest rate variability to be explained by the macro variability at this periodicity. The reduced weight on $\bar{\lambda}$ in the no-smoothing model formalizes this reduced dependency on macro fluctuations to explain these low frequency interest rate swings.

4. Identifying structural shocks using model-based measures

According to the evidence of section 3, a large fraction of the interest rate variance for all maturities is accounted for by macroeconomic impulses. However,

⁶In the no-smoothing case, this eigenvalue is estimated to be $0.9890 \pm .0213i$, almost identical to $\bar{\lambda}$.

unless substantially more structure is imposed on the VAR innovations in equation (19) this description of the data's conditional second moment properties represents an incomplete characterization of the economic determinants of the nominal yield curve. According to equation (19), identification of the structural shock vector ε_t requires restricting matrix A . We propose in this section an approach that closely ties the identifying restrictions to specific economic theories. In particular, as in Prescott (1986) and Hall (1997), we exploit the ability of economic models to guide directly the construction of empirical measures of fundamental economic impulses, such as technology shocks, fiscal policy shocks, and shocks to households' marginal rate of substitution (MRS) between consumption and leisure. As a result, few prior restrictions are placed on the covariance structure of the VAR innovations u_t . As much as possible, we allow the model-based measures to dictate the VAR identification of macroeconomic shocks ε_t .

Let η_t denote the vector of observable model-based measures. (In section 5 we describe in detail how these measures are constructed from data.) We assume that these measures represent noisy measures of the true underlying shocks ε_t . Specifically,

$$\eta_t = D \varepsilon_t + w_t \quad (5)$$

where D is a non-singular ($n \times n$) matrix and w_t is a vector of measurement errors independent of ε_t (and therefore of u_t). To identify the model, we must uniquely determine the matrices A and D . To that end, substitute equation (19) into equation (21) to get

$$\eta_t = C u_t + w_t \quad (6)$$

where

$$C \equiv D A \quad (7)$$

or, equivalently,

$$A = D^{-1} C \quad (8)$$

This condition is important: Since w_t is uncorrelated with u_t , the matrix C can readily be estimated from equation (22) by ordinary least squares. Therefore, A could be identified with no *a priori* restrictions if the n^2 elements of D were known. In effect, this shifts identifying restrictions from the matrix A to matrix D .

Using equations (19), (24), and the fact that $E[\varepsilon \varepsilon'] = I$, one obtains

$$D D' = C \Sigma_u C'. \quad (9)$$

The C and Σ_u matrices on the right-hand side of equation (25) can be estimated directly, so equation (25) imposes $n(n+1)/2$ restrictions on D . Identification of D then requires an additional $n(n-1)/2$ *a priori* restrictions. Arguably, restrictions on D are easier to justify than restrictions on A , since the former maps underlying structural shocks into their empirical counterparts, while the latter maps the underlying shocks to the VAR residuals. For example, D may be diagonal, in which

case the η measures are contaminated only by classical measurement error. Alternatively, theory and measurement limitations may indicate that some η measures are linear combinations of the underlying shocks. In that case, D would have some non-zero off-diagonal elements. We discuss the specific identifying assumptions we impose on D in section 5, below.

When the system is exactly identified, D can be computed directly from equation (25) as the unique factorization of $C\Sigma_u C'$ satisfying the identifying restrictions. When the system is overidentified, neither equation (20) nor equation (25) will hold exactly in finite samples. Nevertheless, one can still estimate D by using the maximum likelihood procedure described in Hamilton (1994, pp.331-332). Once D is determined, matrix A can be computed using equation (24).

5. Model-based measures of structural shocks

To implement the model-based identification strategy described above in section 4, we must obtain model-based measures of macroeconomic driving shocks. In this section we describe four quarterly model-based measures that we use: technology, preference, fiscal policy and monetary policy.

5.1. Technology Shocks

Since Prescott (1986), the driving process for aggregate technology shocks in real business cycle models has been calibrated to empirical measures of Solow residuals. A large literature, including Prescott (1986), has noted that a portion of the fluctuations in standard Solow residual measures is endogenous, responding to macro shocks.⁷ Basu, Fernald, and Shapiro (2001b) provide a recent estimate of technology innovations that attempts to reduce these influences. Ignoring industry composition effects, their aggregate analysis specifies production as follows:

$$Y_t = z_t g_t F(v_t K_t, e_t N_t)$$

$$\ln z_t = \mu + \ln z_{t-1} + \varepsilon_{Tech,t} \quad (10)$$

where Y , z , v , K , e , and N are the levels of output, technology, capital utilization rate, capital stock, labor effort, and labor hours. The object g_t represents costs of adjusting employment and the capital stock. It is an explicit function of observable data, and is calibrated from econometric estimates in the literature (see Shapiro (1986) and Basu, Fernald, and Shapiro (2001a,b)). F is a production function that is homogeneous of degree $\gamma \geq 1$, allowing for the possibility of increasing returns. Basu, Fernald, and Shapiro specify an economic environment where the unobserved variables v and e can be measured as proportional to the workweek of labor and capital. Assuming $\gamma = 1$ — constant-returns-to-scale — Basu, Fernald, and Shapiro (2001b) use time-varying cost shares to compute a quarterly, aggregate measure of the technology innovation.

⁷For example, see Burnside, Eichenbaum and Rebelo (1993) and Braun and Evans (1998))

We use Basu, Fernald, and Shapiro's (2001b) quarterly, aggregate measure of technology for our model-based empirical measure η_{Tech} ⁸ of the aggregate technology shock ε_{Tech} . Although this quarterly measure includes controls for many latent, endogenous features, data limitations prevent controlling for industry compositional effects. This potentially introduces measurement error into this series. The data begin in 1965:II and end in 2000:IV.

5.2. Marginal-Rate-Of-Substitution Shocks

A shock to the marginal rate of substitution between consumption and leisure can potentially shift aggregate demand for goods and services. Hall (1997), Shapiro and Watson (1988) and Baxter and King (1990) find substantial business cycle effects from empirical measures of intratemporal marginal rates of substitution between consumption and leisure. To generate a model-based empirical measure of an MRS shock, we generalize Hall's (1997) procedure to allow for time-nonseparable preferences.⁹ Consider a representative consumer with the following utility specification that includes external habit persistence

$$U(C_t, N_t) = \xi_t \frac{(C_t - b\bar{C}_{t-1})^{1-\gamma}}{1-\gamma} - \frac{N^{1+\phi}}{1+\phi}$$

$$\ln \xi_t = \rho(L) \ln \xi_{t-1} + \eta_{MRS,t} \quad (11)$$

where C is consumption of the representative agent, \bar{C} represents the per-capita aggregate consumption level, N is labor hours, ξ is a serially correlated preference shifter, and ε_{MRS} is a serially independent shock. The first-order conditions for consumption and labor hours lead to the following intratemporal Euler equation (or MRS relationship)

$$\frac{\xi_t (C_t - b\bar{C}_{t-1})^{-\gamma}}{N_t^\phi} = 1/W_t \quad (12)$$

where W is the real wage. Taking logs, one obtains

$$\ln \xi_t = \phi \ln N_t - \ln W_t + \gamma \ln [C_t - b\bar{C}_{t-1}]. \quad (13)$$

In equilibrium, the per-capita aggregate consumption equals the consumption levels of the representative agent, so $\bar{C} = C$.

We use equation (13) to obtain an empirical measure of $\ln \xi_t$. We then estimate autoregression (11) to obtain our model-based measure $\eta_{MRS,t}$ of the MRS shock. Our data are quarterly and extend from 1964:I to 2000:IV. Consumption is measured by per capita nondurables and services expenditures in chain-weighted 1996 dollars. Labor hours correspond to hours worked in the business sector per capita. The real wage corresponds to nominal compensation per labor hour worked in the business sector deflated by the personal consumption expenditure chain price

⁸Throughout this paper, we omit the time subscript t if no ambiguity is implied.

⁹Holland and Scott (1998) study a similar MRS shock for the United Kingdom economy.

index. The hours and compensation data are reported in the BLS productivity release. The utility function parameters are taken from previous studies. First, to ensure balanced growth we set $\gamma = 1$, corresponding to log utility for consumption services. Second, we use Hall's (1997) value for $\phi = 1.7$, corresponding to a compensated elasticity of labor supply of 0.6. Finally, we set the habit persistence parameter $b = 0.73$ as estimated by Boldrin, Christiano and Fisher (2001).

We measure η_{MRS} as the residual in equation (11). We estimate a sixth-order polynomial for $\rho(L)$. In addition, the MRS measure ξ exhibits noticeable low frequency variation, so we also include a linear time trend in the regression to account for demographic factors that are beyond the scope of this analysis. In order to allow for serially-correlated measurement errors in ξ_t , we use an instrumental variables estimator to estimate $\rho(L)$.¹⁰

Macroeconomic researchers have offered differing interpretations for the random marginal rate of substitution shifter in ξ_t in equation (12).¹¹ First, the home production literature due to Benhabib, Rogerson, and Wright (1991) and Greenwood and Hercowitz (1991), among others, suggests that ξ_t could be a productivity shock to the production of home goods. Second, inertial wage and price contracts will distort the simple intratemporal Euler equation as it is specified in (12). In particular, in the Calvo pricing environments considered by Christiano, Eichenbaum, and Evans (2001) and Galí, Gertler, Lopez-Salido (2001), alternative versions of (12) hold. Third, Chari, Kehoe, and McGrattan (2002) and Mulligan (2002) interpret ξ_t as reflecting wedges or distortions, such as changes in tax rates or union bargaining power. To the extent that these alternative explanations have different theoretical implications for impulse response functions, an empirical analysis of our MRS shock can help shed light on which explanation seems to be consistent with the aggregate data.

5.3. Fiscal Policy Shocks

The modern business cycle literature that includes fiscal policy effects has focused primarily on exogenous specifications of government spending and tax rates.¹² To relate these theoretical studies to aggregate data requires distinguishing between the exogenous and endogenous components of fiscal policy. Blanchard and Perotti (2000) construct a quarterly series of exogenous fiscal shocks by using regression methods to control for the systematic response of fiscal policy. We use their measures to identify fiscal policy shocks.

¹⁰Our shock identification strategy assumes that the measurement errors in our model-based shocks are independent of the VAR innovations. Consequently, we use real GDP, the GDP price index and commodity prices as instruments.

¹¹As Hall (1997) pointed out, the greatest amount of evidence against Eichenbaum, Hansen, and Singleton's (1988) preference specifications surrounded the intratemporal Euler equation for consumption and leisure.

¹²Baxter and King (1993) and Christiano and Eichenbaum (1992) study permanent and transitory changes in exogenous government purchases. Braun (1994) and McGrattan (1994) study transitory changes in exogenous tax rates. An alternative approach is taken by Leeper and Sims (1994): they allow tax rates to respond systematically to the state of the economy.

Blanchard and Perotti (2000) start with measures of GDP, government spending excluding transfers, and tax receipts net of transfers. The latter two variables include federal, state, and local measurements. Blanchard and Perotti control for the automatic responses of spending and taxes to changes in GDP, using measures of the elasticity of different types of taxes, transfers, and spending to output. Additional restrictions are imposed to identify exogenous shocks to taxes and government spending.¹³ We construct our model-based empirical measure η_{Fiscal} as a shock to the government deficit, defined as the difference between Blanchard-Perotti's government spending and tax shocks.¹⁴ We treat η_{Fiscal} as a noisy measure of the underlying fiscal policy shock ε_{Fiscal} .

5.4. Accounting for Monetary Policy Shocks

The effects of monetary policy shocks on the term structure have been studied elsewhere,¹⁵ and are not the focus of this paper. However, to isolate the effects of technology, MRS, and fiscal shocks, we control for monetary policy impulses so that the effects of monetary policy shocks are not incorrectly ascribed to these other shocks. To do so, we introduce an empirical measure of monetary policy shocks, denoted η_{MP} . We use an updated version of the monetary policy shock measure in Christiano, Eichenbaum, and Evans (1996). This measure is derived from an identified VAR using the following variables: the logarithm of real GDP; the logarithm of the GDP chain-weighted price index; the smoothed change the index of sensitive materials prices used in section 3; the Federal funds rate; the logarithm of nonborrowed reserves; and the logarithm of total reserves. The data run from 1959:I through 2000:IV.

5.5. Correlation Structure of the Model-Based Measures

Table 2 displays the contemporaneous correlation matrix for the model-based shock measures $\eta \equiv (\eta_{MP}, \eta_{MRS}, \eta_{Tech}, \eta_{Fiscal})'$ described in sections 5.1 - 5.4. . Note that the correlations are fairly low, with the exception of $corr(\eta_{Tech}, \eta_{Fiscal})$, which exceeds 0.30.

According to equation (22), the model-based measures only provide useful information for identifying A if they are correlated with the VAR residuals u_t . Table 3 provides evidence on these correlations for the data we use. It displays the R^2 s for the *OLS* regressions in system (22) using the measures of $\eta = (\eta_{MP}, \eta_{MRS}, \eta_{Tech}, \eta_{Fiscal})'$ described in sections 5.1 - 5.3.. The variables in our macro VAR block are quarterly analogues to the monthly measures used in section 3: real GDP, the GDP price deflator, the commodity price index *PCOM*, and the Federal funds rate. The

¹³Blanchard and Perotti (2001) estimate their VAR under two different trend assumptions. First, they incorporate deterministic time trends; second, they allow for stochastic trends. We have done our analysis with fiscal shocks computed both ways. The results are very similar, so we only display the results for the model with deterministic time trends.

¹⁴We have also performed the analysis with the individual tax and spending shocks. The results are qualitatively unchanged, although the impulse responses to these individual shocks are smaller.

¹⁵See Gordon and Leeper (1994), Bernanke, Gertler, and Watson (1997), and Evans and Marshall (1998).

only problematic shock measure in Table 3 is the fiscal shock, whose R^2 is only 8.7%.¹⁶ This suggests that our fiscal shock measure η_{Fiscal} may not provide strong identification for an underlying fiscal shock in the context of our VAR system. As a result, caution should be exercised in interpreting the responses to the fiscal shock implied by this exercise.

5.6. Identifying restrictions

Given the empirical estimates of η_t , the key step in the identification is to specify restrictions on D , the mapping from the model-based measures η to the true underlying shocks ε . A straightforward approach would be to assume that each element of η_t equals the corresponding element of ε_t plus measurement error. In this case, D is diagonal. We find that the data strongly reject the overidentifying restrictions implied by this model.¹⁷ Alternatively, some η measures may be linear combinations of the underlying shocks, perhaps due to mismeasurement in the way the series in η were computed. This could account for the correlation structure among η_{it} elements, described in Table 2, and would imply non-zero off-diagonal elements of D .

As we noted in section 5.1, there is a large literature on possible mismeasurement of technology shocks. Evans (1992) points to possible contamination of technology shocks by monetary policy; Burnside, Eichenbaum, and Rebelo (1993) discuss the problem of unobserved labor hoarding, and Burnside and Eichenbaum (1993) note the problem of variable capital utilization. While the Basu-Fernald-Shapiro technology measure that we use in this paper attempts to correct for many of these sources of mismeasurement, it may do so imperfectly. Consequently, we wish to allow for the possibility that η_{Tech} may be a linear combination of several shocks. To this end, we selected the following specification of system (21):

$$\begin{bmatrix} \eta_{MP} \\ \eta_{MRS} \\ \eta_{Tech} \\ \eta_{Fiscal} \end{bmatrix} = \begin{bmatrix} d_{11} & 0 & 0 & 0 \\ 0 & d_{22} & 0 & 0 \\ d_{31} & d_{32} & d_{33} & 0 \\ d_{41} & d_{42} & d_{43} & d_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_{MP} \\ \varepsilon_{MRS} \\ \varepsilon_{Tech} \\ \varepsilon_{Fiscal} \end{bmatrix} + w. \quad (14)$$

Specification (14) is overidentified: it imposes seven zero restrictions, whereas exact identification requires only six restrictions. Specification (14) assumes that our monetary policy and MRS measures equal the true underlying shock plus classical measurement error. In contrast, η_{Tech} is allowed to incorporate the influence of the true underlying monetary policy and MRS shocks. Note that we do not permit any of these three measures to be contaminated by the underlying fiscal shocks. The reason for this assumption is that the fiscal policy shock measure has the smallest correlation with the VAR innovations u_t of all of our η_{it} elements. (See Table 3.)

¹⁶Our measure of η_{Fiscal} is the difference between the government spending shock and the tax shock, both estimated by Blanchard and Perotti (2000). When we estimate regression (22) using the spending shock or the tax shock individually, the R^2 s are all below 6%.

¹⁷The likelihood ratio statistic comparing the diagonal- D model to the model where D is exactly identified is $\chi^2(6)$. We obtain a value of 718.9 for this statistic.

Consequently, this row of the matrix C is likely to be estimated imprecisely, so we wish to limit the influence of the fiscal policy measure on the other analyses.¹⁸ Furthermore, Blanchard and Perotti's (2000) approach uses only three variables: GDP, government spending, and government taxes. So this measure has not been projected onto innovations from omitted variables that would be included in larger systems. These considerations motivate us to restrict the potential influence of the fiscal policy measure on the identification of the other shocks.

6. Empirical Results

In this section we explore how macroeconomic shocks affect the term structure using the identification strategy described in sections 4 and 5. The results are displayed in Figures 1 and 3. These figures display the responses of macroeconomic variables and yields to the MRS and technology shocks. In addition, we plot the responses of the one-month real rate¹⁹ and the 60-month term premium.²⁰ The dashed lines give 90% probability error bands for the impulse responses, computed using 500 Monte Carlo draws from the posterior distribution of the model's parameters. Table 4 displays the decomposition of the variance of 5-year ahead macroeconomic forecast errors implied by the ε vector of shocks, also with 90% probability error bands. We compute the posterior distribution using the approach described in Evans and Marshall (2002), which extends the Bayesian methods described in Sims and Zha (1999), Zha (1999), and Waggoner and Zha (2003) in a natural way to account for uncertainty in regression (22).²¹

6.1. Responses to MRS shock

Figure 1 gives the responses to the ε_{MRS} shock. Upon impact, real GDP rises immediately with a persistent effect that lasts several years. The fraction of output variance accounted for by ε_{MRS} is 39% at the 5-year horizon in Table 4. Inflation rises, peaking after one year, although the estimates are imprecise. The transitory nature of the inflation response implies that the MRS shock contributes only a small portion of the total price level variation. According to Table 4, the fraction of inflation variation accounted for by ε_{MRS} is only 9% at the 5-year horizon.

Consider now the systematic response of monetary policy to ε_{MRS} . The nominal federal funds rate responds to an ε_{MRS} impulse with a persistent and significant increase. The Taylor (1993) principle is evident in this response: the real funds rate

¹⁸With specification (14), the coefficients in the regression of η_{Fiscal} on u_t only affect the identification of ε_{Fiscal} , not the other elements of ε .

¹⁹In the figures, the one-month real rate denotes the real return to the one-month nominal bond.

²⁰If y_t^k denotes the k -month yield, the 60-month term premium equals $y_t^{60} - \frac{1}{60} \sum_{s=0}^{59} E_t [y_{t+s}^1]$.

²¹Because this system is overidentified, the standard Bayesian procedure (Doan, 2000) is inappropriate. (See Sims and Zha, 1999). In our analysis, matrix D in equation (14) is estimated by maximum likelihood for each Monte Carlo draw. Sims and Zha (1999) refer to this as the "naive Bayesian procedure". However, when we implement similar, exactly identified systems, the error bands are virtually identical to those displayed in the figures.

rises in response to a shock that increases both deviations of output and inflation from their target levels. Table 4 indicates that ε_{MRS} is an important driver of systematic monetary policy, accounting for 34% of the 5-year ahead variance of the Federal funds rate. Together, these results depict shocks that shift the aggregate economy's demand for goods and services; the Fed responds by "leaning against the wind."

Turning to the responses of yields to the MRS shock, note that ε_{MRS} has substantial, persistent and significant effects on individual nominal yields. In particular, the three yields respond to a one-standard deviation positive ε_{MRS} shock by increasing between 25 and 38 basis points on impact. These responses are long-lived, remaining well above zero over four years after the initial impulse. These responses indicate that aggregate demand shocks can lead to substantial variation in nominal yields. Table 4 confirms this conclusion: the ε_{MRS} accounts for 37%, 37%, and 26% of the 5-year ahead forecast variance of the one-month, 12-month, and 60-month yields, respectively.

Since the responses of the three yields to ε_{MRS} are similar, the MRS shock induces a parallel shift in the level of the yield curve. The reason for this pronounced response of the yield curve is that ε_{MRS} shifts inflation and real rates in the same direction. We discussed above the positive response of one-month inflation. The positive response of the one-month real rate is displayed in the lower left-hand graph of Figure 1. It peaks at 48 basis points in the quarter following the impulse, decaying gradually. The pronounced real rate response following an MRS shock is consistent with our interpretation of these shocks as transitory impulses to the marginal utility of consumption. The responses (not displayed) of the longer-term inflation and real rates are quantitatively similar to their short-term counterparts. These positive inflation and real-rate responses at all maturities result in a significant upward shift in the level of the yield curve.

The lower right-hand graph in Figure 1 gives the response of the 5-year term premium. As can be seen, there is no evidence that this term premium responds to ε_{MRS} . Thus, movements in the long yield due to the MRS shock appear to be well-described by the expectations hypothesis of the term structure.

6.2. Responses to the technology shock

Figure 2 shows responses to ε_{Tech} , the technology shock. This shock induces an increase in output, a fall in the inflation rate, and a decline in all three bond yields. The response of output is pronounced and long-lived after about 5 quarters, but the initial output response is negligible. This delayed response to a technology shock is consistent with several recent empirical and theoretical analyses.²² The ε_{Tech} shock accounts for 20% of the five-year ahead output variance. (See Table 4.)

The ε_{Tech} shock induces a pronounced short-run decline in the one-month-ahead

²²Galí (1999) and Basu, Fernald and Kimball(2000) interpret the delayed response of output to technology shocks as evidence of inertial aggregate demand due to price stickiness. In Boldrin, Christiano, and Fisher (2001) and Francis and Ramey (2001), this delay is consistent with inertial aggregate demand due to habit persistence in consumption and investment adjustment costs.

inflation rate: the response peaks at -55 basis points three quarters after the impulse. This inflation response is transitory, dissipating over the next three to four years. According to Table 4, the ε_{Tech} shock accounts for 55% of the 5-year ahead variance of the inflation rate. The inflation responses are consistent with an economy in which monetary policy allows falling real marginal costs to show through to smaller price increases.

The technology shock induces pronounced, persistent, and significant declines in nominal yields of all maturities. For ε_{Tech} , the responses to a one-standard deviation shock bottom out in three to four quarters at -43, -47, and -38 basis points for the 1-, 12-, and 60-month yields respectively. Overall, ε_{Tech} accounts for 39%, 46%, and 57% of the one-month, 12-month, and 60-month yield variance at the 5-year horizon. Since these negative responses of the three yields are similar in magnitude, the level of the yield curve falls significantly. This decline is economically important: a one-standard deviation shock to ε_{Tech} induces approximately a 30 basis point decline in Cochrane's (2001) measure of the yield curve level (not displayed).

Again, it is useful to decompose these responses of nominal yields into their expected inflation and real-rate components. The initial responses of the real rate to these shocks are positive, as one would expect from a positive impulse to the marginal product of capital. However, the large deflationary impact of these shocks overwhelms the contribution of the real rate, hence the negative initial responses of nominal interest rates. Interestingly, the initial positive response of the real rate turns negative in about seven quarters, so the real rate response actually serves to prolong the negative response of nominal rates. A key factor driving this reversal is the systematic response of monetary policy. In response to ε_{Tech} , the monetary authority reduces the nominal Federal funds rate by a total of nearly 70 basis points over the next three quarters. This policy response is quite persistent. According to Table 4, ε_{Tech} accounts for 46% of the 5-year ahead variance of the Federal funds rate. This response is consistent with the literature on Taylor (1993) rules: ε_{Tech} moves inflation below the target inflation rate and reduces (or leaves unchanged) the output gap.²³ Note that after the first two quarters the funds rate response exceeds the inflation response. Consequently, the real Federal funds rate falls. This decline in the real funds rate is consistent with the stability condition of the Taylor rule literature, that the nominal interest rate respond more than one-for-one with inflation.

An additional factor that shifts the level of the yield curve is the response of term premiums. According to last graph in Figure 2, the ε_{Tech} shocks tend to induce an increase in the five year term premium over the first six months, with a subsequent decline after three years. So, conditional on a technology shock, the expectations hypothesis fails and we see time-varying term premia. Notice that the real rate level response mimics the response of the five year term premium. If

²³The sign of the output gap turns on whether potential output rises immediately with the expansionary technology shock (as in Galí, 1999 and Basu, Fernald, and Kimball, 2000) or is delayed due to adjustment costs and habit persistence (as in Boldrin, Christiano, and Fisher, 2001, and Francis and Ramey, 2001).

this term premium response were flat, the real-rate response would be shorter-lived. Together, the monetary policy reduction in the Federal funds rate and the negative term premiums three to four years after impact tend to pull the level of the real yield curve down.

6.3. Responses to the fiscal shock

Another fundamental macroeconomic impulse comes from exogenous shifts in fiscal policy. The responses to ε_{Fiscal} are displayed in Figure 3. Output displays a delayed response to ε_{Fiscal} , peaking about seven quarters after the shock impact. This delayed response of economic activity to fiscal shocks was also noted by Ramey and Shapiro (1998). Somewhat surprisingly, we find little evidence of significant yield responses to the fiscal shock. In particular, the error bands for all impulse responses are quite wide. Perhaps this reflects the low R^2 in the regression of the VAR innovations u_t onto the model-based measures $\eta_{Fiscal,t}$.

7. Conclusion

This paper presents empirical evidence that macroeconomic factors account for most of the movement in nominal Treasury yields of maturities ranging from one month through five years. Technology shocks and shocks to the marginal rate of substitution between consumption and leisure strongly influence the level of the yield curve. In particular, our MRS shock acts like an aggregate demand shock, in that it simultaneously increases real GDP and prices. This shock increases both real interest rates and inflation, which together serve to raise the level of the nominal yield curve. Our measures of technology shocks also lead to nominal yield curve level effects. These shocks produce competing influences: positive shocks increase real GDP and real interest rates but lower inflation. The overall transmission to nominal yields is attenuated by these contrasting influences, but the effect is still quite large, with the inflation effect playing the dominant role.

Our results differ from those of Ang and Piazzesi (2003), who find a smaller response of the five-year yield to macroeconomic impulses. A key difference between our approach and that of Ang and Piazzesi (2003) is that we allow interest rates to depend on their lagged values, providing additional pathways for the transmission of macroeconomic shocks.

The role of systematic monetary policy is critical for understanding the way macro impulses jointly affect interest rates and the real economy. The Taylor principle seems to be a feature of these empirical responses: if inflationary expectations rise above the inflation target, the Federal funds rate increases by more than the inflation gap. In addition, the funds rate rises in response to real GDP above its potential level. Our macroeconomic shocks induce changes in output and inflation gaps, and systematic monetary policy adjusts the funds rate accordingly. Long term interest rates move in anticipation of these systematic policy responses.

We also found evidence that term premiums respond to the technology shock. Changes in term premiums are associated with time-variation in the market price

of risk. Consistent implications for term premium responses may help macroeconomists and financial economists further integrate macroeconomic facts into asset-pricing models. More generally, by matching our economic factors with the latent factors that have been the focus of much of the term structure literature in empirical finance, it should be possible to further integrate this literature into the analysis of dynamic general equilibrium models.

8. Appendix A: Eigenvalue decomposition of impulse responses

System (2) with l lags can be written as a companion-form VAR system:

$$X_t = \Phi X_{t-1} + \Omega v_t, \quad (15)$$

where X_t is an $q \times 1$ vector ($q \equiv (n+m)l$) that stacks $\{Z_t, FF_t, R_t\}$ and their lags, v_t is the companion-form disturbance vector whose first seven elements are $\{\varepsilon'_t, \gamma'_t\}'$, and Φ is a $q \times q$ matrix of companion-form coefficients. Consider the eigenvalue decomposition of Φ :

$$\Phi = P \Lambda P^{-1} \quad (16)$$

where Λ is a diagonal matrix with the q eigenvalues $\{\lambda_1, \dots, \lambda_q\}$ of Φ along its diagonal, and P is the associated eigenvector matrix. Use the following notation:

$$P_i \equiv i^{th} \text{ column of } P; \quad \tilde{P}_i \equiv i^{th} \text{ row of } P^{-1}; \quad \Omega_j \equiv j^{th} \text{ column of } \Omega; \quad (17)$$

and let $e_k \equiv k^{th}$ column of the $q \times q$ identity matrix. The h^{th} -period-ahead response of variable k to the j^{th} shock is $e'_k \Phi^h \Omega_j$, which can be written

$$\equiv \sum_{i=1}^q \left[w_i^{(k,j)} \right] \lambda_i^h \quad (18)$$

where $w_i^{(k,j)} \equiv e'_k P_i \tilde{P}_i \Omega_j$;

The impulse responses in the no-smoothing case are not quite captured by equations (15) and (18), since, with no lagged interest rates in the system, Φ is singular. However, an expression analogous to equation (18) still holds. Let \hat{X}_t denote a $\hat{q} \times 1$ vector (where $\hat{q} \equiv nl$) that stacks $\{Z_t, FF_t\}$ and their lags. The no-smoothing model can then be written in the following companion form:

$$\begin{aligned} \hat{X}_t &= \hat{\Phi} \hat{X}_{t-1} + \hat{\Omega} \hat{v}_t, \\ R_t &= \hat{\alpha}_0 \hat{X}_t + \hat{\alpha}_1 \hat{X}_{t-1} + H^{-1} \gamma_t \end{aligned}$$

where \hat{v}_t is the companion-form disturbance vector whose first 4 elements are ε_t , $\hat{\Phi}$ is a companion-form coefficient matrix for the dynamics of \hat{X}_t , and both $\hat{\alpha}_0$ and $\hat{\alpha}_1$ are $m \times \hat{q}$ companion form coefficient matrices. Let $\hat{\Phi}_{1:n}$ denote the first n rows of $\hat{\Phi}$, and use definitions (16) and (17) with Φ replaced by $\hat{\Phi}$. Then for $h \geq 1$,

the h^{th} -period-ahead response of the k^{th} yield $R_{k,t+h}$ to the j^{th} macro shock $\varepsilon_{j,t}$ is $\sum_{i=1}^{\hat{q}} \left[\hat{w}_i^{(k,j)} \right] \lambda_i^{h-1}$, where now

$$\hat{w}_i^{(k,j)} \equiv e'_k \left[\alpha_0 \hat{\Phi}_{1:n} + \alpha_1 \right] P_i \tilde{P}_i \Omega_j$$

The contemporaneous response, i.e. for $h = 0$, is $e'_k \alpha_0 \Omega_j$.

9. Appendix B: Bayesian inference when identification uses model-based measures of fundamental shocks

This Appendix is a self-contained derivation of the Bayesian posterior distribution of the impulse responses when the VAR is identified using the model-based approach of section 4.. As is typical in VARs, it is assumed that there is a linear mapping from fundamental shocks to VAR residuals. That is, if u_t denotes the $(n \times 1)$ vector of VAR residuals and ε_t denotes the $(n \times 1)$ vector of fundamental shocks then there exists a matrix of constants A such that

$$A u_t = \varepsilon_t. \quad (19)$$

In addition, it is assumed that the elements of ε_t are mutually uncorrelated. If the variance-covariance matrix of ε_t is normalized to the identity matrix, then equation (19) implies

$$A \Sigma_u A' = I \quad (20)$$

where $\Sigma_u \equiv E u_t u_t'$. Identification then requires imposing sufficient restrictions such that there is a unique matrix A satisfying equation (20).

As described in section 4., one can use model-based measures of the fundamental shocks to achieve identification. Suppose the econometrician has an $(n \times 1)$ time series η_t consisting of noisy measures of the true shock vector ε_t . We shall assume that the noisy measures are related to the fundamental shocks ε_t by

$$\eta_t = D \varepsilon_t + w_t \quad (21)$$

where D is a non-singular $(n \times n)$ matrix and w_t is a vector of measurement errors independent of ε_t (and therefore of u_t). Under this structure, identification of A can be achieved by imposing restrictions on D , rather than restricting A directly. Substitute equation (19) into equation (21) to get

$$\eta_t = C u_t + w_t \quad (22)$$

where

$$C \equiv D A \quad (23)$$

or, equivalently,

$$A = D^{-1} C. \quad (24)$$

Since w_t is uncorrelated with u_t , the matrix C can readily be estimated from equation (22) by ordinary least squares. Given this estimate of C , A could be identified from equation (24) with no *a priori* restrictions if D were known. To estimate D , equation 20 can now be restated as

$$D D' = C \Sigma_u C' \quad (25)$$

As in the typical identified VAR, at least $n(n-1)/2$ *a priori* identifying restrictions on D are required.

In this Appendix, we derive an algorithm to conduct Bayesian inference when this sort of model-based identification is used. In subsection 9.1., we review the basic Zellner (1971) approach to Bayesian inference in vector autoregressions with a diffuse prior. Subsection 9.2. then extends the Zellner approach to encompass identification using model-based measures.

9.1. A review of Zellner's (1971) approach to Bayesian inference in multivariate time series

In this section, we review Zellner's (1971) derivation of the posterior distribution for the VAR slope coefficients and the covariance matrix of the reduced-form residuals. Let \tilde{Y} ($[T + l] \times n$) denote the data used in the VAR. (Here, T denotes the number of usable observations, l denotes the number of lags in the VAR, and n denotes the number of series in the VAR.) To write the VAR in regression notation, let $\nu \equiv nl + 1$, the number of regressors per equation, let the $(T \times n)$ matrix of dependent variables in the VAR be denoted Y ,²⁴

$$Y \equiv \begin{bmatrix} \tilde{Y}_{l+1,1} & \cdots & \tilde{Y}_{l+1,n} \\ \vdots & & \vdots \\ \tilde{Y}_{l+T,1} & & \tilde{Y}_{l+T,n} \end{bmatrix}$$

and let the $(T \times \nu)$ matrix of VAR regressors be denoted X ,

$$X \equiv \begin{bmatrix} 1 & \tilde{Y}_{l,1} & \tilde{Y}_{l-1,1} & \cdots & \tilde{Y}_{1,1} & \tilde{Y}_{l,2} & \tilde{Y}_{l-1,2} & \cdots & \tilde{Y}_{1,2} & \tilde{Y}_{l,3} & \cdots & \tilde{Y}_{1,n} \\ 1 & \tilde{Y}_{l+1,1} & \tilde{Y}_{l,1} & \cdots & \tilde{Y}_{2,1} & \tilde{Y}_{l+1,2} & \tilde{Y}_{l,2} & \cdots & \tilde{Y}_{2,2} & \tilde{Y}_{l+1,3} & \cdots & \tilde{Y}_{2,n} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 1 & \tilde{Y}_{l+T-1,1} & \tilde{Y}_{l+T-2,1} & \cdots & \tilde{Y}_{T,1} & \tilde{Y}_{l+T-1,2} & \tilde{Y}_{l+T-2,2} & \cdots & \tilde{Y}_{T,2} & \tilde{Y}_{l+T-1,3} & \cdots & \tilde{Y}_{T,n} \end{bmatrix}$$

The reduced form of the VAR is given by the regression equation

$$Y = XB + U \tag{26}$$

where U stacks the $n \times 1$ i.i.d. error process u_t as $U = (u_1, u_2, \dots, u_T)'$, and it is assumed that

$$u_t \sim N(0, \Sigma_u). \tag{27}$$

In equation (26), the coefficient matrix B has dimension $(\nu \times n)$. The rows of B correspond to the regressors X ; the columns correspond to the n equations.

The parameters of model (26) are $\{B, \Sigma_u\}$. Zellner (1971) derives the joint posterior distribution of $\{B, \Sigma_u\}$, given the data $\{X, Y\}$. For the exactly identified case, A is an exact function of $\{\Sigma_u\}$ and the impulse responses are exact functions

²⁴Note that our usage of the notation Y differs from that used above in equation (1).

of $\{A, B\}$, so we can take Monte Carlo draws from this joint posterior to derive Monte Carlo estimates of the posterior distribution of the impulse responses.

Zellner (1971, pp. 224-227) shows that under the Jeffreys priors²⁵

$$\text{prior}(B) = \text{constant} \quad (28)$$

$$\text{prior}(\Sigma_u) \propto |\Sigma_u|^{-(n+1)/2} \quad (29)$$

the joint posterior for $\{B, \Sigma_u\}$ is given by

$$p(B, \Sigma_u | Y, X) \propto |\Sigma_u|^{-(n+T+1)/2} \exp \left\{ -\frac{1}{2} \text{trace} \left[S \Sigma_u^{-1} + (B - \hat{B})' X' X (B - \hat{B}) \Sigma_u^{-1} \right] \right\} \quad (30)$$

where \hat{B} denotes the matrix of OLS estimates of the VAR slope coefficients

$$\hat{B} \equiv (X'X)^{-1} X'Y \quad (31)$$

and S denotes T times the sample covariance matrix of the VAR disturbances

$$S \equiv (Y - X\hat{B})' (Y - X\hat{B}).$$

Note that both \hat{B} and S are functions of the data only, not of any model parameters.

Equations (26) and (27) imply that the likelihood of Y conditional on $\{B, \Sigma_u, X\}$ is simply the multivariate normal density, which is proportional to

$$|\Sigma_u|^{-T/2} \exp \left\{ -\frac{1}{2} \text{trace} [(Y - XB)' (Y - XB) \Sigma_u^{-1}] \right\} \quad (32)$$

The likelihood of $\{B, \Sigma_u\}$ conditional on $\{Y, X\}$ is proportional to the object in (32), treated as a function of $\{B, \Sigma_u\}$. The posterior $p(B, \Sigma_u | Y, X)$ is the likelihood multiplied by the joint prior (the product of $\text{prior}(B)$ and $\text{prior}(\Sigma_u)$):

$$p(B, \Sigma_u | Y, X) \propto |\Sigma_u|^{-(n+T+1)/2} \exp \left\{ -\frac{1}{2} \text{trace} [(Y - XB)' (Y - XB) \Sigma_u^{-1}] \right\} \quad (33)$$

To derive (30) from (33), note that

$$\begin{aligned} (Y - XB)' (Y - XB) &= (Y - X\hat{B} + X\hat{B} - XB)' (Y - X\hat{B} + X\hat{B} - XB) \\ &= (Y - X\hat{B})' (Y - X\hat{B}) + (\hat{B} - B)' X' X (\hat{B} - B) \\ &\quad + (Y - X\hat{B})' X (\hat{B} - B) + (\hat{B} - B)' X' (Y - X\hat{B}). \end{aligned} \quad (34)$$

Equation (30) follows from equation (34) by applying the identity

$$(Y - X\hat{B})' X \equiv 0 \quad (35)$$

²⁵Here, and throughout this paper, we express densities using the “ \propto ” sign. This simply means that we have omitted the normalizing constant that ensures that the density integrates to unity.

which follows directly from equation (31) as a matter of algebra. Equation (35) is simply the normal equation of the OLS estimator. It holds exactly because it *defines* the OLS estimator. Note that if \widehat{B} were replaced by the true parameter B , it would only hold in expectation.

The marginal posterior density $p(\Sigma_u)$ can be derived by integrating the right-hand side of equation (30) with respect to B :

$$\begin{aligned} p(\Sigma_u) &\propto \int |\Sigma_u|^{-(n+T+1)/2} \exp \left\{ -\frac{1}{2} \text{trace} \left[S \Sigma_u^{-1} + (B - \widehat{B})' X' X (B - \widehat{B}) \Sigma_u^{-1} \right] \right\} dB \\ &= |\Sigma_u|^{-(T-\nu+n+1)/2} \exp \left\{ -\frac{1}{2} \text{trace} [S \Sigma_u^{-1}] \right\} \\ &\quad \times \int |\Sigma_u|^{-\nu/2} \exp \left\{ -\frac{1}{2} \text{trace} \left[(B - \widehat{B})' X' X (B - \widehat{B}) \Sigma_u^{-1} \right] \right\} dB \\ &\propto |\Sigma_u|^{-(T-\nu+n+1)/2} \exp \left\{ -\frac{1}{2} \text{trace} [S \Sigma_u^{-1}] \right\} \end{aligned} \quad (36)$$

because $|\Sigma_u|^{-\nu/2} \exp \left\{ -\frac{1}{2} \text{trace} \left[(B - \widehat{B})' X' X (B - \widehat{B}) \Sigma_u^{-1} \right] \right\}$, being proportional to the normal density, integrates to a constant.²⁶ The distribution in equation (36) is inverse Wishart with parameter S . Equations (30) and (36) give us the conditional posterior for B

$$p(B|\Sigma_u) = \frac{p(B, \Sigma_u)}{p(\Sigma_u)} \propto |\Sigma_u|^{-\nu/2} \exp \left\{ -\frac{1}{2} \text{trace} \left[(B - \widehat{B})' X' X (B - \widehat{B}) \Sigma_u^{-1} \right] \right\} \quad (37)$$

It is more conventional to write the slope coefficients in B as a column vector. Let B_s denote the vector formed by stacking the columns of coefficient matrix B (with \widehat{B}_s similarly denoting the stacked \widehat{B}). Then the density in equation (37) can be written

$$p(B_s|\Sigma_u) \propto |\Sigma_u|^{-\nu/2} \exp \left\{ -\frac{1}{2} \text{trace} \left[(B_s - \widehat{B}_s) [\Sigma_u^{-1} \otimes X' X] (B_s - \widehat{B}_s) \right] \right\} \quad (38)$$

implying that the vector of slope coefficients B_s is multivariate normal with mean \widehat{B}_s and variance $\Sigma_u \otimes (X' X)^{-1}$, precisely the distribution one would obtain from a classical statistical analysis.

²⁶ There are two subtleties here. First, $|\Sigma_u|^{-\nu/2} \exp \left\{ -\frac{1}{2} \text{trace} \left[(B - \widehat{B})' X' X (B - \widehat{B}) \Sigma_u^{-1} \right] \right\}$

is indeed proportional to the multivariate normal density for B with mean \widehat{B} and variance $\Sigma_u \otimes (X' X)^{-1}$, but the constant of proportionality involves $|X' X|$. This is legitimate, since in Bayesian statistics X is treated as a known constant, not a random variable. Second, Zellner's prior implies a "degrees of freedom" correction in the exponent for $|\Sigma_u|$. Sims and Zha (1999) and Doan (2000) omit the degrees of freedom correction, which is equivalent to choosing a different prior for $|\Sigma_u|$.

9.2. Inference when identification uses noisy measures of the fundamental shocks

In this section, we apply the Zellner (1971) approach, described in section 9.1., above, to this model where exactly $n(n-1)/2$ identifying restrictions are imposed on D . In that case, there exists a unique D satisfying equation (25).

To put the notation in the same format as in section 9.1., let the $(T \times n)$ matrix H contain the model-based shocks measures $\{\eta_t\}_{t=1}^T$. Note that we are treating H as *data*, even though H may contain constructed variables that depends on estimated parameters. We treat H as data for two reasons. First, a researcher may obtain a model-based shock measure from some outside source without receiving all the underlying data used to generate the measure. Second, some of these model-based measures are generated by nonlinear models, which would make it very difficult to derive the posterior distribution of the parameter estimates.

The model now consists of two regression equations: equation (26) and equation (22), which can be written

$$H = UC + W. \quad (39)$$

In equation (39), W stacks the $n \times 1$ i.i.d. measurement error process w_t as $W = (w_1, w_2, \dots, w_T)'$, and it is assumed that

$$W \sim N(0, \Sigma_w). \quad (40)$$

The parameters of the model given by equations (26) and (39) are $\{C, \Sigma_w, B, \Sigma_u\}$. Analogously with the task in section 9.1., we wish to derive the joint posterior distribution of $\{C, \Sigma_w, B, \Sigma_u\}$, given the data $\{X, Y, H\}$. We then will use this posterior distribution to compute error bands for impulse responses. For the exactly identified case, A is an exact function of $\{C, \Sigma_u\}$ and the impulse responses are exact functions of $\{A, B\}$, so we can take Monte Carlo draws from this joint posterior to derive Monte Carlo estimates of the posterior distribution of the impulse responses.

We can write the joint posterior density $p(C, \Sigma_w, B, \Sigma_u)$ as²⁷

$$p(C, \Sigma_w, B, \Sigma_u) = p(C|\Sigma_w, B, \Sigma_u) p(\Sigma_w|B, \Sigma_u) p(B|\Sigma_u) p(\Sigma_u) \quad (41)$$

Under the Jeffreys priors (28) and (29), the densities $p(\Sigma_u)$ and $p(B|\Sigma_u)$ are as derived in section 9.1. in equations (36) and (37). It remains to derive the final two conditional densities: $p(C|\Sigma_w, B, \Sigma_u)$ and $p(\Sigma_w|B, \Sigma_u)$. To do so, we follow exactly the same steps as we used to derive $p(\Sigma_u)$ and $p(B|\Sigma_u)$, except that we condition on B . (It turns out that Σ_u does not directly affect the conditional distribution of C and Σ_w .) For a given B , let us write

$$U(B) \equiv Y - XB$$

$$\hat{C}(B) \equiv (U(B)'U(B))^{-1} U(B)' H.$$

²⁷ All densities in equation (41) are conditional on the data $\{Y, X, H\}$. This dependency is not noted explicitly.

and

$$V(B) \equiv \left(H - U(B)\hat{C}(B) \right)' \left(H - U(B)\hat{C}(B) \right)$$

The interpretation of these objects is as follows: $U(B)$ is the matrix of residuals implied by equation (26) for the given B . $\hat{C}(B)$ is the estimate of C that one would obtain from $U(B)$ and H if one estimated equation (39) via OLS. $V(B)$ is the moment matrix of the residuals from this OLS estimation of equation (39). Conditional on B , $U(B)$, $\hat{C}(B)$, and $V(B)$ are functions of the data, so derivations analogous to those for equations (30), (36), and (37) can be performed in which we replace X by $U(B)$, Y by H , \hat{B} by $\hat{C}(B)$, and S by $V(B)$. Note that we use the analogue to equation (35), which is

$$\left(H - U(B)\hat{C}(B) \right)' U(B) \equiv 0 \quad (42)$$

Noting that the number of estimated parameters per equation in the regression of η_t on u_t is n , one obtains the following conditional posterior densities:

$$p(C, \Sigma_w | B, \Sigma_u, Y, X, H) \propto |\Sigma_w|^{-(n+T+1)/2} \times \exp \left\{ -\frac{1}{2} \text{trace} \left[V(B) \Sigma_w^{-1} + \left(C - \hat{C}(B) \right)' U(B)' U(B) \left(C - \hat{C}(B) \right) \Sigma_w^{-1} \right] \right\} \quad (43)$$

$$p(\Sigma_w | B, \Sigma_u, Y, X, H) \propto |\Sigma_w|^{-(T+1)/2} \exp \left\{ -\frac{1}{2} \text{trace} [V(B) \Sigma_w^{-1}] \right\} \quad (44)$$

$$p(C_s | \Sigma_w, B, \Sigma_u, Y, X, H) \propto |\Sigma_w|^{-n/2} \times \exp \left\{ -\frac{1}{2} \text{trace} \left[\left(C_s - \hat{C}_s(B) \right)' [\Sigma_w^{-1} \otimes U(B)' U(B)] \left(C_s - \hat{C}_s(B) \right) \right] \right\} \quad (45)$$

where the subscript “ s ” denotes a matrix stacked columnwise, as above. Note that the conditional distribution in (44) is of the inverse-Wishart form, and the conditional distribution in (45) is Gaussian.

Together, equations (36), (37), (44), and (45) give the posterior distribution for $\{C, \Sigma_w, B, \Sigma_u\}$. One can obtain a draw from this distribution using the following Monte Carlo procedure:

1. Draw Σ_u from the inverted Wishart density given by equation (36);
2. Given this draw of Σ_u , draw B from the multivariate normal distribution given by equation (37);
3. Given this draw of B , draw Σ_w from the inverted Wishart density given by equation (44);
4. Given these draws of B and Σ_w , draw C from the multivariate normal distribution given by equation (45);

5. We now have one draw from the joint density given by equation (41). In the exactly-identified case we can use this draw of $\{C, \Sigma_u\}$ to compute D as the unique matrix satisfying equation (25). A draw of A is then computed from D and the draw of C using equation (24). The draws of A and B can then be used to generate one draw of the impulse responses.

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Table 1: Fraction of 5-Year Ahead Yield Variance Attributable to Macro Shocks under Recursive Orthogonalization**Panel A: Baseline Case**

	One-month Yield	12-month Yield	60-month Yield
Shock to IP	0.263 (0.083,0.495)	0.260 (0.077,0.522)	0.189 (0.032,0.443)
Shock to P	0.066 (0.014,0.268)	0.073 (0.011,0.282)	0.119 (0.014,0.358)
Shock to PCOM	0.467 (0.240,0.647)	0.518 (0.273,0.684)	0.535 (0.218,0.666)
Total Macro Shocks	.796	.751	.843

Panel B: No-Smoothing Case

	One-month Yield	12-month Yield	60-month Yield
Shock to IP	.081	.071	.045
Shock to P	.140	.154	.202
Shock to PCOM	.249	.261	.286
Total Macro Shocks	.470	.486	.533

Panel A gives the fraction of 5-year ahead forecast variance of the 1-, 12-, and 60-month zero-coupon Treasury yields explained by each of the three orthogonalized shocks (listed in the left-hand-most column) according to the baseline model. Numbers in parentheses are lower and upper 90% error bands, computed using 500 Monte Carlo draws from the Bayesian posterior distribution of the model parameters. Panel B gives the point estimates for the corresponding variance fractions implied by the model without interest rate smoothing. The bottom row in each panel sums the point estimates in the first three rows.

Table 2: Correlation Matrix of $\{\eta_{MP}, \eta_{MRS}, \eta_{Tech}, \eta_{Fiscal}\}$

	η_{MP}	η_{MRS}	η_{Tech}	η_{Fiscal}
η_{MP}	1.0			
η_{MRS}	0.11	1.0		
η_{Tech}	-0.11	0.05	1.0	
η_{fiscal}	-0.02	0.15	0.31	1.0

The model-based shocks to monetary policy, preferences, technology, and fiscal policy are denoted η_{MP} , η_{MRS} , η_{Tech} , and η_{fiscal} , respectively. The derivation of these shocks is described in section ??.

Table 3: R^2 Estimates in Regressions of Model-Based Shocks on VAR Residuals

Shock	R^2
η_{MP}	66.0%
η_{MRS}	22.7%
η_{Tech}	36.7%
η_{fiscal}	8.7%

This table displays the R^2 s from the regression of each of the model-based shocks $\{\eta_{MP}, \eta_{MRS}, \eta_{Tech}, \eta_{Fiscal}\}$ on the VAR residuals u_t , as in equation

(22).

Table 4: Variance Decompositions at 5-Year Horizon Using Identified Shocks

	ε_{MRS}	ε_{Tech}	ε_{Fiscal}	ε_{MP}
Real GDP	39 (18 - 62)	20 (2 - 46)	1 (0 - 17)	40 (15 - 59)
Inflation	9 (2 - 36)	55 (10 - 71)	23 (7 - 62)	14 (4 - 43)
PCOM	14 (3 - 36)	45 (4 - 67)	22 (2 - 73)	19 (6 - 39)
Fed Funds	34 (9 - 65)	46 (5 - 57)	4 (2 - 51)	16 (9 - 36)
1-month yield	37 (13 - 63)	39 (4 - 50)	3 (1 - 44)	11 (6 - 31)
12-month yield	37 (12 - 66)	46 (4 - 57)	3 (1 - 52)	6 (3 - 25)
60-month yield	26 (6 - 57)	57 (4 - 64)	4 (1 - 55)	3 (2 - 22)

For each of the four macro variables $\{GDP, price, PCOM, Fed Funds\}$ and each of the three yields, the table gives the percentage of the 5-year ahead forecast error variance attributable to each of the four identified shocks $\{\varepsilon_{MRS}, \varepsilon_{Tech}, \varepsilon_{Fiscal}, \varepsilon_{MP}\}$. (Percentages for the yields do not add up to unity, due to the effects of the yield shocks γ_t in equation (1).) The shocks use the identification described in section 5.6. Numbers in parentheses are lower and upper 90% error bands, computed using 500 Monte Carlo draws from the Bayesian posterior distribution of the model parameters.

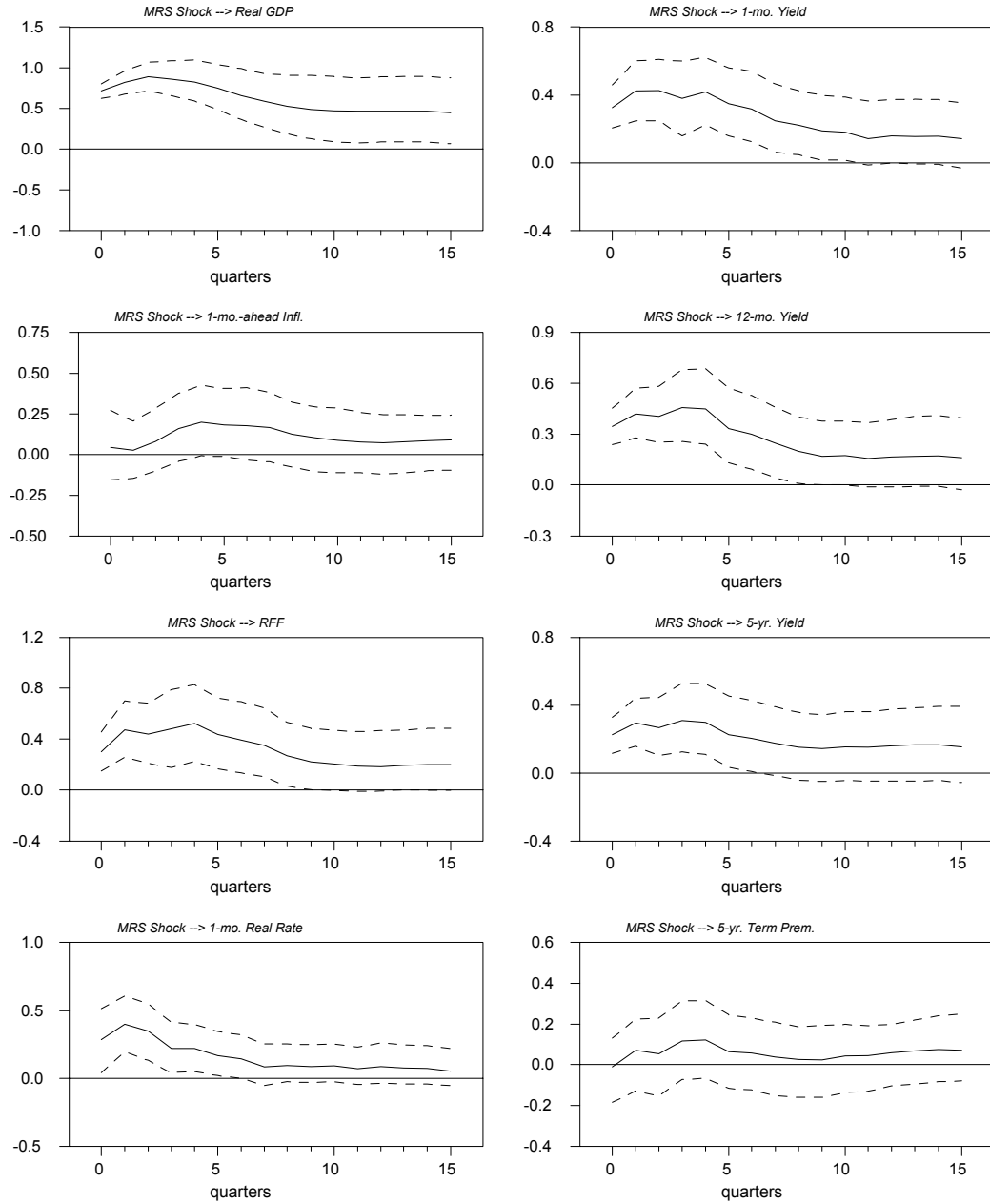
Figure 1: Responses to MRS Shock

Figure 1: This figure displays the responses of eight endogenous variables to a unit shock to the marginal rate of substitution shock ε_{MRS} . The dashed lines give 90% probability error bands implied by the posterior distribution of the model's parameters.

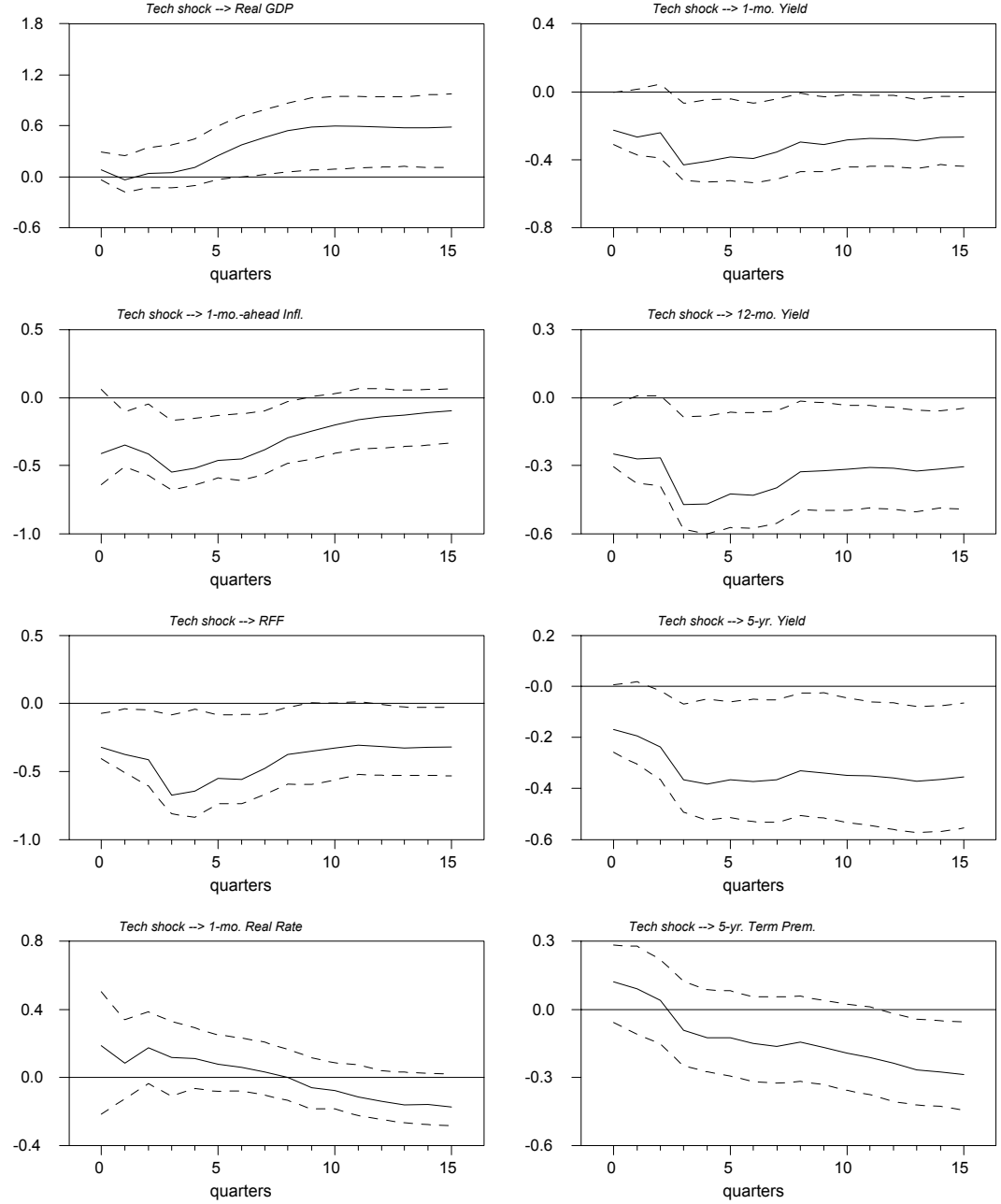
Figure 2: Responses to Technology Shock

Figure 2: This figure displays the responses of eight endogenous variables to a unit shock to the technology shock ε_{Tech} . The dashed lines give 90% probability error bands implied by the posterior distribution of the model's parameters.

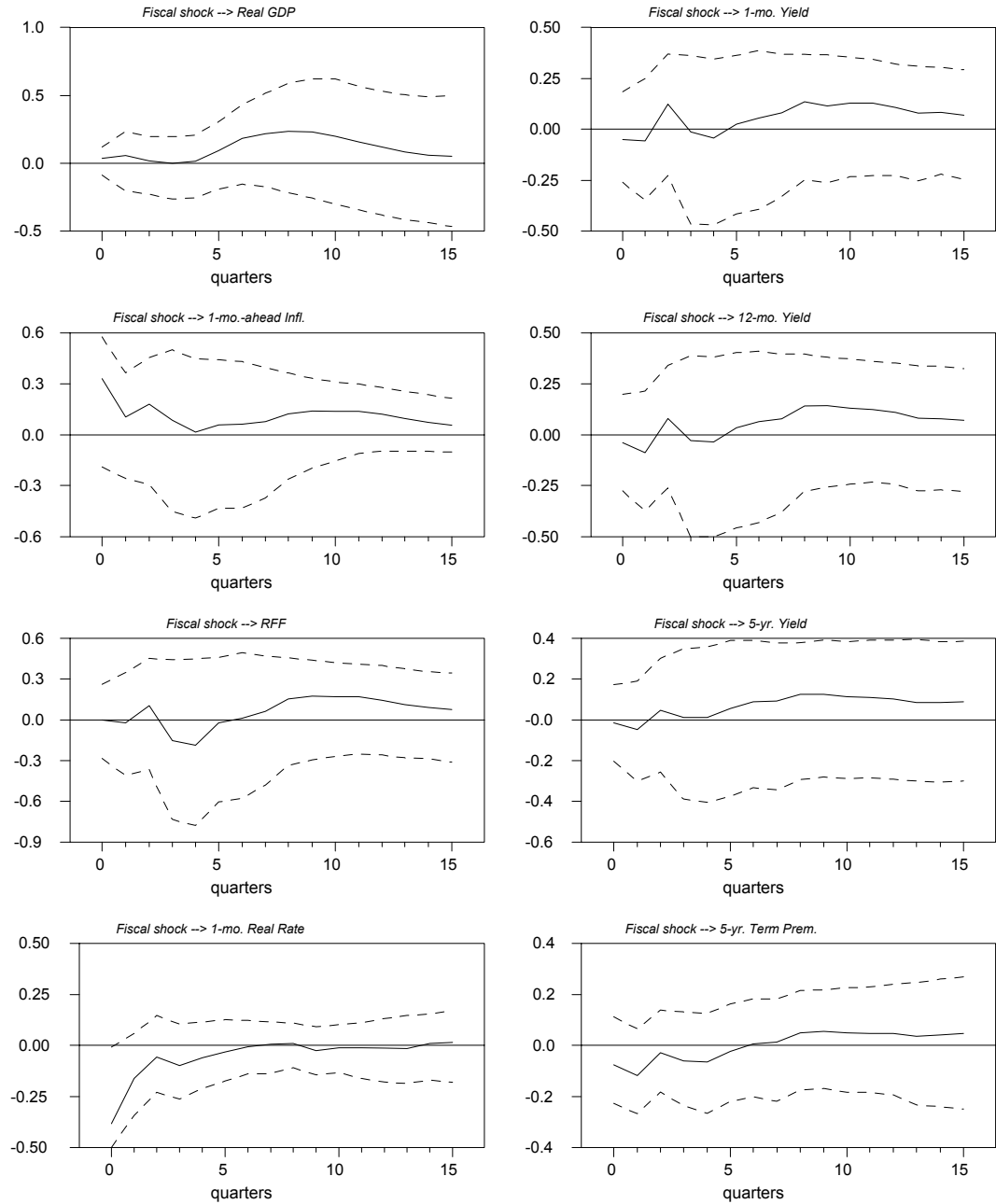
Figure 3: Responses to Fiscal Shock

Figure 3: This figure displays the responses of eight endogenous variables to a unit shock to the fiscal policy shock ε_{Fiscal} . The dashed lines give 90% probability error bands implied by the posterior distribution of the model's parameters.

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