

Master Thesis

Modelling the Yield Curve - Factoring in the Macroeconomy

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under the supervision of

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To Prof. Dr. Helmut Kramer, who not only taught me how to be a considerate economist, but, more importantly, what it means to be a loving father.

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1 Introduction

Over the past decade, people have become increasingly concerned about personal health. One's own health is constantly monitored and assessed using different metrics, while potential risks are forecasted and mitigated as good as possible. For the masses, simplicity is key in this context as easy to understand metrics offer a good indication of one's current health status. In a similar fashion, economists, finance professionals and laypeople alike have some key metrics to assess the health of the economy. Especially in times of turmoil, simple albeit highly relevant metrics often enter the center stage. One of the most well-known and most studied among those key metrics is the yield curve. Its various shapes, slopes and spreads are analysed, forecasted and interpreted in order to assess the health of the overall economy as well as the market's expectations of the future. Beware an inverted yield curve, which the market usually interprets as the onset of a recession, similar to a doctor facing elevated inflammation markers and inferring the onset of illness. But analogous to a doctor, who is faced with an individual with various idiosyncratic characteristics, economic researchers are faced with a tremendous amount of information from which they try to infer a signal in order to predict future economic activity.

Though the yield curve is a key indicator for economic agents, it is often not well understood, either in a general sense and especially in regards to which factors determine the shapes and movements of the yield curve. Ultimately, correctly assessing and predicting the impact of movements in the underlying factors on the yield curve enables central bank policymakers to calibrate their policies to current market expectations as well as giving asset managers the opportunity to position their portfolios in order to maximize returns.

In previous research, the yield curve has been studied in a rather dichotomous fashion, either from the lens of a finance professional, or from the perspective of an economist. However, yields are known to contain a tremendous amount of information about the economy, be it on the current stance of monetary or fiscal policy as well as expectations of the future (Evans and Marshall, 2007). Equivalently, ever since the introduction of the expectation theory by Hicks (1946), monetary policy (expectations) seem to be one of the main drivers of yield curve movements (Evans and Marshall, 1998). But how exactly is the yield curve related to the economy? Is the yield curve a leading indicator for economic activity, or are economic variables the reason for shifts in the yield curve? Could there even be a bidirectional link? This thesis is aimed at answering these questions.

Largely following Diebold, Rudebusch, and Aruoba (2006), a Nelson and Siegel (1987) decomposition of the US and EA yield curves is conducted, yielding factors representing the level, slope and curvature. In a second step, a Bayesian sVAR

is estimated using said factors while also incorporating various economic variables in order to disentangle this messy relationship using distinct shocks and analysing the responses from each variable deducing potential interpretations regarding their relationship.

The thesis is structured in the following way. Chapter 2 offers a brief overview of the literature, highlighting past findings regarding the yield curve's relevance and connection to macroeconomic factors. Chapter 3 introduces the methodological approach as well as the data. The next section presents an empirical analysis of the US (Section 4.1) & Euro Area yield curves (Section 4.2), where the relevance of macroeconomic variables as drivers of yield curve movements is assessed, while simultaneously examining the impacts shifts in the yield curve have on economic variables. Thereafter, a conclusion summarizing the main findings is provided.

2 Literature Review

There has historically been an abundance of literature about the relationship between the term structure of interest rates and economic activity. This section aims at narrowing down this wealth of research, providing an overview of the hitherto available and relevant literature, focusing on identifying the potential one- or bi-directional link between the yield curve and the economy. Thus, it follows the distinction provided by Diebold, Rudebusch, and Aruoba (2006), who divide this research into yields-to-macro versus and macro-to-yields studies.

Some of the central contributions regarding the yields-to-macro relationship have come from Harvey (1988), who shows that the expected real term structure contains information about future consumption growth. Among other researchers, Estrella and Hardouvelis (1991), Estrella and Mishkin (1995) and Estrella and Mishkin (1996), use a probit model approach, modelling the probability of a recession as a function of the term spread¹, through which the authors have provided some remarkable results corroborating the hypothesis in regard to the relevance of the yield curve (slope) for macroeconomic activity, at least for the US and other major European economies such as Germany, the UK and France. In a similar fashion, Bernard and Gerlach (1998) extend the analysis to other major economies such as Canada, Japan, Belgium and the Netherlands, confirming the finding that the term spread offers significant information regarding future recessions. Solely for Japan the significance is limited, which can possibly be attributed to differences in financial regulation and the fact that interest rates in Japan did not fully reflect market participants' expec-

¹The term spread is mainly known as the difference between the 10-year and 3-month government bond yields

tations of the future path of the economy (Bernard and Gerlach, 1998). Evgenidis, Papadamou, and Siriopoulos (2018) offer a meta-analysis of the yields-to-macro view, again focusing on the yield spread's ability to forecast economic activity. Although they again corroborate the findings of the previous 30 years, they note that the modelling strategy should take non-linearities into account as well as underlining the importance of incorporating monetary policy variables into the models, as some predictive power of the yield spread is attributed to expectations about the future path of monetary policy. The authors also note that there still is no widely accepted theory explaining the usefulness of the yield curve. Apart from these rather promising studies affirming one's confidence in the yields-to-macro view, authors such as Dotsey (1998) and Stock and W.Watson (2003) note that yields (spreads) have somewhat lost their predictive ability ever since the 1980s.

On the spectrum of authors leaning towards a macro-to-yields approach, central research has come from Evans and Marshall (1998), Ang and Piazzesi (2003) and Evans and Marshall (2007), all using some variation of a VAR framework.

Using three different identification strategies within a VAR setting, Evans and Marshall (1998) find that monetary policy shocks primarily have a significant effect on short-term interest rates while also reducing expected inflation, implying a rise in the real interest rate. The authors interpret this as monetary policy having a significant effect on the yield curve through a liquidity, rather than an expected inflation effect. Though delivering promising results of a potential macro-to-yields relationship, one key weakness is the sole focus on monetary policy shocks alone, whilst excluding other macro variables. Counteracting these shortcomings and extending their previous research, Evans and Marshall (2007) include other macro variables into their model, such as technology, fiscal policy and preferences for current consumption, an identification set predominantly derived from previous research on DSGE models. As before, they conclude that, apart from fiscal policy shocks, macroeconomic factors such as technology are able to explain a large part of yields movements along the short- to medium- end of the curve.

By including multiple macro variables into a multi-factor model of the term structure, using the assumption of no-arbitrage as identifying restrictions, Ang and Piazzesi (2003) find that macro variables explain a tremendous amount of the variation in bond yields, especially at the short-end of the curve, where the main drivers are shocks to inflation. This is consistent with the previous findings by Evans and Marshall (1998). The authors also conclude that incorporating macro factors in latent factor term structure models enhances out-of-sample forecasts.

Taking the stance of the opposite direction of causality, i.e. yields-to-macro, Ang, Piazzesi, and Wei (2006) offer a comparison to previous research of Estrella and Hardouvelis (1991) etc. Through estimating a relatively simple VAR only including

a set of yields and GDP growth, they conclude that the nominal short rate dominates the slope of the yield curve in predicting future economic activity.

Some other notable contributions in the realm of linking the macroeconomy and the yield curve have come from Dewachter and Lyrio (2006) and Rudebusch and T. Wu (2008). Both sets of authors extend the latent factor model of the term structure with macroeconomic factors. The former specifically reference to Ang and Piazzesi (2003) in the context of a possible model misspecification, since previous research has apparently failed to correctly model the long end of the yield curve. Through their slightly modified approach, the authors introduce long-run inflation expectations in their model and show that not only the short-end of the yield curve, but also longer maturities are indeed driven by macroeconomic variables. They also suggest interesting and seemingly plausible interpretations to the latent factors, chiefly among them finding that the level factor is closely related to the aforementioned long-run inflation expectations. Rudebusch and T. Wu (2008) combine a small scale macro model, assuming that the short-term rate is represented as a monetary policy reaction functions, such as the well known function developed by Taylor (1993), with a standard no-arbitrage latent factor model of the term structure, where the short-term interest rate is modelled as a function of latent factors often interpreted as a level and/or slope factor, the authors are able to synthesize both the finance and the economists approach to modelling interest rates. Obtaining promising results, they offer insightful and intuitive interpretations, e.g. how the level factor and the central banks inflation target are linked, underlining the significance of a holistic approach.

Among the major contributions regarding the application of the Nelson and Siegel (1987) method in a macroeconometric framework are Diebold and Li (2006). Since other prominent theoretical models of the yield curve, chiefly among them are the no-arbitrage as well as equilibrium models, fail to provide the tools necessary for accurate predictions, the authors try to fill this gap using the aforementioned and well-known Nelson-Siegel decomposition. More precisely, the authors compare the term structure forecasting performance of said strategy with that of various benchmark forecasts (e.g. random walk) in order to test their hypothesis that a Nelson-Siegel approach yields superior out-of-sample forecasting results. Through decomposing a set of yields, representing the term structure, into three factors, which are then assumed to be evolving as an AR(1) process over time, Diebold and Li (2006) obtain forecasts of the yield curve based on the forecasts of said factors. The authors find that this approach appears to yield superior forecasts, especially at longer horizons of one year and beyond. Their promising results are one of the main reasons, why a Nelson-Siegel factor model approach has been chosen for this thesis. Another central contribution of the authors is their theoretical as well as empirical argumentation regarding the interpretation of the Nelson-Siegel factors as representing the level, slope and curvature of the decomposed yield curve. This interpretation is

conveniently used in Section 4 to get a first glimpse how well the estimated factors approximate their theoretical counterparts.

One of the central papers serving as an inspiration and template alike for this thesis is the seminal work by Diebold, Rudebusch, and Aruoba (2006). Extending upon the bipartite available literature at that time, the authors are among the most significant to investigate the joint dynamics of both macro and financial variables (i.e., level, slope and curvature factors representing the yield curve). They build upon the approach used by Diebold and Li (2006), using a state-space representation of the Nelson-Siegel model, estimated via maximum-likelihood using the Kalman filter, which simultaneously models the dynamics of the yield curve for each point in time as well as the dynamics of other variables. Through this one-step approach the authors investigate the relationship between the macroeconomy and yields, laying forth the research question of this thesis, how bond yields and the macroeconomy are linked and, more interestingly, if there even is a bi-directional relationship. Diebold, Rudebusch, and Aruoba (2006) conclude that there is strong evidence for a macro-to yield effect and a somewhat weaker evidence for the yield curve to affect (future) macroeconomic dynamics.

Using the same two-step approach as in this thesis, some interesting contributions for emerging economies such as India and Chile, have come from Kanjilal (2011) and Morales (2010), respectively. Both authors find evidence of a potential bi-directional link between macroeconomic variables and the yield curve. Interestingly, Kanjilal (2011) finds a stronger evidence for the yields-to-macro direction, which supports the literature by authors such as Estrella and Hardouvelis (1991) and somewhat contrasts the findings by Diebold, Rudebusch, and Aruoba (2006). Suitably for this thesis, Morales (2010) compares the two-step estimation introduced by Diebold and Li (2006) with the more complicated state space representation (one-step estimation process) by Diebold, Rudebusch, and Aruoba (2006) and concludes that the simplified estimation methodology does not contradict the basic intuition of the results. Furthermore, the author finds evidence for a two-way relationship between yield curve factors and macroeconomic variables for the Chilean economy.

Notable theoretical contributions have come from Hicks (1946), who is among the economists credited with developing the well-known expectations theory, which is also tested in Diebold, Rudebusch, and Aruoba (2006). Kessel (1971) offers a broad overview of common (past) theories explaining the term structure of interest rates. Some of the latest contributions in the corresponding literature have mainly focused on how and how-well a Nelson-Siegel approach can be implemented in a machine learning framework, where, among others, Pedersen and Swanson (2019) offer a comprehensive examination.

3 Methodology and Data

This section is aimed at outlining the methodology applied in this thesis. After giving a comprehensive primer on the aforementioned model of the yield curve introduced by Nelson and Siegel (1987), the identification strategy for the yield curve model containing macroeconomic variables is introduced. Lastly, an overview of the data and its sources is offered.

The first step of the methodological approach involves modelling the yield curve using the Nelson-Siegel three factor model. The genius of the Nelson-Siegel decomposition lies within its flexibility as well as its parsimony. Based on a set of yields, the model is able to capture a wide range of yield curve shapes. Following the notation of Diebold, Rudebusch, and Aruoba (2006), the yield curve at any time t is represented as the Nelson and Siegel (1987) model:

$$y_t(\tau_N) = \beta_{1t} + \beta_{2t} \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_{3t} \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right), \quad (1)$$

where $y_t(\tau_N)$ is a set of N yields with different maturities τ , used for the decomposition, β_{1t} , β_{2t} , β_{3t} and λ are the parameters to be estimated. The regressors are also known as the factor loadings of the β coefficients, where λ shows the rate of decay of said factor loadings.

As described in Nelson and Siegel (1987), the estimation is conducted in the following way. For a provisional value λ , sample values for the factor loadings are calculated. Based thereupon, the best-fitting values of the β coefficients are estimated. This procedure is repeated over a grid of values for λ , yielding the best-fitting values for λ , β_{1t} , β_{2t} and β_{3t} .

As demonstrated by Diebold and Li (2006), the factors β_{1t} , β_{2t} and β_{3t} can have an economically meaningful interpretations, i.e. they are the time-varying level, slope and curvature factors, L_t , S_t , C_t , respectively. Thus, the estimated model² of the yield curve used in this thesis is represented by:

$$y_t(\tau_N) = L_t + S_t \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + C_t \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right), \quad (2)$$

For each time period t , a level, slope and curvature factor is estimated based on the

²The estimation of the yield curve factors is conducted using the Nelson-Siegel method from the R package YieldCurve

prevailing yields $y_t(\tau_N)$ at that time. Consequently, the L_t , S_t and C_t factors are assumed to be an approximate representation of the yield curve at any given time t and, together with the macroeconomic variables, are included in a VAR(N) model aimed at studying the link between the economy and the yield curve.

Aforesaid VAR(N) model forms the second step of the analysis and is represented in the following way:

$$\begin{pmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{pmatrix} + \begin{pmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{pmatrix}, \quad (3)$$

After the estimation of the VAR(N) model, the orthogonal shocks are used to identify the relationship between the macroeconomy and the yield curve. They can be represented in the following way:

$$\begin{pmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{pmatrix} + \begin{pmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{pmatrix}, \quad (4)$$

A comprehensive overview of the potential methodological approaches is offered in Diebold and Rudebusch (2013).

The sample for the United States contains monthly data ranging from January 1973 to December 2022. Except for data on US yields, the excess bond premium and the S&P 500 stock market index, all data has been obtained from the FRED. A more detailed description of the data and its sources is provided in the Appendix A.

Of the highest significance for any analysis involving a Nelson-Siegel decomposition are the yields data. So far, the literature, heavily focused on the US, has primarily used unsmoothed Fama and Bliss (1987) Treasury forward rates obtained via the CRSP³, which are then converted to unsmoothed Fama-Bliss zero yields. This thesis uses zero-coupon US Treasury yields obtained via a novel and improved approach kindly provided by Liu and J. C. Wu (2021). Following Diebold, Rudebusch, and Aruoba (2006), the maturities used are 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 months.

The data used for the analysis of the Euro Area consists of monthly data, spanning from October 2004 to December 2022. Again, the bulk of the dataset containing macro variables has been obtained via the FRED. Solely the Eurostoxx 50 as well

³<https://www.crsp.org/>

as the VSTOXX indices, have been obtained via Yahoo Finance and the Bloomberg Terminal, respectively. The Euro Area yield curve data consists of spot rates derived from bonds with finite maturity denominated in EUR and issued by a euro area central government with an (issuer) rating of triple A. The data has been obtained via the ECB and, due to somewhat limited data availability, includes yields with maturities of 3, 6, 9, 12, 24, 36, 48, 60, 72, 84, 96 and 120 months.

4 Analysis

4.1 The Yield Curve and the Macroeconomy in the USA

4.2 The Yield Curve and the Macroeconomy in the Euro Area

5 Conclusion

A possible extension of this thesis would be to actually use the estimated slope factor in a probabilistic setting like the one proposed by Estrella and Hardouvelis (1991) and see if said factor, being an approximation of the term spread, is able to correctly predict the onset of past recessions. Furthermore, testing the performance in yield curve movement forecasting of a general Nelson-Siegel decomposition approach as well as an extended model including macro variables in a machine learning setting vis-a-vis various other benchmark models could be further enlightening.

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A Appendix

Data US

- Industrial Production (FRED: INDPRO)
- CPI (FRED: CPIAUCSL)
- 3M T-Bill Rate (FRED: TB3MS)
- S&P 500 (Yahho Finance: GSPC)
- Excess Bond Premium (EBP)⁴
- β factors based on yields data provided by Liu and J. C. Wu (2021)⁵

Data EA

- Industrial Production (FRED: INDPRO EA)
- Inflation (FRED: Inflation EA)
- 3M Interbank Rate (FRED: 3M Rate EA)
- Eurostoxx 50 (Yahho Finance: GDAXI)
- VSTOXX Index (source: Bloomberg Terminal)
- β factors based on yields data provided by the ECB⁶

⁴source: Fed Note Data

⁵source: Liu-Wu Yield Data

⁶source: dataset “All years - AAA”