ELSEVIER

Economic Modelling

Volume 29, Issue 2
March 2012

ISSN 0264-9993

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(contents continued on outside back cover)

(This is a sample cover image for this issue. The actual cover is not yet available at this time.)

Economic Modelling 31 (2013) 739–751

Contents lists available at SciVerse ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Factors causing movements of yield curve in India

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ARTICLE INFO

Article history:
Accepted 10 January 2013
Available online xxxx

Keywords:
Estimation of term structure of interest rate
Financial reforms
Latent factor extraction
Level
Monetary policy
Yield curve movements

ABSTRACT

The article identifies principal reasons underlying the movements of yield curve for government debt market in India for the period Jul '97 to Dec '11. The study finds that though statistically Svensson's (SV) (1994) model outperforms Nelson and Siegel's (NS) (1987) model in yield curve estimation, 99% of the movements in yield curves in India are explained by three factors which are 'level' (long-term factor), 'Slope' (short-term factor) and 'Curvature' (medium-term factor) with 'level' contributing more than 90% of its variations. This implies that in more than 90% of cases, the yield curves move parallel either in upward or in downward direction bringing similar effects to all maturity spectrums. This means that yield curve movements in India mainly reflect the monetary policy changes of central bank. Hence, NS's three parameter model is probably more than sufficient to capture all possible shapes of yield curves in India. This finding also suggests that a simple 'duration and convexity' hedging strategy should be appropriate to cover maximum risk exposure of government debt market investors in India.

1. Introduction

The Indian debt markets are relatively less developed compared to its equity markets. Government debt market is mostly restricted to the trading in government securities only. Bonds and other fixed income instruments issued by the public sector units, financial institutions and corporate are the main trading instruments in the non-government (NG) debt market. Government debt market in India has developed much faster over the course of time in terms of liquidity, transparency and volume of transactions as compared to NG debt market after the deregulation of financial markets since 1991. As of 2005, about 91% of all debt securities accounts to the government, 7–8% to financial institutions, and less than 2% to corporate debt. The government debt market stands at 25% of GDP while the corporate debt to GDP is 5% as of 2009. The total market loans of central government securities in 2003 was Rs 6.8 billion as opposed to Rs 1.4 billion in 1996. The outstanding loan for 2009 was Rs 13.6 billion.

The estimation of term structure of interest rates or a benchmark zero coupon yield curve for government debt market has been possible after the liberalisation of the Indian financial market. In the pre-reform days, administered interest rates had been the dominant feature of the Indian market. Yield curve estimation and factors underlying its movements were not meaningful until late nineties due to the gradual course of financial deregulation in India. A 'zero coupon yield curve' (ZCYC) or 'the term structure of interest rate' gives the relationship among the market interest rates with different maturities at a particular time period. This has been used as a benchmark for evaluating investment strategies in the local and global financial markets for more than a decade. Central Banks often use it as an indicator of future inflation. The credibility, sustainability, and the conduct of monetary policy strategies can be assessed through the yield curve developments. The estimate can be used to price all non-sovereign fixed income instruments with an addition of an appropriate credit spread. Principal drivers or determinants of the yield curve help investors to develop hedging strategies for debt instruments. This also helps to assess the maturity and independent functioning of government debt market.

This study aims at identifying key factors underlying the movements of yield curve in government debt market in India in order to provide a guideline to the risk-averse debt market investors and to see if the yield curve movements reflect market expectations. The article begins with estimation of term structure of interest rates for government debt market in India post the financial deregulation in 1990–91 for Indian debt market using parametric parsimonious families of models like Nelson and Siegel (1987) and Svensson (1994) to capture all possible variations of yield curves during the period Jul '97 to Feb '04. The study then moves on to examine the justification of the choice of number of parameters used in the parsimonious model by extracting the unobserved factors underlying the movements of term structure of interest rates or yield curves with no prior assumptions of exact relationship between factors and the changes in bond price for the period Jul '97 to Feb '04 and Mar '04 to Dec '11. Instead of estimating monthly yield curve by 3 or 4 parameter-based parsimonious models (NS or SV), the study extends the extraction of key determinants of yield curve movements for the period Mar '04 to Dec '11 to see if the structural policy changes in government debt market in the past 5–7 years have changed

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http://dx.doi.org/10.1016/j.econmod.2013.01.018

the composition of key factors driving the yield curve movements as eventually the article shows the latter justifies the former.

This article is organised as follows: Section 2 provides a brief overview of Indian debt market. Section 3 has literature survey. Section 4 gives data description. Section 5 explains econometric methodology. Estimation results are discussed in Section 6. Section 7 concludes the study.

2. Overview: Indian debt market

Indian debt market was very thin in terms of breadth and depth before 1990s. Interest rates were administered. Coupon rates were artificially low to reduce the cost of borrowing of the government. It was a captive market dominated by banks, insurance companies, provident funds and other financial institutions. The risk-free government debt market has grown much faster as compared to non-government debt market since the financial deregulation started in the year 1991–92 (July–June). A series of reform initiatives that eventually helped to develop the government debt market is worth mentioning.

The abolition of automatic monetisation through ‘ad hoc’ Treasury Bills and the introduction of a system of Ways and Means Advances for the Central Government are some of the major steps in this direction. RBI has introduced the system of primary dealers (PDs) on March 29, 1995. Subsidiary General Ledger (SGL) account was opened up to disseminate market information on daily trading of government securities in Sep '94. Coupon rates were increased gradually. Foreign institutional investors were allowed to invest in government securities markets starting January 30, 1997. Gradual reduction of reserve requirements both in SLR (Statutory Liquidity Ratios) and CRR (Cash Reserve Ratios) and the introduction of liquidity adjustment facility (LAF) on June 2000 are some of the major achievements towards reform. In February 2002, Negotiated Dealing System (NDS) was operationalised. Securities with call and put option were introduced in July 2002. Interest rate derivatives have been introduced in June 2003. STRIPS or zero coupon bonds are also introduced for trading in secondary market recently.

These gradual phases of reform measures have a profound impact in the improvement of market depth and liquidity over time as reflected in various parameters shown in Table 1. The outstanding stock of central government securities has gone up from Rs 769 billion in 1992 to Rs 13,589 billion in 2009, an increase of 190%. As a proportion of GDP, it has increased substantially from 15% in 1992 to 26% in 2009. The average maturity of securities issued during the year has elongated from around 6 years in 1996 to 15 years in 2003. The weighted average cost of securities has declined from 14% in 1996 to 8% in 2009.

Table 3 shows the bond holding pattern of some of the dominant players in the government debt market. RBI, Life Insurance Corporation (LIC), banks, and provident funds have been the dominant players in government debt market. As seen in Table 3, the stake of other players like mutual funds, and foreign investors have increased to 22% in 2007 as compared to 13% in 2001, a 70% jump. The investor base for Federal securities in the US has been banks, financial institutions, provident funds (PFs), insurance and pension funds. The Major owners are International official Entities and Investors along with the Federal Reserve System. Some of the major investors in the US debt can be listed as Goldman Sachs, Citigroup Companies, Merrill Lynch, Bank of America and AIG. The maturity distribution of dated central Government securities transactions in the secondary market as shown in Fig. 10a,b,c and d from 2008–09 to 2011–12 suggests the buyers' preference of holding long-term securities. As seen in the pie diagrams in Fig. 10a to d, securities in the maturity range of 7–10 years and above 10 years account for 80 to 90% of the trading activity. The securities in the maturity range of 7–10 years represent the highest share of trading at 54.0% in 2008–09 and 56% in the year 2011–12. Fig. 11 also suggests that the RBI does not carry out open market operations with Treasury Bills. In June 2012, the RBI has infused Rs 12,000-crore into the market via Open Market Operations (OMOs) to relax the liquidity situation by purchasing the government securities 8.19% bonds maturing in 2020, 8.79% bonds maturing in 2021, 8.08% bonds maturing in 2022 and 7.35% bonds maturing in 2024. Table 2 provides a comparative picture of bond market capitalization as a percentage of GDP for Indian bond market in the global setting in terms of public versus private ownership. As observed, similar to other developing nations like China, Brazil, and Indonesia India's bond market is dominated by the public sector participation.

Table 1
Sources: RBI, Report on Currency and Finance, Various Issues.

	1992	1996	2002	2003	2009
1. Outstanding Stock (Rs in billions)	769	1375	5363	6739	13,589
2. Outstanding Stock as ratio of GDP (percent)	14.68	14.2	27.89	27.29	25.54
3. Turnover/GDP (percent)	–	34.21	157.68	202.88	332.61
4. Average maturity of the securities issued during the year (in years)	–	5.7	14.9	15.32	13.81
5. Weighted average cost of the securities issued during the year (percent)	11.78	13.77	9.44	7.34	7.69
6. Minimum and maximum maturities of stock issued during the year (years)	–	2–10	5–25	7–30	4–30
7. PDs' share in the turnover					
A. Primary market	–	–	70.46	65.06	45.4
B. Secondary market	–	–	22.04	21.72	18.77
8. Transactions on CCIL (face value Rs in billions)	–	–	548	15,323	62,545

Note: Outstanding Stock represents the total market loans of Central Government. Turnover is the total of outright and repo turnover in G-secs. Outright turnover and repo turnover are calculated as twice and four times the transactions volume respectively.
Data includes development but include MSS and Non-competitive Bids.

3. Literature survey

The history of empirical literature of estimation of term structure of interest rates is very vast. Empirical studies in this direction started in the late 20s with Guthmann's (1929) work in developed countries. Fisher (1966) and Cohen et al. (1996) investigate yield curve fitting using functional forms which can be estimated by the Ordinary Least Square (OLS) regression. However, a seminal work in this field is done by McCulloch (1971, 1975). McCulloch (1971) in his yield curve analysis constrained the cash flows from different bonds of same maturity rates or due at the same time to be discounted at the same rate of return and estimated a discount function from which a term structure can be derived. There has been a significant development in the literature contributed by Schaefer (1977, 1981), Vasicek and Fong (1982), Shea (1985), Nelson and Siegel (1987), Svensson (1994), and Fisher et al. (1995) since then. Schaefer (1981) uses

Table 2
Public versus private bond market (percent of GDP).
Source: World Bank.

Country	Private bond market			Public bond market		
	1991	2001	2007	1991	2001	2007
Argentina	0.1	5.0	5.6	5.6	10.8	23.7
Brazil	0.0	9.4	16.9	0.0	48.7	46.1
China	3.1	7.5	14.5	2.4	9.3	29.4
Indonesia	0.1	1.4	2.0	0.0	31.0	17.0
India	0.7	0.4	2.7	20.6	25.3	31.0
Mexico	1.6	9.7	17.1	17.2	14.3	20.3
Korea	30.2	60.0	58.8	13.0	25.5	48.1
Australia	14.1	28.7	57.4	22.1	18.1	13.1
Canada	12.9	27.3	29.6	69.3	60.0	51.3
Germany	38.6	54.1	34.5	20.2	31.9	39.9
France	55.4	39.8	48.5	22.8	44.9	51.4
United Kingdom	14.0	18.3	15.8	24.8	29.5	32.1
Italy	28.0	34.3	54.8	80.8	86.2	79.1
Japan	40.4	48.5	38.8	44.0	89.4	159.9
United States	72.3	105.7	125.1	55.8	–	–

Table 3
Government bond holdings of major players: post reform.
Source: Handbook of Statistics on the Indian Economy, 2008–09, RBI.

Year	RBI (percent)	Banks (percent)	LIC (percent)	Others (percent)
1991	20.3	59.4	12.3	8.0
1995	2.0	69.6	16.2	12.2
2000	7.0	60.9	18.1	14.1
2001	7.7	61.0	18.3	13.1
2005	5.2	52.4	20.5	21.9
2006	5.0	46.5	22.2	26.4
2007	7.5	46.9	22.8	22.8

'Bernstein polynomials' to estimate the discount functions in a similar manner to McCulloch. Vasicek and Fong (1982) used a third order exponential spline to estimate discount function. Nelson and Siegel used a parsimonious functional form to estimate term structure. Their work differs from the previous works in the sense that they estimated a forward yield curve. Svensson's contribution in this field is an addition of a second humped shaped component to Nelson Siegel's model. Anderson et al. (1996) mentioned that for macroeconomic analysis a parsimonious model such as that of Nelson and Siegel (1987) and hence Svensson (1994) should give desired results. These models overcome the problem of spline model which is based on choosing best 'knot' points.

The parametric parsimonious families of functions like Nelson and Siegel (1987) and Extended Nelson and Siegel or Svensson (1994) models are based on the assumptions about the number of sources of uncertainty and their structure. These sources of variation in interest rates could originate from innumerable number of economic forces like the supply and demand for loans, announcements of unemployment, inflation, growth or recession indicators, monetary policies, changes in market participants' activities, etc. So, many authors have turned the process around and questioned: how many factors could influence the movements of the term structure, without specifying the exact nature of the relation between the factors and the changes in the bond prices beforehand. Some pioneering studies in this field are Vasicek (1977)'s and Cox, Ingersoll and Ross (1985a, 1985b)'s single-factor model, Longstaff and Schwartz (1992)'s and Gultekin and Rogalski (1984)'s multifactor model, and Heath et al. (1990, 1992)'s arbitrage model.

Studies by Litterman and Scheinkman (1991), Knez et al. (1994), Duffee (1996) and Bliss (1997) have used factor analysis to extract the underlying factors causing the movements in the term structure of interest rates. Knez et al. (1994) found that four factors could summarize the possible sources of variation of different money market instruments. Bliss (1997) applied factor analysis in order to capture the movements of the prices of the US Treasury securities of various maturities and the stock returns. The author re-established the fact that 'the three-factor decomposition of movements in the interest rates, first uncovered by Litterman and Scheinkman (1991) is robust.

In the Indian context, the literature of the estimation of yield curve has started in the last decade. Studies by Nag and Ghose (2000), Susan (2000), Darbha et al. (2000), and Subramanian (2001) are worth mentioning in this connection. Nag and Ghose (2000) estimated monthly ZCYC starting from July 1997 to December 1999. Susan (2000) and Darbha et al. (2000) estimated daily spot rate curve starting from December 2000 using National Stock Exchange (NSE) data. Subramanian (2001) has also estimated daily spot rates for the period April 1, 1999 to April 30, 2000 using NSE data. Susan's (2000) and Darbha et al.'s (2000) study reveal that the yield curve follows an inverted shape from the end of 2000 to the mid of 2002. An inverted yield curve implies that long-term rates are smaller than the short-term rates. Subramanian (2001) shows that Svensson's estimation does not at all over estimate or over fit the yield curve. Bose and Mukherjee (2005) have estimated a constant maturity yield curve. Darbha (2002) enhanced his earlier exercise by minimising the weighted price error, which he however, terms as "Idiosyncratic Nature of the Gilt Market in India".

This study is a unique contribution to the existing literature. It intends to identify the key determinants of the underlying movements of yield curves by extracting latent factors causing the movements. It also validates the choice of parametric parsimonious model used for the estimation of term structure of interest rates here. The study seeks to provide logical reasoning of its finding with the study by Kanjilal (2011) which establishes dynamic linkage of inflation, growth and term structure of interest rates for government debt market facilitated by the central bank of India. Estimation of term structure of interest rates is different from the earlier studies in several aspects: data source, data span and estimation methodologies. The article uses both Nelson and Siegel's (1987) and Svensson's (1994) functional form to estimate the spot rates or term structure of interest rates. Unlike most of the previous studies it uses weighted yield error minimization, weighted by the modified duration[1] instead of simply minimising yield error to capture the variation sensitive to the yield errors.

4. Data description

Reserve Bank of India (RBI) Subsidiary General Ledger (SGL) data is used for the estimation of yield curve for the period Jul '97 to Feb '04. The logic behind using this data source is that RBI provides more comprehensive information of the trading compared to the other primary source of data National Stock Exchange (NSE). However, there has been an issue of not differentiating the settlement date and contract date till the beginning of May 2001 in RBI's SGL data. Starting from Jul '97 to Feb '04, term structure of interest rates has been estimated monthly. Monthly series have been constructed taking the last trading date of every month. However, at the initial years of trading, transactions were very irregular in the secondary market. So, for those cases, dates on the last week of every month have been chosen. The occurrence of such cases is, of course, very rare. The distortion that could arise due to the difference of settlement date and the contract date could be expected to be negligible. State government securities are not taken into consideration as they are not default-free and their transactions are very irregular in nature. On the same logic we have omitted the government of India floating bonds, callable bonds, and indexed bonds in our study. The data used to extract key factors underlying the yield curve movements is month-to-month movements of yield to maturities (YTM) of 1, 2, 3, …, 15 years for Jul '97 to Feb '04. These are Reserve Bank of India (RBI) quoted end-of-month yields to maturities rates. Exclusion set of illiquid securities remains same as mentioned above. To see if the changes happened in government debt market post Feb '04 till date, have changed the number and composition of principal factors causing the yield curve movements, factor extraction exercise is repeated for Mar '04 to Dec '11. Set of instruments, data source chosen for this task remains same.

5. Estimation methodology

5.1. Spot rate estimation model

The yield curve is a plot of the interest rate yields on bonds with differing terms to maturity, but with the same risk, liquidity, and tax considerations. In other words, yield curve is a description of the term structure of interest rates. The term structure of interest rates is defined as the relationship between the remaining term to maturity and the interest rate on zero coupon bonds or the yield to maturity (YTM) on a zero coupon bond. YTM on a zero coupon

[1] This study is a part of the thesis titled 'Term Structure of Interest Rates: Some Macroeconomic Aspects in India'. The aim of the thesis was to explore dynamic linkages of yield curves and macroeconomic fundamentals of the Indian economy in the presence of central bank's intervening monetary policy. Hence, yield error minimization is preferred to price error minimization.

bond is also called 'spot rate'. So, a yield curve is a graphical representation of spot rates and the term to maturity.

There are some specific challenges that every market faces while estimating a zero-coupon yield curve. They are mostly dependent on the market structure, practice and conventions of individual markets. Some of the major hurdles of estimation of term structure of interest rates for Indian sovereign debt market are:

a) Zero-coupon bonds are not traded in abundant.
b) 'Quoted-price' of an instrument fails to reflect the true price.
c) Instruments are traded more than once in a day with different prices.
d) Some instruments have 'tax-effects.'
e) Instruments having options, indexed features are illiquid.

Term structure of interest rates estimation is successful if the effect of these idiosyncratic factors can be removed from its valuation. Majority of government securities promise coupon payments on a regular interval. In practice there are very few markets where zero-coupon bonds are traded widely or even if they are traded, they are small in quantity, hence they lack liquidity. So, yield curve for such markets is estimated with coupon bearing bonds. YTM of coupon bonds is not identical with the YTM of zero-coupon yields, as the YTM of coupon bearing bonds is calculated using the same discount rate for all the future cash-flows till the maturity period. So, the appropriate estimation process should discount each cash flow with the spot rate appropriate for the period. A coupon bond can be seen as a portfolio of zero-coupon bonds of different maturity. This means each future coupon payment correspond to a zero-coupon bond. Hence, yields on a coupon bond are the average yields on zero-coupon bonds of maturity starting from the first coupon payment till the payment of redemption and the last coupon payment. Yield curves showing the YTM on coupon bonds for different maturity are an imprecise estimation of term structure of interest rates as they underestimate the true steepness of the term structure of interest rates (Schaefer, 1977). Thus the principal chore in estimating the term structure is to extract the spot rates from the coupon holding bonds.

So, each coupon bearing bond needs to be converted into multiple zero-coupon bonds to develop a zero-coupon yield curve. There are also other practical hurdles. Quoted-price of an instrument requires adjustment with accrued interest rate to obtain the 'Dirty-price' as the securities are traded on any day other than the coupon payment dates only. In instruments which are traded multiple times, a single price needs to be obtained following the market convention. Tax-effects on coupon-payment of a security require appropriate adjustment to remove bias in its valuation. Also, it is not advisable to club these instruments with other regular ones in estimating the spot rate as the investors perceive the valuation of such instruments very differently. Finally, the choice of instruments in estimating the yield curve should be preferably based on its liquidity.

This study uses both Nelson and Siegel (1987) and extended Nelson and Siegel or Svensson's (1994) parsimonious model for the estimation of monthly zero coupon yield curve or spot rate curve. The choice of the functional form is very crucial as it ultimately determines the trade-off between 'smoothness' and 'flexibility' (Anderson et al., 1996). The choice of the functional form is subjective. The objective function of yield curve estimation for Macroeconomics policy makers differs from the financial or market practitioners. For macroeconomic analysis, parsimonious families of models like Nelson and Siegel (1987) are appropriate, while highly flexible non-parametric approaches may be better suited for pricing, though market practitioners also use parsimonious models for pricing.

The forward rate f(m;b) described by Svensson's formulation is

$$f(m;b) = \beta_0 + \beta_1 \exp(-m/\tau_1) + \beta_2 (m/\tau_1) \exp(-m/\tau_1) + \beta_3 (m/\tau_2) \exp(m/\tau_2); \quad (1)$$

where $b = [\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2]$ are the parameters to be estimated and ' m' is the maturity.

β_0 represents a level of the curve or a long-term component of the model. The second term $\beta_1 \exp(-m/\tau_1)$ which is an exponential term captures the monotonic decreasing (or increasing when β_1 is negative) effect. Hence, it can be considered as a slope component. The third term $\beta_2 (m/\tau_1) \exp(-m/\tau_1)$ generates a hump shaped (or U-shaped if β_2 is negative) curve. So, this can be considered as curvature component. τ_1 is the speed at which slope and curvature change or decay to zero. Svensson (1994) has brought the flexibility and thereby improved the goodness of fit by adding a second humped shaped (or U-shaped) term $\beta_3 (m/\tau_2) \exp(-m/\tau_2)$ where $\tau_2 > 0$. τ_2 is the speed at which the second curvature part of the model changes or decays to zero.

Forward rate of Nelson and Siegel model is given by:

$$f(m;b) = \beta_0 + \beta_1 \exp(-m/\tau) + \beta_2 (m/\tau) \exp(-m/\tau); \quad (2)$$

where $b = [\beta_0, \beta_1, \beta_2, \tau]$ are the parameters to be estimated and 'm' is the maturity. Parameters β_0, β_1, and β_2 represent level, slope and curvature components of the model. τ is the speed at which slope and curvature of the yield curve change or decays to zero.

The spot rate function r(m;b) can be obtained by integrating Eq. (1) and is given by:Svensson's model:

$$\begin{aligned} r(m;b) = \beta_0 &+ \beta_1[\{1-\exp(-m/\tau_1)\}/(m/\tau_1)] \\ &+ \beta_2[\{1-\exp(-m/\tau_1)\}/(m/\tau_1) - \exp(-m/\tau_1)] \\ &+ \beta_3[\{1-\exp(-m/\tau_2)\}/(m/\tau_2) - \exp(-m/\tau_2)]. \end{aligned} \quad (3)$$

Nelson Siegel's model:

$$r(m;b) = \beta_0 + (\beta_1 + \beta_2)[\{1-\exp(-m/\tau)\}/(m/\tau)] - \beta_2 \exp(-m/\tau); \quad (4)$$

where, $\beta_0 > 0$, $\tau_1 > 0$ and $\tau_2 > 0$ for Eq. (3) and $\beta_0 > 0$, $\tau > 0$ for Eq. (4). As m tends to zero spot rate tends to $(\beta_0 + \beta_1)$ and as m tends to infinity r(m) tends to β_0 for both the models. So, one of the underlying assumption of these models is that spot rates and forward rates asymptote to a constant level as maturity increases.

5.2. Estimation methodology

5.2.1. Spot rate extraction methodology

Spot rate function r(m;b) can be estimated either by minimising 'price errors' or by 'yield errors'. The task is to obtain a set of theoretical price and yields for the traded securities on a particular day. The price of a 'm' period bond with 'n' coupon payments of size c payable at time t_k, $k = 1, 2, \ldots, m$, is defined as:

$$P + ai = C\sum_{1}^{m} \delta(t_k) + R\delta(t_m)$$

where

P	market quoted price/clean price
ai	accrued interest rate
C	coupon rate
R	redemption payment
p	purchase price

$\delta(t)$ is the discount function and can be expressed as

$$\delta(t;b) = \exp(-r(t;b) * t/100) \quad (6)$$

where r(t;b) is the spot rate, b is the set of parameters.

As a first step, discount function has to be estimated. Spot rates r(t;b) are estimated taking some initial seed values for the parameters $b = [\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2]$ for Svensson's model and then discount function is calculated to get theoretical price and yields. The final step is to minimise the objective function. The process continues until the set of parameters attains global minima. The estimation algorithm is shown in Appendix A. The same algorithm is applied for Nelson and Siegel model. The term structure of interest rates thus estimated generates a continuous zero coupon yield curve. This means that the estimated yield curve helps to find out the price and YTM of the dated instruments which are not traded on a particular day. This also helps to remove the skewed pattern of maturity distribution towards long-term, medium-term or short-term bonds of any market.

The selection of the objective function in estimating zero coupon yield or spot rate curve is very crucial, as the shape of the yield curve is dependent on the choice of the proper objective function. There is always a trade-off between the minimisation of price errors or yield errors. Generally, price error minimisation yields relatively high yield errors and vice versa. The ultimate decision depends on the final purpose the curve would be used for. Bond market traders generally prefer a fair or smooth price curve for their trading purpose. In that case, price-error minimization should be the choice. On the other hand, monetary policy makers are generally interested in yields; hence for monetary or macroeconomic policy analysis yield error minimisation is preferred.

Weighted price or yield error minimisation is preferred to the simple minimisation of price or yield error. This helps to capture the variation in bond prices or yields sensitive to the price or yield errors. This can be best understood with the concept of duration. The duration is defined as the elasticity of the price with respect to one plus the yield. This relates to the yield to maturity change of a bond to its price change,

$$\Delta P/P = -D * \Delta Y \tag{8}$$

where P is the bond price, Y is the yield and D^* is modified duration. For a given change in the yield, the price change for a long-term (short-term) bond, hence a long (short) duration is much larger (smaller) than that of short-term (long-term) instruments with small (large) duration. Thus, it appears that the change in the bond price for a given change in yield is dependent on the respective instrument's duration. Therefore minimising un-weighted price errors or in other words, putting equal weights to each instrument will over fit the long-term yields at the expense of the short-term yields. To remove this time effect from the minimisation process, one weighting process is to weight the price errors of each bond by a value derived from the inverse of its duration or weight the yield errors of each bond by its duration. In this study, weighted yield error minimisation is used.

5.2.2. Latent factor extraction methodology

The variation in shapes of yield curves could originate from innumerable number of internal or external economic factors which could be systematic or uncertain specific to that economy. Interpretation of the extracted factors by relating them to possible causal economic events sometimes becomes a challenge. To identify the factors underlying the movements of yield curve with no prior assumptions of the number of parameters to capture the various shapes of the yield curve, factor analysis is deployed. Factor analysis assumes that the relationships between the variables are due to effects of underlying factors and that observed correlations are the result of variables sharing common factors. Each variable in factor analysis may be expressed as a linear combination of factors that are not actually observed. In this case, the interest rate movements can be explained by the following equation.

$$\begin{aligned}&(\text{Month}-\text{to}-\text{Month changes in interest rates})\\&= a(\text{Factor1}) + b(\text{Factor2}) + c(\text{Factor3});\end{aligned} \tag{9}$$

a, b, and c indicate the extent to which different factors influence the changes in the interest rates. The following equation shows that factors can be calculated to explain the variability in the data.

$$\text{Factor } X = \beta_1 \text{Var1} + \beta_2 \text{Var1} + \ldots + \beta_k \text{Var1}; \tag{10}$$

where Var1, Var2, …, VarK are changes in yield to maturities for maturities 1, 2, …, k years. $\beta_1, \beta_2, \ldots \beta_k$ are their standardized regression coefficients. Principal Component Analysis (PCA) is used to extract the unobserved factors. The mathematical details of PCA are given in Appendix C.

6. Estimation results

6.1. Yield curve estimation

Nelson–Siegel and Svensson's models are used to estimate zero coupon yield curves from Jul '97 to Feb '04. Basic statistics of estimated short-term, medium-term and longer-term spot rates by Svensson's model are shown in Fig. 1a and b. Fig. 1a shows that there is a downward parallel shift of interest rates in government debt market in India. Fig. 1b shows that there is no major volatility in the interest rate across the maturity spectrum though the short-term fluctuation in 1998 was high. The basic characteristics of the spot rates are not significantly different for Nelson–Siegel estimation, and hence not reported to avoid duplication. Figs. 2(a–b), 3(a–b), 4(a–b), 5(a–b), 6(a–b) and 7(a–b) show zero-coupon yield curves for Nelson–Siegel's (NS) and Svensson's (SV) models. These curves are so chosen that they depict different shapes of the yield curves in estimation sample span. The yield curves primarily show an increasing shape, where long rates are greater than the short rates. This implies that the economy is in 'growing-phase'. It is important to note that even if the maturity distribution of government of India dated securities has buyers' preference towards long-term securities, the estimated yield curves based on Nelson–Siegel–Svensson's parsimonious model have eliminated that bias. This implies that such estimation of term structure of interest rates will help the investors, market practitioners or policy makers to identify the price and YTM of dated securities for any maturity profiles of their interest.

Mean absolute error (MAE) and Root Mean Square Error (RMSE) are chosen as measures of goodness-of-fit. Non-tested tests are conducted to select the appropriate model between NS and SV. The in-sample MAE of Svensson's model is 16 basis points as compared to 22 basis points for Nelson and Siegel's model, as shown in Table 4. The out-of-sample test is conducted for seven random data points. Out-of-sample test, as shown in Table 5, is conducted by dropping the traded securities one at a time for any data month and then estimating yield and price of those securities. MAE and RMSE for out-of-sample test also suggest that Svensson's model gives a better fit than the Nelson–Siegel model. Tables 6 and 7 show the results for non-nested tests for two randomly chosen data points. Both Akaike and Schwartz Bayesian criteria suggest that Svensson's model performs better than Nelson–Siegel model.

Market traders and investors use zero coupon yield curves to evaluate the prices of non-government debt securities and the securities not traded on a particular day. Central banks in India can use yield curves as a leading indicator to predict inflation and growth in the economy provided the yield curves are not the reflection of central banks' monetary policy change.

6.2. Extraction of latent factors

6.2.1. Factor extraction: Sample period Jul '97 to Feb '04

Principal component analysis is applied to extract latent factors behind the month-to-month movements of YTMs of 1, 2, 3, ..., 15 years. The objective is to capture the numerous sources of variations causing the changes in the interest rates by a few variables, called factors.

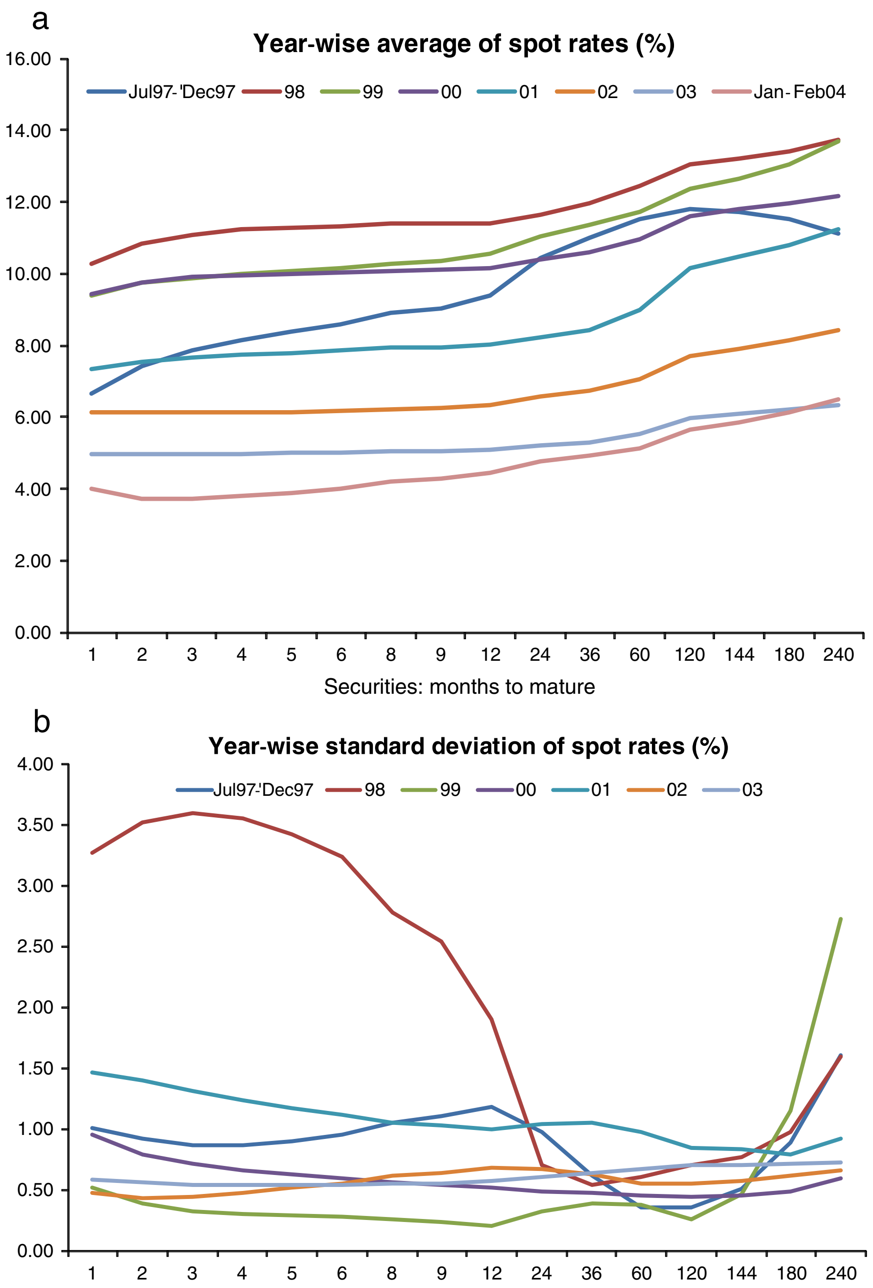


Fig. 1. Basic statistics.

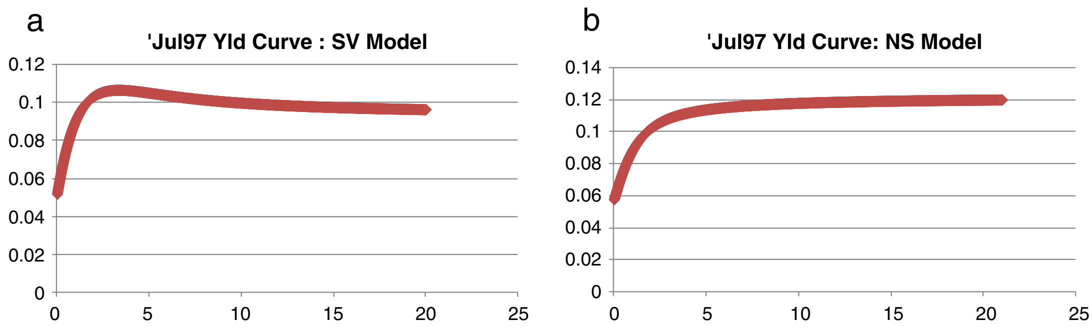


Fig. 2. Yield curves.

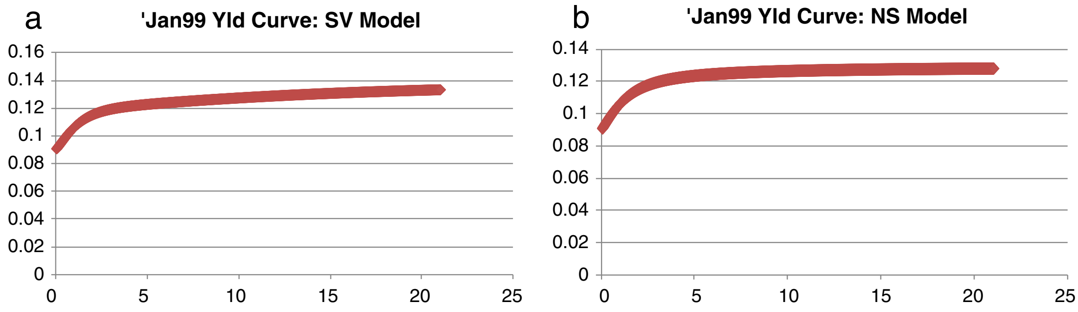


Fig. 3. Yield curves.

Fig. 8a shows the percent variation explained by each individual factor represented by 'BARs'. The dotted line above the 'BAR' represents the cumulative percent variation. As shown in Fig. 8a, first factor accounts for 92% of the total variation. Additional 6.2% is explained by second factor and 1.2% by third factor. Fourth factor accounts for 0.3% of the total variation. The first four factors account to almost 100% variability of the movements in the term structure. Factor loadings on each factor are shown in Fig. 9a. This helps us to interpret each factor more clearly. A factor loading defines the weight or the correlation of each variable on each of the individual factor. For example, as shown in Fig. 9a, first security has 92% weight on first factor. The loadings for all other instruments are highly correlated with Factor1. The loadings of each instrument under every factor are correlated within the instruments, but the correlation across the factors is close to zero. So, each factor captures identical set of information. The average factor loading of Factor1 is 0.95. So, the loading on the first factor seems a constant, hence, in the long term, does not decay to zero. It would represent a 'parallel shift' to higher or lower across all set of instruments. Also, the first factor has equal effects for all the instruments. So, this is a long-term factor. Hence, Factor1 can be called 'Level'. Factor2 can be called 'Steepness' or 'Slope' as it increases from relatively large negative values at shorter maturities to positive values at longer-term maturities, with the same values in absolute term at both ends. A positive shock to this factor would lower the short-term rates while increasing the long rates. The loading on the second factor decays monotonically and approaches to zero; hence this is considered as a short-term factor. It accounts for additional 6.2% of total variation in the Spot rates. Factor3 can be called 'Curvature' or 'Hump'. A positive shock to this factor would lower spot rate of medium term securities while increasing the rates at both shorter and longer-term rates, providing a hump shaped effect to the term structure. However, it accounts to only 1.5% of the total variation in the yield curve. Loadings on Factor4 are close to Factor3; however, it cannot be interpreted as typical 'Curvature' effect. It accounts only for 0.3% of total variation. First three factors: 'Level', 'Slope' and 'Curvature' explain almost 99% variation in the yield curve. Three factors 'Level', 'Slope' and 'Curvature' underlying the movements of the yield curve can be related to 'long-term', 'short-term' and 'medium-term' factors of interest movements for debt market in India. The finding suggests that interest rate movements depend on long-term, short-term and medium-term factors of yield curve though the variation predominantly depends on the 'long-term' factor or 'level' or a factor which brings a 'parallel-shift' effect to all maturity spectrum for the sample period Jul '97 to Feb '04. This finding goes in favour of the parametric estimation of zero coupon yield curve through parsimonious model of Nelson and Siegel (1987) whose functional relationship to estimate the forward rates or spot rates is dependent on three major factors; level, slope and curvature. This implies that though statistically SV yields the best fit model over NS in yield curve estimation, the contribution of second hump shaped component of SV model in capturing different shapes of yield curve is negligible. NS model should capture all possible variations in yield curves for government debt market in India.

6.2.2. Factor extraction: Sample period Mar '04 to Dec '11

Indian debt market has witnessed many structural changes in the past few years as mentioned in Section 2 as a continuous effort by the central bank to widen its depth and to allow its function more independently. The variations in the interest rates or the yield curve are a reflection of market's expectation about a number of economic incidents like the supply and demand for loans, announcements of unemployment, inflation, growth or recession indictors, monetary policies changes, changes in market participants' activities, etc. Hence, the extracted factors obtained from the movements in term structure of interest rates are the summarized effects of changes in economic events and should have causal relationship with key economic factors; inflation, growth, unemployment or monetary policy changes in an economy. Economic downturn which infected the most economies in the world starting end of 2007 could not slow down the growth path of Indian economy. But in the past two and half years Indian economy fell behind the growth projections primarily due to a dozen of hikes in key interest rates by the central bank to reduce

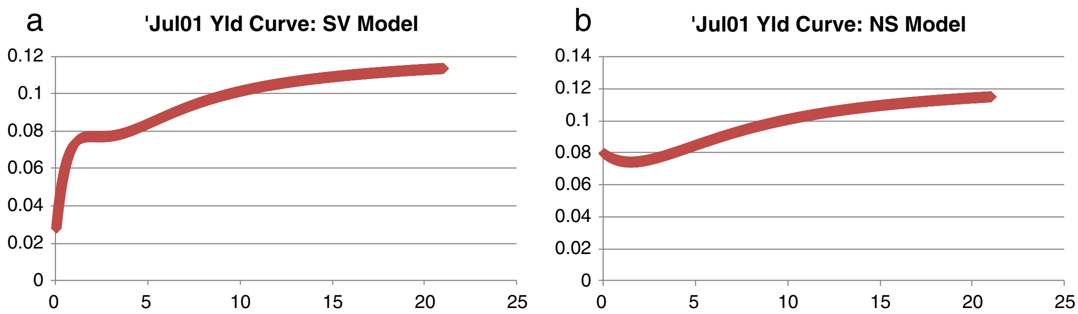


Fig. 4. Yield curves.

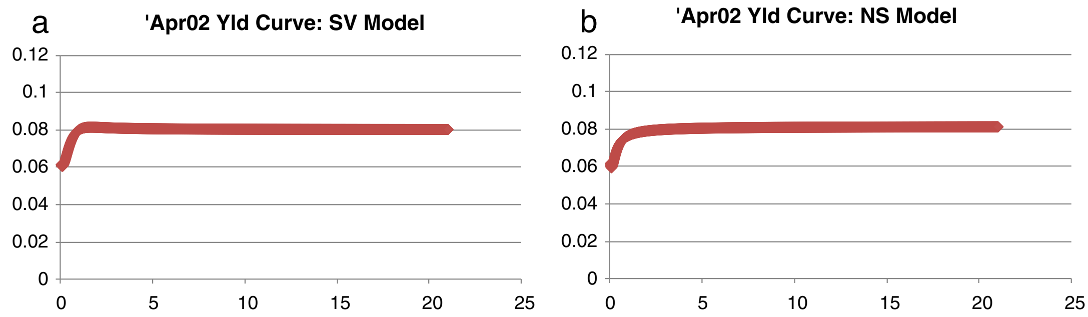


Fig. 5. Yield curves.

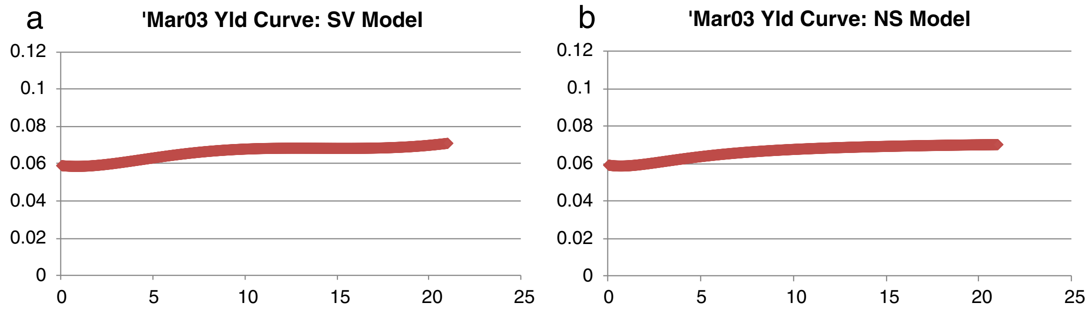


Fig. 6. Yield curves.

the supply constrained double digit inflation. These changes along with the continuous development in debt market might have some impact on the shape and movements of yield curves. Hence, it is quite interesting to explore if the sources of variation behind the yield curve movements are still restricted to the long-term factor or level component of the yield curve as observed during the period Jul '97 to Feb '04. Towards that objective, principal component analysis is applied further to find out the latent factors driving yield curve movements during the period Mar '04 to Dec '11. Figs. 8b and 9b show the percent variation explained by each factor and factor loadings of individual securities on each factor respectively. Comparing Figs. 8b with Figs. 8a and 9b with 9a, it can be concluded that more than 90% variation in yield curve movements for government security market in India depends on the long-term factor which would bring either upward or downward movements across yield curve maturity. Also, three factors represent the total variation in yield curves. This suggests that NS model should be the preferred parametric model for the period Mar '04–Dec '11. Similar study by Bliss (1997) for the US shows that yield curve movements or different possible shapes of yield curves can be explained by 'level', 'Slope' and 'Curvature'; level or long-term component being the predominant source of variation in yield curve movements contributing approximately 80% in the total variations. This means that other sources of variations slope and curvature do have significant impact in the movements of yield curve.

Dominance of long-term factor or 'level' in the movements of yield curve has further implications. The association of level with inflation and slope with growth is investigated extensively in the literature. Empirical study by Diebold et al. (2003) finds that the association between level with inflation and slope with capacity utilization for the US economy is approximately 40%. Kanjilal (2011) finds that the association of level with inflation and slope with growth is approximately 10 to 15%. This indicates that movements in the yield curve may represent the key macroeconomic indicators in an economy. However, researchers have established empirically that the dynamics of yield curve and macroeconomic factors cannot independently be determined without a monetary policy indicator as in most cases it is found that monetary policy plays a major role to these movements and the predictive content of the yield curve is primarily due to the conduct of monetary policy (Rudebusch and Wu, 2003, Wu 2002). Study by Kanjilal (2011) has investigated the role of central bank's intervention after incorporating a monetary policy variable in the dynamics of yield curve and macroeconomic variables. Findings suggest that the central banks facilitate the linkage between yield curve and macro variables by first responding to inflation which brings an impact to the level of the yield curve. Study finds that the role of central banks has become more profound after the launch liquidity adjustment factor in June 2000. The finding that the long-term factor of yield curves can only affect macro fundamentals through an intervening role of central bank suggests that the movements in yield curves are mostly parallel and linked with the monetary policy changes of central bank. So, parallel changes in yield curves mean the yield curve variations relate to the increase or decrease in the policy interest rates in the economy which is determined by the central bank in India. In other words, movements in the yield curves are greatly a reflection of the monetary policy changes in India.

This finding can be leveraged by fixed income security investors. An investor investing in fixed income security markets aims at hedging the value of a fixed income portfolio because it is exposed to interest or market risk. For example, a bond with annual cash-flows up-to a 5-year maturity could be affected by potential changes in 5 zero-coupon rates. This means the investor needs a hedging strategy against possible changes in 5 interest rates across the maturity of 5 years. In the Indian scenario, as the study finds that the changes in interest rates are majorly exposed to a long-term factor, an investor does not require to deploy complex hedging strategy like factor duration hedging or 'level–slope–curvature' hedging. Instead, investors could hedge maximum risk-exposure of the fixed income portfolio deploying just 'duration and convexity' hedging.[2] Duration hedging assumes that the

[2] For details on hedging strategy please refer to Lionel Martellini, Philippe Priaulet and Stéphane Priault (Jul 22, 2003). Fixed-Income Securities: Valuation, Risk Management and Portfolio Strategies (The Wiley Finance Series).

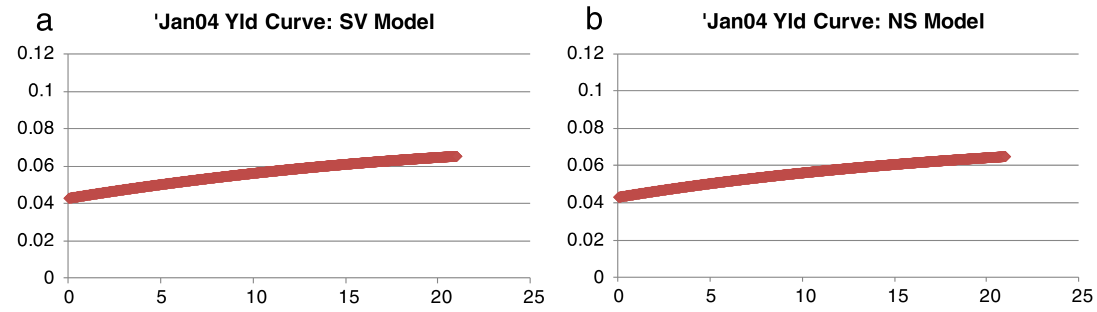


Fig. 7. Yield curves.

interest rate changes are very small, and convexity hedging relaxes this assumption and is capable of capturing larger changes in interest rates. But both duration and convexity hedging strategies assume that yield curve movements are affected by the 'parallel-shift'. Factor based hedging strategy is summarized in Appendix D.

7. Conclusion

This study aims at identifying key factors underlying the movements of yield curve in government debt market in India in order to provide a guideline to the risk-averse debt market investors and to see if the yield curve movements reflect market expectations. The article begins with estimation of term structure of interest rates for government debt market in India post the financial deregulation in 1990–91 for Indian debt market using parametric parsimonious families of models like Nelson and Siegel (1987) and Svensson (1994) to capture all possible variations of yield curves during the period Jul '97 to Feb '04. The estimation uses weighted yield-error minimization technique. The study then examines the justification of the choice of number of parameters used in the parsimonious model by extracting the unobserved factors underlying the movements of yield curves with no prior assumptions of exact relationship between factors and the changes in bond price for the period Jul '97 to Feb '04 and Mar '04 to Dec '11. Instead of estimating monthly yield curve by 3 or 4 parameter-based parsimonious models (NS or SV), the study extends the extraction of key determinants of yield curve movements for the period Mar '04 to Dec '11 to see if the dynamism in government debt market after Feb '04 have changed the composition of key factors driving the yield curve movements as the extraction of number of latent factors justifies the choice of parameters in a parametric model.

The study suggests that Svensson's model best fits the sample for Indian debt market for the period Jul '97 to Feb '04. The result goes in favour of the finding obtained by Subramanian (2001). In spite of the best efforts put to provide a reliable estimate of zero coupon yield curves certain shortfalls which could have originated because of the still slightly dormant nature of the debt market in initial years of the sample period cannot be firmly overruled. Yield curves estimated for the sample period have increasing shapes reflecting the encouraging environment for investments. These estimates of yield curves might help central government, policy makers or Central Banks (RBI) to take decision on the stances of monetary policy, inflation rate targeting or increase overall growth in the economy on the assumption that the yield curves reflect the macro fundamentals and vice versa (Kanjilal, 2011).

Factor analysis is used to extract the possible sources of variation in yield curve movements for the period Jul '97 to Feb '04 and Mar '04 to Dec '11. Month-to-month changes of end-of-month YTMs of 1, 2, 3, …, 15 years maturities are used. Both the periods show similar results suggesting that the number and composition of unobserved factors capturing the yield curve changes are same. Results show that 3 factors 'Level', 'Slope' and 'Curvature' explain the yield curves movements in India with level playing the dominant role. 92–94% of variation comes from the long-term factor or level which brings a parallel-shift-effect across all maturities in yield curve. Rest 4–6% and 6–4% are explained by 'Slope'/'Short-term' factor and 'Curvature'/ 'Medium-term' factor respectively. This finding goes in favour of the parametric estimates of parsimonious model of Nelson and Siegel (1987) whose functional relationship to estimate the forward rates or spot rates is dependent on three major factors; level, slope and curvature. This suggests that NS model should be the preferred parametric model for the period Mar '04–Dec '11.

The variations in the interest rates are a reflection of market's expectation about a number of economic incidents like the supply and demand for loans, announcements of unemployment, inflation, growth or recession indictors, monetary policies changes, changes in market participants' activities, etc. The study finds the dominance of long-term factor or 'level' or 'Parallel-shift-effect' in the movements of yield curve. Yield curves move upward (downward) with the increase (decrease) in the policy interest rates by the central bank in India. This means the yield curve variations in India essentially reflect the changes in monetary policies. This finding further supports the study by Kanjilal (2011) where the author finds that inflation has an impact on the 'level' factor of yield curve through

Table 4
Goodness-of-fit; Svensson and Nelson–Siegel.

In-sample fit			
Jul-'97–Feb-'04	Mean absolute yield error	Standard deviation of yield error	RMSE of yield error
Svensson	0.1635	0.0788	1.0191
Nelson–Siegel	0.2173	0.1150	1.2679

All errors are in basis points.

Table 5
Goodness-of-fit; Svensson and Nelson–Siegel.

Out-of-sample fit			
	Mean absolute yield error	Standard deviation of yield error	RMSE of yield error
Svensson	0.1703	0.085	1.053
Nelson–Siegel	0.246	0.103	1.3069

All errors are in basis points (for seven randomly chosen data points).

Table 6
Results for non-nested tests for Dec-'97.

M1 = Yld on Sv, M2 = Yld on Ns, Dec-'97		
Dependent variable is X1 30 observations used from 1 to 30		
Regressors for model M1: X2		
Regressors for model M2: X3		
Test statistic	M1 against M2	M2 against M1
N-Test	−.17127 [.864]	−5.5548 [.000]
NT-Test	−.16830 [.866]	−5.4610 [.000]
W-Test	−.16571 [.868]	−3.5366 [.000]
J-Test	.16299 [.871]	4.6052 [.000]
JA-Test	.16299 [.871]	4.6052 [.000]
Encompassing	F1, 28.026567 [.872]	F1, 28 21.2078 [.000]
Model M1: DW 2.1210; R-Bar-Squared .94174; Log-likelihood −9.1868		
Model M2: DW 1.2584; R-Bar-Squared .89770; Log-likelihood −17.6303		
Model M1 + M2: DW 2.1155; R-Bar-Squared .93971; Log-likelihood −9.1726		
Akaike's Information Criterion of M1 versus M2 = 8.4435 favours M1		
Schwarz's Bayesian Criterion of M1 versus M2 = 8.4435 favours M1		

the changes in policy rate. In other words, central banks signal plays a vital role in establishing this causality. Additionally, this finding has a great bearing on hedging strategies in government debt market in India. The changes in interest rates in government debt market being primarily exposed to a long-term factor, an investor does not require to deploy complex hedging strategy like factor duration hedging or 'level-slope-curvature' hedging. Instead, 'duration and convexity' hedging strategy should cover maximum risk-exposure.

Government debt market in India has come a long way since the financial deregulation started in 1990. It has passed through many structural changes which helped its growth significantly. But as this study unfolds, government debt market still has a long way forward to travel to improve its overall base, active retail market participation, liquidity and most importantly to operate independently; where yield curve becomes a 'benchmark' for investors and reflect 'market expectations' to guide macroeconomic policy makers in framing key policy decisions.

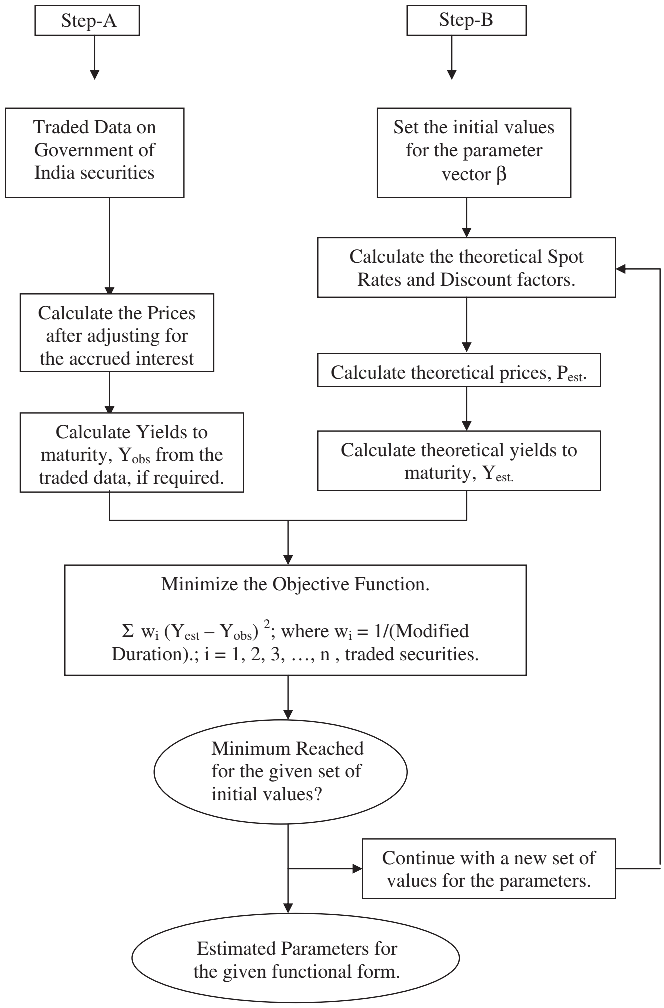
Acknowledgements

The author would like to extend sincere thanks to Mr. Karan Bhalla for arranging some recent facts of government debt market in India. The author is also indebted to the anonymous referees for their valuable comments and suggestions which enhanced the excellence of the article.

Table 7
Results for non-nested tests for two Dec-'03.

M1 = Yld on Sv, M2 = Yld on Ns, Dec-'03		
Dependent variable is X1 66 observations used from 1 to 66		
Regressors for model M1: X2		
Regressors for model M2: X3		
Test statistic	M1 against M2	M2 against M1
N-Test	−.1738E-3 [1.00]	−1.6481 [.099]
NT-Test	−.1633E-3 [1.00]	−1.6355 [.102]
W-Test	−.1633E-3 [1.00]	−1.5746 [.115]
J-Test	.1711E-3 [1.00]	1.5932 [.111]
JA-Test	.1711E-3 [1.00]	1.5932 [.111]
Encompassing	F1, 64.2928E-7 [1.00]	F1, 64 2.5381 [.116]
Model M1: DW 2.0330; R-Bar-Squared .94689; Log-likelihood 38.3973		
Model M2: DW 1.9751; R-Bar-Squared .94478; Log-likelihood 37.1138		
Model M1 + M2: DW 2.0330; R-Bar-Squared .94606; Log-likelihood 38.3973		
Akaike's Information Criterion of M1 versus M2 = 1.2834 favours M1		
Schwarz's Bayesian Criterion of M1 versus M2 = 1.2834 favours M1		

Appendix A. Yield error minimization estimation algorithm



Appendix B. Non-nested test statistic: Mathematical details

The test would examine if the alternative model, in this case yield curve has any explanatory power over the set of error terms of preferred model under null hypothesis.

$$
\begin{aligned}
&H_0 : Yi = \alpha SV_{(i)} + u_i(\text{Svensson's model})\\
&H_1 : Yi = \alpha YNS_{(i)} + v_i(\text{Nelson}-\text{Siegel})\\
&Y_i = (1-\alpha)YNS_{(i)} + \alpha YSV_{(i)} + u_i(i = 1, 2, \ldots, n; \text{Set of'n'instruments})
\end{aligned}
$$

The null hypothesis to be tested is $\alpha = 0$. α is asymptotically distributed as N(0,1) and a significant value suggests that the yield curve model under the null hypothesis is accepted.

Appendix C. Mathematical details of PCA

Let $X = (X_1, X_2, \ldots, Xp)$ be a p-random vector with a known variance–covariance matrix Ψ and $E(X) = 0$. The objective is to maximise the variance of a linear combination $\alpha'_1 X$ ($\alpha_1 \in R^P$); however the problem is not well defined yet, because it is possible to make this variance

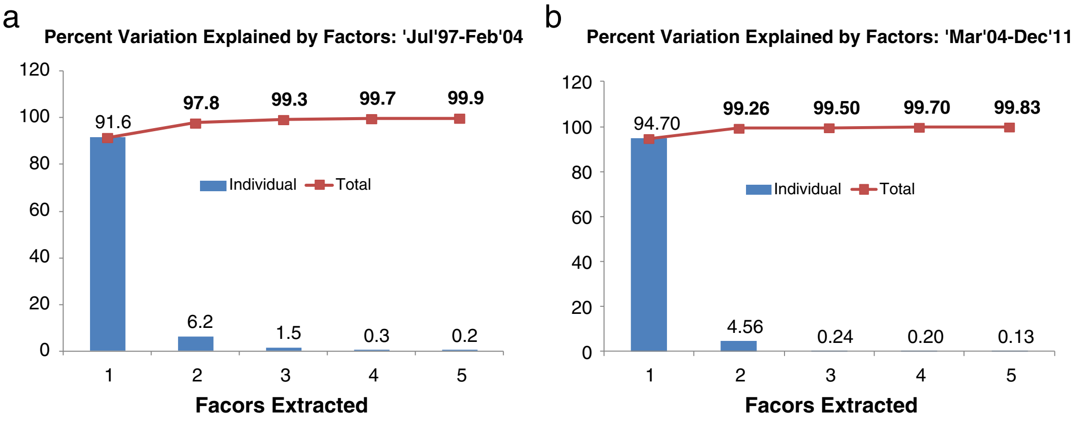


Fig. 8. Factor extraction.

arbitrarily high for an arbitrary α_1. Imposing the normalisation constraint that $\alpha'_1\alpha_1=1$, then we have

$$\text{s.t.}\alpha'_1\alpha_1 = 1.$$

Solving the problem through a Lagrangian

$$\underset{\alpha}{\text{Max}}\ \text{Var}\left[\alpha'_1 X\right] = \alpha'_1 \Psi \alpha_1$$
$$L = \alpha'_1 \psi \alpha_1 - \lambda_1\left(1 - \alpha'_1\alpha_1\right).$$

First order condition yields,

$$\psi\alpha_1 = \lambda_1\alpha_1.$$

α_1 reveals to be an eigenvector of the variance–covariance matrix ψ, while λ_1 is its corresponding eigenvalue. Furthermore, from above equation it is easy to calculate the variance of α'_1 X

$$\alpha'_1\psi\alpha_1 = \alpha'_1\lambda_1\alpha_1$$

which we want to maximise. By using the normal condition ($\alpha'_1\alpha_1=1$), we have that the maximised variance of α'_1 X is equal to the characteristic root λ_1.

The same procedure can be repeated i-times with the additional condition that α_I must be orthogonal with the $i-1$ previous characteristic vectors of ψ. For the case of $i=2$, this is

$$\underset{\alpha}{\text{Max}}\ \text{Var}\left[\alpha'_1 X\right] = \alpha'_1 \Psi \alpha_1$$
$$\text{s.t.}\alpha'_2\alpha_2 = 1\&$$
$$\alpha'_1\alpha_2 = 0$$

which requires at most solving a 2×2 system of equations. Still α_2 is a characteristic vector of ψ and λ_2 is its second largest eigenvalue. Flury presents a generalisation for the case in which there are no multiple roots ($\lambda_1>\lambda_2>\ldots>\lambda p$).

Therefore, we define the i-th component of X, U_i as

$$U_i = \alpha'_1 X. \tag{1}$$

In general, if the columns of a matrix A are the characteristic vectors α_i of ψ, then its components are

$$u = \begin{bmatrix} U_1 \\ U_2 \\ U_p \end{bmatrix} = A'X. \tag{2}$$

It is usual to arrange the columns of $A=(\alpha_1, \alpha_2, \ldots\ \alpha_p)$, such that α_1 is the characteristic vector corresponding to the largest eigenvalue λ_1, α_2 is the eigenvector corresponding to the second largest λ_2 and so on so forth.

Appendix D. Factor duration hedging/level, slope, curvature factor hedging/hedging interest-rate risk with 3 risk variables/factors

Many empirical studies of the dynamics of the interest rate of term structure of interest rates show that two or three factors account for most of the yield curve changes. The factors are generally interpreted as 'Level', 'Slope' and 'Curvature'. This suggests that a multifactor approach should be used for pricing and hedging fixed-income securities. To identify the factors underlying the movements of interest rates or market risks, one needs to leverage the parsimonious models of Nelson and Siegel (1987) or Svensson's (1994) or factor models of

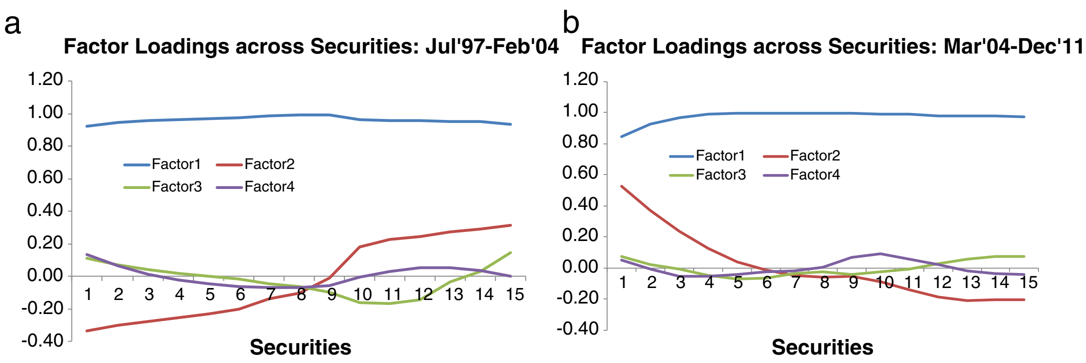


Fig. 9. Factor loadings.

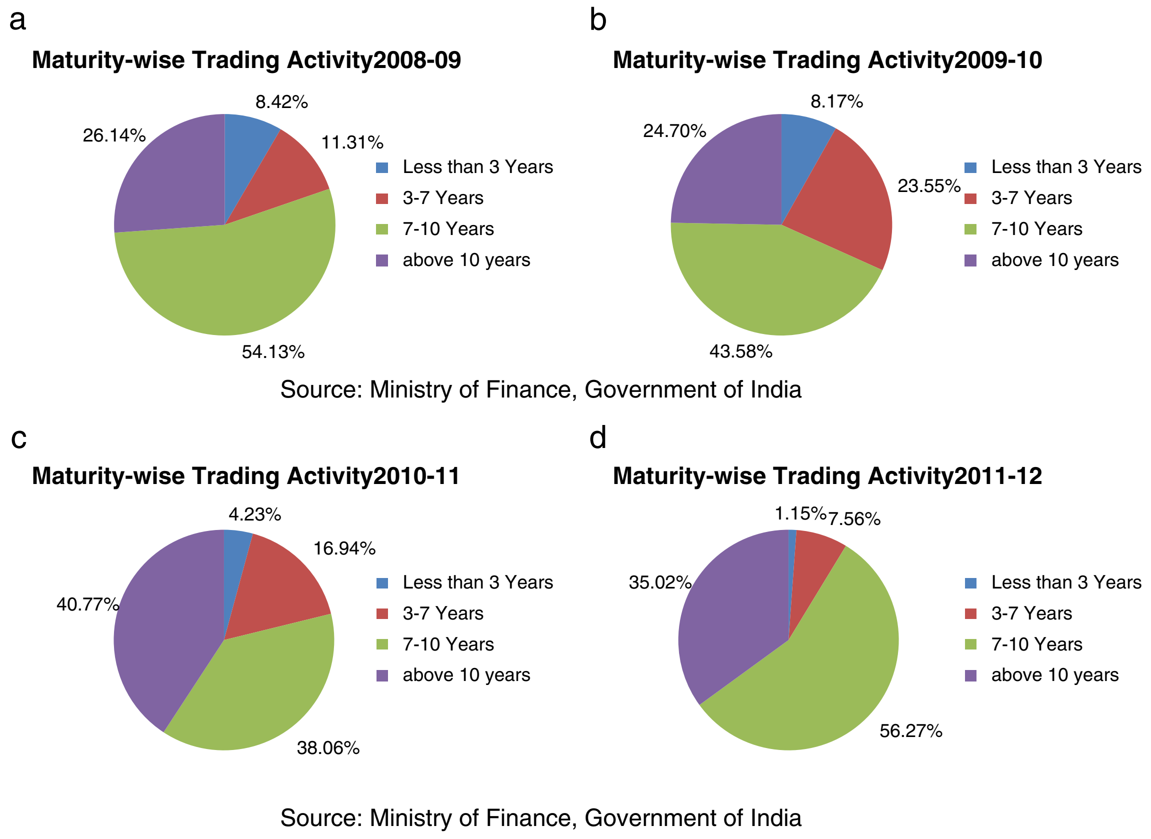


Fig. 10. Maturity-wise trading activity.

Vasicek (1977) or Cox et al. (1985a,b) or others used to estimate the zero-coupon yield curve.

In order to hedge 3 factors driving the interest rate movements by Nelson–Siegel model, we will have to introduce three hedging instruments with prices H1, H2 & H3 and YTMs by y1 y2 & y3 respectively.

The continuously compounded spot rate or zero coupon interest rate by Nelson and Siegel (1987) model is given by:

$$r(m) = \beta_0 + \beta_1[(1-\exp(m/\tau))/(m/\tau) + \beta_2(1-\exp(m/\tau))/(m/\tau) - \exp(-m/\tau)]. \tag{I}$$

β_0, β_1 and β_2 represent 3 factors 'Level', 'Slope' & 'Curvature' respectively; m is the maturity, τ is the speed at which slope, curvature reaches to zero.

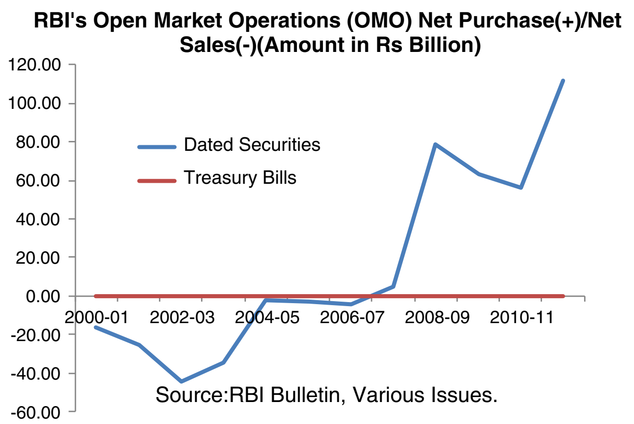


Fig. 11. Open market operations.

Aim is to obtain a global portfolio P* composed of the portfolio to be hedged plus the hedged portfolio which are "Level-neutral", "Slope-neutral" & "Curvature-neutral".

$$\text{Global Portfolio} = P^* = P(r) + \varphi_1 H_1(r_1) + \varphi_2 H_2(r_2) + \varphi_3 H_3(r_2).$$

$P(r)$ is the price of the portfolio to be hedged.
$H(r_1)$, $H(r_2)$ and $H(r_3)$ are the prices of hedged portfolio.
r_1, r_2 & r_3 are continuously compounded spot rates or zero-coupon interest rates of three assets which can be approximated by the functional form of Nelson–Siegel given in Eq. (I).
φ_1, φ_2 & φ_3 are hedged quantity.

As the spot rates or interest rates of all these assets at any point in time would be impacted primarily by three factors, so, aim is to construct a global portfolio insensitive to the 3 factors 'Level', 'Slope' and 'Curvature'.

The optimal hedge quantity of these three assets (φ_1, φ_2, φ_3) can be obtained by solving the following linear equations, at each date.

$$\text{"Level}-\text{Neutral" Equation} = dP^*/d\beta_0 = 0$$
$$\text{"Slope}-\text{Neutral" Equation} = dP^*/d\beta_1 = 0$$
$$\text{"Curvature}-\text{Neutral" Equation} = dP^*/d\beta_2 = 0$$

$$\begin{aligned}
\text{Level}-\text{Neutral} &= \varphi_1 dH_1(r_1)/d\beta_0 + \varphi_2 dH_2(r_2)/d\beta_0 + \varphi_3 dH_3(r_2)/d\beta_0 = -dP/d\beta_0;\\
\text{Slope}-\text{Neutral} &= \varphi_1 dH_1(r_1)/d\beta_1 + \varphi_2 dH_2(r_2)/d\beta_1 + \varphi_3 dH_3(r_2)/d\beta_1 = -dP/d\beta_1;\\
\text{Curve}-\text{Neutral} &= \varphi_1 dH_1(r_1)/d\beta_2 + \varphi_2 dH_2(r_2)/d\beta_2 + \varphi_3 dH_3(r_2)/d\beta_2 = -dP/d\beta_2;
\end{aligned} \tag{II}$$

where

$$D_0 = -dP/d\beta_0 = -d/d\beta_0\left(\sum c_i * [\exp(m_i * (-r(m_i)))]\right);$$

$r(m_i)$ being the continuously compounded interest rate; summation is over i; $i=1, 2, \ldots, k$; C_i cash-flows/coupon payments at ith time

$$= -d/d\beta_0 \sum C_i * [\exp(m_i * \{\beta_0 + \beta_1((1-\exp(m/\tau))/(m/\tau))$$
$$+\beta_2[(1-\exp(m/\tau))/(m/\tau) - \exp(-m/\tau)\}\})] = -\sum m_i C_i \exp(-m_i r(m_i));$$
$$D_1 = -dP/d\beta_1 = -\sum m_i((1-\exp(m_i/\tau))/(m_i/\tau))C_i \exp(-m_i r(m_i))$$
$$D_2 = -dP/d\beta_2 = -\sum m_i[(1-\exp(m/\tau))/(m/\tau) - \exp(-m/\tau)]C_i \exp(-m_i r(m_i)).$$

D_0, D_1 & D_2 are \$Duration of bond portfolio P with respect to the factors β_0, β_1 & β_3.

Or Eq. (II) can be written as,

$$A\varphi = P \text{ or, } \varphi = A^{-1}P.$$

A: hedged portfolio; P: initial portfolio.

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