# Master Thesis

# Modelling the Yield Curve in the United States and the Euro Area - Factoring in the Macroeconomy

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under the supervision of

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To Prof. Dr. Helmut Kramer, who not only taught me how to be a considerate economist, but, more importantly, what it means to be a loving father.

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#### 1 Introduction

Over the past decade, people have become increasingly concerned about personal health. One's own health is constantly monitored and assessed using different metrics, while potential risks are forecasted and mitigated as well as possible. For the masses, simplicity is key in this context as easy-to-understand metrics offer a good indication of one's current health status. In a similar fashion, economists, finance professionals and laypeople alike rely on some key metrics to assess the health of the economy. Especially in times of turmoil, simple albeit highly relevant metrics often enter the center stage. One of the most well-known and most studied among those key metrics is the yield curve. Its various shapes, slopes and spreads are analysed, forecasted and interpreted in order to assess the health of the overall economy as well as market expectations of the future. Beware an inverted yield curve, which the market usually interprets as the onset of a recession, similar to a doctor facing elevated inflammation markers and inferring the onset of illness. But analogous to a doctor, who is faced with the challenge of an accurate diagnosis in spite of an individual's various idiosyncratic characteristics, economic researchers are faced with a tremendous amount of information from which they try to infer a signal in order to predict future economic activity. In this context, understanding the underlying dynamics is an essential means to the this end.

That said, the global economy has become increasingly complex in recent years, with the start of an unprecedented global monetary policy tightening, geopolitical tensions, and unexpected shocks such as the COVID-19 pandemic or the war in Ukraine dramatically influencing economic conditions. This has amplified the need to refine the hitherto available tools and methods available in the economist's toolbox in order to infer as precise economic signals as possible even in such volatile times. Rooted in decades of economic research, the yield curve has oftentimes offered simplistic yet highly accurate predictions of the future path of the economy. However, though the yield curve - mapping interest rates to specific maturities seemingly is a key indicator for economic agents, it is often not well understood, either in a general sense and especially in regards to its dynamics with various economic variables. A contributing factor to this could be that the yield curve has been studied in a rather dichotomous fashion, either from the lens of a finance professional aiming to maximize the returns of his/her portfolio, or from the perspective of an economist concerned with predicting the future state of the economy as accurately as possible. Yet yields are known to contain a tremendous amount of information useful in both realms, be it on the current stance of monetary or fiscal policy as well as expectations of future economic activity and inflation (Evans and Marshall, 2007). Vice versa, ever since the introduction of the expectation theory by Hicks (1946), monetary policy (expectations) seem to be one of the main drivers of yield curve movements (Evans and Marshall, 1998). Thus, correctly assessing the impacts of future developments in both macroeconomic factors such monetary policy, as well as the term structure seems invaluable for both portfolio managers and economists alike. Ultimately, a precise understanding of the underlying drivers, be they of economic or financial nature, enables central banks to calibrate their policies to current market conditions and expectations, as well as giving asset managers the opportunity to position their portfolios profitably. But how does one combine those two worlds? Is the yield curve really a leading indicator for economic activity? Are economic conditions determining the shapes and movements of the yield curve? Could there even be a bidirectional link, where the causality runs in both directions? This thesis is aimed at answering these questions.

Stemming from the methodology introduced by Diebold, Rudebusch, and Aruoba (2006), a decomposition of the United States and Euro Area yield curves using the well-known model introduced by Nelson and Siegel (1987) is conducted, yielding the three latent factors representing the level, slope and curvature of the respective yield curve. In a second step, a structural vector autoregression (sVAR) model is estimated including said factors along with various economic variables. In order to disentangle this obscure relationship, distinct structural shocks are obtained using short-term restrictions via a Cholesky decomposition of the reduced form variance-covariance matrix of the error term. The resulting impulse responses are utilized for the sake of deducing potential interpretations regarding the dynamic interactions between the macroeconomy and the yield curve.

The thesis is structured in the following way. Chapter 2 offers a brief overview of the literature, highlighting past findings regarding the yield curve's relevance and connection to macroeconomic factors. Chapter 3 introduces the methodological approach implemented as well as the data used. The subsequent section presents the empirical analysis of the US (Section 4.1) & Euro Area yield curves (Section 4.2), where the relevance of macroeconomic variables as drivers of yield curve movements is assessed, while simultaneously examining the impacts changes in the yield curve factors have on economic variables. Thereafter, a conclusion summarizing the main findings is provided.

## 2 Literature Review

Historically, there has been an abundance of literature about both the term structure of interest rates per se, as well as its connection to economic activity. This section aims at narrowing down this wealth of research, providing an overview of the hitherto available and relevant literature, focusing on identifying the potential one- or bidirectional link between the yield curve and the economy. Thus, it follows the

distinction provided by Diebold, Rudebusch, and Aruoba (2006), who divide this research into yields-to-macro versus and macro-to-yields studies.

Some of the central contributions regarding the yields-to-macro relationship have come from Harvey (1988), who shows that the expected real term structure contains information about future consumption growth. Among other researchers, Estrella and Hardouvelis (1991), Estrella and Mishkin (1995) and Estrella and Mishkin (1996), use a probit model approach, estimating the probability of historical recessions as a function of the term spread<sup>1</sup>, through which the authors have provided some remarkable results corroborating the hypothesis in regard to the relevance of the yield curve (slope) for future macroeconomic activity, at least for the US and other major European economies such as Germany, the UK and France. In a similar fashion, Bernard and Gerlach (1998) extend the analysis to other major economies such as Canada, Japan, Belgium and the Netherlands, confirming the finding that the term spread offers significant information regarding future recessions. Solely for Japan the significance is limited, which can possibly be attributed to differences in financial regulation and the fact that interest rates in Japan did not fully reflect market participants expectations of the future path of the economy (Bernard and Gerlach, 1998). Evgenidis, Papadamou, and Siriopoulos (2018) offer a meta-analysis of the yields-to-macro view, again focusing on the yield spread's ability to forecast economic activity. Although they again corroborate the findings of the previous 30 years, they note that the modelling strategy should take non-linearities into account as well as underlining the importance of incorporating monetary policy variables into the models, as some predictive power of the yield spread is attributed to expectations about the future path of monetary policy. The authors also note that there still is no widely accepted theory explaining the usefulness of the yield curve. Apart from these rather promising studies affirming one's confidence in the yieldsto-macro view, authors such as Dotsey (1998) and James H. Stock and W. Watson (2003) note that yields (spreads) have somewhat lost their predictive ability ever since the 1980s.

On the spectrum of authors leaning towards a macro-to-yields approach, central research has come from Evans and Marshall (1998), Ang and Piazzesi (2003) and Evans and Marshall (2007), all using some variation of a VAR framework. Using three different identification strategies within a VAR setting, Evans and Marshall (1998) find that monetary policy shocks primarily have a significant effect on short-term interest rates while also reducing expected inflation, implying a rise in the real interest rate. The authors interpret this as monetary policy having a significant effect on the yield curve through a liquidity, rather than an expected inflation effect. Though delivering promising results of a potential macro-to-yields relationship, one

<sup>&</sup>lt;sup>1</sup>The term spread is generally known as the difference between a long-term and a short-term interest rate, e.g. the difference between the 10-year and 3-month government bond yields

key weakness is the sole focus on monetary policy shocks alone, whilst excluding other macro variables. Counteracting these shortcomings and extending their previous research, Evans and Marshall (2007) include other macro variables into their model, such as technology, fiscal policy and preferences for current consumption, an identification set predominantly derived from previous research on DSGE models. As before, they conclude that, apart from fiscal policy shocks, macroeconomic factors such as technology are able to explain a large part of yields movements along the short- to medium- end of the curve. By including multiple macro variables into a multi-factor model of the term structure, using the assumption of no-arbitrage as an identifying restriction, Ang and Piazzesi (2003) find that macro variables explain a tremendous amount of the variation in bond yields, especially at the short-end of the curve, where the main drivers are shocks to inflation, which is consistent with previous findings by Evans and Marshall (1998). The authors also conclude that incorporating macro factors in latent factor term structure models enhances outof-sample forecasts. In contrast, now taking the stance of the opposite direction of causality, i.e. yields-to-macro, Ang, Piazzesi, and Wei (2006) offer a comparison to previous research such as Estrella and Hardouvelis (1991). Through estimating a relatively simple VAR only including a set of yields and GDP growth, they conclude that the nominal short rate dominates the slope of the yield curve in predicting future economic activity.

Some other notable contributions in the realm of linking the macroeconomy and the yield curve have come from Dewachter and Lyrio (2006) and Rudebusch and T. Wu (2008). Both sets of authors extend the latent factor model of the term structure with macroeconomic factors. The former specifically reference to Ang and Piazzesi (2003) in the context of a possible model misspecification, since previous research has apparently failed to correctly model the long end of the yield curve. Through their slightly modified approach, the authors introduce long-run inflation expectations in their model and show that not only the short-end of the yield curve, but also longer maturities are indeed driven by macroeconomic variables. They also suggest interesting and seemingly plausible interpretations to the latent factors, chiefly among them finding that the level factor is closely related to the aforementioned long-run inflation expectations. Rudebusch and T. Wu (2008) combine a small scale macro model, assuming that the short-term rate is represented by a monetary policy reaction function, such as the well known function developed by Taylor (1993), with a standard no-arbitrage latent factor model of the term structure, where the short-term interest rate is modelled as a function of latent factors often interpreted as a level and/or slope factor. Through this approach, the authors are able to synthesize both the finance and the economists approach to modelling interest rates. Obtaining promising results, they offer insightful and intuitive interpretations, e.g. how the level factor and the central banks inflation target are linked, underlining the significance of a holistic approach.

Among the major contributions regarding the application of the Nelson and Siegel (1987) method in a macroeconometric framework are Diebold and Li (2006). Since other prominent theoretical models of the yield curve, chiefly among them are the no-arbitrage as well as equilibrium models, fail to provide the tools necessary for accurate predictions, the authors try to fill this gap using the aforementioned and wellknown Nelson-Siegel decomposition. More precisely, the authors compare the term structure forecasting performance of said strategy with that of various benchmark forecasts (e.g. random walk) in order to test their hypothesis that a Nelson-Siegel approach yields superior out-of-sample forecasting results. Through decomposing a set of yields, representing the term structure, into three factors, which are then assumed to be evolving as an AR(1) process over time, Diebold and Li (2006) obtain forecasts of the yield curve based on the forecasts of said factors. The authors find that this approach appears to yield superior forecasts, especially at longer horizons of one year and beyond. Their promising results are one of the main reasons, why a Nelson-Siegel factor model approach has been chosen for this thesis. Another central contribution of the authors is their theoretical as well as empirical argumentation regarding the interpretation of the Nelson-Siegel factors as representing the level, slope and curvature of the decomposed yield curve. This interpretation is conveniently used in Section 4 to get a first glimpse how well the estimated factors approximate their theoretical counterparts.

One of the central papers serving as an inspiration and template alike for this thesis is the seminal work by Diebold, Rudebusch, and Aruoba (2006). Extending upon the bipartite available literature at that time, the authors are among the most significant to investigate the joint dynamics of both macro and financial variables (i.e., level, slope and curvature factors representing the yield curve). They build upon the approach used by Diebold and Li (2006), using a state-space representation of the Nelson-Siegel model, estimated via maximum-likelihood using the Kalman filter, which simultaneously models the dynamics of the yield curve for each point in time as well as the dynamics of other variables. Through this one-step approach the authors investigate the relationship between the macroeconomy and yields, laying forth the research question of this thesis, how bond yields and the macroeconomy are linked and, more interestingly, if there is a one- or bi-directional relationship. Diebold, Rudebusch, and Aruoba (2006) conclude that there is strong evidence for a macro-to yield effect and a somewhat weaker evidence for the yield curve to affect (future) macroeconomic dynamics.

Using the same two-step approach as in this thesis, some interesting contributions for emerging economies such as India and Chile, have come from Kanjilal (2011) and Morales (2010), respectively. Both authors find evidence of a potential bi-directional link between macroeconomic variables and the yield curve. Interestingly, Kanjilal (2011) finds a stronger evidence for the yields-to-macro direction, which supports the literature by authors such as Estrella and Hardouvelis (1991) and somewhat

contrasts the findings by Diebold, Rudebusch, and Aruoba (2006). Suitably for this thesis, Morales (2010) compares the two-step estimation approach introduced by Diebold and Li (2006) with the more complicated state space representation (one-step estimation process) by Diebold, Rudebusch, and Aruoba (2006) and concludes that the simplified estimation methodology does not contradict the basic intuition of the results. Furthermore, the author finds evidence for a two-way relationship between yield curve factors and macroeconomic variables for the Chilean economy.

Notable theoretical contributions have come from Hicks (1946), who is among the economists credited with developing the well-known expectations theory, which is also tested in Diebold, Rudebusch, and Aruoba (2006). Kessel (1971) offers a broad overview of common (past) theories explaining the term structure of interest rates. Some of the latest contributions in the corresponding literature have mainly focused on how and how-well a Nelson-Siegel approach can be implemented in a machine learning framework, where, among others, Pedersen and Swanson (2019) offer a comprehensive examination.

## 3 Methodology and Data

This section is aimed at outlining the methodology applied in this thesis. After giving a comprehensive primer on the aforementioned model of the yield curve established by Nelson and Siegel (1987), the identification strategy for the VAR model containing macroeconomic and yield curve variables is introduced. Lastly, an overview of the data and its sources is offered.

The first step of the two-step methodological approach involves modelling the yield curve using the Nelson-Siegel three factor model. The genius of the Nelson-Siegel decomposition lies within its flexibility as well as its parsimony. Based on a set of observable yields, the model is able to capture a wide range of yield curve shapes. One of its core strength is its ability to enable analysts to inter- and extrapolate between yields within the sample, thus providing yields for all maturities along the curve. As mentioned in section 2, the Nelson-Siegel model performs relatively well in out-of sample forecasting. What is more, though it does not explicitly ensure the absence of arbitrage, Coroneo, Nyholm, and Vidova-Koleva (2011) show that the Nelson-Siegel model aligns with the assumption of no-arbitrage. Given these highly promising attributes, the Nelson-Siegel approach appears to be well suited for the task at hand.

Following the representation of Diebold, Rudebusch, and Aruoba (2006), the yield curve at any time t is thus assumed to be represented as the Nelson and Siegel (1987) model:

$$y_t(\tau) = \beta_{1t} + \beta_{2t} \left( \frac{1 - e^{-\lambda \tau}}{\lambda \tau} \right) + \beta_{3t} \left( \frac{1 - e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau} \right), \tag{1}^2$$

where t = 1, ..., T,  $y_t(\tau)$  is a set of N yields, each with a distinct maturity  $\tau$ , used for the decomposition, and  $\beta_{1t}$ ,  $\beta_{2t}$ ,  $\beta_{3t}$  and  $\lambda$  are the parameters to be estimated. The regressors are also known as the factor loadings of the  $\beta$  coefficients, where  $\lambda$  shows their respective rate of decay. These factor loadings are central for ensuring the flexibility to represent various yield curve shapes. The loading on  $\beta_{1t}$  is 1 for all maturities and is thus, interpreted as the long-term factor, while the loading on the second factor,  $\beta_{2t}$  starts at 1 and decays monotonically towards 0, and thus can be viewed as a short-term factor. Finally, the loading on the third factor,  $\beta_{3t}$ , starts at 0, increases, but then decays back to 0, hence, it may be thought of as the medium-term factor (Diebold and Li, 2006). The contribution of each factor loading to the overall shape of the estimated yield curve is then given by each respective factor, e.g.  $\beta_{1t}$  shows the contribution of the long-term factor,  $\beta_{2t}$  that of the short-term component, while  $\beta_{3t}$  illustrates the contribution of the medium-term component (Nelson and Siegel, 1987).

As described in Nelson and Siegel (1987), the estimation can be conducted in the following way. For a provisional value  $\lambda$ , the sample values for the factor loadings are calculated. Based thereupon, the best-fitting values of the  $\beta$  coefficients are estimated. This procedure is repeated over a grid of values for  $\lambda$ , yielding the overall best-fitting values for  $\lambda$ ,  $\beta_{1t}$ ,  $\beta_{2t}$ , and  $\beta_{3t}$ .

Conveniently, as demonstrated by Diebold and Li (2006), the factors  $\beta_{1t}$ ,  $\beta_{2t}$ , and  $\beta_{3t}$  can have economically meaningful interpretations, i.e. they are the time-varying level, slope<sup>3</sup> and curvature factors,  $L_t$ ,  $S_t$ ,  $C_t$ , respectively.

Thus, the estimated<sup>4</sup> model of the yield curve used in this thesis is represented by:

$$y_t(\tau) = L_t + S_t \left( \frac{1 - e^{-\lambda \tau}}{\lambda \tau} \right) + C_t \left( \frac{1 - e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau} \right), \tag{2}$$

In a more comprehensive matrix representation, the estimation of the factors is

<sup>&</sup>lt;sup>2</sup>Mathematically, this representation can be thought of as a constant plus a Laguerre function (Nelson and Siegel, 1987)

<sup>&</sup>lt;sup>3</sup>As shown by Diebold and Li (2006),  $\beta_{2t}$ , and hence  $S_t$ , equals the negative slope, that is, the difference between short-term and long-term yields

 $<sup>^4</sup>$ The estimation of the yield curve factors is conducted using the Nelson.Siegel method from the R package YieldCurve

done using the following system of linear equations, where each row corresponds to equation 2 with a specific yield  $y_t$  and a corresponding maturity  $\tau_N$  as well as an error term  $\varepsilon_t(\tau_N)$ :

$$\begin{pmatrix} y_t (\tau_1) \\ y_t (\tau_2) \\ \vdots \\ y_t (\tau_N) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1 - e^{-\tau_1 \lambda}}{\tau_1 \lambda} & \frac{1 - e^{-\tau_1 \lambda}}{\tau_1 \lambda} - e^{-\tau_1 \lambda} \\ 1 & \frac{1 - e^{-\tau_2 \lambda}}{\tau_2 \lambda} & \frac{1 - e^{-\tau_2 \lambda}}{\tau_2 \lambda} - e^{-\tau_2 \lambda} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1 - e^{-\tau_N \lambda}}{\tau_N \lambda} & \frac{1 - e^{-\tau_N \lambda}}{\tau_N \lambda} - e^{-\tau_N \lambda} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t (\tau_1) \\ \varepsilon_t (\tau_2) \\ \vdots \\ \varepsilon_t (\tau_N) \end{pmatrix}$$
(3)<sup>5</sup>

Thus, for each time period t, a level, slope and curvature factor is estimated based on the prevailing yields contained in vector  $y_t$  with distinct maturities  $\tau_N$ . Consequently, the  $L_t$ ,  $S_t$  and  $C_t$  factors are assumed to be an approximate representation of the yield curve at any given time t and, together with the macroeconomic variables, are included in a VAR(p) model aimed at studying the link between the economy and the yield curve.

Aforesaid VAR(p) model forms the second step of the analysis and is represented in the following way:

$$\mathbf{Y}_{t} = \mathbf{c} + \mathbf{A}_{p} \mathbf{Y}_{t-p} + \varepsilon_{t}, \ \varepsilon_{t} \sim \mathcal{N} (0, \Sigma_{\varepsilon}),$$

$$(4)$$

where  $Y_t$  denotes the  $(K \times 1)$  matrix containing K endogenous variables, c is a  $(K \times 1)$  vector of intercept terms, p denotes the maximum lag length,  $A_p$  is a  $(K \times K)$  matrix of the autoregressive coefficients for lag length p and  $\varepsilon_t$  is a  $(K \times 1)$  matrix of the reduced-form error terms.

The identification strategy is based on a structural VAR approach using contemporary recursive restrictions via a Cholesky decomposition of the variance-covariance matrix of the reduced-form errors  $\Sigma_{\varepsilon}$ . Largely following the notation of Kilian and Lütkepohl (2017), the representation of the structural VAR, ignoring the intercept vector c, as well as the relationship between the reduced-form and structural form VAR can be seen by:

$$\mathbf{B}_{0}\mathbf{Y}_{t} = \mathbf{B}_{p}\mathbf{Y}_{t-p} + \omega_{t}, \ \omega_{t} \sim \mathcal{N}\left(0, \mathbf{I}\right),$$

$$\underbrace{\mathbf{B}_{0}^{-1}\mathbf{B}_{0}}_{\mathbf{I}}\mathbf{Y}_{t} = \underbrace{\mathbf{B}_{0}^{-1}\mathbf{B}_{p}}_{\mathbf{A}_{p}}\mathbf{Y}_{t-p} + \underbrace{\mathbf{B}_{0}^{-1}\omega_{t}}_{\varepsilon_{t}},$$
(5)

<sup>&</sup>lt;sup>5</sup>This corresponds to the measurement equation in Diebold, Rudebusch, and Aruoba (2006)

where  $\omega_t$  is the serially uncorrelated vector of the structural shocks. Since these structural shocks permit to deduce causal conclusions based on the correlations within the data, the central aim of this thesis is the identification of the structural shocks, whereupon the relationship between the model variables are examined. As can be seen in equation 5, identification depends upon identifying the inverse of matrix  $B_0$ , which governs the contemporaneous relationship between the model variables. In this thesis, it is obtained applying the aforementioned Cholesky decomposition of the variance-covariance matrix of the reduced-form errors.  $B_0^{-1}$ shows how each (correlated) reduced-form shock is a linear combination of specific (uncorrelated) structural shocks. To be precise,  $B_0^{-1}$  is a lower-triangular matrix, where each element above the main diagonal is zero, and it depicts how a structural shock to each variable affects each of the models variables on impact, thus allowing for the inference of potential relationships among the variables in the model. Since the identification of  $B_0^{-1}$  is not unique w.r.t. the ordering of the variables, assessing the economic adequacy of the chosen ordering is crucial. In particular, as all elements above the main diagonal are zero, implying that one assumes that the variables ordered first tend to be affected by the other variables with a delay of at least one period, it would generally be reasonable to order the variables from the slowest to the fastest reacting. In summary, following the estimation of a VAR(p) model containing specific macroeconomic and yield curve variables and applying the identification strategy described above, the orthogonal shocks, based on a certain variable ordering, are used to identify the relationship between the macroeconomy and the yield curve.

Based on a standard identification set proposed by the likes of Furlanetto, Rayazzolo, and Sarferaz (2017), the empirical analysis presented in section 4 is conducted using K=8 variables, where the relevant macroeconomic variables are industrial production  $(IP_t)$ , inflation  $(\pi_t)$ , and a short-term interest rate  $(i_t)$ , while the financial variables include an indicator for financial stress  $(FS_t)$  and a variable representing the stock market  $(M_t)$ . The variables representing the yield curve are the three factors obtained from the first step via the Nelson-Siegel decomposition  $(L_t, S_t, C_t)$ . Drawing from Martins and Afonso (2010), who argue that financial variables may more prone to be affected instantaneously by shocks to the macroeconomy while the latter may react more slowly to shocks to the former, the variables are ordered from the most exogenous to the least exogenous. Thus, the ordering of vector  $Y_t^{\mathrm{T}}$  is given by  $Y_t^{\mathrm{T}} = [IP_t, \pi_t, i_t, FS_t, L_t, S_t, C_t, M_t]$ . This ordering is also consistent with, among others, Ang and Piazzesi (2003), who order their macro variables before the included yields in their VAR approach. While the macroeconomic variables being ordered first appears plausible given the fact that they often tend to react in a lagged manner, the stock market being ordered last broadly follows Kilian and Park (2009), assuming that the stock market reacts to shocks to the other variables contemporaneously, which seems reasonable as it is generally assumed that stocks instantaneously incorporate any new information available<sup>6</sup>, while, following the previously mentioned assumption that macro variables tend to react inertly, it does affect all other variables only with a delay of at least one period.

Though the selected methodology applied in the thesis has, in various forms and alterations, been applied quite often in the literature, a tremendous amount of alternative approaches is available in the economist's toolbox. A comprehensive overview of these numerous potential methodological approaches modelling the yield curve and studying it's relationship with the macroeconomy is offered in Diebold and Rudebusch (2013).

As to the concrete data used, the sample for the United States contains monthly data ranging from January 1975 to December 2022. All macroeconomic time series (industrial production, consumer price index, federal funds rate) have been obtained from the FRED website. The S&P 500 stock market index has been acquired via Yahoo Finance. The excess bond premium (ebp) has been chosen as the financial stress indicator, which is based on the FED note by Favara et al. (2016), and provides a measure of risk in the corporate bond market (Gilchrist and Zakrajšek, 2012). Of the highest significance for any analysis involving a Nelson-Siegel decomposition of the yield curve are the yields data used. So far, the literature - heavily focused on the US - has primarily used unsmoothed Fama and Bliss (1987) Treasury forward rates obtained via the CRSP<sup>7</sup> database, which are then converted to unsmoothed Fama-Bliss zero yields. However, in this thesis zero-coupon US Treasury yields obtained via a novel and improved approach freely provided by Liu and J. C. Wu (2021) are utilized. Following Diebold, Rudebusch, and Aruoba (2006), the maturities used are 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 months.

The data for the analysis of the Euro Area consists of monthly data, spanning from October 2004 to December 2022. Again, the bulk of the dataset has been obtained via the FRED website. Solely the Eurostoxx 50 index, representing the European stock market, as well as the VSTOXX index, again serving as the indicator for financial stress in the European economy, have been obtained via Yahoo Finance and the Bloomberg Terminal, respectively. The Euro Area yield curve data consists of spot rates derived from bonds with finite maturity denominated in EUR and issued by a euro area central government with an (issuer) rating of triple A. The data has been obtained via the ECB website and, due to somewhat limited data availability, includes yields with maturities of 3, 6, 9, 12, 24, 36, 48, 60, 72, 84, 96 and 120 months. A more detailed description of the data and its sources is provided in the Appendix A.

<sup>&</sup>lt;sup>6</sup>See, for example, Pearce and Roley (1984), Beaudry and Portier (2006), and Ormos and Vázsonyi (2011) for a discussion about the response of stock prices to economic news

<sup>&</sup>lt;sup>7</sup>https://www.crsp.org/



Figure 1: Estimated level, slope and curvature factor, USA (in %)

## 4 Analysis

This last section lays out the main corpus of the thesis, presenting the empirical analysis, which tries to answer the research question how the macroeconomy and the yield curve are related in the US and the Euro Area based on the methodology outlined in section 3. Subsection 4.1 is concerned with the United States, while subsection 4.2 focuses on the Euro Area. Each subsection starts in a descriptive manner, focusing on the respective time series of interest, especially the estimated yield curve factors and their characteristics in relation to the macroeconomic variables, after which interpretations of the potential link between the macroeconomy and the yield curve are presented based on the resulting impulse responses obtained via a structural vector autoregression model.

## 4.1 The Yield Curve and the Macroeconomy in the USA

In order to get an initial sense of comparability, Figure 1 depicts all three estimated yield curve factors obtained via the Nelson-Siegel model for the US from January

1973 until December 2022. Among other things, one can see that the level factor,  $L_t^{US}$ , has consistently been positive and well above the level of 5\% for most of the time. Being above 10%, the estimated level factor is fairly high during the second oil price crisis induced by the Iranian Revolution in 1979, which caused world oil prices and thus, also US inflation to spike - an interesting observation that is further discussed when looking at Figure 2. The spike in October 2008 could potentially be attributed to the onset of the financial crisis, where the bursting of the real estate bubble in the US led to the biggest financial crisis since the Great Depression an idiosyncratic shock that might reveal possible shortcomings of the Nelson-Siegel model during times of turmoil. Though a decreasing trend in the estimated level can be observed since the 1990s, the pace of the decrease has been the highest with the level being the lowest during the 2010s - a time marked by QE and deflationary risks. As for the estimated slope  $(\hat{S}_t^{US})$  and the curvature  $(\hat{C}_t^{US})$  factors, both factors assume positive and negative values during the sample period. It is apparent that the curvature factor is the most volatile, especially during the 1970s and 1980s, while from the 2000s onwards, the correlation between the slope and the curvature factors is fairly high<sup>8</sup>. As a mirror image to the likely erroneous spike of the level factor during the financial crisis of 2008, the (negative) slope factor displays a similar spike, only downwards, which, since one would expect an inverted yield curve and thus, an increase of the negative slope at times of a looming a recession, contradicts economic theory as well as countless empirical analyses outlined in the literature review of section 2, again underlining the possible inaptitude of the Nelson-Siegel approach when faced with an idiosyncratic event.

Figures 2-4 show the estimated factors together with their empirical proxies. These proxies are insofar relevant as they offer a first indication of the gap between the estimated coefficients and the actually realized yields (Kanjilal, 2011). Figure 2 depicts the estimated level factor,  $\hat{L}_t^{US}$  and the proxy variable proposed in the literature being the arithmetic average of the 10-year, 2-year and 3-month yield. It is apparent that the empirical proxy is fairly close to the estimated level factor, especially in the beginning of the sample, while a stronger divergence can be observed from the financial crisis onwards, though the level proxy has been close to the estimate in recent years. Over the whole sample period, the correlation between the estimated level factor and the respective proxy is 82%, being in line with the findings of Rudebusch and T. Wu (2008), and validating the estimate as being a good representation for level factor, i.e. the long-run component of the yield curve. Since it is the first indication of a link between the macroeconomy and the yield curve, a highly interesting observation for the task at hand can also be drawn when comparing the level factor with the inflation rate over time. From the Fisher equation one can derive how nominal interest rates and inflation expectations could be linked. Based on

 $<sup>^8</sup>$ The correlation between estimates of the slope and curvature factor is about 32% from 01-1973 until 12-1999, while it is around 77% from 01-2000 until the end of the sample



Figure 2: Level factor, empirical proxy and inflation, USA (in %)

Fisher (1930), one would expect that nominal interest rates move one-to-one with a change in expected inflation, with the real interest rate being unaffected. In order to study this proposition Figure 2 also includes the year-on-year change in the CPI via the dotted green line. Though this inflation measure is concerned with actual and not expected inflation, one can see the spikes in the inflation rate during the second oil crisis beginning in 1979, where both inflation and the level factor have increased tremendously - a first indication that the inflation rate is indeed linked with the long-run factor of the yield curve. In a similar fashion,  $\hat{L}_t^{US}$  first spikes in October 2008 and then decreases strongly, a pattern that is almost perfectly coinciding with the inflation rate during that time. When looking at the recent period of inflationary pressures induced by supply bottlenecks during the COVID-19 pandemic in 2021 as well as the Russian aggression in Ukraine in 2022, one can see that again the level factor, which has of course been flanked by increasing interest rates through monetary tightening by the FED beginning in 2022, has moved in accordance with actual inflation, though not rising as sharply as the inflation rate. Overall, the estimated level factor and inflation exhibit a rather strong co-movement in the US. In fact, the correlation between  $\hat{L}_t^{US}$  and inflation over the sample pe-



Figure 3: Slope factor and empirical proxy, USA (in %)

riod is 44% and significant<sup>9</sup>, a finding that is consistent with the interpretation of the level factor as being related to inflation expectations, as described in, among others, Dewachter and Lyrio (2006), Rudebusch and T. Wu (2008), and Diebold, Rudebusch, and Aruoba (2006).

Looking at Figure 3, one can see that the estimated slope factor,  $\hat{S}_t^{US}$  is extremely close to its empirical proxy being the (negative) slope of the yield curve, i.e. the difference between the 3-month and 10-year yield. In fact, the correlation is strikingly high and significant with 81%. As often done in the literature, the slope factor is said to be correlated, in fact, even able to predict economic activity. Thus, to again get a glimpse of a possible link between the macroeconomy and the yield curve represented by the slope factor, Figure 3 also depicts economic activity in the form of the year-on-year growth rate industrial of production over time. Evidently, there is some co-movement between the slope and industrial production, where most economic downturns are preceded by an upward spike in the term spread, corroborating the findings related to the yields-to macro literature outlined in section 2 and again strengthening the case of a link between the yield curve and the macroeconomy.

 $<sup>^9\</sup>mathrm{From}$  hereon, a significance level of 5% is postulated with regards to statistical hypothesis testing

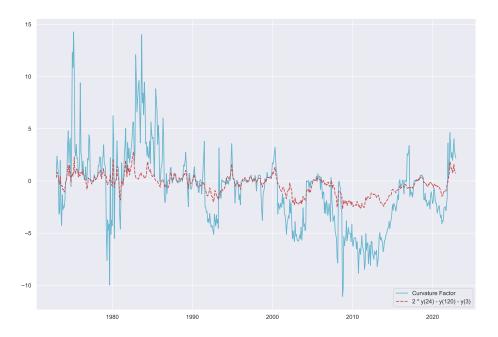


Figure 4: Curvature factor and empirical proxy, USA (in %)

Similarly in Figure 4, with a correlation of 79%, the curvature factor tends to be somewhat close to its proxy, where they generally have similar trends over time, though it unambiguously has a very high volatility relative to it's proxy variable.

In summary, on the one hand the observations described above tend to validate the indication that the estimated Nelson-Siegel factors accurately represent the yield curve and are thus, fitted for studying the relationship between the macroeconomy and the term structure. On the other hand, Figures 2 and 3 have also offered preemptive evidence confirming that there indeed seems to be a relationship consistent with economic theory as well as the literature, namely that the level factor seems to indeed be an accurate representation of the bonds market long-run inflation gauge, as well as an increasing slope factor potentially indicating an upcoming recession.

Consequently, the next step involves analysing macroeconomic and yield curve variables together in a comprehensive structural VAR framework<sup>10</sup> outlined in section 3. However, before fitting a VAR model, it is first important to understand the nature of the variables involved. Thus, Table 1 shows the results of an Augmented Dickey-Fuller test of each variable included in the model, testing whether it is stationary,

 $<sup>^{10}</sup>$ Note that the VAR models in this thesis are estimated via Ordinary Least Squares (OLS)

	t-Statistic	Critical value	p-value
$IP_t^{US}$	-5.6906	-2.8665	0.0000
$\pi_t^{US}$	-3.0096	-2.8665	0.0340
$i_t^{US}$	-2.7681	-2.8665	0.0630
$FS_t^{US}$	-5.4727	-2.8664	0.0000
$\hat{L}_t^{US}$	-1.7930	-2.8665	0.3839
$\hat{S}_t^{US}$	-3.0791	-2.8665	0.0281
$\hat{C}_t^{US}$	-2.9236	-2.8665	0.0427
$M_t^{US}$	-4.8305	-2.8665	0.0000

Table 1: Augmented Dickey-Fuller (ADF) unit root test, US

i.e. it possesses a unit root at the 5% significance level. Apart from the Federal Funds rate  $(i_t^{US})$  and the level factor  $(\hat{L}_t^{US})$ , one can reject the null hypothesis of the existence of a unit root at the 5% level for all remaining model variables. This is insofar not surprising given the fact that the year-on-year growth rates for industrial production, the price level and stock prices are used, which results in detrended time-series. Taking the first differences of both the Federal Funds rate as well as the level factor would result in stationary time series, though due to consistency considerations as well as previous methodologies used in the literature, both time series are included in levels. As noted by Morales (2010), due to parsimony reasons, the order of the vector autoregression process is selected based on both the Bayesian (BIC) and Hannan-Quinn (HQIC) information criteria selecting a lag length of 1. Table 3 presents the estimated VAR(1) coefficients of the comprehensive yields-macro model along with the log-likelihood and various information criteria. Looking at the individual time series on the main diagonal, one can immediately infer that the three yield curve factors are highly persistent, especially the level and slope factor, while in the macroeconomic realm industrial production and inflation display a high persistence. Additionally, the financial market variables stock prices as well as financial stress exhibit a high persistence. Only the short-term interest rate has a strikingly low persistence, which can be seen as confirming the finding by Rudebusch (2005) that incorporating the term structure when modelling the monetary policy reaction function results in refuting the notion of monetary policy inertia.

Since the aim of this thesis is to understand the dynamic interactions between the yield curve and the macroeconomy in the United States, Figure 5 depicts the orthogonal impulse responses to a one unit shock in each of the model's variables over a three year period (36 months) with 90% confidence intervals. Each row shows the response of a certain variable i to a specific structural shock of variable j, where each column displays to which variable j the respective shock occurred. For example, the second row shows the responses of inflation  $(\pi_t^{US})$  to various structural

shocks to the model variables, while the third column illustrates the responses of each variable to a structural monetary policy shock in the form of the short-term interest rate  $(i_t^{US})$ . With this in mind, there are four types of impulse responses to consider: macro-to-macro, macro-to-yields, yields-to-macro and yields-to-yields.

As a first assessment of validity, the macro-to-macro impulse responses are considered. It is reassuring that the results seem to broadly match those of similar small scale macro models oftentimes used in the literature (e.g. Galí (1992), James H Stock and Watson (2001)). Specifically, a shock to industrial production, i.e. an aggregate demand shock, increases both inflation (second row, first column) and the short-term interest rate (third row, first column), while an inflation shock decreases aggregate demand over time. The Federal Funds rate increases on impact as well as over time to a positive inflation shock, which is not only consistent with economic theory, but, given the dual mandate of the FED, also with real-world central bank behavior. The last macro-to-macro shock to consider is a monetary policy shock (third column). Since the short-term restrictions described in chapter 3 are based on the assumption that both industrial production and inflation react with a certain time lag, both variables do not react on impact, but over time - as one would expect based on economic reasoning - industrial production as well as inflation decrease as a response to a surprise increase in the short-term rate, where the decrease of inflation is more persistent, while industrial production decreases more strongly during the first year.

Examining the first direction of the link between the macroeconomy and the yield curve, i.e. yield curve responses to macro shocks, offers some interesting insights. For example, an aggregate demand shock increases the level factor, which, when keeping in mind the interpretation of the level factor as the bond's market long-run inflation expectation - via the Fisher effect - seems reasonable as higher aggregate demand should generally lead agents to expect a higher future price level according to standard economic theory. Similarly, the slope factor increases after an aggregate demand shock over time, meaning that the short-end of the yield curve rises relative to the long-end. Again, this is in line with standard monetary policy responses to surprises in aggregate demand since bond investors would expect central banks to increase interest rates in the near term as they anticipate higher inflation due to the aggregate demand shock, likely pushing up short-run yields relative to the long-end of the term structure (Diebold, Rudebusch, and Aruoba, 2006). A similar response can be observed when looking at aggregate supply shocks (inflation shocks). Specifically, the level, curvature as well as the slope factor increase after a surprise increase in US inflation. The former result can be explained via the aforementioned Fisher effect, where the bond market likely expects higher future inflation when facing a surprise upward tilt in the current inflation rate leading to an increase in yields. This increase in the level factor would correspond to the assumption that inflation expectations are not firmly anchored Diebold, Rudebusch, and Aruoba (2006). The latter effect is again linked to monetary policy, where higher short-term rates are priced in by investors due to present and future expected price level increases, leading to a more negatively sloped yield curve. Interestingly, both phenomena have occurred only recently during the last two years where inflationary pressures induced by the war in Ukraine led to significant increases in US yields due to the markets expectation of an imminent monetary tightening by the FED, while the yield curve inverted as short-end yields exceeded long-end yields in anticipation of an upcoming recession. The last macro-to-yields shock to consider is a monetary policy shock. While the curvature factor responds only marginally, the level factor decreases as a response to a surprise increase in the Federal Funds rate (third column). In this context, contrasting explanations regarding the level factors response to a monetary policy shock are offered. Given a central bank has a high degree of credibility and transparency, a surprise monetary tightening could indicate a lower inflation target and thus, induce bond investors to revise their inflation expectations downwards, potentially lowering the level factor, while a surprise tightening could also signal that a central bank is concerned about an overheating economy and an overshooting inflation rate, which would likely result in higher expected inflation and thus, an increasing level factor (Diebold, Rudebusch, and Aruoba, 2006). Apparently, the former effect has dominated the latter over the sample period, which does not seem implausible given the high credibility of the FED.

The other direction of causality to consider is the yields-to-macro channel. Whereas the short-term interest rate does not react to the long-run level factor, its reaction to the slope factor is strikingly high, increasing after a surprise increase in the slope factor, confirming that there is a close connection between the monetary policy instrument and the short-run factor of the yield curve. In the literature, two distinct explanations are offered in this context. On the one hand, the FED might respond to current yields when setting the short-term rate, whilst on the other hand, market yields very likely respond to new information regarding the macroeconomy in anticipation of monetary policy decisions, hence, shifts in fixed-income markets presumably precede central bank actions (Diebold, Rudebusch, and Aruoba, 2006; Morales, 2010). This would, for example, lead to short-end yields to increase in anticipation of an imminent monetary policy tightening. Noteworthily, the federal funds rate also responds with an increase to a shock in the curvature factor, though not by the same order of magnitude as the yield curve slope does. Another compelling case to consider is a shock to the level factor, which increases aggregate demand. This is insofar relevant from the economists lens when once again keeping in mind the level factors interpretation as a proxy for inflation expectations. In this case, an increase in inflation expectations lowers the ex-ante real interest rate,  $i_t^{US} - \hat{L}_t^{US}$ , leading to a surge in economic activity - a finding that is consistent with standard DSGE models. In contrast, some rather implausible responses raise questions about the validity of the yields-to-macro results. For example, the response of industrial production to a surprise increase in the yield curve slope does contradict previous findings in the literature. Specifically, an increase in the slope - equivalent to a flattening and potentially inversion of the yield curve - is generally associated with an imminent recession rather than an economic boom. Similarly, one would anticipate inflation to decrease in response to a shock to the slope factor, whereas it is expected to increase with an increasing level factor, i.e. due to the decreasing ex-ante real interest rate. In this context, it could be argued that an increase in the level of yields possibly has dampening effects on the economy - similar to a contractionary monetary policy - inducing a decrease in inflation, though this effect likely is in conjunction with a decrease in industrial production, and not an increase as the impulse response of industrial production suggests. Based on these dubious results regarding the slope one could argue that these findings confirm the conclusions of Ang, Piazzesi, and Wei (2006), where the short-term interest rate dominates the slope of the yield curve when predicting economic activity, at least when comparing the plausibility of the results for the short-term interest rate vis-a-vis the slope in Figure 5. Furthermore, it could be argued that the modelling strategy based on OLS estimates applied certainly has some limitations and leads to partly improbable results.

Generally, it has to be noted that based on the impulse responses the yields-to macro responses have wider confidence intervals most of the time, hence, it seems that there is more uncertainty involved in the macro responses to yields shocks, whereas there is a stronger evidence for the macro-to-yields channel since those responses are associated with less uncertainty as well as broadly being consistent with economic theory as well as results from the literature.

The remaining impulse responses to consider are the yields-to-yields shocks. Apart from the high persistence in the level and slope factor noted above, some noteworthy observations include the slope factors response to a level factor shock. Consistent with the findings of Diebold, Rudebusch, and Aruoba (2006), a surprise increase in the level factor, i.e. long-run inflation expectations, is associated with loose monetary policy conditions represented by a lowering of the short-end of the yield curve relative to the long-end, synonymous with a steepening of the yield curve. Similarly, a surprise increase in the slope factor, possibly as anticipation to a monetary tightening, is associated with an increase of the level factor, corresponding to higher future inflation expectations.

Finally, the link between the yield curve factors and the macroeconomic variables are assessed using block Granger causality tests, where one can test if a set of variables Granger cause another set of variables in the model. Conveniently, this method enables to determine whether there is a one- or bi-directional relationship present, e.g. testing whether solely a set macro variables Granger cause the yield curve factors or if both the macro variables do Granger cause and are Granger caused by

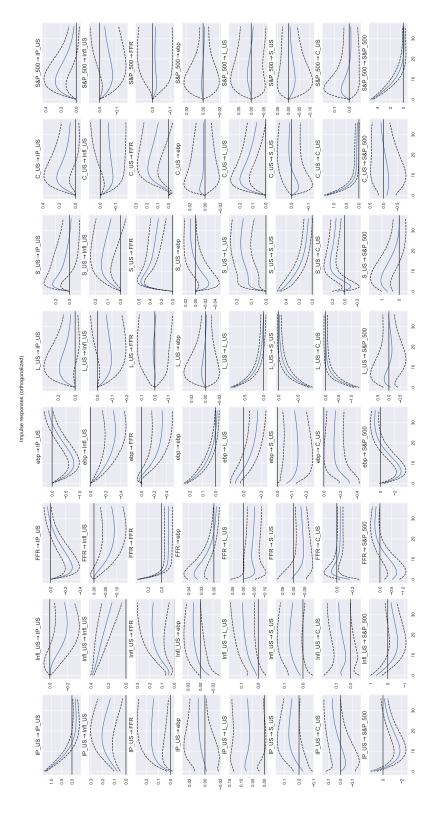


Figure 5: Impulse Responses, US

	t-statistic	Critical value	p-value
Macro-to-Yields		1.8818	0.0000
Yields-to-Macro		1.8818	0.0000

Table 2: Block Granger causality tests, US

the yield curve factors. In the present setting, the macro-to-yields test examines whether the set of macro variables in the model  $(IP_t^{US}, \pi_t^{US}, i_t^{US})$  do Granger cause the estimated yield curve factors  $(\hat{L}_t^{US}, \hat{S}_t^{US}, \hat{C}_t^{US})$ , whilst the yields-to-macro test evaluates the presence of a Granger causality in reverse order, namely, if the yield curve factors Granger cause the set of macro variables. Based on Table 2, depicting the resulting block Granger causality tests, one can conclude that there seems to be a bi-directional link present in the United States, where both macroeconomic variables seem to contain useful information regarding the behavior of the yield curve and, vice versa, the yield curve seems to affect macroeconomic fluctuations.

# 4.2 The Yield Curve and the Macroeconomy in the Euro Area

Though, starting in January 2004, the sample period is markedly smaller compared to that of the United States, the analysis of the Euro Area nevertheless reveals some further interesting insights into the relationship between the yield curve and the economy. Again Figure 6 offers a comparison between all three estimated Nelson-Siegel factors over time. Just like in the US, the level factor has been positive during the whole sample period, while also exhibiting a decreasing trend for most of the time, especially since the mid 2010s, a period, as mentioned before, marked by QE, negative interest rates, and in the case of the Euro Area, the European debt crisis. Since the start of 2022, where residual inflationary pressures due to the COVID-19 pandemic as well inflation induced by soaring energy prices due to the conflict in Ukraine also severely affected the European economy, the level factor has increased quite strongly, surpassing the 2% mark - an interesting observation that is not only in line with the above mentioned Fisher effect regarding the relationship between the level effect and inflation, but this co-movement is also best seen in Figure 7. With a correlation of 75%, the slope factor displays a strong co-movement with the curvature factor. Evidently, the curvature factor has the highest volatility, especially due to the spike in May 2011, which could again be due to an idiosyncratic shock in the form of the Euro crisis, but could also be attributed to the fact that the Euro Area yields are based on a rather diverse pool of issuers with possible unobserved heterogeneity, which could potentially have confounding effects when using the Nelson-Siegel model.

	$IP_t^{US}$	$\pi_t^{US}$	$i_t^{US}$	$FS_t^{US}$	$\hat{L}_t^{US}$	$\hat{S}_t^{US}$	$\hat{C}_t^{US}$	$M_t^{US}$
2	-0.1651	0.2026	-0.0501	-0.0345	0.5002	-0.5581	-0.7174	1.5318
	(0.1744)	(0.0529)	(0.0471)	(0.0318)	(0.1138)	(0.1314)	(0.2473)	(0.6817)
$IP_{t-1}^{US}$	0.8816	0.0132	0.0086	0.0005	0.0046	0.0025	-0.0134	-0.4305
1	(0.0137)	(0.0042)	(0.0037)	(0.0025)	(0.0089)	(0.0103)	(0.0194)	(0.0535)
$\pi_{t-1}^{US}$	-0.0477	0.9795	0.0126	0.0058	0.0102	0.0223	0.0039	-0.4072
	(0.0260)	(0.0079)	(0.0070)	(0.0047)	(0.0169)	(0.0196)	(0.0368)	(0.1014)
$i_{t-1}^{US}$	-0.2730	-0.0024	0.5393	0.0220	-0.0801	-0.0106	0.3227	-0.1502
	(0.0632)	(0.0192)	(0.0171)	(0.0115)	(0.0413)	(0.0476)	(0.0897)	(0.2471)
$FS_{t-1}^{US}$	-0.6609	-0.1468	0.0521	0.8966	0.0685	-0.1480	-0.2175	-4.4019
	(0.1260)	(0.0382)	(0.0340)	(0.0230)	(0.0822)	(0.0949)	(0.1787)	(0.4925)
$\hat{L}_{t-1}^{US}$	0.3604	-0.0011	0.5155	-0.0237	1.0214	0.0493	-0.2522	0.5345
	(0.0739)	(0.0224)	(0.0199)	(0.0135)	(0.0482)	(0.0557)	(0.1048)	(0.2888)
$\hat{S}_{t-1}^{US}$	0.2571	0.0431	0.5407	-0.0305	0.1206	0.9161	-0.2928	0.4040
	(0.0734)	(0.0223)	(0.0198)	(0.0134)	(0.0479)	(0.0553)	(0.1041)	(0.2869)
$\hat{C}_{t-1}^{US}$	0.0303	-0.0163	-0.0158	0.0035	0.0426	-0.0156	0.8149	-0.0219
	(0.0191)	(0.0058)	(0.0052)	(0.0035)	(0.0125)	(0.0144)	(0.0271)	(0.0747)
$M_t^{US}$	0.0148	-0.0030	-0.0009	0.0001	0.0001	-0.0013	0.0049	0.9086
	(0.0040)	(0.0012)	(0.0011)	(0.0007)	(0.0026)	(0.0030)	(0.0057)	(0.0157)
Log-Likelihood	-5443.2304							
AIC	-4.2882							
BIC	-3.7599							
HQIC	-4.0825							

Table 3: Vector Autoregression estimation results, US



Figure 6: Estimated level, slope and curvature factor, EA (in %)

Once more, in order to get an indication of how well the factor estimates represent the yield curve, Figures 7-9 display the estimated factors,  $\hat{L}_t^{EA}$ ,  $\hat{S}_t^{EA}$ ,  $\hat{C}_t^{EA}$  with their respective empirical proxies. All three estimates exhibit a very strong and significant correlation with their proxies ranging from 74% ( $\hat{L}_{t}^{EA}$ ) to 86% ( $\hat{C}_{t}^{EA}$ ), which underlines the suitability of the estimates as accurately representing the level, slope and curvature of the Euro Area yield curve. Most interestingly, the correlation between inflation and the level factor, though not being significant, is negative during the sample period, which of course contradicts the Fisher effect and the interpretation, that the level factor is an approximation for the expected inflation rate. Notably, not only is the sample period rather short, four highly idiosyncratic shocks have occurred during that short period of time, i.e. the financial crisis of 2007/08, the European debt crisis starting in 2010, the COVID-19 pandemic starting in 2020 and the ongoing war in Ukraine since February 2022, which likely stretches the rather simplistic approach used in this thesis to its limits. Adding to that the comparatively high heterogeneity of the Euro Area, it is by no means surprising to obtain counterintuitive results.

Apart from the estimated slope factor and it's proxy variable, Figure 8 also includes Euro Area industrial production representing economic activity. The first thing that stands out is the very high variability of industrial production, especially during the

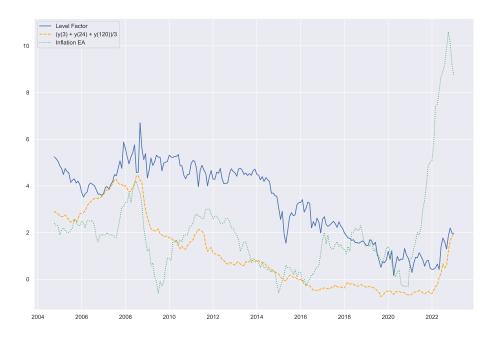


Figure 7: Level factor, empirical proxy and inflation, EA (in %)

onset of the COVID-19 pandemic in early 2020, ranging from -19,91% to 23,90%. Though the recession following the financial crisis of 2007/08 was preceded by an upward trend of the estimated slope factor - being consistent with economic theory - the number of economic downturns simply is too small during the short sample to infer any substantial conclusions regarding the validity of the relationship between the term spread and economic activity in the Euro Area.

In short, Figures 6-9 show that the obtained estimates are very close to their respective proxy variables, and are thus well suited to represent the level, slope and curvature of the Euro Area yield curve. Nevertheless, they have also indicated that the limited sample period of 220 months marked by various distinct exogenous shocks impede to draw viable conclusions.

Nonetheless, in order to study the dynamics between the yield curve and the macroe-conomy in the Euro Area, orthogonalized impulse responses have been obtained using the modelling strategy outlined in Section 3. In order to examine the stationary of the time series involved, Table 4 shows the results of respective ADF tests for each variable, testing the presence of a unit root at the 5% significance level. The inflation rate in the Euro Area is not stationary, which is not surprising since the years after the financial crisis of 2007/08 have been marked by a decreasing trend



Figure 8: Slope factor and empirical proxy, EA (in %)

in inflation, while inflation has surged since the start of 2022 due to the ongoing Ukraine war. Similarly, the short-term Euro Area interest rate - the 3-month interbank rate - is also non-stationary. In contrast to the United States, all three yield curve factors do possess a unit root and are thus, not stationary. Equivalently to the model that has been fitted for the US data, a VAR of order one has been chosen for the Euro Area based on the BIC and HQIC, respectively. Table 6 shows the estimation results. All coefficients on the main diagonal point to the fact that each time series exhibits a rather high persistence, most notably, the short-term interest rate as well. Hence, while the hypothesis of monetary policy inertia in the context of Rudebusch (2005) seems to be rejected for the United States, the ECB seems to act more sluggishly based on the high persistence of the short-term rate even when controlling for the term structure via the Nelson-Siegel factors.

Now, taking Figure 10 into consideration, depicting the orthogonal responses to a one unit impulse of each variable involved in the model over a period of 36 month with 90% confidence bands, there are again four different types of dynamics to evaluate. Among those, the macro-to-macro responses seem to be consistent with standard impulse responses from macro-only models. An aggregate demand shock - a shock to  $IP_t^{EA}$  - is associated with an increase in inflation  $(\pi_t^{EA})$  and the short-term interest rate  $(i_t^{EA})$ , while an aggregate supply shock in the form of an increasing inflation



Figure 9: Curvature factor and empirical proxy, EA (in %)

rate decreases industrial production and leads to an increase in the interest rate, thus, being in accordance with a monetary policy response to surprise increases in prices. Vice versa, a surprise monetary tightening induces both a decrease in aggregate demand and prices.

Continuing with the macro-to-yields responses, an unexpected increase in aggregate demand as well as inflation induces an increase in the yield curve level. Here again, since one likely expects inflation to rise in the future after a surge in either aggregate demand or prices, the interpretation of the level factor as the long-run gauge of inflation proves convenient. Seemingly, neither in the United States, nor in the Euro area inflation expectations are firmly anchored since in both economies an unanticipated upward tilt in prices leads to inflation expectations (the level factor) to increase. Analogously, the slope factor clearly increases as a response to innovations in economic activity over time, which would again be consistent with a monetary policy response to an increase in aggregate demand, increasing the short-term interest rate and thus, the short-end of the yield curve relative to the long-end. Interestingly, though a similar response of the slope factor in the US can be observed with regards to a price shock, the yield curve slope in the Euro Area seems to not respond to inflation shocks. Also, an aggregate demand as well as an aggregate supply shock both lead to an increase in the medium-term curvature factor. As mentioned in

	t-Statistic	Critical value	p-value
$IP_t^{EA}$	-3.6453	-2.8757	0.0050
$\pi_t^{EA}$	-2.1955	-2.8757	0.2079
$i_t^{EA}$	-1.9314	-2.8756	0.3174
$FS_t^{EA}$	-5.0570	-2.8748	0.0000
$L_t^{EA}$	-1.0338	-2.8751	0.7407
$S_t^{EA}$	-1.3915	-2.8751	0.5863
$C_t^{EA}$	-1.5053	-2.8751	0.5309
$M_t^{EA}$	-3.5289	-2.8757	0.0073

Table 4: Augmented Dickey-Fuller (ADF) unit root test, EA

Section 4.1, the response of the yield curve level to a monetary policy shock can have two distinct interpretations. Either, the level factor increases after a surprise monetary tightening due to agents assuming that monetary policy authorities fear an overheating economy and thus, adjust their inflation expectations upwards, or, in light of high credibility of a central bank, the level factor decreases since the tightening could imply a lower inflation target (Diebold, Rudebusch, and Aruoba, 2006). Whereas the latter effect dominated in the United States, i.e. the yield curve level decreased after an upward shift in the short-term rate, pointing to a high credibility of the FED, the level factor responds with an increase to a monetary policy tightening in the Euro Area, indicating that the former effect prevails. As was the case in the United States, the slope factor increases on impact after a surprise increase in the interest rate suggesting that the short-term factor of the yield curve seems to be intricately linked to the monetary policy instrument. However, by order of magnitude, the response of the slope factor to a monetary policy shock is markedly higher in the United States than in the Euro Area. A similar response can be observed from the curvature factor, which increases in impact after a monetary policy shock.

Coming to the yields-to-macro responses it is immediately noticeable that the macro variables have negligible responses to shocks in the slope factor. Though economic activity decreases after the yield curve becomes less steep, which is consistent with the yields-to-macro literature in the manner of Estrella and Hardouvelis (1991), the response seems to be insignificant. Industrial production increases after a shock to the level factor, which is consistent with the finding in the United States that a surprise jump in expected inflation leading to a decrease in the ex-ante real interest rate,  $i_t^{EA} - \hat{L}_t^{EA}$ , resulting in an economic expansion. However, inflation does not exhibit a response compatible with this interpretation since one would expect inflation to increase rather than decrease after a fall in the real interest rate due to a positive shock to the level factor. As is the case in the US, it could be argued that

	t-statistic	Critical value	p-value
Macro-to-Yields		1.8854	0.0002
Yields-to-Macro		1.8854	0.0005

Table 5: Block Granger causality tests, EA

inflation decreases as an overall increase in level of yields potentially has contractionary effects on the economy similar to a monetary policy tightening, though this should likely be accompanied by a decrease in industrial production. Also, the decrease of the short-term interest rate as a response to an increase in the level factor seems implausible as economic reasoning predicts a rise of the monetary policy rate as a response to higher inflation expectations approximated by the yield curve level. Interestingly, the macro variables respond quite strongly to curvature factor shocks, inducing an increase in all three variables, most notably the persistent increase of the short-term interest rate over the whole three years. Overall, as in the US, the yields-to-macro responses in the Euro Area do not always conform with economic theory as well as previous findings in the literature potentially highlighting some drawbacks of the methodology employed.

Taking into account the yields-to-yields responses, the slope factor seems to only marginally affect the other two term structure factors, while a shock to the long-run level factor decreases both the slope and curvature factor, respectively. The surprise increase in the medium-term curvature factor leads to an increase in the level and slope, where the increase of the level is clearly more pronounced.

Finally, Table 5 shows two block Granger causality tests based on the estimation results. Once again, the macro-to-yields block tests whether the macro variables  $(IP_t^{EA}, \pi_t^{EA}, i_t^{EA})$  Granger cause the yield curve variables  $(\hat{L}_t^{EA}, \hat{S}_t^{EA}, \hat{C}_t^{EA})$ , while the yields-to-macro tests the Granger causality in the opposite direction. Apparently, the null hypothesis of no Granger causality present can be rejected for both sets of variables, leading to the conclusion that again a bi-directional link between the macroeconomy and the yield curve is present in the Euro Area. However, when considering the rather implausible and/or insignificant yields-to-macro responses in Figure 10, this conclusion seems unconvincing. As was the case in the United States, the responses concerned with the macro-to-yields channel seem more convincing and consistent with the literature in the Euro Area.

	$IP_t^{EA}$	$\pi_t^{EA}$	$i_t^{EA}$	$FS_t^{EA}$	$\hat{L}_t^{EA}$	$\hat{S}_t^{EA}$	$\hat{C}_t^{EA}$	$M_t^{EA}$
$\mathcal{O}$	1.2566	0.1349	0.1196	3.3488	0.0514	0.1131	-0.3529	8.4328
	(0.6388)	(0.0985)	(0.0355)	(1.5143)	(0.1072)	(0.11115)	(0.2697)	(2.2158)
$IP_{t-1}^{EA}$	0.8892	0.0156	0.0007	-0.1784	-0.0027	0.0017	-0.0137	0.1437
	(0.0422)	(0.0065)	(0.0023)	(0.1001)	(0.0071)	(0.0074)	(0.0178)	(0.1465)
$\pi_{t-}^{EA}$	-0.0933	0.9983	0.0272	0.3455	-0.0007	0.0135	0.1053	-0.7083
	(0.0989)	(0.0152)	(0.0055)	(0.2343)	(0.0166)	(0.0172)	(0.0417)	(0.3429)
$i_{t-1}^{EA}$	-0.5740	-0.0884	0.9627	1.5284	0.0275	0.0842	0.1549	-3.4139
	(0.8192)	(0.1263)	(0.0455)	(1.9419)	(0.1375)	(0.1429)	(0.3458)	(2.8416)
$FS_{t-1}^{EA}$	-0.0348	0.0030	-0.0056	0.7429	0.0042	-0.0123	-0.0093	-0.2087
	(0.0230)	(0.0036)	(0.0013)	(0.0546)	(0.0039)	(0.0040)	(0.0097)	(0.0799)
$\hat{L}_{t-1}^{EA}$	0.4550	0.0404	0.0188	-1.2571	0.9504	-0.0918	-0.1435	3.3983
	(0.8775)	(0.1353)	(0.0488)	(2.0801)	(0.1473)	(0.1531)	(0.3704)	(3.0438)
$\hat{S}_{t-1}^{EA}$	0.0760	0.0605	-0.0022	-1.0454	-0.0330	0.8605	0.0540	2.0348
	(0.8234)	(0.1269)	(0.0457)	(1.9517)	(0.1382)	(0.1436)	(0.3476)	(2.8559)
$\hat{C}_{t-1}^{EA}$	0.3245	0.0258	0.0317	-0.5248	0.0417	0.0015	0.7142	1.1557
	(0.1359)	(0.0210)	(0.0070)	(0.3222)	(0.0228)	(0.0237)	(0.0574)	(0.4715)
$M_{t-1}^{EA}$	-0.0024	0.0002	0.0013	0.0383	0.0011	-0.0002	0.0051	0.8038
	(0.0126)	(0.0019)	(0.0007)	(0.0299)	(0.0021)	(0.0022)	(0.0053)	(0.0437)
Log-Likelihood	-1759.3817							
AIC	-5.9781							
BIC	-4.8639							
HQIC	-5.5281							

Table 6: Vector Autoregression estimation results, EA

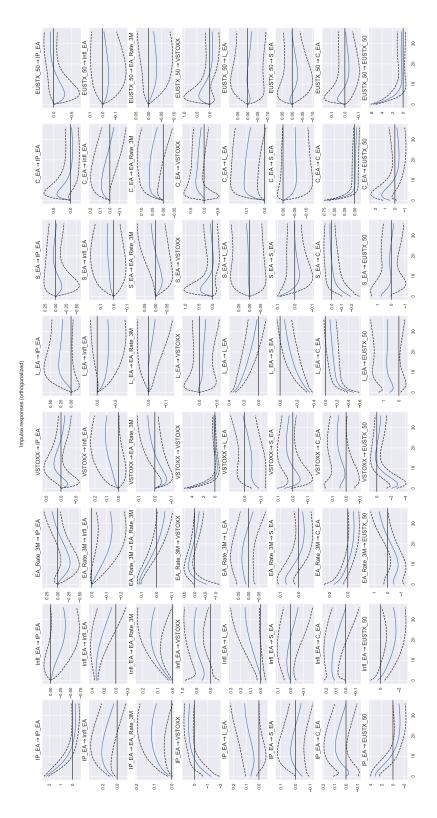


Figure 10: Impulse Responses, EA

#### 5 Conclusion

To summarize, this thesis has investigated how the yield curve and the macroeconomy are related in the United States as well as the Euro Area, especially focusing on the question, if there is a one or bi-directional link. By using a two step approach similar to Diebold, Rudebusch, and Aruoba (2006), first modelling the yield curve using the Nelson-Siegel model to obtain a level, slope and curvature factor and subsequently including said factors in a VAR model containing various other macroeconomic as well as financial variables, the resulting structural shocks have given an indication of said relationship.

A possible extension of this thesis would be to actually use the estimated slope factors in a probabilistic setting like the one proposed by Estrella and Hardouvelis (1991) and see if said factors, being an approximation of the term spread, are able to correctly predict the onset of past recessions - a finding that would be consistent with the literature. Furthermore, testing the performance in yield (curve) forecasting of a general Nelson-Siegel model versus the extended model including macro variables used in this thesis could be further enlightening in regards to the usefulness of a yields-macro model.

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## A Appendix

#### Data US

- Industrial Production (FRED: INDPRO)
- CPI (FRED: CPIAUCSL)
- Federal Funds Effective Rate (FRED: DFF)
- S&P 500 (Yahho Finance: ^GSPC)
- Excess Bond Premium (EBP)<sup>11</sup>
- $\bullet$   $\beta$  factors based on yields data provided by Liu and J. C. Wu  $(2021)^{12}$

#### Data EA

- Industrial Production (FRED: EA19PRINTO01GYSAM)
- Inflation (FRED: CPHPTT01EZM659N)
- 3M Interbank Rate (FRED: IR3TIB01EZM156N)
- Eurostoxx 50 (Yahho Finance: ^STOXX50E)
- VSTOXX Index (source: Bloomberg Terminal as of 23.01.2024)
- $\beta$  factors based on yields data provided by the ECB<sup>13</sup>

<sup>&</sup>lt;sup>11</sup>source: Fed Note Data

<sup>&</sup>lt;sup>12</sup>source: Liu-Wu Yield Data

<sup>&</sup>lt;sup>13</sup>source: dataset "All years - AAA"