

Variant hash of the message  
 ②  
 $m = 5$   
 $y^2 = x^3 - 3x - 6$   
 $G = (1; 5)$   
 $n = 11$   
 $Q = (6; 4)$   
 $K = 2$   
 $P = 11$   
 gen. dig. sig.

1 page

Shumilov  
Alexander

$$1) \text{ First we find } (X_C, Y_C) = K \cdot G = 2G = G + G$$

$$G = (1; 5)$$

$$\text{Slope: } \lambda = \frac{3G_x^2 - p}{2G_y} = \frac{3 \cdot 1 - 3}{2 \cdot 5} = 0$$

$$\Rightarrow X_C = 0 - 1 - 1 = -2 \pmod{p} \quad (\text{no need to calc. inverse})$$

$$Y_C = 0(G_x - X_C) - 5 = -5 \pmod{p}$$

$$\Rightarrow X_C = -2 \pmod{11} = 9$$

$$Y_C = -5 \pmod{11} = 6 \Rightarrow (X_C, Y_C) = (9, 6) = KG \quad (+\text{tangent line})$$

$$G = (X_C, X_C \cdot d + km) \pmod{p}, \text{ where } d: \frac{Q}{P} = d \cdot G$$

$$\Rightarrow (6; 4) = \underbrace{G + \dots + G}_{d \text{ times}} \quad \begin{matrix} \text{open key} \\ \text{secret key} \end{matrix}$$

We know  $2G \Rightarrow$ , let's compute  $3G$  and hope that we are lucky:

$$3G = 2G + G = \overbrace{(9, 6)}^P + \overbrace{(1, 5)}^Q = R$$

$$\text{Slope: } \lambda = \frac{3 \cdot Q_y - P_y}{Q_x - P_x} \Rightarrow \begin{cases} Q_y - P_y = -1 \pmod{11} = 10 \\ Q_x - P_x = -8 \pmod{11} = 3 \end{cases}$$

modulo inverse, not division

$$\Rightarrow \lambda = 10 \cdot 3^{-1} \pmod{11}$$

$$3^{-1} \cdot 3 \pmod{11} = 1 \quad (3 \text{ and } 11 \text{ are coprime})$$

$$\Rightarrow \text{if } 3^{-1} \pmod{11} = x, \text{ then } 1 = 3x \pmod{11} \Rightarrow x = 4$$

$$\Rightarrow x = 40 \pmod{11} = 7 \Rightarrow \begin{cases} R_x = \lambda^2 - Q_x - P_x = (49 - 1 - 9) \pmod{11} = 6 \\ R_y = \lambda(P_x - R_x) - P_y = 7(9 - 6) - 6 \pmod{11} = 4 \end{cases} \quad \begin{matrix} 39 \\ 21 \\ 3 \\ 15 \end{matrix}$$



Пожалуйста, пользуйтесь темно-синей или черной ручкой, не пишите за пределами клеточек и на оборотах листов, не мните листы и не складывайте их пополам.

Variant

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$$\Rightarrow \text{indeed } Q = 3G \Rightarrow d = \cancel{6} \ 3$$

$$\Rightarrow S = (x_c, x_c \cdot d + k \cdot m) \bmod p =$$

$$\therefore (g, g^{27} \cdot 3 + 2 \cdot 5) = (g, 37) \bmod 11 = \underline{\underline{(g, 4)}}$$

This pair is signature.

## ④ Digital right management.

It's actually great coincidence that I got dig. rights, because 2 days ago I've read that Spotify used blockchain to solve its problems with copyrighting, so the use case from real life is ready.

a) First let's describe the environment. Most common (probably) example of dig. right manag. problem is media (content: music, videos, apps, ...) So such content is distributed among people. I believe that in this system there are multiple consumers and multiple content producers. So here is the answer to the a):

a) Based on env. overview - no. There could be some platform that maintain the work of the system, but we cannot say that it is trusted

probably in any case  
there is one

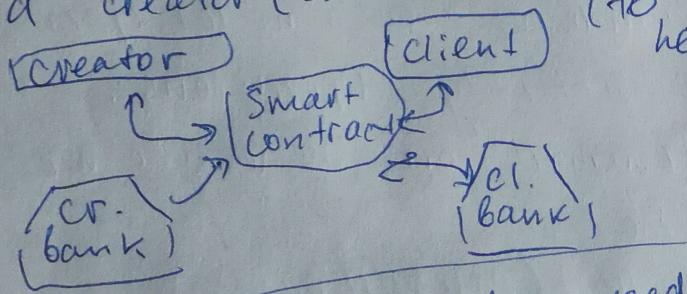
## ④ Continuation of ④.

b) There are multiple things to track down: creation of some content is followed by its submission to the system, then gaining rights on it. From consumer point of view there is paying for "content" that he/she wants to buy (so there is transactions). Also payment to the creator is also a transaction that could be tracked.

c) In my opinion the system is insecure. The work of submission of creation lies on shoulders of creator, so he could make some intentional or non-intentional mistakes (so "paper contract" could be invalid) - could raise a problem in any part of following chain to consumer. Also example - consumer pays and because some internal error caused by human factor he waists money and does not get anything.

And last - data insecurity. At any point system is vulnerable to attacks. So user data could be stolen (appshacking.)

d) Smart contracts here could be used to reduce the influence of 1 person (human) on any transactions. For example between platform that connects creator and consumer, and creator (creator's bank) and consumer (consumer's bank) (to manage txs, so to say)

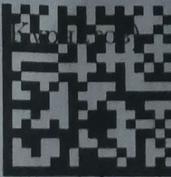


help  
Answer is yes (6/6)

??

e) On 1<sup>st</sup> lecture we discussed 6 crucial points for it:

- business problem: yes, for example media content sphere
- partners with diff. interest: yes, definitely (clients earn money, clients spent, for ex.)
- date exch. between partners: yes, I described it
- Many partners modify data: yes, there are many clients (agents)
- no easy fix w/ Blockchain: yes, Blockchain seems to help the most (privacy, management)
- no way to rely on cent-authority: yes, discussed above.



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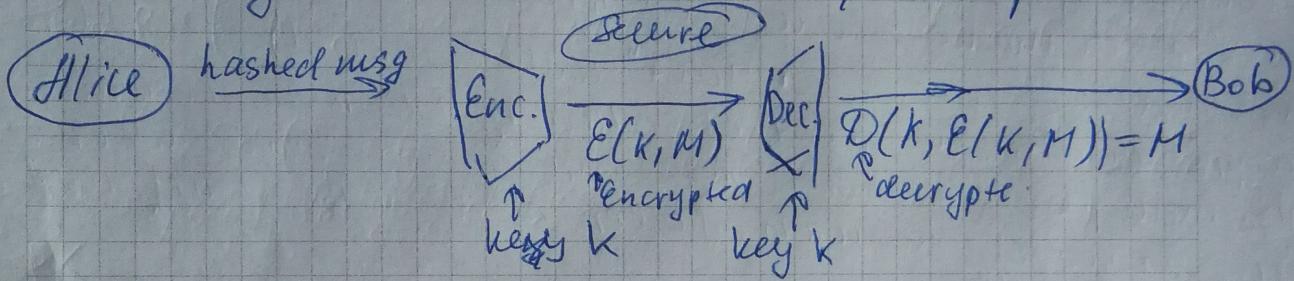
Variant

Shumakov A.I.

## (1) Block ciphers, build blocks, example.

The question is connected with secret key cryptography. In problem of encryption & decryption we considered so-called block ciphers.

What is block? Block - is a group of bits with fixed length. Let's recall encr/decr. process:



So when we talk about transf. on a block, it is defined by the symmetric key.

Idea of block cipher, (developed by mathematician Shannon)

All modern and currently-used bl-ciph are based on so-called "iterated product cipher" idea. Shannon suggested to combine some similar operations to strengthen the security. Operations: substitution and permutation. On lecture we discussed Feistel example.

Substitution

Let's assume we have a message (input.)

We present first it in bits form. Then we assume general "permutations" on mem-substitute

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a substitution



Variant

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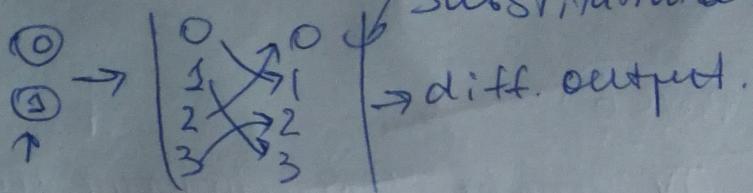
part

Shumilov

one message with another message. ~~then~~  
On slide we had a picture, I'll try to recreate it

Alexander

Substitution box



2-bits example.

Note, that in ideal situations such box should be very large, but complexity in memory consumption is pretty high (biggest problem): if we have  $m$  bits, substit. to be stored -  $2^m$  (~~is~~ large).

Permutation Idea - combine permutation with substitutions. Permutation is easier: we just change the order of bits in message. Difference with previous operation: in substitution we should check all possible messages in order to reconstruct actual substitution. So permutation is ~~faster~~ (in a sense of comp. memory consump.), but less lighter

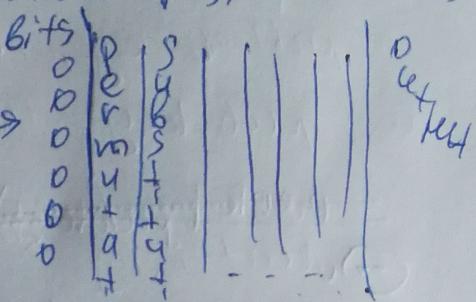
Secure.

Subst. box + Permut. box : message  $\rightarrow$

Also we talked about role of key in choice of substitution (2 variants)

That depends on a key:  $|S| = |S_0 \cap S_1|$

key chooses

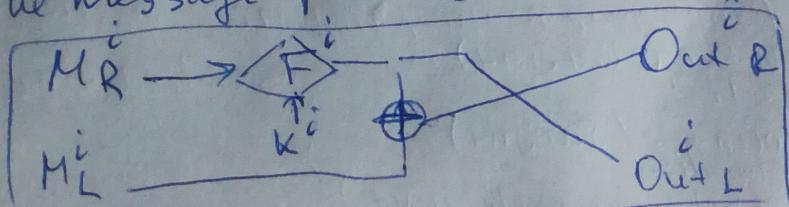


→ (PSPSPS... such type)

Example: Feistel cipher

> Message is been splitted in 2 parts: MR, ML

> Feistl. Function - for so-called Feistel cell, that depends on the message part (non-linear) and the key



→ 1 block → applic. segm.

Out R - new MR

Out L - new ML



Variant

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Shumilov R.

③ Bitcoin tx-s. Def.: TX - transfer of Bitcoin value that is broadcast to the network and collected into blocks

a) TX types  
④ coinbase tx - 1<sup>st</sup> trans. present in block that is created by a miner

⑤ P2PKH (pay to public key hash) } for sending to

⑥ P2SH (pay to script hash) } for more

⑦ P2PK (pay to public key) } Bitcoin addresses contains hash of public key / as it said in file name)

P2PKH - majority of txs nowadays

P2PK - as far as I understand, it's form is simpler than previous one (shorter because of a way of storing the public key, in P2PKH we store public key hash, not pub. key itself). Generally less secure, aren't used in modern systems

~~⑧ P2PKH (pay to public key hash)~~  
⑧ Multisig - requires before output could be spent, multisig requires a number of signatures - so-called  $m$ -of- $n$

$N$  of given pub. keys

min. number of sig. which should match with pub. key

for success  
⑨ Null data and ⑩ KeyWitness (P2WPKH, P2WSH), W = witness

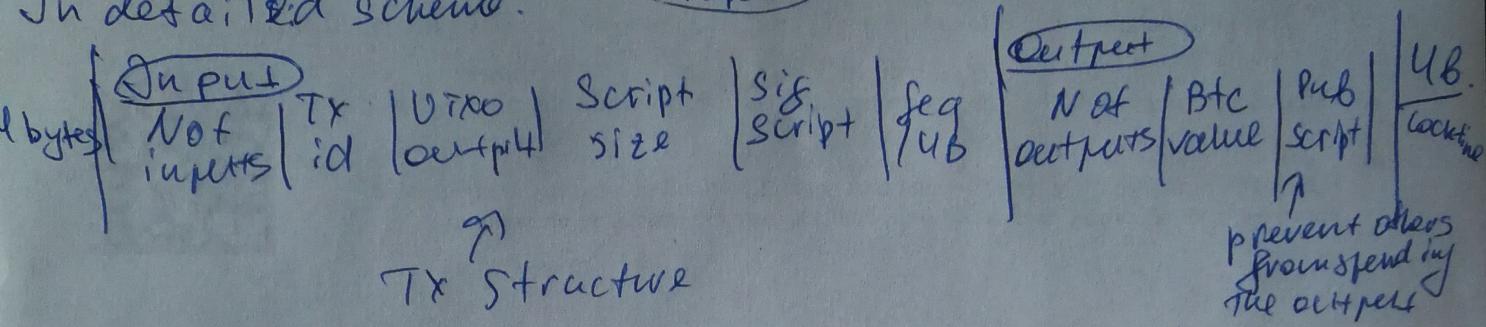
b) TX structure. Let's briefly look at structure:

TX consists of:

- version (is 5 currently)
- flag (indicates the presence of data)
- in-counter (varInt)
- Inputs list
- out-counter (varInt)
- Output list
- Witnesses (for each input)

In detailed scheme:

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### c) Bitcoin emission and circulation

First, let's compare Btc with normal physical currency.

In normal case currency is controlled by central bank organization and should match the increase in amount of goods to exchange/trade in order to organize stable good prices. It means, that CB could actually print money. It was the case of centralized economy.

In Btc case there's no authority to regulate it. Currency is created by the nodes of a peer-to-peer network. Rate of it is defined by Btc algorithms between the nodes. The thing with Btc is that number of Btc in existence will not exceed 21 million, because of geometrical nature of decrease in No. Btc generated per block (and Btc are created each time when a user discovers a block). It happens because of adjustment of block creation rate. P generation and circulation

A hand-drawn graph on the left shows a curve starting at a point labeled '21M' on the y-axis and decreasing rapidly towards the x-axis, representing the decreasing rate of BTC generation over time.