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Liquidity and Asset Prices

Empirical analysis of the Norwegian Stock Market

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Abstract

This thesis investigates time-varying characteristics of illiquidity and the pricing of its risk using liquidity-adjusted capital asset pricing model(LCAPM). Collecting data from Norwegian stock market between 1998 and 2017, we employ multivariate GARCH model to assess the persistence of illiquidity shocks. The pricing of liquidity risk and its implications on expected returns are empirically tested using the conditional LCAPM. We show that various sources of liquidity risk that affect asset returns are time-varying. We find some support for our conditional LCAPM, but our results are not robust to alternative specifications and estimation techniques. The total annualized illiquidity premium found in the Norwegian stock market is 1.75%.

Acknowledgements

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1. Introduction

Liquidity for securities varies over time and contributes to the volatility of equity returns. This implies that liquidity is an important source of risk in financial markets. Empirical studies, such as Hasbrouck & Seppi (2001);Chordia et al. (2005); and Huberman & Halka (2001) find commonality in liquidity and liquidity risk which affects the market as a whole[1]. This means that various measures of liquidity for different securities are positively correlated with each other. Liquidity risk for securities is therefor hard to diversify and contributes to the systematic risk in the market. Traditional asset pricing models such as CAPM -which model compensation for undiversifiable risk taken by investors- fail to incorporate liquidity risk borne by investors. Therefore, additional asset pricing models may be improved by including components to account for illiquidity and the liquidity risk in the market.[2].

Acharya & Pedersen (2005) presents an extended framework for the CAPM model which includes liquidity risk. In the liquidity-adjusted CAPM they show that illiquidity premium depends on the expected transaction cost in the end of the holding period, E(s^j), and three additional sources of illiquidity risk, β_2^j,β_3^j, and β_4^j. Each beta captures different source of liquidity risk.

(i) The first liquidity beta, β_2^j, measures the co-variance between illiquidity of a stock j and the market illiquidity (s^M). When the market becomes illiquid, the value of holding a stock that remains liquid is valued more. It is also a way to hedge a drop in asset values by holding liquid stocks in an illiquid market. These stocks should generally have higher prices and offer lower risk premium. On the other hand, investors would want to be compensated for holding illiquid stocks when the overall market becomes illiquid, therefor stocks with a high β_2 implies a higher returns than average. This beta is also positive due to the commonality in liquidity.

(ii) The second liquidity beta, β_3^j, measures the exposure of an assets gross return(R^j) to market-wide illiquidity (s^M). A high β_3^j means that these stocks offer high returns even when the market becomes illiquid. Holding stocks with a high β_3^j is a way to hedge against a drop in the market illiquidity. Investors are willing to accept lower risk premium in favour of liquidity

[1] Liquidity refers to the ability to trade an asset that closely corresponds to its consensus value.

[2]In this thesis, we will use liquidity and illiquidity interchangeably. Both terms infer that an investor should receive a premium for the associated risk of holding assets with illiquidity cost and risk

for stocks in times of market illiquidity. β_3^j is usually negative because a rise to illiquidity in the market leads to reduced asset value.

(iii) Last liquidity beta, β_4^j measures the co-movement between illiquidity of a stock s^j, and gross return on the market (R^M). In most cases this variable is also negative. A high β_4^j means that stocks remain liquid in a down market. These stocks require lower expected return because investors can easily sell these stocks at lower transaction cost in adverse market phase.

The market microstructure literature that explores the relation between liquidity and asset returns is vast. From the initial works of Amihud & Mendelson (1986) with asset specific focus, to subsequent studies with market-wide perspective(Pastor & Stambaugh (2003);Acharya & Pedersen (2005); Amihud et al. (2015)),previous studies investigate liquidity as a stock characteristic and as an aggregate risk factor. Given the repeated examples of financial crises and market turmoil, illiquidity in the market is still of significant interest both for investors and researchers. The persistence of illiquidity shocks in the market and the implications of these shocks on the pricing of liquidity risk still remain largely unexplored in the Norwegian stock market.

The purpose of this study is to examine the relationship between stocks returns and liquidity risk while taking into account the time-varying characteristic of illiquidity in the Norwegian stock market. Thus, we contribute to existing literature in the following ways. First, we use a modified version of Amihud (2002) illiquidity proxy to reflect illiquidity in the market across time. Second, we study the time series relation between liquidity and returns. This will show whether or not returns are affected by liquidity shocks, and the notion of flight to liquidity.Third, we will employ the liquidity-adjusted CAPM derived by Acharya & Pedersen (2005),where stock returns are cross-sectionally dependent on market risk and three additional risk betas that capture different aspects of illiquidity and its risk. Last,this study estimates the conditional version of LCAPM that allows liquidity risk and market risk to change over time conditionally on the illiquidity in the market.

While the topic of time-varying characteristic of illiquidity has been covered in previous studies, the econometric models used in those studies fail to adequately account for the importance of the illiquidity attribute. Current literature use the autoregressive(AR) process to model time-varying illiquidity(Amihud (2002);Acharya & Pedersen (2005)). However, using these models with AR process are restrictive because one assumes that the illiquidity shocks are temporary. These models are useful when analyzing the time-series of illiquidity during tranquil periods, but they fail to capture important feature of illiquidity during periods of long lasting uncertainty and turmoil in the market. This is consistent with the studies of Brunnermeir & Pedersen (2009), who finds empirically that illiquidity shocks lead to prolonged periods of market-wide illiquidity. They describe this as liquidity spiral. Therefore, we estimate the conditional version of LCAPM where we employ diagonal vech multivariate generalized autoregressive conditional heteroskedsticity(DVECH MGARCH) model developed by Bolleslev et al. (1988). In contrast to unconditional LCAPM, investors update their expectations and beliefs based on the available information in the market with regards to the shocks to contemporaneous regimes of illiquidity and returns. Therefore, the market risk premium vary conditionally on the state of the market, reflecting updated beliefs about future illiquidity and the pricing of the unexpected shocks. This is an important feature that is not captured in the unconditional LCAPM.

We wish to investigate the cross-sectional predictions of the conditional LCAPM in the Norwegian stock market. We use stocks covering the period from 1998 to 2017. The Amihud (2002) measure for illiquidity has been used as a proxy for illiquidity s^j. We use equal-weighted method as our primary test, while also considering other properties and specifications for robustness.The attributes of liquidity risk and the expected returns are considered using portfolios instead of individual stocks on a monthly basis.

Using Amihud (2002) illiquidity measure we find that illiquid stocks also have high liquidity risk, which is consistent with the notion of "flight to liquidity" in times when market is down or generally when market-wide illiquidity increases. More precisely, stocks that have high average illiquidity, s^j, also tend to have high commonality in liquidity with market-wide illiquidity and high liquidity sensitivity to market returns. However, these stocks have lower return sensitivity to market-wide illiquidity compared to the most liquid stocks. Stocks that are illiquid are also characterised as having low turnover, small size, and high volatility of illiquidity. Furthermore, illiquid stocks offer higher returns compared to the more liquid stocks.

To consider the economic impact of liquidity level and each of the liquidity betas derived by Acharya & Pedersen (2005), we evaluate their contribution to returns by applying cross-sectional generalized methods of moments(GMM) regression. Furthermore, we find that liquidity risk contributes on average about 0.25% annually to the market risk premium between stocks with high average illiquidity and low average illiquidity. We decompose the effects of liquidity risk into the contribution from each of the liquidity risks:

(i) We estimate that the return premium due to co-variance between illiquidity and market illiquidity, $\text{cov}(s^j, s^M)$, of 0.26%. Thus, it seems like the return premium required by investors to hold illiquid asset when market illiquidity increases is high.

(ii) The effect of the sensitivity of returns to market illiquidity, $\text{cov}(R^j, s^M)$, is -0.02%. This risk premium affects the returns in a positive direction mainly due to monotonically decreasing relation between liquid and illiquid stocks. Pastor & Stambaugh (2003) and Acharya & Pedersen (2005) find a relatively small and significant value for this effect in their empirical studies.

(iii) And the third liquidity beta return premium, which measures co-movement between stocks illiquidity and market returns $\text{cov}(s^j, R^M)$, is 0.002%. Having the ability to sell off stocks in illiquid market seem to have less effect for the market risk premium. Acharya & Pedersen (2005) found that the return premium for this liquidity betas was by far the biggest contributor for liquidity risk.

Illiquidity premium that depends on the expected transaction cost in the end of holding period for investors corresponds to 1.50%. This makes the overall illiquidity premium 1.75%. These estimates and the overall importance of liquidity level and liquidity risk depends on our model-implied restrictions of a constant market risk premium and a fixed transaction cost. However, since we have constructed our LCAPM conditionally, we are able to relax these model-implied constraints and estimate different liquidity risk premiums, while also allowing transactions cost to be a free parameter. Using this unrestricted model, we find that the overall illiquidity premium corresponds to 1.76%. Out of the each estimated liquidity risk premiums, we find support for the third liquidity beta, which measures the co-movement between stocks illiquidity and market returns($\text{cov}(s^j, R^M)$), while the other liquidity betas do not seem to be significant. We also find support for market risk premium associated with the third liquidity risk beta when we use alternative specifications and estimation methods.

The rest of this thesis is organized as follows. In Section 2 we review the empirical literature. Section 3 described the data processing and characteristics of the Norwegian stock market. Section 4 derives the liquidity-adjusted CAPM and the rest of methodology approach. Section 5 provides the results. Section 6 concludes.

2. Literature Review

2.1 Illiquidity and Asset Prices

Studies about liquidity are primarily concentrated on analyzing the impact of individual assets liquidity on returns. Amihud & Mendelson (1986) were the first to examine the relationship between liquidity, asset prices and how this is interlinked with investors holding periods. In their study they found that investors that trade more frequently would prefer to hold assets that have lower transaction cost, i.e stocks that have higher liquidity. On the other side, investors who prefer to invest once in a while and hold those securities over a given period, would be more willing to invest in the illiquid stocks(they only incur the transaction cost once in a given time period).

Amihud & Mendelson (1986) call this the clientele effect, and examine the expected returns and liquidity(using bid-ask spread as a proxy for illiquidity) with different investor types. They test this theory on NYSE and AMEX stock data for 1961-80 while including year dummies, and find that a 1-percentage point increase in bid-ask spread for a stock increase the monthly expected return by 0.211 percent and 2.5 percent yearly. Based on their estimation, they find that the marginal investor only trades once every five month. Later studies conducted by Eleswarapu & Reinganum (1993) extended the sample period in Amihud & Mendelson (1986) by 10 years and found that the liquidity premium is only limited to January.Goldreich et al. (2005) find that on-and-off the run securities yield often narrows as one is approaching off-the-run date[3]. Interpretation of this is that investors are forward looking and care about future liquidity and liquidity risk of assets.

Brennan & Subrahmanyam (1996) examine the relation between illiquidity premium and returns while measuring for alternative liquidity proxy that measures price impact and market depth[4]. Using intraday data from NYSE in a period of 1984-1991 and controlling for price, size and book-to-market ratio[5] they find positive relationship between illiquidity and returns. Using time-series instead of cross-sectional data, Jones (2002) finds evidence that when the spread is large, so are the expected returns. While using turnover ratio as a measure of liquidity, he finds that a high

[3]On-the-run treasuries are often used by trades making a short bet, meaning lenders of these types of securities can earn lending fees from short sellers. These fees also contribute to the difference in yields for on and off- the run securities.

[4]Also known as Kyle's lambda, derived in Kyle (1985)

[5] These are the Fama & French (1993) factors that affect stock returns.

turnover ratio leads to lower stock returns.

Using daily data Hasbouck (2005) gets mixed results. While using different liquidity measure techniques such as posted spreads, effective cost, and dynamic models based on trade direction he finds that the relationship between returns and liquidity varies considerably in significance and direction. The reasoning behind these conflicting findings is due to the difficulty in handling daily data. Using daily data, it is hard to distinguish between the impact of liquidity factors and that of volatility, in addition to other factors such as noise traders sell and buy orders[6].

According to the study done by Amihud & Mendelson (1986) the relationship between returns and spread is concave, meaning that as the slope of the relationship decline as the spread increase.While controlling for other factors, such as firm size and market-to-book ratio, they also find the so called small firm effect, where small firms offer higher returns than the bigger firms, even if they have the same risk. They point out that a larger spread leads to increase in required returns which translate to a reduction in asset valuation. Therefore, if firms engage in financial policies to increase their liquidity, they will have a higher payoff in terms of asset valuation. Studies done by Foester & Karolyi (1998) provide evidence that companies list in more liquid stock exchanges in order to reduce their cost of capital, and Miller (1999) finds that market reaction to positive news is highest for firms that are listed on more liquid and better known stock exchanges such as NYSE and NASDAQ.

2.2 Liquidity Risk

The studies conducted in the previous section have mainly considered liquidity as a stock characteristic rather than an aggregate risk factor of concern to investors. In most recent academic literature, the existence of commonality in liquidity leads to a redefined role of liquidity in asset pricing. Therefor, this commonality in liquidity could represent a source of risk that is not diversifiable. If this is the case, the sensitivity of stocks to liquidity shocks should induce a premium, i.e higher average return. Said differently, the returns of stocks are not only dependent on the level of future expected liquidity, but also the uncertainty over its level. Consistent with this proposition,empirical studies such as Huberman & Halka (2001);Chordia et al. (2000); and Hasbrouck & Seppi (2001) found a positive relationship between various measures for liquidity(bid-ask, price impact etc.) of stocks, and conclude that there is a commonality in market-wide illiquidity, or liquidity risk. This commonality varies over time both for individual stocks and for the market as a whole. Since there is co-movement in liquidity, it is hard to diversify this risk away, thus contributing to the systemic risk.

[6] Uninformed traders that base their buy and sell orders on irrational beliefs.

Acharya & Pedersen (2005) proposed an extended version of the traditional CAPM that accounts for liquidity risk. By expanding on the premise where investors require compensation for liquidity risk, they created additional component linking price and liquidity of financial assets. They included three betas, which are called liquidity betas that measure the liquidity risk in the market. In their liquidity-adjusted CAPM(LCAPM), the expected return of a security depends on its expected liquidity, as well as co-movement of its own returns and liquidity with market return-and liquidity. While measuring for the Amihud (2002) illiquidity ratio[7] for all common stocks listed on NYSE and AMEX from 1962-1999 and obtain three main results.First, they find that illiquid stocks are more exposed to liquidity risk. Second, liquidity risk seem to be priced in the market(meaning that investors expect higher returns for holding illiquid stocks). Last,they find that their extended model of the CAPM adjusted for liquidity is better suited to control for returns than the traditional CAPM in terms of goodness of fit. They find that the overall various sources of liquidity risk results in 1.1 percentage difference in annual returns for liquid and illiquid portfolios of stocks. This difference is mainly due to investors value of securities that remain liquid when the market is down.

Several other studies found similar results while using the Acharya & Pedersen (2005) model to study the relation between liquidity risk and returns. Lee (2011) used this model on a global level and find evidence that support LCAPM, both in terms of goodness of fit and the expected returns. Liquidity risks seem to be priced independently of market risk in most international financial markets.

Li et al. (2014) tests whether liquidity risk is priced in Japanese equity market. Consistent with the findings of Acharya & Pedersen (2005) for stock exchanges in US, Li et al. (2014) find evidence that LCAPM outperforms CAPM in terms of goodness of fit. However, they obtain weak evidence for liquidity risk. Liquidity level(discussed in previous section) and market risk(beta in regular CAPM) seem to be be more important than market-wide illiquidity(i.e liquidity risk) for investors.

Pastor & Stambaugh (2003) were among the first to study the relation between liquidity risk and returns. In contrast to Acharya & Pedersen (2005), they created a single variable which measures market-wide illiquidity. This measure was constructed based on the observations that illiquid stocks showed traits of large reversals in returns[8]. Their liquidity risk is based on co-movement between stocks return and a measure of market-wide illiquidity, which equivalent of second liquidity beta in Acharya & Pedersen (2005).

Pastor & Stambaugh (2003) find that stock returns are cross-sectionally related to the fluctuations in liquidity. Stocks that have higher exposure to illiquidity risk outperform those that have lower illiquidity by 7.5 percent annually adjusted for exposure to the market return, size, value and mo-

[7]Measuring daily returns on daily volume, presented in by Amihud (2002) and applied in in Section 4.
[8]Short-term return reversals where high returns are followed by low returns in the following month.

mentum factors[9]. Furthermore, Sadka (2006) looked at liquidity risk as a measure of co-variation between fund returns when there are unexpected changes in aggregate liquidity. Using aggregate liquidity risk they find that funds that are significantly more exposed to liquidity risk outperform funds that have lower-loading by 8 percent annually over a period of 13 years. However, this performance is independent of the illiquidity of a fund due to lockups and redemption notice periods.

A couple of papers also looked at the interactions between illiquidity and the fundamental business risk. Vayanos (2004) studied the relationship between fund managers and and investors. If the performance of the fund falls beyond a certain threshold, investors will most likely withdraw their investment. During times when the market is volatile they, the investors will liquidate, and to prevent such liquidation in a down market they must offer higher illiquidity premium. Therefor, the risk of liquidation is linked to market volatility. On the other hand, Favero et al. (2010) find negative relationship between illiquidity premium and volatility. They conclude that when there is high volatility of returns, there is also less investment opportunities. Investors will care less about liquidity in the market in times of financial crisis and market turmoil.

2.3 Related Empirical Evidence

The research on liquidity and asset pricing at Oslo Stock Exchange(OSE) is mostly conducted by Bernt Arne Ødegård and Randi Næs. Through several papers, they have examined the relation between investors holding period, liquidity and returns. In Ødegaard, Næs & Skjeltorp (2008) they use different measures of liquidity that captures various dimensions of liquidity over time and across industry groups. They find that these liquidity measurements help to explain the state of Norwegian economy(business cycle analysis). In subsequent studies, such as Ødegaard & Næs (2009) and Ødegaard (2017), they focused on analyzing the role of various liquidity measures and how these affect investors holding period and required returns. More precisely, Ødegaard (2017) find that equity owners have different holding periods consistent with Amihud & Mendelson (1986). They find that institutional investors holding period is 4months shorter than average investors.

2.4 Contribution to Existing Literature

In previous studies, such as described above, the main goal has been to examine the relation between liquidity, asset pricing and holding period. The focus has been on analyzing the role of

[9]These are the traditional factors that effect returns in the classical empirical asset pricing. Derived in Fama & French (1993), Fama & French (1996) and Fama & French (2015). Size refers to the returns being affected by the size of a company. There is evidence that show smaller stocks offer higher returns than the bigger stocks. Value refers to companies that have high book equity value compared to their market value. Companies with high value have been proven to offer higher returns. Momentum refers to stocks returns moving in the same direction over a given period,i.e if stock is perform well then it will continue offer higher returns.

individual liquidity in asset pricing, i.e the liquidity level of securities. Consistent with previous studies such as Chordia et al. (2005) they also find commonality in liquidity the Norwegian market across stocks and different liquidity measurements.

Our study is different from that of Bernt Arne Ødegård and Randi Næs in the following aspects. First, we focus on the Amihud (2002) illiquidity measure. Various proxies for liquidity has been used in previous studies, but most recent studies have confirmed that this is one of the better proxies for measuring liquidity[10] This study further modifies the Amihud (2002) illiquidity measurement based on suggestions of Hasbouck (2005) and Acharya & Pedersen (2005) in order to reflect illiquidity in the market across time. Second, we examine the time series relation between liquidity and returns. This will show whether or not returns are affected by liquidity shocks, and the notion of flight to liquidity. Third, we use the liquidity adjusted CAPM derived by Acharya & Pedersen (2005)to test whether liquidity risk is priced(related to returns). As mentioned earlier, these authors argue that the uncertainty of liquidity(liquidity risk) should be priced in addition to the level of liquidity. Therefore, we will examine a model which includes both liquidity level and liquidity risk. Last, the statistical model used in this study considers the illiquidity shocks in the market conditionally, while Acharya & Pedersen (2005) assumes they are unconditional. Unconditional LCAPM is more appropriate when there are short periods of illiquidity in the market, while it fails to capture liquidity shocks that that are long lasting. It also assumes a constant risk premium in the market, while in reality there might be different risk premiums conditionally on the state of illiquidity in the market.

[10]Evidence is provided in Section 4.

3. Data

As we have shown in the last section, the literature on liquidity and asset pricing is vast. Most of this research focuses on North-American Stock exchanges. Our study differs where we try to analyze these effects on the Oslo Stock Exchange, which is the centralized trading platform for securities in Norway. Compared to other major stock exchanges in Europe and North-America, the OSE is merely a medium-sized trading platform.

3.1 Filtering and Selection of Data

The data used in this thesis are collected from several sources. Daily frequency data on all common stocks that are available on OSE is collected from the Amadeus database, which is a source of financial data in the *Børsprosjektet* at NHH. The data set used in our study covers the period from January 1998 to December 2017. The extracted data set contains information regarding firms market cap, turnover rate, and stock returns. We choose to include only ordinary shares. These shares are adjusted for dividends, splits and other cash payouts.

In previous literature, such as Ødegaard (2018), Amihud (2002) and Acharya & Pedersen (2005), they remove stocks that trade below NOK 10 and above NOK 10 000[11] in order to avoid extreme outlines. However, since we are already operating with limited stocks on OSE(on average 220 companies in our sample period) applying this condition, we loose 33% of our observations. Moving forward as we apply more restrictions to our data sample we end up with too small of data sample. Therefor we decide to only remove penny stocks, i.e stocks that trade for less than NOK 1. Even though keeping all the stocks might cause extreme price and returns movements that are driven by the most liquid securities, we use equal-weighted model as our primary test.

From the extracted data market capitalization is calculated from stock prices(AdjLast)and shares issued(ShareIssued). The overall market capitalization is taken as total of firms market capitalization in a given day, month and a year. When applying value-weighted results we treat firms market capitalization to total market capitalization as the weight. Also, market capitalization is lagged by one month.

[11] Many studies employ criteria where they use \$5 and \$1000 sample restriction, to reflect OSE in NOK, we convert and adjust this criteria slightly.

Returns are computed based on the adjusted stock prices. We use daily data to construct monthly returns for stock prices and require the stocks to be actively traded on the market. For it to eligible to be included the stock must have at least 15 days of return and volume data in a given month. Daily returns for stock j are based on the adjusted stock prices P^j are computed as follow:

$$r_t^j = \frac{P_t^j - P_{t-1}^j}{P_{t-1}^j} \tag{1}$$

Since return outlines can affect our returns, we follow Ødegaard (2018) and remove observations below 0.1% quintile and above 99.9 % to reduce these.

Daily and monthly frequency data on risk free rates and market returns are extracted from the Ødegaard (2018) database[12]. The risk free rates are based on the NIBOR[13] data extracted from Norwegian Central Bank. Market returns are extracted on daily and monthly basis, both equally and value weighted in addition to all shares traded on OSE[14].

[12] `http://finance.bi.no/~bernt/financial_data/index.html`

[13] Norwegian Interbank Offered Rate, this rate is constructed based on the interest banks have to pay other banks for borrowing liquidity over night plus a risk premium.

[14] We also tried to collect the companies equity data in order to construct book-to-market ratio and double sort this variable based on size, but with the Amadeus client still being in its developing stage with regards to balance sheet statistics and Blomberg database having far too many missing companies, this data was unavailable.

3.2 Characteristics of Oslo Stock Exchange

In this sub-section we present some data and key features from OSE that are relevant for this thesis. Over the past decades, OSE has experienced a transformation, from having 100 companies listed in 1982 to reaching its peak in 2009 with 278 active traded companies. Currently there are 198 active shares at OSE each month(Ødegaard (2018)). Table 1 describes the ten most traded stocks per September 2018. We see that these top firms account for 62% of the market value and 56% of the stocks traded on OSE. This implies that OSE is affected by the firm size and liquidity for the major companies listed.

Table 1: Turnover and market equity of the ten most traded stocks Sep.2018

This table reports the summary statistics for the ten most traded stocks in Oslo Stock Exchange. Turnover for these stocks are reported in NOK Millions and as a percentage compared to the total value of stocks listed on the exchange. The market value of these companies are also reported in similar matter[15].

Most traded Sep.2018	Turnover(Mill NOK)	%	MV(Mill NOK)	%
Equinor	14,878.8	15.04	76,223	25.94
Norsk Hydro	7,404.3	7.48	101,091	3.42
Marine Harvest	5,207.1	5.26	92,487	3.13
Telenor	5,142.7	5.19	234,109	7.93
DNB	5,099.7	5.15	274,748	9.30
Yara International	4,778.0	4.83	109,205	3.70
Subsea 7	4,341.2	4.38	39,415	1.33
Aker BP	3,753.2	3.79	124,383	4.21
Orkla	2,295.5	2.32	70,062	2.37
TGS-NOPEC Geophysical Company	2,222.3	2.24	34,048	1.15
Sum top 10	55,122.7	55.68	1,845,771	62,49
Total	99,006.1	100.00	2,953, 838	100.00

The liquidity at OSE has also seen a transformation over the past couple of decades. The average number of trading days per stock has gone from 146 in 1998 to 226 in 2017[16]. Liquidity measured by transaction cost(bid-ask spread) has seen a increase in the same period as well. From Figure 1 we see that this liquidity proxy played a significant role when investing in different securities from early 1980s and until the end of 1990s. Because of the uncertainty in investing smaller capitalization firms it would lead to a significantly higher transaction cost compared to the bigger stocks.[17]

[16]`https://www.oslobors.no/Oslo-Boers/Statistikk`

[17]This is also the main reason we have chose to start our sample period in the 1998. We want this study to reflect more current environment with regards to the liquidity and asset prices.

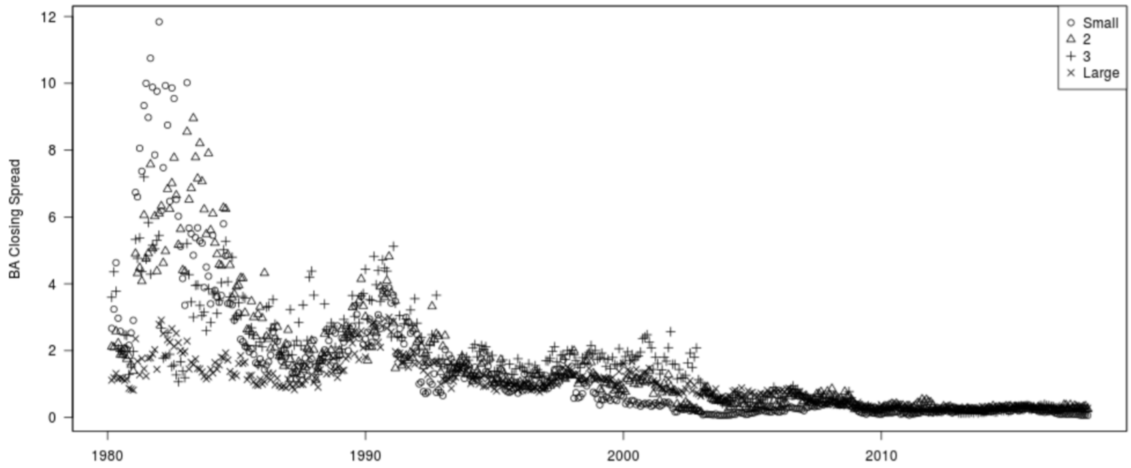

Figure 1: Bid-Ask spread of Size sorted Portfolios. Time series 1980-2017 (Source:Ødegaard,2019)

Though illiquidity as a whole seems to be reduced over this period, there are still significantly differences between stocks and industries(Ødegaard (2018). One of the major reasons for the improved liquidity can be contributed to the electronic trading system. However, Pedersen (2015) states that the electronic market has contributed to improved liquidity with lower bid-ask spreads, but the amount of stocks you can buy at different firm size levels are limited, mainly due to the small size of trading platform such as OSE. This means that if we classify each company based on their market capitalization and put them in portfolios, such as done in Figure 1, we are left with too few companies to choose from. Furthermore, there have been periods over the past decades that are described as liquidity crisis, which is why we see a spike in closing bid-ask spread for most companies in some time periods.

4. Method

4.1 Liquidity Measurement

4.1.1 Amihud Illiquidity Ratio

Though there are many proxies for measuring illiquidity, such as bid-ask spread, Rolls measure and others, we use the Amihud (2002) measure of illiquidity in this thesis. Amihud (2002) illiquidity ratio measures price impact of trading securities. This is one of the most widely used proxies in empirical asset pricing. Each stocks absolute returns are divided by its NOK turnover,

$$Illiq_{jt} = \frac{1}{D_{jm}} \sum_{t=1}^{D_{jmt}} \frac{|r_{jtm}|}{Turnover_{jtm}} \tag{2}$$

where $|r_{jtm}|$ is the absolute return for a stock j on a day t of a month m, *Turnover* is the corresponding volume traded in NOK. This illiquidity ratio explains how the price responds for each NOK of transaction, i.e prime impact. If the stock prices moves a lot in response to a small traded volume then it will lead to a high *Illiq* value, i.e the asset is illiquid, and vice versa.

In the liquidity factors section(Appendix B) we discussed how illiquidity arises from several sources and how these different cost of selling assets works in real markets. We assume that *Illiq* is a valid proxy for measuring illiquidity. The Amihud (2002) measure have received strong support with respects to its ability to accurately measure illiquidity. Goyenko et al. (2009) showed empirically that this ratio is one of the better proxies for price impact, since there is a strong correlation between this ratio and high-frequency measures of price impact. Subsequent studies, such as Amihud et al. (2015) applied this proxy globally and found similar results as Goyenko et al. (2009). Amihud (2002) showed that this ratio also does a good job of explaining stock returns, both cross-sectionally and over time.

Next we follow Acharya & Pedersen (2005) and normalize this variable to make it stationary and to reflect the cost of a trade in the the stock market. As Acharya & Pedersen (2005) point out, using Amihud (2002) illiquidity measurement in its standard form has two problems: i) It is measured in percent per dollar, wheres the LCAPM is specified in terms of dollar cost per dollar invested. This is a problem because it will ignore inflation, thus making it non-stationary; ii) *Illiq* is specified in terms of the cost of selling, but it does not directly measure the cost of trading securities. This leads us to create our version of illiquidity for individual stock, s_t^j, as

$$s_t^j = \min(0.25 + 0.30 Illiq_t^j P_{t-1}^M, 30.00) \tag{3}$$

where P_{t-1}^M is a ratio of lagged monthly capitalization of the market portfolio and our first observation of market capitalization, which is January 1998(we will explain more of this ratio in section. 4.4).

This adjustment solves the stationary problem. We set parameters of 0.25% and 0.30% to ensure that the normalized illiquidity(s_t^j) has the same level and variance as the effective half-spread(i.e transaction cost minus midpoint of bid-ask spread) for our stocks. We also set the maximum value to 30%, since a transaction cost more than this would seem unreasonable and to avoid extreme values of our illiquidity measurement. This adjusted illiquidity is our main source of measuring illiquidity for stocks, market and portfolios.

4.2 Liquidity Adjusted Capital Asset Pricing Model

In this section we present our main model for examining the relation between liquidity, investors exposure to liquidity risk and returns. Liquidity varies over time for securities, contributing to the volatility of returns[18]. This leads to uncertainty for investors with regards to the future transaction cost and optimal time to sell off an asset. Therefor, this uncertainty contributes to creating liquidity risk for investors. This is the idea presented by Acharya & Pedersen (2005). By normalizing the investors holding period to h=1 for asset j we get:

$$R^j \approx r^j + s^j \tag{4}$$

where R^j is gross return, r^j is net return and $s^j$is the transaction cost(i.e illiquidity cost) , where both are assumed to be random. Since investors care about net return the CAPM will need to be adjusted:

$$E(R^j - s^j) = r^f + \beta_j[E(R^M - s^M) - r^f] \tag{5}$$

where R^M is the gross return on the market, s^M is a measure of illiquidity in the market, r^f is the risk-free rate. If we weight each stocks illiquidity by its fraction value of the market portfolio, we derive the following betas:

$$\begin{aligned}\beta_j \equiv \frac{\text{cov}(\text{r}^j, r^M)}{\text{var}(R^M \text{ - } s^M)} &= \frac{\text{cov}(R^j - s^j, R^M - s^M)}{\text{var}(R^M\text{- } s^M)} \\ = \underbrace{\frac{\text{cov}(R^j, R^M)}{\text{var}(R^M \text{ - } s^M)}}_{\beta_1^j} + \underbrace{\frac{\text{cov}(s^j, s^M)}{\text{var}(R^M \text{ - } s^M)}}_{\beta_2^j} - \underbrace{\frac{\text{cov}(R^j, s^M)}{\text{var}(R^M \text{ - } s^M)}}_{\beta_3^j} &- \underbrace{\frac{\text{cov}(s^j, R^M)}{\text{var}(R^M \text{ - } s^M)}}_{\beta_4^j}\end{aligned} \tag{6}$$

Therefore, based on the this expression, the required risk premium on stock j can be expressed as

$$E(R^j - r^f) = \beta_1^j \lambda^M + \underbrace{E(s^j) + \beta_2^j \lambda^M - \beta_3^j \lambda^M - \beta_4^j \lambda^M}_{\text{illiquidity premium}} \tag{7}$$

[18]Brunnermeier & Pedersen (2008) show that there are variation in liquidity over time and link it to funding conditions of market makers.

where the required excess return, $E(R_j - r^f)$, is the expected relative illiquidity cost, $E(s^j)$, plus four betas(or co-variances) times the risk premium. $\lambda^M \equiv E(R^M - s^M - r^f)$ is the market risk premium. This decomposition yields a model where the four betas are dependent liquidity and asset payoff. This liquidity adjusted CAPM tests the relation between the percentage changes of midquote prices(described in last section) instead of the changes in transaction prices(such as bid-ask spread). It is also worth noting that the betas are not the typical slope coefficients of regular CAPM, represented by $\frac{Cov(R^j,R^M)}{Var(R^M)}$. To account for illiquidity in the market, Acharya & Pedersen (2005) employ a common denominator for all betas using the variance of the difference between market illiquidity and-returns.

Equation (7) shows that illiquidity premium depends on the expected transaction cost in the end of the holding period, $E(s_j)$, and three additional sources of illiquidity risk, β_2^j,β_3^j, and β_4^j. Each beta captures different source of illiquidity risk.

(i) The first liquidity beta, $\beta_2 j$, measures the co-variance between illiquidity of an asset j and the market illiquidity (s_M). When the market becomes illiquid, the value of holding an asset that remains liquid is valued more. It is also a way to hedge a drop in asset values by holding liquid stocks in an illiquid market. These stocks should generally have higher prices and offer lower risk premium. On the other hand, investors would want to be compensated for holding illiquid stocks when the overall market becomes illiquid, therefor stocks with a high β_2 implies a higher returns than average. This beta is also positive due to the commonality in liquidity.[19]

(ii) The second liquidity beta, β_3^j, measures the exposure of an assets gross return(R^j) to market-wide illiquidity (s^M). A high β_3^j means that these stocks offer high returns even when the market becomes illiquid. Holding stocks with a high β_3^j is a way to hedge against a drop in the market illiquidity. Investors are willing to accept lower risk premium in favour of liquidity for stocks in times of market illiquidity. β_3^j is usually negative because a rise to illiquidity in the market leads to reduced asset value.[20].

(iii) Parameter β_4^j measures the co-movement between illiquidity of an asset j and gross return on the market portfolio(R^M). In most cases this variable is also negative.[21] A high β_4^j means that stocks remain liquid in a down market. These stocks require lower expected return because investors can easily sell these stocks at lower transaction cost in adverse market phase

[19] As discussed in chapter 2.3.

[20] Evidence presented in Amihud (2002) and Pastor & Stambaugh (2003)

[21] Evidence in Acharya & Pedersen (2005) and Chordia et al. (2005).

4.3 Portfolio Construction and Evaluation

To consider out relation between liquidity risk and asset prices we will be using portfolios instead of individual stocks that differ in their liquidity attributes for our liquidity adjusted CAPM. Empirically, using portfolios is a way to reduce the noise that can be caused by analyzing individual stocks. More specifically, our illiquidity measure is noisy in terms of volatility between stocks. This makes it harder to distinguish the relation between return and illiquidity, and will lead to imprecise results.

We use daily illiquidity-and returns measurements to construct these portfolios.We start off by forming a market portfolio for both of these variables in a given month t. As mentioned in previous section, our monthly market return data is already extracted and implemented without further need of processing. For market illiquidity in a given month, we average the daily illiquidities in a month t.

Next, we form quintile portfolios for each year during the period of 1998 to 2017. First we create 5 illiquidity portfolios at the beginning of each year$t$ based on the previous years*t-1* daily illiquidity measurements. We do this by averaging *t-1* daily illiquidity measurements described in Section 4.1. Portfolio 1 contains the most liquid stocks, meaning the largest companies, while portfolio 5 includes the most illiquid stocks. Similarly, we form 5 illiquidity-variation(denoted as σ(Illiquidity)) portfolios based on the standard deviation of daily illiquidities in the previous year and finally 5 size portfolios sorted by market capitalization at the beginning of the year t.

We compute monthly return for portfolio,p, such as

$$r_t^p = \sum_{j \ in \ p} w_t^{jp} r_t^j \tag{8}$$

where we take sum of stocks included in portfolio p in month t of either value-weighted of equally-weighted weights w^{jp}_t. Market capitalization at the end of month t is used to value-weight the portfolios. We also compute the normalized illiquidity of portfolio p in similar matter:

$$s_t^p = \sum_{j \ in \ p} w_t^{jp} s_t^j \tag{9}$$

where s_t^{jp} represents illiquidity of either value-weighted or equal-based weights, w_t^{jp}.

Our main test is specified in terms of equal-weighted for returns and illiquidity for the market portfolio. Using equal-weighted averages, such as done by Amihud (2002) and Chordia et al. (2000), is a way to reduce the impact of large liquid securities. Our model is fragile in terms of value-weighting since the returns on a long enough time scale are driven by the biggest and most liquid stocks. Therefor we consider equal-weighted market portfolio, but we also test value-weighted model for robustness. For our testing portfolios we will consider both equal-weighted and value-weighted returns and illiquidity averages.

4.4 Innovations in Illiquidity

4.4.1 Time-varying Illiquidity

In this section we show that there is persistence in liquidity, which implies that liquidity can predict future returns and co-movements with contemporaneous returns.

The empirical evidence provided in Section 2 also found that liquidity is persistent and varies over time(Amihud (2002); Chordia et al. (2000); Hasbrouck & Seppi (2001); Huberman & Halka (2001); Jones (2002);Pastor & Stambaugh (2003)). The LCAPM developed by Acharya & Pedersen (2005) show that the persistence in liquidity can help to predict future returns based on their assumptions. Intuitively, if we observe high illiquidity today, this usually predicts high illiquidity in the future, leading to higher required returns for investors. Jones (2002) finds empirically that the annual stock market returns increases if we observe a high bid-ask spread in previous year. On the other side, the expected return decreases if we observe a low turnover. Amihud (2002) analysed the time-series effects of illiquidity and stock returns, finding that expected illiquidity helps to predict expected returns, while unexpected illiquidity(i.e liquidity shocks) lead to a negative effect on expected returns.

Predictability of liquidity also implies a negative conditional covariance between contemporaneous returns and illiquidity. If illiquidity is high, then the required returns will also be high, which will depress in the current price leading to lower returns.This is true as long as there is persistence in liquidity, which is consistent with the results of Chordia et al. (2000), Jones (2002), and Pastor & Stambaugh (2003). Therefor, any framework that relies on examining the relation between liquidity risk and returns need to consider the persistence of illiquidity(Acharya & Pedersen (2005)).

4.4.2 Conditional LCAPM

In Section 4.2 we provided the explanation for how the liquidity adjusted CAPM works and how it is constructed. Testing LCAPM as it is specified in Equation(6) would give us the static version. Meaning that we ignore all time-varying elements of liquidity. As we have shown in previous sections, market liquidity and liquidity risk are not constant in time. The effects of liquidity shocks have important implications for the pricing of liquidity risk. Testing the static version would give us unrealistic results of liquidity risk and returns.

To asses the implications of persistence in liquidity we develop a dynamic conditional LCAPM where the betas are conditional on the state of illiquidity in the market and returns. Since time series in financial markets have unstable variance and covariances, the conditional LCAPM gives us an opportunity to investigate the relation between liquidity risk(β_2^j, β_3^j β_4^j), market risk(β_1^j)

and asset prices that vary over our sample period. With this in mind, our LCAPM model can be written as:

$$E(r_t^j - r_t^f) = \beta_1^j \lambda_t^M + E(s_t^j) + \beta_j^2 \lambda_t^M + \beta_j^3 \lambda_t^M + \beta_j^4 \lambda_t^M \tag{10}$$

where

$$\beta_1^j = \frac{\operatorname{cov}(R_t^j - E_{t-1}(R_t^j), R_t^M - E_{t-1}(R_t^M))}{\operatorname{var}(R_t^M - E_{t-1}(R_t^M) - [s_t^M - E_{t-1}(s_t^M)])} \tag{11}$$

$$\beta_2^j = \frac{\operatorname{cov}(s_t^j - E_{t-1}(s_t^j), s_t^M - E_{t-1}(s_t^M))}{\operatorname{var}(R_t^M - E_{t-1}(R_t^M) - [s_t^M - E_{t-1}(s_t^M)])} \tag{12}$$

$$\beta_3^j = \frac{\operatorname{cov}(R_t^j - E_{t-1}(R_t^j), s_t^M - E_{t-1}(s_t^M))}{\operatorname{var}(R_t^M - E_{t-1}(R_t^M) - [s_t^M - E_{t-1}(s_t^M)])} \tag{13}$$

$$\beta_4^j = \frac{\operatorname{cov}(s_t^j - E_{t-1}(s_t^j), R_t^M - E_{t-1}(R_t^M))}{\operatorname{var}(R_t^M - E_{t-1}(R_t^M) - [s_t^M - E_{t-1}(s_t^M)])} \tag{14}$$

and $\lambda^M = \mathrm{E}(\lambda_t^M) = \mathrm{E}(\mathrm{R}_t^M \text{-} \mathrm{s}_t^M \text{-} \mathrm{r}_t^f)$. Asset-specific and market-wide illiquidity are represented by s_t^j and s_t^M computed using Amihud (2002) illiquidity proxy.

4.4.3 Diagonal Vech Multivariate Generalized Autoregressive Conditional-Heteroskedasticity (DVECH MGARCH)

To compute the innovations for market portfolio as well as the test portfolio we start off defining defining the un-normalized illiquidity for a portfolio

$$\overline{Illiq_t^p} = \sum_{j \ in \ p} w_t^{jp} min(Illiq_t^j \frac{30.00 - 0.25}{0.30 P_{,t-1}^M}) \tag{15}$$

where average portfolio illiquidity, $\overline{Illiq_t^P}$, is the sum of all stocks in a portfolio p that is weighted, w_t^{jp} by market capitalization, and our illiquidity is normalized to make it stationary and to put it on a scale to reflect the cost of a single trade. $P_{t-1}^M = \frac{MkCap_{t-1}}{MkCap_{t0}}$ is a ratio included as a detrending factor which controls for time trend in market illiquidity. $MkCap_{t-1}$ is the market capitalization on month *t-1* (stocks used to calculate $MkCap_{t-1}$ are those admitted in in month t) and $MkCap_{t_0}$ is the market capitalization of the first month of our sample.

Next, we use diagonal vech multivariate generalized autoregressive conditional heteroskedsticity(DVECH MGARCH) model developed by Bolleslev et al. (1988). This model allows conditional mean and variance to follow a dynamic vector-autoregressive(VAR) structure. Thus,we are able to obtain the conditional variances and covariances of innovations in illiquidity and returns over time. The general DVECH multivariate GARCH model is given by

$$\begin{aligned} \mathbf{y}_t &= \mathbf{C}\mathbf{x}_t + \epsilon_t \\ \epsilon_t &= \mathbf{H}_t^{1/2} \nu_t \\ \mathbf{h}_t = \mathbf{s} + \sum_{i=1}^{m} (\mathbf{A}_i \operatorname{vech}(\epsilon_{t-i}\epsilon_{t-i}') &\sum_{j=1}^{n} \mathbf{B}_j \mathbf{h}_{t-j} \end{aligned} \tag{16}$$

where $\mathbf{y}_t$ is an m x 1 vector of independent variables; $\mathbf{C}$ is an m x 1 matrix of parameters;
$\mathbf{x}_t$ is an k x 1 vector of independent variables, which may contain lags of $\mathbf{y}_t$;
$\mathbf{H}_t^{1/2}$ is the Cholesky factor of the time-varying conditional covariance matrix $\mathbf{H}_t$;
ν_t is an m x 1 vector of zero-mean, unit variance, and identically distributed innovations;
$\mathbf{h}_t$ = vech($\mathbf{H}_t$); the vech() function stacks the lower diagonal elements of a symmetric matrix into a column vector.

We apply a the diagonal vech(DVECH) model by restricting $\mathbf{A}$ and $\mathbf{B}$ to be diagonal and positive definite for each t. This ensures the stationarity condition of the model is met. We use the DVECH MGARCH model with one ARCH term where (i,j)th element of conditional covariance matrix is

$$h_t^{ij} = s^{ij} + a^{ij}\epsilon_{t-1}^{i}\epsilon_{t-1}^{j} \tag{17}$$

where s^{ij}, a^{ij} are parameters(return and illiquidity) and ϵ_{t-1} is the vector of errors from previous time period.

This shows that the linear form where elements of current conditional covariance matrix is a function of its past values and its past shocks but not on the past of other conditional variances and covariances. Since we are interested in variance and covariance between innovations in illiquidity and returns, both for portfolios and market, the errors represent the shocks, i.e the innovations. Simply put, the residuals(i.e the error term) ϵ_t represent the unexpected shocks to our parameters(returns and illiquidity for portfolios and market, depending on the specification).

For instance, when we estimate β_4^j, we need the covariance of the innovations in market illiquidity- and returns. We run the DVECH MGARCH model in Equation 17 and interpreted the innovation in market illiquidity such as

$$s_t^M - E_(s_t^M)_{t-1} = \epsilon_{t-1}^{i} \tag{18}$$

and innovations in market returns are

$$R_t^M - E(R_t^M)_{t-1} = \epsilon_{t-1}^{j} \tag{19}$$

The innovations in portfolios illiquidity and returns are computed the same way.

For diagnostic check we analyzed correlograms of the cross-product of our residuals from the DVECH models. The ARCH term results for the conditional variance and covariance had a p-value of 0%. We do not observe any ARCH effects left of our residuals. Meaning that autocorrelation is no longer a problem. This shows that our model is appropriate and adequate in describing conditional heteroskedasticity of returns and illiquidity.

Previous studies,such as Amihud (2002) and Acharya & Pedersen (2005), have used AR(2) specification to control for innovations in market illiquidity. They find that the error term(residual) of the AR(2) process corresponds to the innovations in illiquidity, which is significantly priced in their unconditional model. On a monthly frequency, they find autocorrelation of 0.945 and 0.87, while we find 0.93 in Norwegian stock market. Using the same AR(2) process requires the assumption of linearity of illiquidity in time-series. This results to temporary innovations with the assumption of constant illiquidity over the time period. In addition, with these assumptions, the market risk premium(λ_t^M) is constant, i.e the same for all the betas. We derive the conditional LCAPM, which relaxes these model constrains. We are able to control for different risk premiums for our liquidity betas conditional on the state of illiquidity in the market. If illiquidity shocks are persistent then they generate persistent illiquidity regime. Therefor, investors one period expected future illiquidity is not not a function of the average illiquidity, but their expectation of future illiquidity regime.

4.5 Empirical Estimation

To study the relation between liquidity risk and expected returns, we need a couple of assumptions and set some model constraints. We study this relation by running a cross-sectional regression of our portfolios using the Generalized Methods of Moments(GMM) framework. Running GMM produces similar estimates as the traditional Fama & Macbeth (1983) cross-sectional regression or using pooled OLS, but GMM also allows serial correlation and takes into account the pre-estimation of our betas. The application of GMM in empirical asset pricing is provided in Cochrane (2001). First we set a constraint that the risk premium for the betas is the same, defined as:

$$\beta_{net}^p = \beta_1^p + \beta_2^p - \beta_3^p - \beta_4^p \tag{20}$$

which makes our liquidity adjusted CAPM:

$$E(R_t^p - r_t^f) = \alpha + hE(s_t^p) + \beta_{net}^p \lambda^M \tag{21}$$

where we allow a nonzero intercept, α, even though Acharya & Pedersen (2005) claim that this intercept should be zero.

While testing the model with a single market risk premium does provide some insight, there might be different risk premiums associated with each beta, as discussed above. Therefore, we will also consider the following test, which allows for different market risk premium for our betas.

$$E(R_t^p - r_t^f) = \alpha + \beta_1^p \lambda^1 + hE(s_t^p) + \beta_2^p \lambda^2 + \beta_3^p \lambda^3 + \beta_4^p \lambda^4 \tag{22}$$

As we mentioned briefly in the beginning of Section 4.2, our investors holding period is h=1. The reason for specification is that investors incur the illiquidity cost(i.e transaction cost, s) once for each of our model period. In addition, our illiquidity for portfolios, $E(s^p)$, does not scale with time period since it is an average illiquidity of a specific portfolio. This means that we scale it by h to adjust the difference between estimation period and holding period[22].

We will use a common proxy in empirical literature for our holding period, where inverse of annual turnover is an estimate of holding period for investors

$$\text{Holding Period} = \frac{1}{\text{Nr. of shares traded}} \tag{23}$$

Taking average of this ratio gives us an estimation of how long it takes for all shares to be turned over once. Therefor, in our model we will calibrate h as the average of monthly turnover across all stocks. Our expected illiquidity, $E(s^p)$, will be the average portfolio illiquidity. We will also allow h to be a free parameter and relax these model constraints to test different aspects of our model. Lastly, to run the cross-sectional regression with a fixed h we will treat net return $E(R_t^p - r_t^f) - hE(s_t^p)$ as the dependent variable.

[22]For in depth explanations of the assumptions behind this specification see Acharya & Pedersen (2005)

5. Results

5.1 Liquidity Risk

We start off this part by looking at the characteristics of our liquidity risk model. We focus primarily on our illiquidity sorted portfolios that are equally-weighted. We also report results for size-and volatility sorted portfolios.

We see from Table 2 that sorting on past illiquidity successfully producer portfolios that are monotonically increasing in average illiquidity from portfolio 1 throughout portfolio 5. The most illiquid stocks(portfolio 5) seem to have the highest illiquidity volatility and are characterised by having higher expected returns, lower turnover and small size. It is however worth noticing that these illiquid stocks do not have monotonically increasing volatility of returns. Volatility of returns seem to be highest for the most liquid stocks(i.e portfolio 1) and show no clear pattern. This can be due to the low-volatility phenomena that previous literature such as Ang et al. (2006),Fu (2009) and Ang et al. (2009), have covered in detail. Furthermore, we see that if stocks are illiquid they also tend to have high liquidity risk. They have high values of (β_2^p) and large negative values of (β_4^p). This means that stocks that are illiquid in absolute terms(s^p) have a lot of commonality with market liquidity($\text{cov}(s^p, s^M)$) and high liquidity sensitivity to market returns($\text{cov}(s^p, r^M)$)).

We find however that these stocks have higher(less negative) β_3^p than the most liquid stocks, though the difference is only marginal. Illiquid stocks have lower return sensitivity to market liquidity($\text{cov}(r^p, s^M)$). This is interesting since it shows that when the market becomes illiquid, the returns of these small illiquid stocks remain stable. This is consistent with the theory that small sized companies that are illiquid are less affected than the largest companies. They are unable to provide higher returns in time of financial instability since these companies are already offering higher returns in tranquil periods in order for investors to be willing to invest in them. Portfolio 1 have higher negative value of β_3^p leading to these stocks being most affected when the market turns. These stocks have higher sensitivity to market-wide illiquidity compared to their illiquid counterpart. Investors holding stocks in the illiquid portfolio will be less affected than those who hold stocks in the liquid portfolios in terms of returns when market-wide illiquidity increases.

Table 2: Liquidity Risk for Illiquidity Sorted Portfolios

This table reports summary statistics for equally weighted illiquidity portfolios that are formed each year during 1998-2017. The market beta (β_1^p)and the liquidity betas (β_2^p), (β_3^p), (β_4^p) are calculated using monthly innovations in return and illiquidity for each portfolio. The respective parameters of betas are estimated using DVECH MGARCH model. Our market portfolio is equal-weighted, both for returns and illiquidity. Portfolio 1(5) contains biggest(smallest) and most(least) liquid companies. The average illiquidity is $E(s_p)$, standard deviation of illiquidity is $\sigma(s^p)$, average excess return, $E(r^{e,p})$, turnover(trn) and size (market capitalization) are computed using monthly data of the respective characteristics. Lastly, $\sigma(r^p)$ is the average monthly standard deviation of daily returns for the portfolios stocks computed each month. All of these variables are reported in percentage terms except for Size(Market Capitalization) which is reported in NOK billion. Betas are multiplied with 10 for presentation purpose; t-statistics are reported in parenthesis.

Portf	β_1^p	β_2^p	β_3^p	β_4^p	$E(s^p)$	$\sigma(s^p)$	$E(r^{e,p})$	$\sigma(r^p)$	Trn	Size
	(·10)	(·10)	(·10)	(·10)	(%)	(%)	(%)	(%)	(%)	(NOK Bill)
1	1.58	0.20	-0.50	0.03	0.26	0.09	-0.19	10.15	5.94	219.69
	(12.12)	(4.50)	(-12.10)	(2.11)						
2	1.50	16.45	-0.33	-0.27	0.32	0.03	0.77	4.32	1.29	6.41
	(12.18)	(12.57)	(-8.10)	(-1.87)						
3	1.22	34.20	-0.37	-0.25	0.59	0.89	1.31	3.86	0.48	5.23
	(12.11)	(14.38)	(-9.57)	(-7.32)						
4	1.01	19.45	-0.45	-0.43	1.01	1.37	1.73	6.66	0.19	1.20
	(12.17)	(21.44)	(-12.25)	(-9.97)						
5	0.92	54.94	-0.37	-0.52	2.04	2.80	3.46	4.90	0.21	0.51
	(11.92)	(18.32)	(-7.66)	(-3.61)						

We also see that β_4^p is slightly positive for the most liquid stocks, though this value is small. These stocks remain extremely liquid even when market returns are negative. One of the reason we get a positive value here can be due to our portfolio 1 consist of the biggest companies noted, which in size surpasses the rest of our portfolios combined. In addition, the relation between this portfolios liquidity and market returns are positive(positive correlation which is shown in section below.) The rest of our portfolios have negative values of β_4^p that are decreasing monotonically. This is also consistent with the assumption of flight to liquidity in a illiquid market. Lastly, all of our estimated betas are statistically significant at conventional levels.

Next, we consider the properties of value-weighed returns and illiquidity for our test portfolios reported in Table 3. Using value-weighted test portfolios we get substantially different results. Expected illiquidity of portfolios are monotonically increasing from the most liquid portfolio to least liquid, and their volatility show similar traits. However, expected average excess returns is increasing until portfolio 3 and then decreasing. Illiquid stocks seem to produce negative returns(portfolio 5), while having extremely high volatility and return reversals. Portfolio 1 produces stable returns with low volatility over the sample period.

Looking at our liquidity betas, β_2^p is increasing in illiquidity,while β_3^p starts off with a large negative value and have obscure pattern in the test portfolios. The sign of β_3^p varies from large negative values to small positive values in the portfolios. If we interpret β_3^p in economical sense, the investors expect returns of the stocks in the liquid companies to remain stable in times of illiquidity in the market. Said differently, positive values of β_3^p for portfolio 2 and 4 means that the returns of the portfolios do not react too much to market illiquidity, i.e low sensitivity of returns to market liquidity. The liquid stocks(portfolio 1) seem to have higher sensitivity of returns to market illiquidity.This is interesting on its own, but since the most illiquid portfolios are not supported in terms of statistical significance we should be careful in this consideration.β_4^p seem to be increasing slowly until portfolio 3 and decreasing thereafter. Portfolio 1 and 5 are more sensitive to market returns compared to the rest. With the exception of a couple of values for β_3, none of the values of β_3 and β_4 are significant or consistent with the results of equal-weights. However, β_1 and β_2 are in order with presumed assumptions. These values are also statistically significant. Also worth noting is the magnitude of β_1 for illiquid stocks. Intuitively, smaller companies returns are more sensitive to the overall market returns than the bigger companies.

Table 3: Liquidity Risk for Illiquidity Sorted Portfolios:Value Weighted Returns-and Illiquidity

This table reports summary statistics for value-weighted illiquidity portfolios that are formed each year during 1998-2017. The market beta (β_1^p)and the liquidity betas (β_2^p), (β_3^p), (β_4^p) are calculated using monthly innovations in return and illiquidity for each portfolio. The respective parameters of betas are estimated using DVECH MGARCH model. Our market portfolio is equal-weighted, both for returns and illiquidity. Portfolio 1(5) contains biggest(smallest) and most(least) liquid companies. The average illiquidity is $E(s_p)$, standard deviation of illiquidity is $\sigma(s^p)$, average excess return, $E(r^{e,p})$, turnover(trn) and size (market capitalization) are computed using monthly data of the respective characteristics. Lastly, $\sigma(r^p)$ is the average monthly standard deviation of daily returns for the portfolios stocks computed each month. All of these variables are reported in percentage terms except for Size(Market Capitalization) which is reported in NOK billion. Betas are multiplied with 10 for presentation purpose; t-statistics are reported in parenthesis.

Portf	β_1^p (·10)	β_2^p (·10)	β_3^p (·10)	β_4^p (·10)	$E(s^p)$ (%)	$\sigma(s^p)$ (%)	$E(r^{e,p})$ (%)	$\sigma(r^p)$ (%)	Trn (%)	Size (NOK Bill)
1	0.05	0.56	-7.03	-0.01	0.04	0.00	0.16	0.22	5.94	219.69
	(6.26)	(9.15)	(6.27)	(-0.56)						
2	0.08	1.09	0.09	0.16	0.08	0.00	0.22	57.06	1.29	6.41
	(2.52)	(2.70)	(4.53)	(0.94)						
3	2.30	2.20	-0.25	1.34	0.37	0.00	3.55	5.03	0.48	5.23
	(3.10)	(2.66)	(1.26)	(1.37)						
4	1.93	1.83	0.14	0.51	0.26	0.01	1.75	3.33	0.19	1.20
	(11.13)	(10.71)	(0.44)	(3.57)						
5	43.24	48.13	-0.01	-0.63	4.55	0.17	-23.35	219.15	0.21	0.51
	(39.13)	(12.11)	(-0.11)	(-1.25)						

Even though the results of value-weighted method is not consistent with the model implied assumptions, there might be a couple of reason we observe these characteristics. First, we have not put a restriction for stocks in our methodology, meaning all stocks traded, the most liquid and least liquid are included, with exception of penny stocks(less than NOK 1 are excluded). This causes our model to be driven by the most liquid stocks, meaning the firms with largest size as noted by their market capitalization. We see that the turnover is twice as much for portfolio 1 than the rest of portfolios combined, and the size exceeds eighteen times. Secondly, our time period do us no justice in terms of liquidity. With our dynamic conditional version of liquidity risk, the liquidity crisis in the end of 1990s, 2008 and the oil crash in 2014, might cause us to get these results. We find comfort in that the values obtained from β_3 and β_4 are insignificant, while we find support for β_1 and β_2.

In Table 4 we report the liquidity risk for size sorted portfolios. We see that the attributes of returns and illiquidity and their respective volatility is consistent with the findings of equal-weighted model. However, the values and patterns of our liquidity risk betas,β_2,β_3 and β_4, are weirdly inconsistent. There is only support for β_1, while the rest of our betas are insignificant.

Table 4: Liquidity Risk for Size Sorted Portfolios

This table reports summary statistics for size sorted portfolios that are formed each year during 1998-2017. The market beta (β_1^p)and the liquidity betas (β_2^p), (β_3^p), (β_4^p) are calculated using monthly innovations in return and illiquidity for each portfolio. The respective parameters of betas are estimated using DVECH MGARCH model. Our market portfolio is equal-weighted, both for returns and illiquidity. Portfolio 1(5) contains biggest(smallest) and most(least) liquid companies. The average illiquidity is $E(s_p)$, standard deviation of illiquidity is $\sigma(s^p)$, average excess return, $E(r^{e,p})$, turnover(trn) and size (market capitalization) are computed using monthly data of the respective characteristics. Lastly, $\sigma(r^p)$ is the average monthly standard deviation of daily returns for the portfolios stocks computed each month. All of these variables are reported in percentage terms except for Size(Market Capitalization) which is reported in NOK billion. Betas are multiplied with 10 for presentation purpose; t-statistics are reported in parenthesis.

Portf	β_1^p	β_2^p	β_3^p	β_4^p	$E(s^p)$	$\sigma(s^p)$	$E(r^{e,p})$	$\sigma(r^p)$	Trn	Size
	(·10)	(·10)	(·10)	(·10)	(%)	(%)	(%)	(%)	(%)	(NOK Bill)
1	0.08	6.21.	-0.34	-0.50	0.39	0.10	0.43	11.04	4.61	220.71
	(18.44)	(1.12)	(-1.89)	(-1.30)						
2	0.08	-24.12.	-0.29	-2.07	0.46	0.41	0.86	3.75	1.67	3.65
	(18.35)	(-0.70)	(-1.26)	(-1.02)						
3	0.07	52.71	-0.17	-0.51	0.70	1.01	1.25	4.29	0.78	1.20
	(19.75)	(1.08)	(-0.14)	(-1.79)						
4	0.06	22.29	-0.16	-0.80	1.11	1.53	1.81	3.94	0.43	0.45
	(18.55)	(1.02)	(-1.17)	(-1.23)						
5	0.05	33.46	-0.18	-1.38	1.83	2.48	1.59	4.59	0.23	0.15
	(18.45)	(1.03)	(-1.25)	(-1.30)						

5.1.1 Correlations

Before assessing the relation between liquidity and returns, a natural step is to determine the relation between the different liquidity betas. The way we construct our liquidity betas, β_2^p β_3^p and β_3^p, they are proportional to the product of correlations between their respective characteristics and their volatility. As we showed in the previous section, the more illiquid stocks tend to have higher returns, though the volatility tend to be highest for the most liquid portfolio. More illiquid stocks also have higher volatility in illiquidity, which is increasing monotonically in portfolio illiquidity.

Table 5: Correlation between Market Returns, Market Illiquidity and Portfolio Illiquidity for Illiquidity sorted Portfolios

This table reports correlations of annualized portfolio illiquidity, market returns-and illiquidity. Our testing portfolio is specified in terms of equal-weight. Market portfolio is also equally-weighted. The correlations are formed for each year and then averaged over our sample period. The results are based on monthly data covering the period from January 1998 to December 2017.

	Portf1,s^P	Portf2,s^P	Portf3,s^P	Portf4,s^P	Portf5,s^P
Market Illiquidity, s^M	0.11	0.66	0.89	0.95	0.98
Market Returns, R^M	0.113	-0.137	-0.226	-0.202	-0.246

This variability of returns and illiquidity are not the only contributing factor that leads to a positive relation between illiquidity and liquidity risk.From Table 5 we see that the correlation between market illiquidity,s^M, and portfolio illiquidity is increasing. Portfolio 1 seem to have pretty low correlation compared to the rest. Next, the correlations coefficients between portfolio returns, r^p, and market illiquidity, s^M, reported in Table 6 are decreasing slowly and thereby increasing(this relation would have been more clear with 10 portfolios instead of 5 but we do not report these). Also, something more interesting is the correlation coefficients for portfolio illiquidity, s^p, and market returns, R^M, which are decreasing from portfolio 1 until portfolio 5, but with a positive relation between illiquidity of portfolio 1 and market returns.

Table 6: Correlation Coefficient between Portfolio Returns and Market Illiquidity for Illiquidity Sorted Portfolios

This table reports correlations of annualized portfolio returns and market illiquidity. Our testing portfolio is specified in terms of equal-weight. Market portfolio is also equal-weighted.The correlations are formed for each year and then averaged over our sample period. The results are based on monthly data covering the period from January 1998 to December 2017.

	Portf.1,r^p	Portf.2,r^p	Portf.3,r^p	Portf.4,r^p	Portf.5,r^p
Market Illiquidity, s^M	-0.221	-0.230	-0.197	-0.170	-0.178

Lastly, we look at the correlations between our liquidity betas. The collinearity between different measures of liquidity risk discussed above is verified by looking at the relation between these betas. Table 7 report correlations between betas for illiquidity-sorted portfolio. It is clear that there is a collinearity problem amongst the betas, which is a cause for concern when testing the relation between liquidity and returns.

Table 7: Correlation Coefficient between Betas for Illiquidity Sorted portfolio

This table reports correlations of our computed betas, β_1^p, β_2^p,β_3^p, and β_4^p for equally-weighted illiquidity portfolios. The betas are estimated based on the innovations in illiquidity and returns. DVECH MGARCH model has been used to calculate these innovations. The market portfolio is also equally-weighted for returns and illiquidity. The correlations are formed for each year and then averaged over our sample period. The results are based on monthly data covering the period from January 1998 to December 2017.

	β_1^p	β_2^p	β_3^p	β_4^p
β_1^p	1.000	0.986	-0.997	-0.914
β_2^p		1.000	-0.993	-0.872
β_3^p			1.000	0.902
β_4^p				1.000

Looking at the individual stock level, we see that the collinearity is still persistent and quite high between β_1^j, β_2^j and β_3^j. However, this collinearity has been reduced substantially amongst the rest of betas. One of the reason we get much smaller collinearity for individual stocks might be due to larger estimation errors. This high collinearity between our betas is worrying when we are trying to assess their effect on returns. As Acharya & Pedersen (2005) point out, this makes it harder to separate the individual effects of illiquidity and individual liquidity betas.

Table 8: Correlation Coefficient between Betas for Individual Stocks

This table reports correlations of betas, β_1^j, β_2^j,β_3^j, and β_4^j for individual shares listed on OSE.The betas are estimated based on the innovations in illiquidity and returns. DVECH MGARCH model has been used to calculate these innovations. We compute these correlations annually and average these over our sample period. We use monthly returns and illiquidity for the stocks as well as the market portfolio to calculate the betas.The results are based on monthly data covering the period from January 1998 to December 2017.

	β_1^j	β_2^j	β_3^j	β_4^j
β_1^j	1.000	0.511	-0.987	-0.162
β_2^j		1.000	-0.523	-0.279
β_3^j			1.000	0.154
β_4^j				1.000

5.2 Asset Pricing with Liquidity Risk

In this section we present the results of our liquidity adjusted capital asset pricing model. We start off by examining our primary test, which is illiquidity sorted and from there on move on to test volatility and size sorted portfolios. We also test the robustness of weighted method and control for size-and momentum.

5.2.1 Illiquidity Sorted Portfolios

The average holding period h for illiquidity sorted portfolios is calibrated to 0.015 in some specifications. This implies that it takes $1/0.015 \cong 66$ months for all stocks to be turned over once,which corresponds to investors holding period. We obtain this value by averaging turnover for our test portfolios.

We start off by testing the LCAPM with only one risk premium, λ^M, where our risk factor is the net beta, β^p_{net}. The result for this specification is reported in Equation 1(we will refer to the Equations in a specific Table as Equations 1-8 in this Section) in Table 9. We see that we get a small, but significant value of the risk premium λ^M,while the constant α is insignificant at 5%. Equation 2 produces significantly better results where h is a free parameter. Both, risk premium and expected holding period is significant at 5%.

To isolate the effect of liquidity risk,β^p_2 β^p_3 and β^p_3, over our traditional market risk,β^p_1, and liquidity level, $E(s^p)$, we consider the following model

$$E(R^p_t - r^f_t) = \alpha + hE(s^p_t) + \beta^p_1\lambda^1 + \beta^p_{net}\lambda^M \tag{24}$$

This relation is reported in Equation 4 with h as a fixed parameter. In this specification, β^p_{net} is still significant, but β^p_1 seem to produce a relatively small values while being insignificant. Equation 5 produces quite different results when we allow h to be a free parameter. β^p_{net} has increased in value, while we get a small negative value of β^p_1 and a small reduced value of E(s). All of the parameters in Equation 5 seem to be significant. In Equation 6 we set h=0, which leads to a strong support for β^p_{net}. It is also worth pointing out that the negative value of β^p_1 in Equation 5 and 6 does not mean a negative risk premium λ^M in the market. Since we have included β^p_1 as a part of β^p_{net}, we simply need to add the coefficient of β^p_{net} to get the correct value.

Table 9: Asset Pricing: Model Testing for Illiquidity Sorted Portfolios

This table report the estimated coefficients of a cross-sectional regression for illiquidity sorted portfolios. Portfolios are formed using monthly returns and illiquidity innovations during the period of 1998-2017. We obtain the conditional parameters of the betas using DVECH MGARCH model. Testing portfolios are equal-weighted, and the market portfolio is reported equal-weighted as well. To test the liquidity-adjusted model we consider the relation such as

$$E(R_t^p - r_t^f) = \alpha + \beta_1^p \lambda^1 + hE(s_t^p) + \beta_2^p \lambda^2 + \beta_3^p \lambda^3 + \beta_4^p \lambda^4 + \beta_{net}^p \lambda^M$$

where $\beta_{net}^p = \beta_1^p + \beta_2^p - \beta_3^p - \beta_4^p$. In Equation 1,4,7 we set the holding period(average of monthly turnover) h as a fixed parameter, while Equation 2,5,8 we let it be a free value. We include a nonzero intercept of α. z-statistics are reported in parenthesis and are estimated using Generalized Methods of Moments(GMM) which takes into account pre-estimation of betas and allows serial correlation in error term.

Equation	Constant	$E(s^p)$	β_1^p	β_2^p	β_3^p	β_4^p	β_{net}^p
1	-0.002	0.015					0.004
	(-1.05)	(-)					(1.80)
2	-0.001	0.012					0.008
	(-0.30)	(5.75)					(1.73)
3	-0.040		0.105				
	(-6.08)		(0.38)				
4	-0.03	0.015	0.001				0.004
	(-0.88)	(-)	(0.12)				(1.86)
5	0.006	0.010	-0.003				0.096
	(1.36)	(5.82)	(-1.83)				(2.15)
6	0.024		-0.007				0.08
	(5.44)		(-2.36)				(3.04)
7	0.003	0.015	0.34	0.001	-0.009	-0.045	
	(1.04)	(-)	(0.46)	(0.95)	(-0.47)	(-2.58)	
8	-0.000	0.017	0.037	0.001	-0.008	-0.047	
	(-0.07)	(4.48)	(0.51)	(0.20)	(-0.40)	(-2.64)	

For instance in Equation 5 β_1^p is:

$$E(R_t^p - r_t^p) = 0.006 + 0.010E(s_t^p) - 0.003\beta_1^p + 0.096\beta_{net}^p$$
$$E(R_p^t - r_f^t) = 0.006 + 0.010E(s_t^p) + 0.093\beta_1^p + 0.096(\beta_2^p - \beta_3^p - \beta_4^p) \tag{25}$$

To test the full model where we allow the betas to have different risk premium λ and a fixed h, we run run the unrestricted model obtained in Equation 7. Equation 8 runs the same model with h as a free value. Here we have our generalized relation

$$E(R_t^p - r_t^f) = \alpha + \beta_1^p\lambda^1 + hE(s_t^p) + \beta_2^p\lambda^2 + \beta_3^p\lambda^3 + \beta_4^p\lambda^4 \tag{26}$$

without setting the model restrictions that $\lambda^1 = \lambda^2 = -\lambda^3 = -\lambda^4 = \lambda^M$. We see that all of the betas produce small and insignificant results, with the exception of average portfolio illiquidity, E(s_p), and β_4^p. However, since we have a strong collinearity problem, this evidence should be interpreted with caution. Lastly, we want to point out that the intercept α fluctuates between being significant in some specifications and insignificant in others even though our model implies a zero constant value.

Next, it is probably more interesting to examine the economic significance of our results and the overall liquidity risk. To show the magnitude of our results we compute the annual market risk premium, λ^M, and the market risk premium for different liquidity betas(i.e $\lambda^1, \lambda^2, \lambda^3, \lambda^4$) required to hold illiquid stocks. We calculate this by the product of the market risk premium and the difference between liquidity risk of our most liquid and least liquid portfolios. This is often considered as going long for the first portfolio and short for the last portfolio in empirical literature.

We start off by using our unrestricted model in Equation 8 of Table 9. The effect on annualized expected returns between portfolio 1 and 5 that can be explained by β_2, the commonality of portfolio illiquidity and market illiquidity is

$$\lambda^2(\beta_2^{p_5} - \beta_2^{p_1})12 = 0.05\% \tag{27}$$

and the effect of β_3, the sensitivity of returns to market illiquidity, on yearly returns is

$$-\lambda_3(\beta_3^{p_5} - \beta_3^{p_1})12 = -0.02\% \tag{28}$$

and similarly the effect of β_4, the sensitivity of portfolio illiquidity to the overall market return is

$$-\lambda_4(\beta_4^{p5} - \beta_4^{p_1})12 = 0.03\% \tag{29}$$

This leads to the overall effect of liquidity risk of 0.06% per year. However, out of the estimated liquidity betas in Equation 8, only β_4 is significant. The results produced by this unrestricted model rely heavily on the validity of our model. If we instead use the same market risk premium

, λ^M, of 0.004 from Equation 1 with our calibrated h we get the following results:

$$\lambda^M(\beta_2^{p_5} - \beta_2^{p_1})12 = 0.26\% \tag{30}$$

$$-\lambda^M(\beta_3^{p_5} - \beta_3^{p_1})12 = -0.02\% \tag{31}$$

$$-\lambda^M(\beta_4^{p_5} - \beta_4^{p_1})12 = 0.002\% \tag{32}$$

Which makes the overall effect of liquidity risk of 0.251% per year. Here, the largest economic impact on expected returns seem to be to the β_2. The effect of our significant β_4 from the unrestricted model seem to be relatively small, while interestingly the effect of β_2 seem to be negative.

We note that the effect of our liquidity risk, estimated using betas is relatively small.First, we use the robust standard errors of our estimates of λ^M, and β_2 β_3 β_4 to test if the liquidity risk, β_2 - β_3- β_4, in our restricted model is significantly different from zero. With 95% confidence interval we get [-6%, 8%], which does not make our results significantly different from zero, even when we include the impact of expected illiquidity E(s). The unrestricted model in Equation 8 is also insignificant.

The impact of expected illiquidity, E(s), on the yearly difference in returns between portfolio 1 and 5 is 1.5% using our calibrated coefficient. If we consider our results from using the same market risk premium for all risk premium betas meaning Equation 1, the overall effect of illiquidity and liquidity risk is 1.75%. And if we use the unconstrained model with different market risk premium and a free value of transaction cost, this effect is 1.76%.

The overall liquidity risk characterized as our liquidity betas with a single market risk premium is quite small and barely significant in the restricted model. This can be related to our choice to look at the a small trading platform such as OSE. Our overall liquidity risk value is quite small compared to the findings of Pastor & Stambaugh (2003). However, they employed a different measure of liquidity and sort stocks based on their liquidity risk(which corresponds to β_3) while we have sorted stocks based on securities level(the Amihud (2002) proxy). Similarly, Acharya & Pedersen (2005) found that their liquidity risk to be quite high compared to ours and statistically significant. While their primary illiquidity sorted model is value-weighted for portfolios and equal-weighted for market, we used equally-valued for both. Testing for equally- weighted portfolios and market, they found that their liquidity risk became insignificant similar to ours. Pastor & Stambaugh (2003) also did not control for the illiquidity level, which has been shown from previous litterateur such as Amihud & Mendelson (1986), Brennan & Subrahmanyam (1996),Brennan & Subrahmanyam (1996) to offer a significant risk premium.

Table 10: Asset Pricing:Model Testing for Volatility of Illiquidity Sorted Portfolios

This table report the estimated coefficients of a cross- sectional regression for Volatility of Illiquidity sorted portfolios. Portfolios are formed using monthly returns and illiquidity innovations during the period of 1998-2017. We obtain the conditional parameters of the betas using DVECH MGARCH model. Testing portfolios are equal-weighted, and the market portfolio is reported equal-weighted as well. To test the liquidity-adjusted model we consider the relation such as

$$E(R_t^p - r_t^f) = \alpha + \beta_1^p \lambda^1 + hE(s_t^p) + \beta_2^p \lambda^2 + \beta_3^p \lambda^3 + \beta_4^p \lambda^4 + \beta_{net}^p \lambda^M$$

where $\beta_{net}^p = \beta_1^p + \beta_2^p - \beta_3^p - \beta_4^p$. In Equation 1,4,7 we set the holding period(average of monthly turnover) h as a fixed parameter, while Equation 2,5,8 we let it be a free value. We include a nonzero intercept of α. z-statistics are reported in parenthesis and are estimated using Generalized Methods of Moments(GMM) which takes into account pre-estimation of betas and allows serial correlation in error term.

Equation	Constant	$E(s^p)$	β_1^p	β_2^p	β_3^p	β_4^p	β_{net}^p
1	0.002	0.015					-0.001
	(0.96)	(-)					(-0.23)
2	0.002	0.015					-0.001
	(0.42)	(6.41)					(-0.95)
3	0.014		0.09				
	(2.75)		(0.04)				
4	0.001	0.015	0.010				-0.000
	(0.62)	(-)	(0.74)				(-0.68)
5	0.000	0.016	0.002				-0.000
	(0.06)	(8.49)	(0.18)				(-0.78)
6	0.015		-0.014				0.001
	(2.74)		(-0.99)				(2.20)
7	-0.025	0.015	0.101	-0.000	-0.025	-0.039	
	(-1.03)	(-)	(1.37)	(-0.42)	(-1.18)	(-2.64)	
8	-0.003	0.02	0.090	-0.001	-0.020	-0.0413	
	(-0.45)	(4.50)	(1.15)	(-1.06)	(-0.92)	(-2.48)	

While we have controlled for a single market risk premium for the liquidity betas such, using different risk premium does not yield much higher results. With the unrestricted model which allows different risk premium, we obtain a significant β_4 similar to Acharya & Pedersen (2005), while the rest of the betas are insignificant. The statistical insignificance of our liquidity betas can also be due to the upper and lower bound of stocks included in our sample.

We have included all stocks listed throughout our sample period(with the exclusions of stocks less than NOK 1). We also want to point out that most of our illiquidity premium is run by our proxy for average holding period, h, which is not a perfect estimate for holding period. There are several better estimates that have been derived by Ødegaard (2018) to reflect how long different investors hold on to their stocks in OSE. The holding period used in our estimates have been criticised to severely-overestimating holding period. Lastly, the collinearity between liquidity and liquidity risk implies that our most robust estimates will corresponds to their true overall effect, which is given in Equation 8 in Table 9, i.e the unrestricted model.

5.2.2 Volatility of Illiquidity Sorted Portfolios

Sorting stocks on σ(Illiquidity) does not produce much better results than previously estimated. We see from Table 10 that most of our market premium estimation produces small negative values which are insignificant, with the exception of Equation 6, where we hold $h = 0$. The holding period h seem to have clear relevance when we allow it to be a free parameter. In Equations 2,5 and 8 we get the values that correspond closely to its calibrated values.

The most interesting results are obtained from Equation 7-8. In both instances we get non-significant values of β_2 and β_3 with very small values. Looking at β_4 we see that we get large values compared to other variables with clear significance. Hence, we find further evidence for portfolio illiquidity sensitivity to market returns. While the overall liquidity risk does not seem to matter over and above liquidity level or market risk. β_4 contribution to yearly returns here corresponds to 0.03%. The overall liquidity risk does not seem to be significantly different from zero.

5.2.3 Illiquidity Sorted Portfolios: Robustness of Weighted Method

To check the robustness of weighting portfolios, we decide to test with different specifications and portfolios. In Table 11 we test our model with value-weighted portfolios and equal-weighted market. In this paper we treat this specification as a robustness check, while Acharya & Pedersen (2005) uses this as primary testing model. From Equation 1 we see that our β_{net} is borderline significant at 5% and interestingly we get a negative value. The rest of our equations it does not seem to be significant.

Table 11: Asset Pricing:Model Testing for Illiquidity Sorted Portfolios-Value Weighted Portfolios and Equal Weighted Market

This table report the estimated coefficients of a cross-sectional regression for Illiquidity sorted portfolios. Portfolios are formed using monthly returns and illiquidity innovations during the period of 1998-2017. We obtain the conditional parameters of the betas using DVECH MGARCH model. Testing portfolios are value-weighted, and the market portfolio is reported as equal-weighted. To test the liquidity-adjusted model we consider the relation such as

$$E(R_t^p - r_t^f) = \alpha + \beta_1^p \lambda^1 + hE(s_t^p) + \beta_2^p \lambda^2 + \beta_3^p \lambda^3 + \beta_4^p \lambda^4 + \beta_{net}^p \lambda^M$$

where $\beta_{net}^p = \beta_1^p + \beta_2^p - \beta_3^p - \beta_4^p$. In Equation 1,4,7 we set the holding period(average of monthly turnover) h as a fixed parameter, while Equation 2,5,8 we let it be a free value. We include a nonzero intercept of α. z-statistics are reported in parenthesis and are estimated using Generalized Methods of Moments(GMM) which takes into account pre-estimation of betas and allows serial correlation in error term.

Equation	Constant	$E(s^p)$	β_1^p	β_2^p	β_3^p	β_4^p	β_{net}^p
1	0.003	0.015					-0.022
	(0.04)	(-)					(-1.86)
2	0.016	-0.029					-0.08
	(3.65)	(-0.83)					(-0.77)
3	0.007		-0.050				
	(0.59)		(-1.23)				
4	0.008	0.015	-0.022				-0.014
	(0.48)	(-)	(-1.01)				(-1.07)
5	0.073	-0.022	-0.026				-0.014
	(1.80)	(-0.02)	(-0.57)				(-0.63)
6	0.010		-0.011				-0.012
	(2.36)		(-0.93)				(-0.86)
7	-0.003	0.015	0.025	-0.008	-0.138	0.025	
	(-0.06)	(-)	(0.33)	(-0.90)	(-0.94)	(1.35)	
8	0.012	-0.067	0.030	-0.006	-0.187	0.079	
	(0.38)	(-0.33)	(0.38)	(-0.65)	(-0.35)	(1.41)	

Table 12: Asset Pricing:Model Testing for Illiquidity Sorted Portfolios-Equal Weighted Portfolios and Value-Weighted Market

This table report the estimated coefficients of a cross- sectional regression for Illiquidity sorted portfolios. Portfolios are formed using monthly returns and illiquidity innovations during the period of 1998-2017. We obtain the conditional parameters of the betas using DVECH MGARCH model. Testing portfolios are equal-weighted, and the market portfolio is reported as value-weighted. To test the liquidity-adjusted model we consider the relation such as

$$E(R_t^p - r_t^f) = \alpha + \beta_1^p \lambda^1 + hE(s_t^p) + \beta_2^p \lambda^2 + \beta_3^p \lambda^3 + \beta_4^p \lambda^4 + \beta_{net}^p \lambda^M$$

where $\beta_{net}^p = \beta_1^p + \beta_2^p - \beta_3^p - \beta_4^p$. In Equation 1,4,7 we set the holding period(average of monthly turnover) h as a fixed parameter, while Equation 2,5,8 we let it be a free value. We include a nonzero intercept of α. z-statistics are reported in parenthesis and are estimated using Generalized Methods of Moments(GMM) which takes into account pre-estimation of betas and allows serial correlation in error term.

Equation	Constant	$E(s^p)$	β_1^p	β_2^p	β_3^p	β_4^p	β_{net}^p
1	0.002	0.015					-0.000
	(1.05)	(-)					(-0.67)
2	0.001	0.016					-0.000
	(0.36)	(5.51)					(-0.75)
3	0.034		-0.030				
	(5.20)		(-0.06)				
4	0.002	0.015	0.000				-0.000
	(0.28)	(-)	(0.03)				(-0.63)
5	-0.003	0.018	0.006				-0.000
	(-0.24)	(4.00)	(0.34)				(-0.65)
6	0.042		-0.039				-0.000
	(6.09)		(-3.88)				(-0.88)
7	-0.002	0.015	0.001	-0.000	0.364	-0.004	
	(0.27)	(-)	(0.06)	(-0.45)	(1.22)	(-3.88)	
8	-0.003	0.018	0.006	-0.000	0.353	-0.004	
	(-0.19)	(4.01)	(0.33)	(-0.47)	(1.18)	(-3.88)	

The traditional β_1 is also negative, though insignificant, in most of the cases. Similar conclusion is drawn from the examining the expected illiquidity. Also, the intercept seem to be non-zero and significant in some cases.

In Equations 7 and 8 we can not draw the same conclusion as we did for Illiquidity and σ(Illiquidity) sorted portfolios where we obtained significant β_4. The contribution of returns for β_2 seem to be negative while β_4 positive, which is not in order with the model assumptions, though these are insignificant values.

We also test for equal-weighted portfolios and value-weighted market in Table 12. We see that with value-weighted market produces non-significant results for our risk premium in any of the Equations. However, we do get significant values of expected illiquidity when we allow h to be a free parameter in all cases. The constant(i.e intercept) and holding period seem to have opposite effects when we control for holding period h. We see that holding h fixed leads to significant results for E(s) and relaxing this constraint leads gives plausible results for the intercept being significantly different than zero. Lastly, β_4 and E(s) show promising results in Equation 7-8. The rest of the variables seem to have little relevance at standard levels.

5.2.4 Size Sorted Portfolios

From previous section we know that the small sized stocks usually are illiquid and have higher liquidity risk. We find it interesting to further check if sorting stocks by size will change our results with regards to liquidity risk and returns. Table 13 reports our results where we re-estimate our model sorted on the size. The market risk-premium does not seem to produce any significant values in any cases. The coefficient of β_{net} seem to be negative and zero in all of the Equations. While controlling the effect of liquidity over our traditional β_1 we see that sorting stocks on size gives us a a value closely corresponding to the true value β_1, which has been documented empirically. Next, the effect of expected illiquidity E(s) seem to be significant and positive in the cases with unconstrained model. We also find that the intercept is significantly different from zero only when h=0. Sorting stocks by size does not give any support for any of our liquidity betas at 5%, but provides some significance at 10% for β_4.

5.2.5 Controlling for Size and Momentum

In this last part we test our models ability to explain the size and momentum effects[23]. We run our cross-sectional regression while controlling for size and momentum. We run our main testing model with illiquidity sorted portfolios and report results in Table 14. While including a size and momentum variables leads to higher robust standard errors, the result seem to be similar to those derived previously. Our coefficient for size variable is close to zero and insignificant in all

[23]Size and Momentum variables have proven to have significant effects on investors returns in empirical research. Fama & French (1993), Fama & French (1996),Fama & French (2015).

cases with the exception for Equation 3. Momentum factor seem to be significant in all of the cases.

β_{net} is insignificant in almost all Equations with the exception of Equation 6. Here we get strong support for the market risk premium. The expected illiquidity variable has a strong positive value in most specifications when we allow unrestricted h. Interestingly, our estimated intercept, α is not significantly different from zero in any Equations. As for Equation 7-8, we see that coefficient for β_4 is large compared to our other liquidity betas(β_2 and β_3). This beta is still significant when we control for size and momentum.

5.2.6 Specification Tests

Before we move on to our concluding remarks, we want to perform several specification tests for our liquidity risk betas and the overall liquidity-adjusted CAPM. For this we consider our primary model for testing reported in Table 9. First, our initial model fails to reject that $\alpha = 0$, even though the way we have set up our model, it implies insignificant value of the intercept. These values are only significant in Equation 3 and 6 where we set $h = 0$. Second, we perform Wald test for our unrestricted model in Equation 8. Based on this test we can reject the restrictions of $\lambda^1 = \lambda^2 = \lambda^3 = \lambda^4$, $\alpha = 0$ and $h =$ h where h is our fixed parameter. With this test we get a p-value of 0%, which means that there is some support for our unrestricted model and liquidity betas. If we instead test for the usual CAPM with $\lambda^2 = \lambda^3 = \lambda^4 = 0$ and $h = 0$, we fail to reject with p-value of 75%. For our LCAPM with β_{net} in Equation 1, we fail to reject $\lambda^M = 0$ with p-value of 88%. Thus it seems that the unrestricted model is a better fit than assuming the same market risk premium for all liquidity betas.

Table 13: Asset Pricing:Model Testing for Size sorted portfolios

This table report the estimated coefficients of a cross- sectional regression for Size sorted Portfolios. Portfolios are formed using monthly returns and illiquidity innovations during the period of 1998-2017. We obtain the conditional parameters of the betas using DVECH MGARCH model. Testing portfolios are equal-weighted, and the market portfolio is reported equal-weighted as well. To test the liquidity-adjusted model we consider the relation such as

$$E(R_t^p - r_t^f) = \alpha + \beta_1^p \lambda^1 + hE(s_t^p) + \beta_2^p \lambda^2 + \beta_3^p \lambda^3 + \beta_4^p \lambda^4 + \beta_{net}^p \lambda^M$$

where $\beta_{net}^p = \beta_1^p + \beta_2^p - \beta_3^p - \beta_4^p$. In Equation 1,4,7 we set the holding period(average of monthly turnover) h as a fixed parameter, while Equation 2,5,8 we let it be a free value. We include a nonzero intercept of α. z-statistics are reported in parenthesis and are estimated using Generalized Methods of Moments(GMM) which takes into account pre-estimation of betas and allows serial correlation in error term.

Equation	Constant	$E(s^p)$	β_1^p	β_2^p	β_3^p	β_4^p	β_{net}^p
1	-0.002	0.014					-0.000
	(-1.04)	(-)					(-0.33)
2	0.004	0.008					-0.000
	(0.96)	(2.18)					(-0.08)
3	0.012		-2.194				
	(5.20)		(-0.96)				
4	-0.003	0.014	1.295				-0.000
	(-1.36)	(-)	(2.46)				(-0.98)
5	0.003	0.008	1.11				-0.000
	(0.83)	(2.11)	(2.50)				(-0.71)
6	0.012		0.7951				-0.000
	(5.51)		(1.57)				(-0.35)
7	-0.002	0.014	-0.568	-0.000	0.159	0.003	
	(-0.65)	(-)	(-0.05)	(-0.41)	(0.06)	(0.36)	
8	0.007	0.007	-1.502	-0.000	-0.728	0.013	
	(1.41)	(1.84)	(-0.13)	(0.26)	(-0.28)	(1.56)	

Table 14: Asset Pricing:Controlling for Size and Momentum

This table report the estimated coefficients of a cross- sectional regression for Illiquidity sorted portfolios. Portfolios are formed using monthly returns and illiquidity innovations during the period of 1998-2017. We obtain the conditional parameters of the betas using DVECH MGARCH model. Testing portfolios are equal-weighted, and the market portfolio is reported equal-weighted as well. To test the liquidity-adjusted model we consider the relation such as

$$E(R_t^p - r_t^f) = \alpha + \beta_1^p\lambda^1 + hE(s_t^p) + \beta_2^p\lambda^2 + \beta_3^p\lambda^3 + \beta_4^p\lambda^4 + \beta_{net}^p\lambda^M + ln(size^p)\lambda^5 + MOM^p\lambda^6$$

where $\beta_{net}^p = \beta_1^p + \beta_2^p - \beta_3^p - \beta_4^p$, $ln(size^p)$ is thee time-series average of the log of the ratio of the portfolios market capitalization(size) at the beginning of a monthly to the total market capitalization(total size). And MOM^p is the cumulative returns in a month for the portfolios.In Equation 1,4,7 we set the holding period(average of monthly turnover) h as a fixed parameter, while Equation 2,5,8 we let it be a free value. We include a nonzero intercept of α. z-statistics are reported in parenthesis and are estimated using Generalized Methods of Moments(GMM) which takes into account pre-estimation of betas and allows serial correlation in error term.

Eq.	Const.	$E(s^p)$	β_1^p	β_2^p	β_3^p	β_4^p	β_{net}^p	$ln(size^p)$	MOM
1	-0.002	0.015					0.000	-0.000	0.011
	(-0.26)	(-)					(0.17)	(-0.23)	(2.05)
2	-0.002	0.016					0.000	-0.001	0.011
	(-0.20)	(4.24)					(0.04)	(-0.41)	(2.05)
3	-0.012		0.006					-0.003	0.011
	(-1.22)		(0.45)					(-2.90)	(2.13)
4	-0.003	0.015	0.006				-0.000	0.000	0.011
	(0.21)	(-)	(0.26)				(-0.10)	(0.08)	(2.05)
5	-0.003	0.016	0.006				-0.000	-0.000	0.011
	(-0.31)	(3.87)	(0.29)				(-0.27)	(-0.23)	(2.05)
6	-0.005		-0.025				0.002	-0.002	0.012
	(-0.45)		(-1.25)				(2.39)	(-1.80)	(2.12)
7	-0.004	0.015	0.036	0.001	-0.009	-0.046		-0.000	0.011
	(0.19)	(-)	(0.49)	(0.63)	(-0.45)	(-2.61)		(-0.36)	(2.06)
8	-0.003	0.017	0.037	0.000	-0.008	-0.047		-0.000	0.011
	(0.22)	(4.32)	(0.51)	(0.23)	(-0.43)	(-2.64)		(0.28)	(2.05)

6. Conclusion

We have studied a model which takes into account liquidity risk at OSE. The traditional asset pricing model has been adjusted to reflect the cost of illiquidity and its respective risk over time. We find that investors care about the returns of securities and illiquidity, especially in a down market. They are also willing to trade-off the performance of these assets in favour for liquidity in times when liquidity dries up. Investors returns are positively affected by this liquidity and are increasing in the co-variation between securities illiquidity and market-wide illiquidity. Returns are increasing if the stocks performance is highly sensitive to market-wide illiquidity and decreasing in the co-variation between stocks illiquidity and gross returns on the market. The returns are also increasing for the most illiquid stocks which are characterized by small firm size and low turnover. The volatility of returns however seem to be highest for the most liquid companies.

There is some evidence that show that liquidity risk is priced and some support for the liquidity adjusted CAPM. The impact of liquidity risk on returns is 0.25% when we impose restrictions of a fixed average holding period and a single market risk premium. The overall illiquidity premium is 1.75%. With the unrestricted model the liquidity risk is priced at 0.06% and the illiquidity premium is 1.76%. Out of our liquidity betas, there is only support for the stocks liquidity co-variation to market returns, which affects the expected returns 0.03%. We find no economic significance for the restricted model when sorting on volatility of illiquidity, weighted method, size or when we control for size. In almost all these cases we find support for β_4, which is in line with the findings of Acharya & Pedersen (2005).

This model provide a simple framework when studying the impact of liquidity and expected returns of assets. With the limited stocks available on OSE and with a such small market compared to the one covered by Acharya & Pedersen (2005) we have made several adjustments. We use the conditional version of the LCAPM and use different restrictions for our data sample. However, this model still helps to explain some of the relation between liquidity risk and asset prices.

A. Appendix Alternative Filtering

Table 15: Liquidity Risk for Illiquidity Sorted Portfolios- Removing Observations with Stock Price Under NOK10 and Above NOK10000

This table reports summary statistics for equally weighted illiquidity portfolios that are formed each year during 1998-2017. The market beta (β_1^p)and the liquidity betas (β_2^p), (β_3^p), (β_4^p) are calculated using monthly innovations in return and illiquidity for each portfolio. The respective parameters of betas are estimated using DVECH MGARCH model. Our market portfolio is equal-weighted, both for returns and illiquidity. Portfolio 1(5) contains biggest(smallest) and most(least) liquid companies. The average illiquidity is $E(s_p)$, standard deviation of illiquidity is $\sigma(s^p)$, average excess return, $E(r^{e,p})$, turnover(trn) and size (market capitalization) are computed using monthly data of the respective characteristics. Lastly, $\sigma(r^p)$ is the average monthly standard deviation of daily returns for the portfolios stocks computed each month. All of these variables are reported in percentage terms except for Size(Market Capitalization) which is reported in NOK billion. Betas are multiplied with 10 for presentation purpose; t-statistics are reported in parenthesis.

Portf	β_1^p	β_2^p	β_3^p	β_4^p	$E(s)^p$	$\sigma(s^p)$	$E(r^{e,p})$	$\sigma(r^p)$	Trn	Size
	(·10)	(·10)	(·10)	(·10)	(%)	(%)	(%)	(%)	(%)	(NOK Bill)
1	2.33	0.01	-0.46	-0.04	0.25	0.01	-0.39	8.92	6.47	89.74
	(12.12)	(4.50)	(-9.16)	(2.11)						
2	2.26	0.56	-0.30	-0.14	0.26	0.03	0.53	4.03	1.57	13.02
	(12.18)	(12.57)	(-14.5)	(-1.87)						
3	2.08	5.12	-0.37	-0.35	0.32	0.16	0.76	3.01	0.38	4.97
	(12.11)	(14.38)	(-5.56)	(-7.32)						
4	1.71	10.68	-0.230	-0.08	0.47	0.39	1.30	2.61	0.16	3.28
	(12.17)	(21.44)	(-6.34)	(-9.97)						
5	1.41	28.66	-0.28	-2.06	0.80	0.72	2.25	8.56	0.06	1.33
	(11.92)	(18.32)	(-6.24)	(-3.61)						

Table 16: Asset Pricing: Model Testing for Illiquidity Sorted Portfolios-Removing Observations with Stock Price Under NOK10 and Above NOK10000

This table report the estimated coefficients of a cross-sectional regression for illiquidity sorted portfolios. Portfolios are formed using monthly returns and illiquidity innovations during the period of 1998-2017. We obtain the conditional parameters of the betas using DVECH MGARCH model. Testing portfolios are equal-weighted, and the market portfolio is reported equal-weighted as well. To test the liquidity-adjusted model we consider the relation such as

$$E(R_t^p - r_t^f) = \alpha + \beta_1^p \lambda^1 + hE(s_t^p) + \beta_2^p \lambda^2 + \beta_3^p \lambda^3 + \beta_4^p \lambda^4 + \beta_{net}^p \lambda^M$$

where $\beta_{net}^p = \beta_1^p + \beta_2^p - \beta_3^p - \beta_4^p$. In Equation 1,4,7 we set the holding period(average of monthly turnover) h as a fixed parameter, while Equation 2,5,8 we let it be a free value. We include a nonzero intercept of α. z-statistics are reported in parenthesis and are estimated using Generalized Methods of Moments(GMM) which takes into account pre-estimation of betas and allows serial correlation in error term.

Equation	Constant	$E(s^p)$	β_1^p	β_2^p	β_3^p	β_4^p	β_{net}^p
1	0.002	0.015					0.001
	(0.81)	(-)					(0.74)
2	-0.007	0.046					-0.002
	(-1.45)	(3.75)					(1.64)
3	0.071		0.12				
	(2.05)		(0.77)				
4	-0.02	0.015	0.03				0.001
	(-0.59)	(-)	(1.96)				(1.04)
5	-0.012	0.042	0.034				-0.001
	(-1.84)	(3.37)	(-2.19)				(-1.00)
6	0.044		0.024				0.02
	(1.07)		(1.46)				(2.01)
7	-0.000	0.015	0.247	0.000	-0.098	0.016	
	(-0.00)	(-)	(1.60)	(0.24)	(-3.65)	(1.37)	
8	-0.013	0.055	0.028	-0.003	-0.094	-0.011	
	(-1.95)	(3.92)	(1.83)	(-2.01)	(-3.54)	(-0.84)	

B. Appendix Liquidity Factors

B.1 Liquidity Factors

In most empirical asset pricing litterateur the measurement for liquidity is often taken to be exogenous. It is derived by the securities executed sell and buy orders. Some of these proxies for measuring liquidity are spread(quoted,effective, realized), VWAP(Volume-Weighted Average Price), illiquidity based on price impact or return covariance[24]. In this section we provide some literature evidence on why some stocks are more liquid than others and what role these factors play for the required returns. Please note that we are not going to try to determine the source of these liquidity factor in the Norwegian stock exchange, nor are they relevant for our thesis. We simply wish to provide the reader an understanding on how market microstructure works.

B.1.1 Asymmetric Information

Under perfect market conditions and complete markets all all agents would have the same information regarding the payoff for a risky asset. However, these imperfections exist and agents have access to various sources of information. The work done by Glosten & Milgrom (1985) takes into account trading with more informed agents. The model they present is based on the premise that liquidity suppliers(meaning the dealers in the stock exchanges) are less informed than the investors and thus will require a compensation for this risk taking[25]. Such a compensation leads to increasing illiquidity premium in the likelihood of informed trading. They conclude that in the event where informed traders have more valuable information and trade more often then the illiquidity premium should increase as well. Using a probability of informed trading Easley et al. (2002) also find similar results. Using intraday buy and sell orders, they develop a model for monthly returns in a period of 1984-1998 using NYSE stock data and controlling for Fama & French (1993) factors. They found that a higher probability of informed trading leads to higher predicted asset returns.

[24]Measuring market depth in Stoll (2000) and return covariance first estimated by Roll (1984).

[25]Some of these losses are covered by trading with uninformed traders

B.1.2 Transaction Cost

Liquidity providers usually have explicit cost related to order-processing, such as trading fees, settlement fees etc. This is also additional cost borne by liquidity suppliers which affects the bid-ask spread. In most literature transaction cost is dependent on the size and direction of a trade. Lo et al. (2004) find that a small change in order-processing cost had a significant impact on the frequency of trades and the illiquidity premium. Vayanos & Wang (2009) expanded on this research and find an increase in transaction cost deter liquidity suppliers from trading, hence raising illiquidity. An increase in transaction cost as a cause of underlying illiquidity contributed to the raise of expected asset returns.

B.1.3 Inventory Risk

Inventory holding cost is the most common risk that liquidity suppliers face. Dealers that hold inventory are at all time exposed to the potential devalue of these holdings(i.e when new information arrives) and therefor will require a compensation. Stoll (1978) was one of the first to study this relationship. Following each NASDAQ stock for six days with regards to volume traded by dealers they measured dealers willingness to bear inventory risk and concluded that this variable had a significant effect on the illiquidity measurement and lead to higher stock prices. Most recent study done by Hendershott & Menkveld (2014) looked at the illiquidity in the presence of inventory risk. Using 12 years of data from NYSE they developed a model where they examined the deviation from the fundamental values of securities that is born by inventory risk. A $100 000 inventory shock for dealers lead to 0.98 percent deviation in prices for small-cap stocks and 0.02 percent for large-cap stocks. These results makes sense since fewer people are interested in buying smaller stocks which leads to it taking a longer time for the dealer to unwind his position. The liquidity demand is also less elastic. Dealers are in this scenario exposed to risk for a longer time and thus require higher compensation for their troubles.

Another interesting finding by Huang & Stoll (1997) is the price impact for different trade sizes. They find that cost of liquidity provision is dependent on trade size. During large trades the inventory cost accounts for two-thirds of the spread, while transaction cost impact is reduced to half its value. Explanation for these findings is that large orders lead to dealers taking larger inventory, while fixed transaction cost can be spread over a larger trade. The adverse selection component is unaffected by large trade sizes due to information leak with these orders. Dealers also do their research when large orders arrive in order to avoid loosing out.

Though each liquidity factors discussed above plays a role in contributing to the liquidity of an asset, they are not the sole contributor. Chordia et al. (2000) state that inventory risk and asymmetric information both affect inter temporal changes in stock liquidity. Huang & Stoll (1997) use all these factors in the same setting and find that adverse selection cost accounts for 9.59 percent of the bid-ask spread. Inventory risk and transaction cost accounted for 28.65 percent and 61.76 percent respectively

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