Notes

Week 4

Module 2 Week 4C

ANOVA

One-Way ANOVA

- **An**alysis **o**f **Va**riance
- Recall using a t-test to examine *differences in means between two groups*.
- We will now extend this to the case of k groups (more than 2 groups).
- The entire variation in the outcome of interest will be decomposed into separate components.
- Examples:
    - Is there a difference in income between lawyers, professors, and doctors?
    - Is there a difference in revenue between McDonalds, Burger King, and Wendy's?
    - Is there a difference in GPA between 1st year, 2nd year, junior, and senior students?
- We will focus on the simple case of *One-Way ANOVA*

- *Variances* are used to determine if *means* differ across groups.
- *Assumptions*:
    - Each population from which the sample is taken is normal
    - All samples are random and independent
    - Populations have equal variances
    - Each factor is categorical (e.g. profession, restaurant type)
    - Each response (outcome of interest) is numerical (e.g. income, revenue)
- H_0: all means are equal
- H_a: at least two means differ

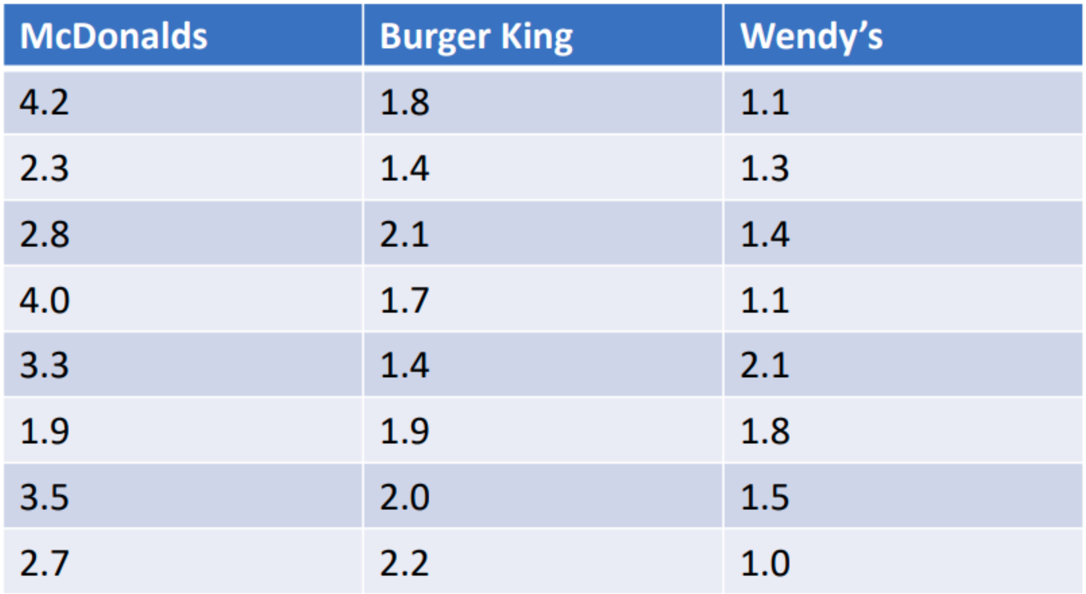
- ANOVA uses the ***F-distribution***
    - Derived from the *Student's t-distribution*
- **F-statistic** is a ratio with numerator df and denominator df
- Variance *between* samples
    - An estimate of overall variance
    - Variance of the sample means *from the overall mean*

    - Also known as *"Variance Due to Treatment"* or *"Explained Variation"*.
- Variance *within* samples
    - An estimate of overall variance
    - Variance of observations *within* a category *from that category’s mean*
    - Also known ads *"Variation due to Error"* or *"Unexplained Variation"*.

- **Sum of squares *total* (SST)** = $\sum_i (X_i - \bar{X})^2$
- **Sum of squares between (SSB)** $= \sum_k n_k(\bar{X}_k - \bar{X})^2$
    - Often called the *explained* or *model sum of squares*
- **Sum of squares within (SSW)** $= \sum_k (n_k - 1)(s_k^2)$
    - Often called the *sum of squares due to error (SSE)*
- **SST = SSB + SSW**
- **Mean squared within (MSW)** $= \frac{SSW}{dfw} = \frac{SSW}{n – k}$
    - Often denoted *MSE for mean squared error*
- **Mean squared between (MSB)** $= \frac{SSB}{dfb} = \frac{SSB}{k – 1}$
- *F-test is all about comparing differences between groups relative to differences within groups.*

- *MSB* can be influenced by differences in population means among the different groups.
- *MSW* is not influenced by differences in population means among the different groups.
- H_0: *populations all have the same normal distribution*
    - Remember, we *assume equal variances and normality*, so if means are equal, the normal distributions for each group are the same.
    - *If H_0 is true, MSB and MSW should be about the same*
- **F-stat = MSB/MSW**
    - If H_0 is true, the F-stat ≈ 1
    - Always a *Right-Tailed Test*

- E.g. Is there a difference in mean sales between McDonald's, Burger King, and Wendy’s?
    - Suppose we have the following random sample of data on annual sales

- 

McDonalds	Burger King	Wendy’s
4.2	1.8	1.1
2.3	1.4	1.3
2.8	2.1	1.4
4.0	1.7	1.1
3.3	1.4	2.1
1.9	1.9	1.8
3.5	2.0	1.5
2.7	2.2	1.0

- $\bar{X} = 2.104$, $\bar{X}_{McDon} = 3.09$, $\bar{X}_{BK} = 1.81$, $\bar{X}_{Wendy's} = 1.41$
- $SST = 18.43$
- $SSB = (8 * (3.09–2.104)^2) + (8 * (1.81–2.104)^2) + (8 * (1.41–2.104)^2) = 12.32$
- $SSW = SST–SSB = 18.43–12.32 = 6.11$
- $MSB = 12.32/(3–1) = 6.16$
- $MSW = 6.11/(24–3) = 0.291$
- $\text{F-stat} = 6.16/0.291 = 21.17$
- Numerator $df = 2$, denominator $df = 21$
- $\text{p-value} = 0.000009$
- $0.000009 < 0.05$ => reject H_0, there is a difference in mean sales between the three restaurants.