

Math Camp

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Where are we going?

Probability Theory:

- 1) Mathematical model of uncertainty
- 2) Foundation for statistical inference
- 3) Continues our development of key skills
 - Proofs [precision in thinking, useful for formulating arguments]
 - Statistical computing [basis for much of what you'll do in graduate school]

Model of Probability

Three parts to our probability model

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- 1) **Sample space**: set of all things that could happen

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Three parts to our probability model

- 1) **Sample space**: set of all things that could happen
- 2) Events: subsets of the sample space
- 3) Probability: **chance** of an event

Sample Spaces: All Things that Can Happen

Definition

*The **sample space** as the set of all things that can occur. We will collect all distinct outcomes into the set S*

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Key point: this defines **all possible realizations**

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Notation:

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Suppose $E = W$, $F = N$. Then $E \cap F = \emptyset$ (there is nothing that lies in both sets)

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 - $E \cap F = (H, H)$
 - $E \cap G = \emptyset$
 - $F \cap G = \emptyset$
- Suppose $S = \mathbb{R}_+$. $E = \{x : x > 10\}$ and $F = \{x : x < 5\}$. Then $E \cap F = \emptyset$.

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Definition

Suppose we have events E_1, E_2, \dots, E_N .

Define:

$$\cup_{i=1}^N E_i = E_1 \cup E_2 \cup E_3 \cup \dots \cup E_N$$

$\cup_{i=1}^N E_i$ is the set of outcomes that occur at least once in E_1, \dots, E_N .

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Define:

$$\cap_{i=1}^N E_i = E_1 \cap E_2 \cap \dots \cap E_N$$

$\cap_{i=1}^N E_i$ is the set of outcomes that occur in each E_i

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 - Domain: all events E

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- 1) *For all events E ,*
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- 3) *For all sequences of mutually exclusive events E_1, E_2, \dots, E_N (where N can go to infinity)*

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- 1) *For all events E ,*
 $0 \leq P(E) \leq 1$
- 2) $P(S) = 1$
- 3) *For all sequences of mutually exclusive events E_1, E_2, \dots, E_N (where N can go to infinity)*
 $P(\cup_{i=1}^N E_i) = \sum_{i=1}^N P(E_i)$

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- Suppose we are flipping a pair of fair coins. Then $P(H, H) = 1/4$

Example: Congressional Elections

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We'll use **data** to infer these things

Properties of Probability

We can derive intuitive properties of probability theory.

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In words: Probability an outcome in E happens is 1— probability an outcome in E doesn't.

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As you add more “outcomes” to a set, it can’t reduce the probability.

Examples in R

Simulation: use pseudo-random numbers, computers to gain evidence for claim

Tradeoffs:

Pro Deep understanding of problem, easier than proofs

Con Never as general, can be deceiving if not done carefully (also, never a monte carlo study that shows a new method is wrong)

Walk through R code to simulate these two results

To the R code!

4.2. *Three different combination rules were used.* We then tried to identify the rules used to combine individual drug predictions into a combination score. Letting $P()$ indicate probability of sensitivity, the rules used are:

$$P(TFAC) = P(T) + P(F) + P(A) + P(C) - P(T)P(F)P(A)P(C),$$

$$P(TET) = P(ET) = \max[P(E), P(T)], \text{ and}$$

5
1

Inclusion/Exclusion

Proposition

Suppose E_1, E_2, \dots, E_n are events. Then

$$\begin{aligned} P(E_1 \cup E_2 \cup \dots \cup E_n) &= \sum_{i=1}^N P(E_i) - \sum_{i_1 < i_2} P(E_{i_1} \cap E_{i_2}) + \dots \\ &\quad + (-1)^{r+1} \sum_{i_1 < i_2 < \dots < i_r} P(E_{i_1} \cap E_{i_2} \cap \dots \cap E_{i_r}) \\ &\quad + \dots + (-1)^{n+1} P(E_1 \cap E_2 \cap \dots \cap E_n) \end{aligned}$$

Proof: Version 1, Intuition

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Inclusion/Exclusion

Corollary

Suppose E_1 and E_2 are events. Then

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

R Code!

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Consider events E_1 and E_2 . Then

$$P(E_1 \cap E_2) = P(E_1) - P(E_1 \cap E_2^c)$$

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Easy Problems

Surprising Probability Facts: Birthday Problem

Probabilistic reasoning pays off for harder problems

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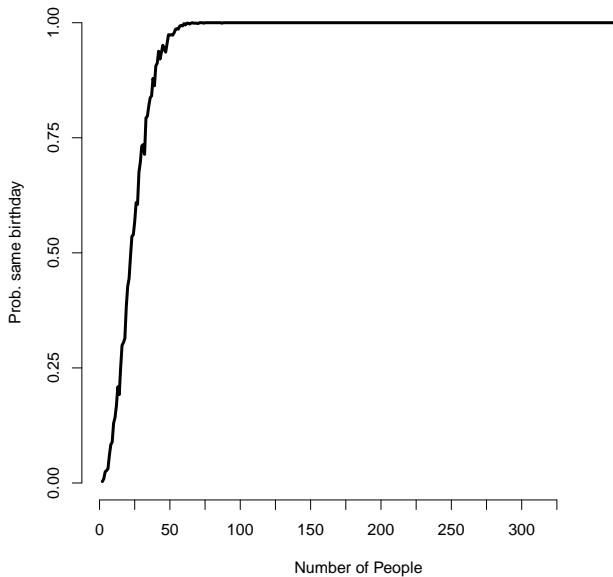
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- Assuming leap year counts, $N = 367$ guarantees at least two people with same birthday (**pigeonhole principle**)
- For $N < 367$?
- Examine via simulation



Surprising Probability Facts: the E-Harmony Problem

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1 in 536,870,912 people

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Across many “variables” (events) agreement is harder

Probability Theory

- Today: Introducing probability model
- Conditional probability, Bayes' rule, and independence