



Analytical Framework for Autonomous Charging Station Enabling Electrification of Transportation System

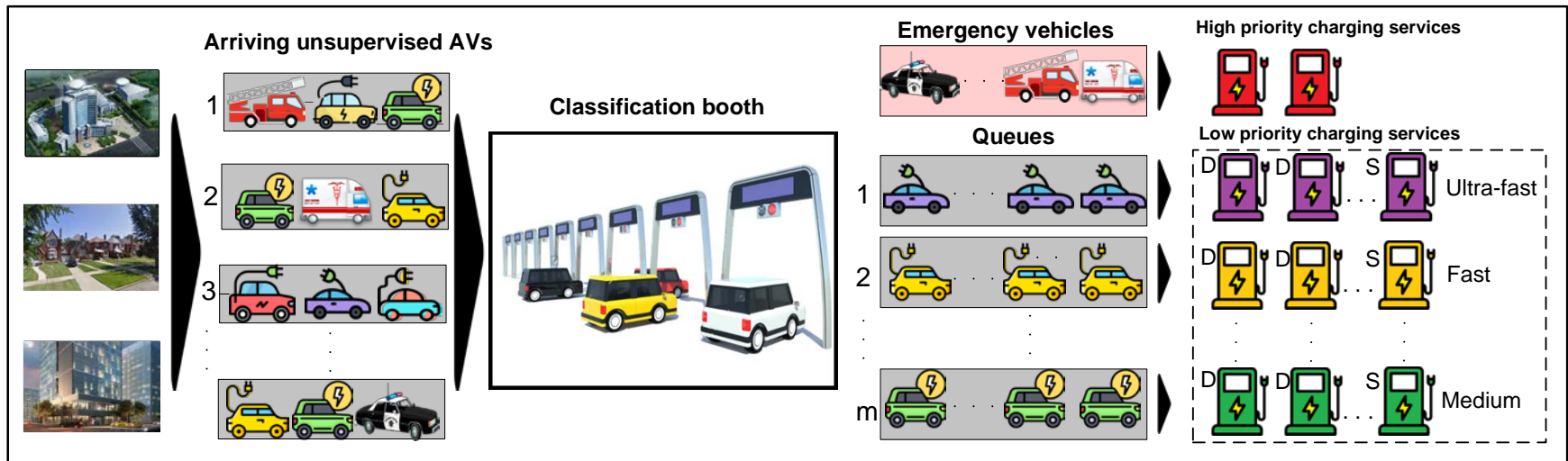
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**Charlotte
2019**

Proposed Autonomous Charging Station

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Servicing Flow



Tier of charge	Speed	Charging rate, kW	Time to full charge*, hours	Application
1	Slow	3	6 – 8	Overnight at home
2	Medium	6	3 – 4	Commercial and public on-street
3	Fast	120	0.25 – 0.6	Commercial and public on-street
4	Ultra-fast	450	0.5 – 0.75	Deployment stage

*Time to full charge is defined based on an average battery size for a given tier of charge



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Proposed Autonomous Charging Station

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Functional Architecture



Physical Infrastructure: power generators, their control and monitoring gear; electric chargers or fuel pumps; security cameras; wireless and/or wired networking equipment; computer servers; and associated hardware;

Communication: information exchange between the AVs and the charging station processing servers;

Classification: matching the needs of the AVs with the capabilities of the charging station;

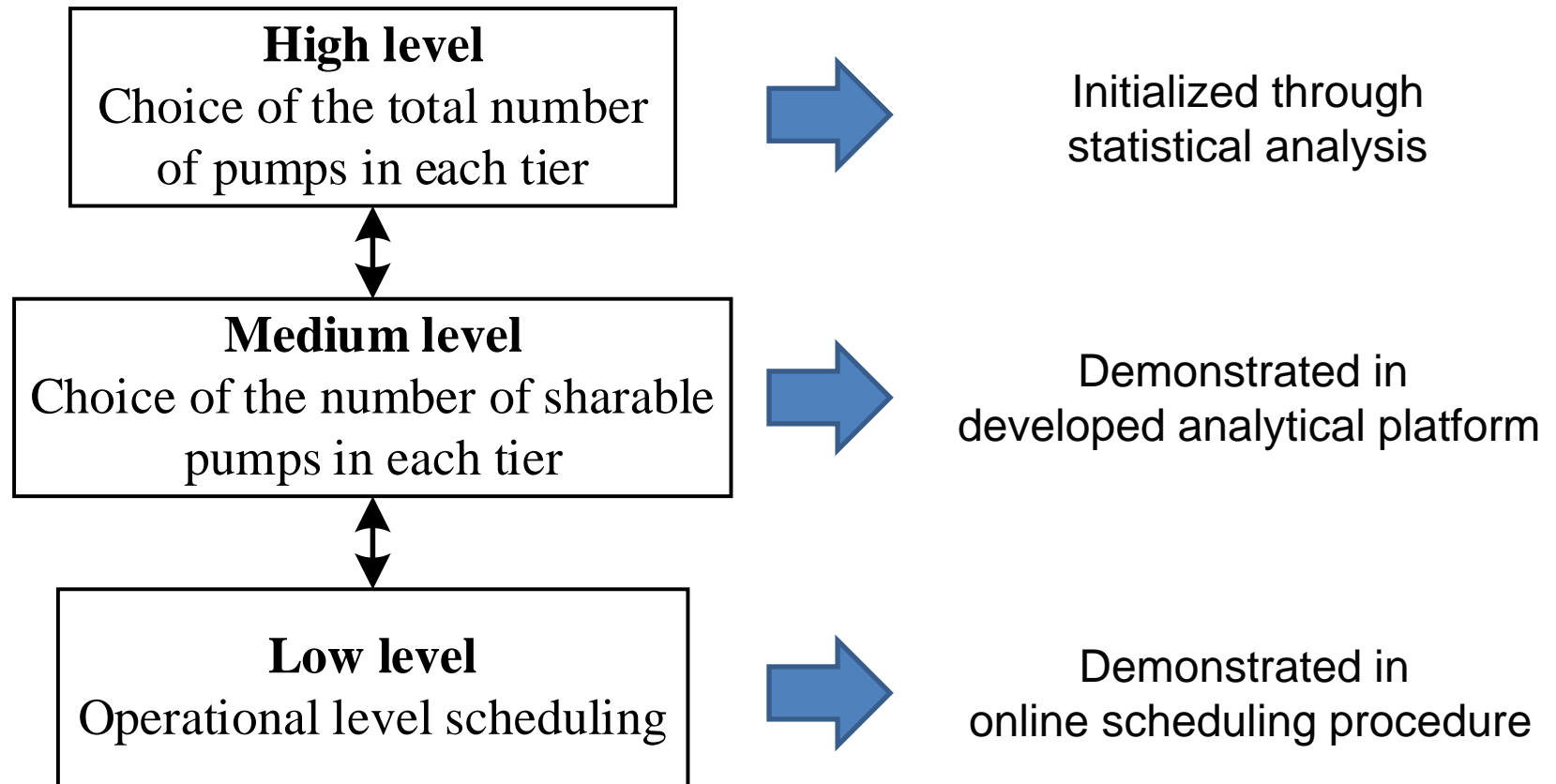
Queueing: flexible queueing system, where incoming classified AVs are paired with the most suitable charging pumps;

Financial: vehicles' authorization and payment transactions;

Services: will charge/fuel AVs based on information exchanged between the lower layers.



Theoretical background and Methodology



Theoretical background and Methodology

- **Discrete-event simulation**
 - scalable for high complexity systems;
 - verification of analytical approach;
- Multiple charging rates → **multi-server queueing system**;
- **Pooled** queueing system
 - queue discipline “first come first serve” (priorities for featured services);
 - priority line for emergency vehicles and services;
- **Classification** based on pump sharing priority rule:
 1. Same tier designated;
 2. Same tier sharable;
 3. Next highest tier sharable;
 4. Opportunistic charging and reject & referral;
- Any existing **dynamic pricing model** can be adapted.



Theoretical background and Methodology

- Flexibility of vehicles' arrivals due to their autonomous nature → homogeneous arrival rates over 24 hours;
- Charging requests are described by Poisson Process with hourly arrival rate:

$$\lambda_a = \frac{f_a N_a}{24 \times 7}$$

N_a and f_a - number of AVs of tier a in the region and the average weekly frequency of their visits per week;

- Inter-arrival time is described by expon. distribution: $IA_a(t) = \lambda_a \cdot e^{-\lambda_a t}$;
- Probability of a single arrival during short time period h : $\lambda_a h$.

Vehicle	Average weekly frequency per vehicle	
	Personal	Service
Tesla Model S	0.6	4.72
Tesla Model X	0.59	4.67
Chevy Volt	0.42	3.3
Ford Fusion Energi	0.32	2.5
Nissan Leaf	1.64	12.9



Input Parameters

- λ = arrival rate of charging requests (veh/hour): described by Poisson Process;
- IA = inter-arrival time between requests (mins): described by exponential distribution;
- n = number of tiers; R_i = ratio of vehicles in tier $i \in n$: obtained from preliminary analysis;
- E_i = energy acceptance rate of tier $i \in n$ (kW); B_i = battery capacity of tier $i \in n$ (kWh);
- N = number of vehicles;
- $ED_j^i = U_j[0.7 \dots 0.9] * B_i$ = energy demand of vehicle $j \in N$ from tier $i \in n$ (kWh);
- $A_j = U[5 \dots 30]$ = earliest arrival time of vehicle $j \in N$ (mins);
- $D_j^i = U_j[0.9 \dots 1.1] * t_i$ = charging due date of vehicle $j \in N$ from tier $i \in n$ (mins) with average waiting time for this tier t_i .

Output Parameters

- N_R, N_S = number of rejected and serviced vehicles (overall and per tier); $N_R + N_S = N$;
- $\sum_{j=1}^{N_S} ED_j^i(t)$ = power consumption profile: amount of power consumed by ACS over time;
- $Ut_{tot}, Ut_1, \dots, Ut_n$ = utilization of pumps (total and for tier $i \in n$);
- Scheduling calendar: timeline of each pump utilization.



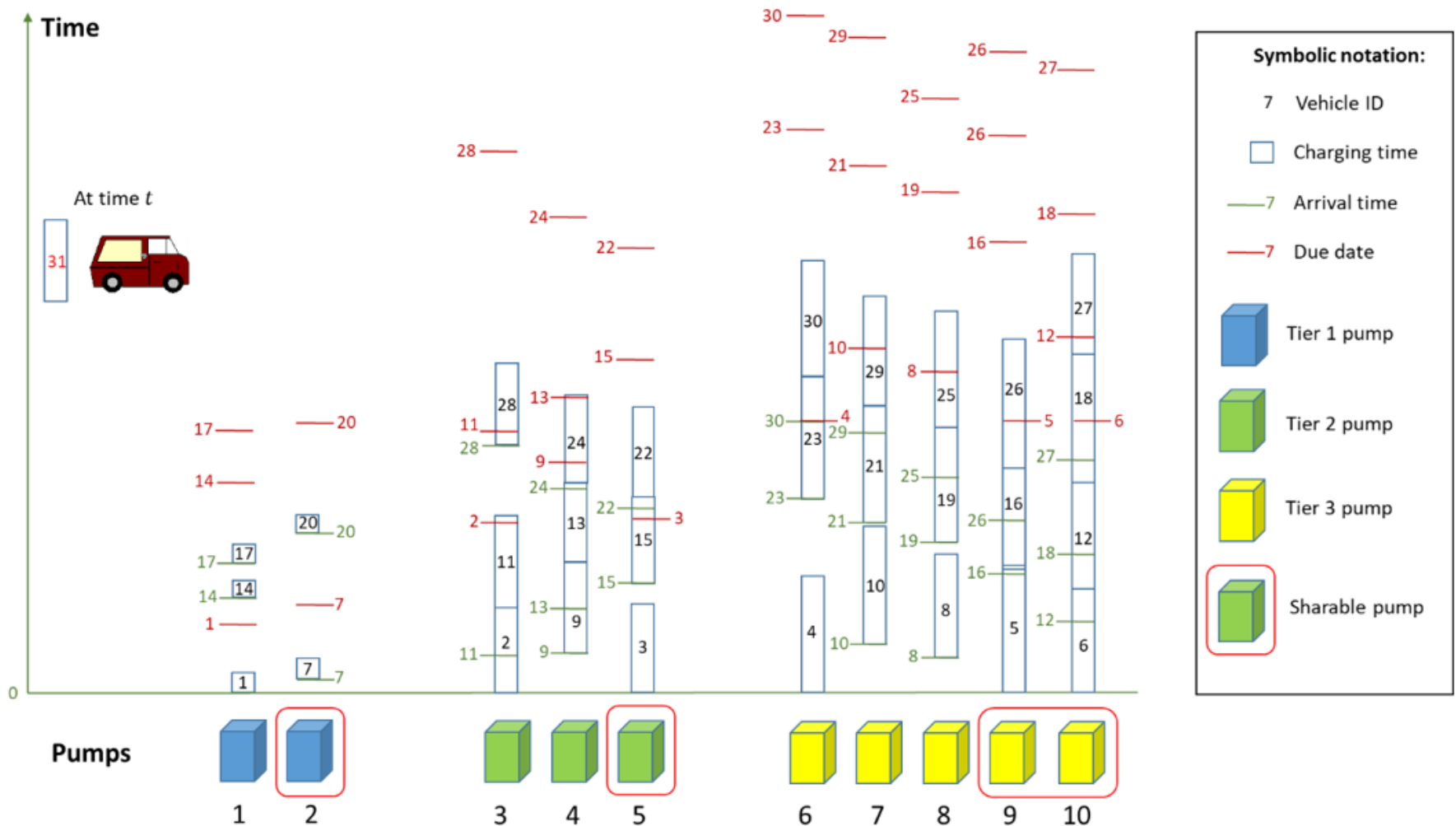
Assumptions

- Focusing on classification and queueing layers and utilizing existing technologies for the other layers of functional architecture;
- Charging rate of each pump in a tier is equal to the power acceptance rate of the vehicles in the same tier;
- Real-time pricing and waiting times are provided by charging stations;
- Vehicles can't be charged at lower tier pumps;
- No queue disruption for scheduled vehicles;
- Guarding time intervals and operation failures are neglected.



Analytical Framework

Scheduling Procedure at ACS (example)



Mathematical Formulation

Input Data:

$N_k = \{1, 2, \dots, n_k\}$ = index set of existing vehicles assigned to pump k at the time of scheduling a new vehicle N . N_k is a totally ordered set, where $i < j$ for $i, j \in N_k$ implies that vehicle i is charged before vehicle j .

a_i = arrival time of vehicle (= 0 if arrived already); d_i = due date of vehicle $i \in N_k$

c_i = charging time of vehicle $i \in N_k$ at the current queue

w_i = penalty imposed on the completion time of vehicle $i \in N_k$

a_N = earliest arrival time of new vehicle; d_N = due date of new vehicle

c_N = charging time of new vehicle at the current pump

w_N = penalty imposed on the completion time of new vehicle

$y_0 = 0$ defined for the sake of simplicity of formulation



Mathematical Formulation

Decision Variables:

x_i = start time of charging vehicle $i \in N_k$

x_N = start time of charging new vehicle

y_i = completion time of charging vehicle $i \in N_k$

y_N = completion time of charging new vehicle

z_i = 1 if new vehicle is placed right before vehicle i for $i \in N_k$; 0 otherwise

z_L = 1 if new vehicle is placed after vehicle n_k ; 0 otherwise



Mathematical Formulation

Sum of charging completion times

$$\begin{aligned} \text{Minimize} \quad & \sum_{i=1}^{n_k} w_i y_i + w_N y_N \\ \text{subject to} \quad & y_i = x_i + c_i \quad \text{for } i \in N_k \end{aligned}$$

$$y_N = x_N + c_N; \quad x_N \geq a_N; \quad y_N \leq d_N$$

$$x_i \geq a_i \quad \text{for } i \in N_k$$

$$y_i \leq d_i \quad \text{for } i \in N_k$$

$$x_i \geq y_{i-1}(1 - z_i) + y_N z_i \quad \text{for } i \in N_k$$

$$x_N \geq y_{i-1} z_i \quad \text{for } i \in N_k$$

$$x_N \geq y_{n_k} z_{last}; \quad \sum_{i \in N_k} z_i + z_{last} = 1$$

$$x_i \geq 0, x_N \geq 0, y_i \geq 0, y_N \geq 0; \quad z_i, z_N \in \{0, 1\}.$$

Optimization Objective:
minimize the sum of charging completion times of all vehicles in the queue including the newly arrived vehicle



Overall Structure and Flowchart

Divided into 3 modules:

- **Vehicle Data Generation**
- **Pump Data Generation**
- **Queueing and Scheduling**

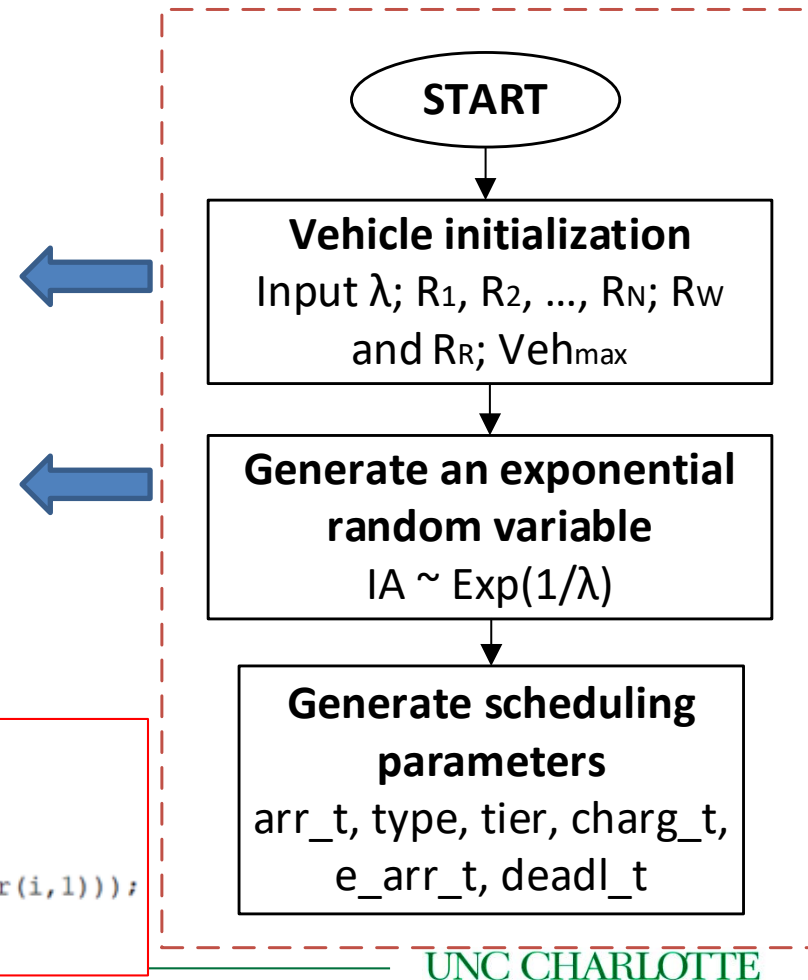
- Charging requests' arrival rate: λ ;
- Ratio in each tier: R_1, R_2, \dots, R_N ;
- Walk-in or reservation requests' ratios: R_W, R_R ;
- Maximum number simulated: Veh_{max} .

```
IA = 60*expnd(1/lambda,Veh_max-1,1);  
e_arr_t=5+25*rand(Veh_max,1);
```

- type: reservation or walk-in.

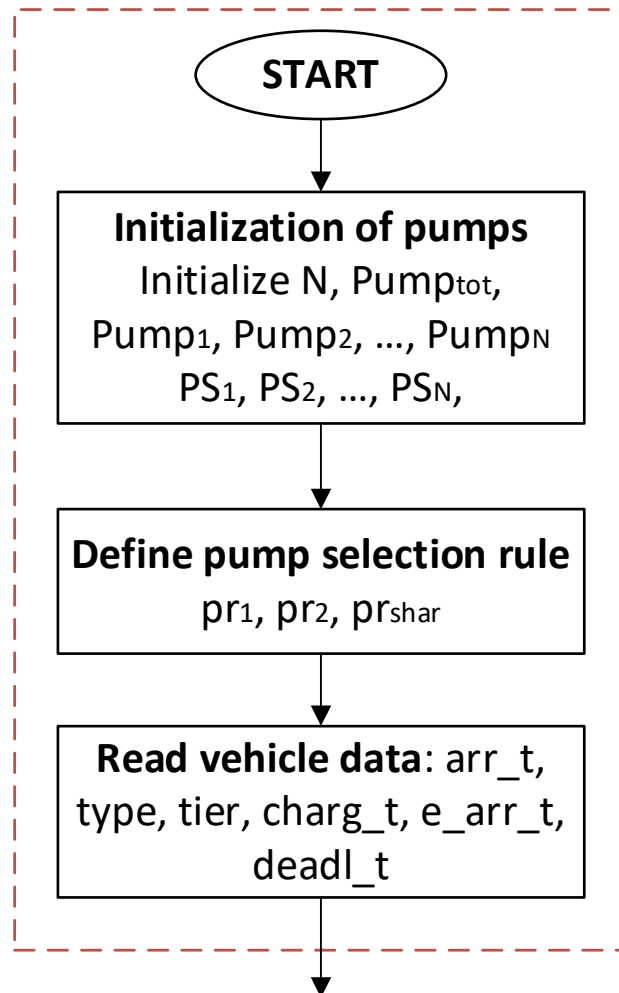
```
for i=1:Veh_max  
    en_dem(i,1)=(0.2*rand+0.7)*Bat_cap(tier(i,1));  
    deadl_t(i,1)=(0.4*rand+0.8)*Tier_Wait_Aver(tier(i,1));  
    for n=1:N  
        charg_t(i,n)=60*en_dem(i,1)/min(Ac_rate(n),Ac_rate(tier(i,1)));  
    end  
end
```

Vehicle Data Generation



Overall Structure and Flowchart

Pump data generation



- The number of tiers: N ;
- The total number of pumps: $Pump_{tot}$;
- Numbers of pumps per tier: $Pump_1, \dots, Pump_N$;
- Numbers of sharable pumps per tier: PS_1, \dots, PS_N ;
- Defining priorities for pump selection rule:
 - pr_1 - designated pumps of the same tier;
 - pr_2 - sharable pumps of the same tier;
 - pr_{shar} is used for sharable pumps of higher tier

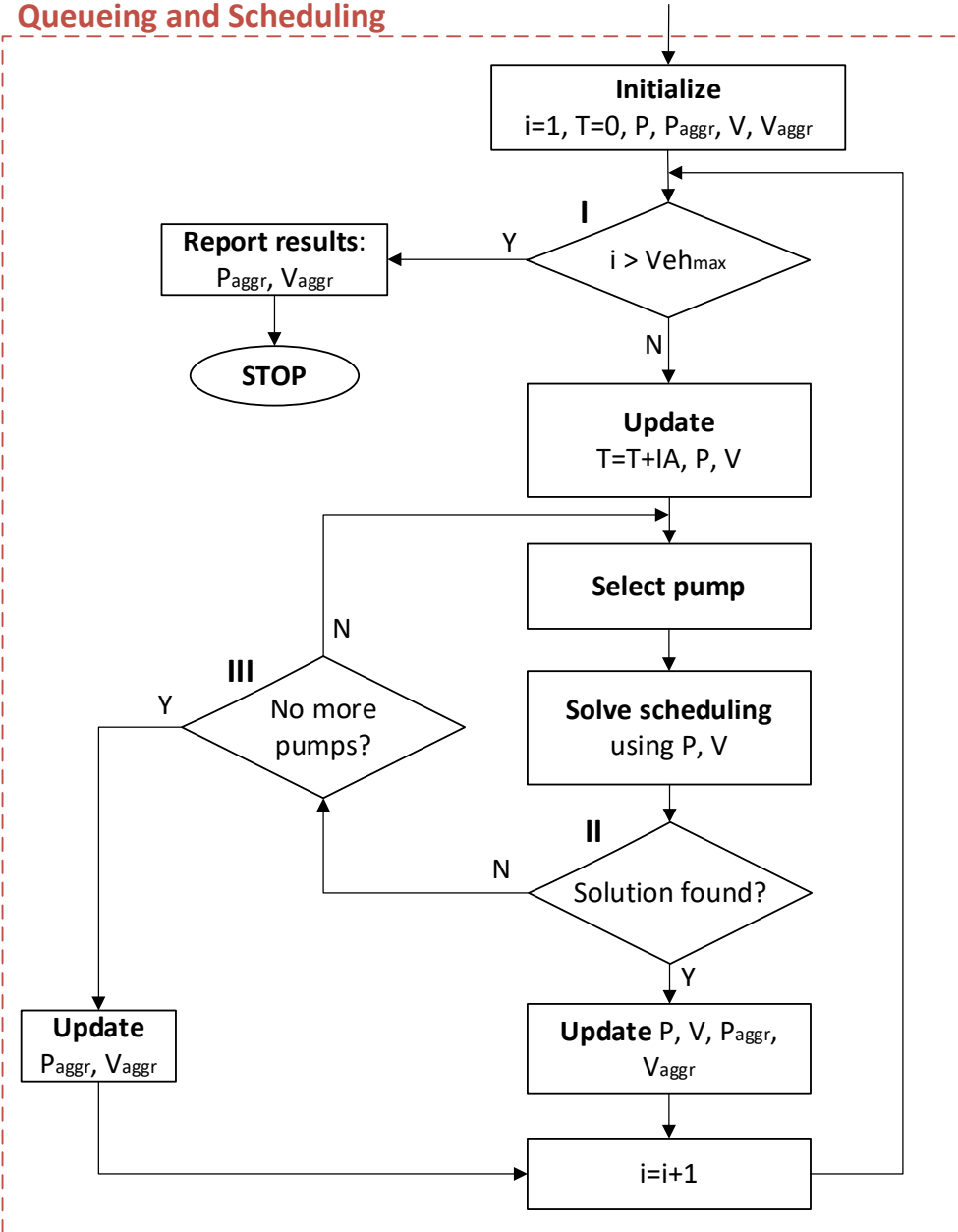
Sharable pumps		Pump selection rule		
Tier 1	Tier 2	Tier 1	Tier 2	Tier 3
0	0	1, 2	3, 4, ..., 11	12, ..., 32
1	3	2, 1	4, ..., 11, 3, 1	12, ... 32, 3, 1

Retrieving results from
vehicle data generation module

Overall Structure and Flowchart

- ❖ Initialization and update of
 - current clock time at each iteration;
 - currently scheduled vehicle;
 - power consumption of ACS;
 - current and overall pump status matrices (P and P_{aggr});
 - current and overall vehicle status matrices (V and V_{aggr});
- ❖ Vehicles' classification by pump selection;
- ❖ Solution of charging scheduling optimization for each vehicle;
- ❖ Reporting results

Queueing and Scheduling



Key Data Structures

V – current state vehicles matrix showing vehicles' scheduling parameters along with their scheduling results for vehicles that are currently in the system;

V												
ID	type	tier	arr_t	e_arr_t	deadl_t	res_arr_t	charg_t	compl_t	Pump#	act_arr_t	act_compl_t	
1	1	3	0.00	25.28	466.00	0.00	228.65	3.65	12	223.18	451.83	
2	1	2	1.77	14.49	326.91	0.00	162.01	21.50	3	16.26	178.27	
3	1	3	2.45	12.98	487.73	0.00	207.75	66.41	12	15.43	223.18	
4	1	2	10.95	29.65	240.60	0.00	152.93	36.76	4	40.60	193.53	
5	1	3	23.21	22.95	561.96	0.00	218.02	18.23	13	226.09	444.11	
6	1	3	32.16	15.33	542.33	0.00	178.60	1.48	13	47.49	226.09	
7	1	2	38.46	7.47	359.66	0.00	155.18	14.65	3	178.27	333.45	
10	1	2	64.21	6.85	331.66	0.00	137.02	11.75	4	193.53	330.55	
11	1	1	67.35	8.01	101.00	0.00	34.87	9.18	1	131.07	165.95	
12	1	3	78.24	29.54	426.06	0.00	202.10	6.75	14	107.78	309.88	
13	1	2	79.30	17.42	278.44	0.00	160.83	4.92	5	96.72	257.54	
14	1	3	90.02	5.56	433.89	0.00	221.84	7.03	15	95.58	317.42	
15	1	3	94.15	6.35	420.15	0.00	222.88	4.58	16	100.50	323.38	
16	1	3	94.15	8.52	390.08	0.00	216.83	0.71	17	102.68	319.51	
17	1	1	100.36	27.34	123.21	9.18	33.55	42.73	1	165.95	199.50	
18	1	3	118.70	16.65	424.81	0.00	197.34	7.35	14	309.88	507.22	
19	1	2	127.96	19.02	257.45	0.00	156.08	19.04	6	146.98	303.06	
20	1	3	133.49	17.36	386.47	0.00	203.08	0.26	18	150.85	353.93	

P – current state pumps matrix showing pumps' tiers, e.g., ultra-fast, fast, and medium, pumps' types: sharable or designated, and vehicles currently scheduled for these pump (i.e., currently in the system);

P						
Pump#	tier	type	CHARGED VEHICLES IDs			
1	1	0	11	17	0	0
2	1	0	0	0	0	0
3	2	0	2	7	0	0
4	2	0	4	10	0	0
5	2	0	13	0	0	0
6	2	0	19	0	0	0
7	2	0	0	0	0	0
8	2	0	0	0	0	0
9	2	0	0	0	0	0
10	2	0	0	0	0	0
11	2	0	0	0	0	0
12	3	0	3	1	0	0
13	3	0	6	5	0	0
14	3	0	12	18	0	0
15	3	0	14	0	0	0
16	3	0	15	0	0	0
17	3	0	16	0	0	0
18	3	0	20	0	0	0

Key Data Structures

V_{aggr} – overall vehicles matrix showing vehicles' scheduling parameters along with their scheduling results for all vehicles that have been served and currently in the system.

Vaggr												
ID	type	tier	arr_t	e_arr_t	deadl_t	res_arr_t	charg_t	compl_t	Pump#	act_arr_t	act_compl_t	
1	1	3	0.00	25.28	466.00	0.00	228.65	3.65	12	223.18	451.83	
2	1	2	1.77	14.49	326.91	0.00	162.01	21.50	3	16.26	178.27	
3	1	3	2.45	12.98	487.73	0.00	207.75	66.41	12	15.43	223.18	
4	1	2	10.95	29.65	240.60	0.00	152.93	36.76	4	40.60	193.53	
5	1	3	23.21	22.95	561.96	0.00	218.02	18.23	13	226.09	444.11	
6	1	3	32.16	15.33	542.33	0.00	178.60	1.48	13	47.49	226.09	
7	1	2	38.46	7.47	359.66	0.00	155.18	14.65	3	178.27	333.45	
8	1	1	45.94	23.36	140.14	0.00	30.19	3.12	1	100.88	131.07	
9	1	1	46.40	20.93	101.06	0.00	33.55	6.73	1	67.33	100.88	
10	1	2	64.21	6.85	331.66	0.00	137.02	11.75	4	193.53	330.55	
11	1	1	67.35	8.01	101.00	0.00	34.87	9.18	1	131.07	165.95	
12	1	3	78.24	29.54	426.06	0.00	202.10	6.75	14	107.78	309.88	
13	1	2	79.30	17.42	278.44	0.00	160.83	4.92	5	96.72	257.54	
14	1	3	90.02	5.56	433.89	0.00	221.84	7.03	15	95.58	317.42	
15	1	3	94.15	6.35	420.15	0.00	222.88	4.58	16	100.50	323.38	
16	1	3	94.15	8.52	390.08	0.00	216.83	0.71	17	102.68	319.51	
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19	1	2	127.96	19.02	257.45	0.00	156.08	19.04	6	146.98	303.06	
20	1	3	133.49	17.36	386.47	0.00	203.08	0.26	18	150.85	353.93	

P_{aggr} – overall pumps matrix showing parameters their tier and type (in a manner similar to matrix P), and all vehicles that have been served and currently in the system.

Paggr							
Pump#	tier	type	CHARGED VEHICLES IDs				
1	1	0	9	8	11	17	
2	1	0	0	0	0	0	
3	2	0	2	7	0	0	
4	2	0	4	10	0	0	
5	2	0	13	0	0	0	
6	2	0	19	0	0	0	
7	2	0	0	0	0	0	
8	2	0	0	0	0	0	
9	2	0	0	0	0	0	
10	2	0	0	0	0	0	
11	2	0	0	0	0	0	
12	3	0	3	1	0	0	
13	3	0	6	5	0	0	
14	3	0	12	18	0	0	
15	3	0	14	0	0	0	
16	3	0	15	0	0	0	
17	3	0	16	0	0	0	
18	3	0	20	0	0	0	

Key Coding Blocks: Solve Scheduling

Objective: Minimize $w^T y + w_N y_N$

```
%% 1. Creating row-vector f (coef of objective function)
```

```
f = zeros(1,5*nk+3); f(nk+2:2*nk+2) = 1;
```

```
%% 2. Creating matrix A (coef of constraints)
```

```
A = A_matr(nk);
```

```
%% 3. Creating column-vector b (rhs constants)
```

```
b = b_vect(nk,Char_time,E_arr_time,Due_date);
```

```
%% 4. Creating column-vector lb (low bound)
```

```
lb = zeros(1,size(f,2));
```

```
%% 5. Creating row-vector Vtype (var type: continuous or binary)
```

```
Vtype(1:5*nk+3) = 'C';
```

```
Vtype(2*nk+3:3*nk+3) = 'B';
```

```
%% 6. Creating row-vector sense ('>,<,' signs for constraints)
```

```
sense = sense_vect(nk);
```

Linearization of
bilinear terms

Creating linear constraints:

$$A * x = b$$

Creating bound
constraints: $x \geq lb$

Defining variable types:
continuous (C) or binary (B)



Key Coding Blocks: MILP model setup for $N_k=2$

X1	X2	XN	Y1	Y2	YN	Z1	Z2	ZL	VN1	V12	VN2	V2L	Description	sense	b_vect
-1	0	0	1	0	0	0	0	0	0	0	0	0	Y1-X1	=	charg_t
0	-1	0	0	1	0	0	0	0	0	0	0	0	Y2-X2	=	charg_t
0	0	-1	0	0	1	0	0	0	0	0	0	0	YN-XN	=	charg_t
0	1	0	0	0	0	0	0	0	0	0	0	0	X2	>	arr_t
0	0	1	0	0	0	0	0	0	0	0	0	0	XN	>	arr_t
0	0	0	1	0	0	0	0	0	0	0	0	0	Y1	<	deadl_t
0	0	0	0	1	0	0	0	0	0	0	0	0	Y2	<	deadl_t
0	0	0	0	0	1	0	0	0	0	0	0	0	YN	<	deadl_t
1	0	0	0	0	0	0	0	0	-1	0	0	0	X1-VN1	>	0
0	0	0	0	0	-1	-10000	0	0	1	0	0	0	VN1-YN-10000*Z1	>	-10000
0	0	0	0	0	-1	0	0	0	1	0	0	0	VN1-YN	<	0
0	0	0	0	0	0	-10000	0	0	1	0	0	0	VN1-10000*Z1	<	0
0	1	0	-1	0	0	0	0	0	0	1	-1	0	X2-Y1+V12-VN2	>	0
0	0	0	-1	0	0	0	-10000	0	0	1	0	0	V12-Y1-10000*Z2	>	-10000
0	0	0	-1	0	0	0	0	0	0	1	0	0	V12-Y1	<	0
0	0	0	0	0	0	0	-10000	0	0	1	0	0	V12-10000*Z2	<	0
0	0	0	0	0	-1	0	-10000	0	0	0	1	0	VN2-YN-10000*Z2	>	-10000
0	0	0	0	0	-1	0	0	0	0	0	1	0	VN2-YN	<	0
0	0	0	0	0	0	0	-10000	0	0	0	1	0	VN2-10000*Z2	<	0
0	0	1	0	0	0	0	0	0	0	-1	0	0	XN-V12	>	0
0	0	1	0	0	0	0	0	0	0	0	0	-1	XN-V2L	>	0
0	0	0	0	-1	0	0	0	-10000	0	0	0	1	V1L-Y1-10000*ZL	>	-10000
0	0	0	0	-1	0	0	0	0	0	0	0	1	V1L-Y1	<	0
0	0	0	0	0	0	0	0	-10000	0	0	0	1	V1L-10000*ZL	<	0
0	0	0	0	0	0	1	1	1	0	0	0	0	Z1+Z2+ZL	=	1
1	0	0	0	0	0	0	0	0	0	0	0	0	X1	>	arr_t



Key Coding Blocks: Solve Scheduling

```
%% Running optimization routine
model.obj = f;
model.A = sparse(A);
model.rhs = b;
model.lb = lb;
model.sense = sense;
model.vtype = Vtype;
result = gurobi(model);

if strcmp(result.status, 'INFEASIBLE')
    display(result.status);
    run=1; pr_curr(m)=0; %zeroing occupied pump in pr_curr list
    if sum(pr_curr)==0; % condition for all available pumps
        V_aggr = V_rej(V_aggr, veh_num, V_read);
    end
else
    x = result.x % Recording results
    fval = result.objval; % optimal value of objective function
    V_aggr = V_a_sch(V_aggr, veh_num, V_read, x);
    V = V_sch(V, veh_num, V_read, x);
    P_aggr = P_a_sch(P_aggr, veh_num, x);
    P = P_sch(P, veh_num, x);
end
```

Using mixed-integer linear programming (MILP) solver from Gurobi

Checking result status and updating Vaggr, V, Paggr, P based on it



Initialization of Pumps

Tier	Battery size, kWh	Power acceptance rate, kW
1	81	120
2	20	6.6
3	14	3.3

Tier	Balanced	Lower first tier	Higher first tier
1	0.2	0.1	0.3
2	0.3	0.4	0.2
3	0.5	0.5	0.5

Tier	Average charging time per vehicle, hours	Service rate per pump, veh/hour	Number of pumps	Pump ID
1	0.54	1.85	2	1, 2
2	2.42	0.413	9	3, 4, ..., 11
3	3.39	0.295	21	12, 13, ..., 32

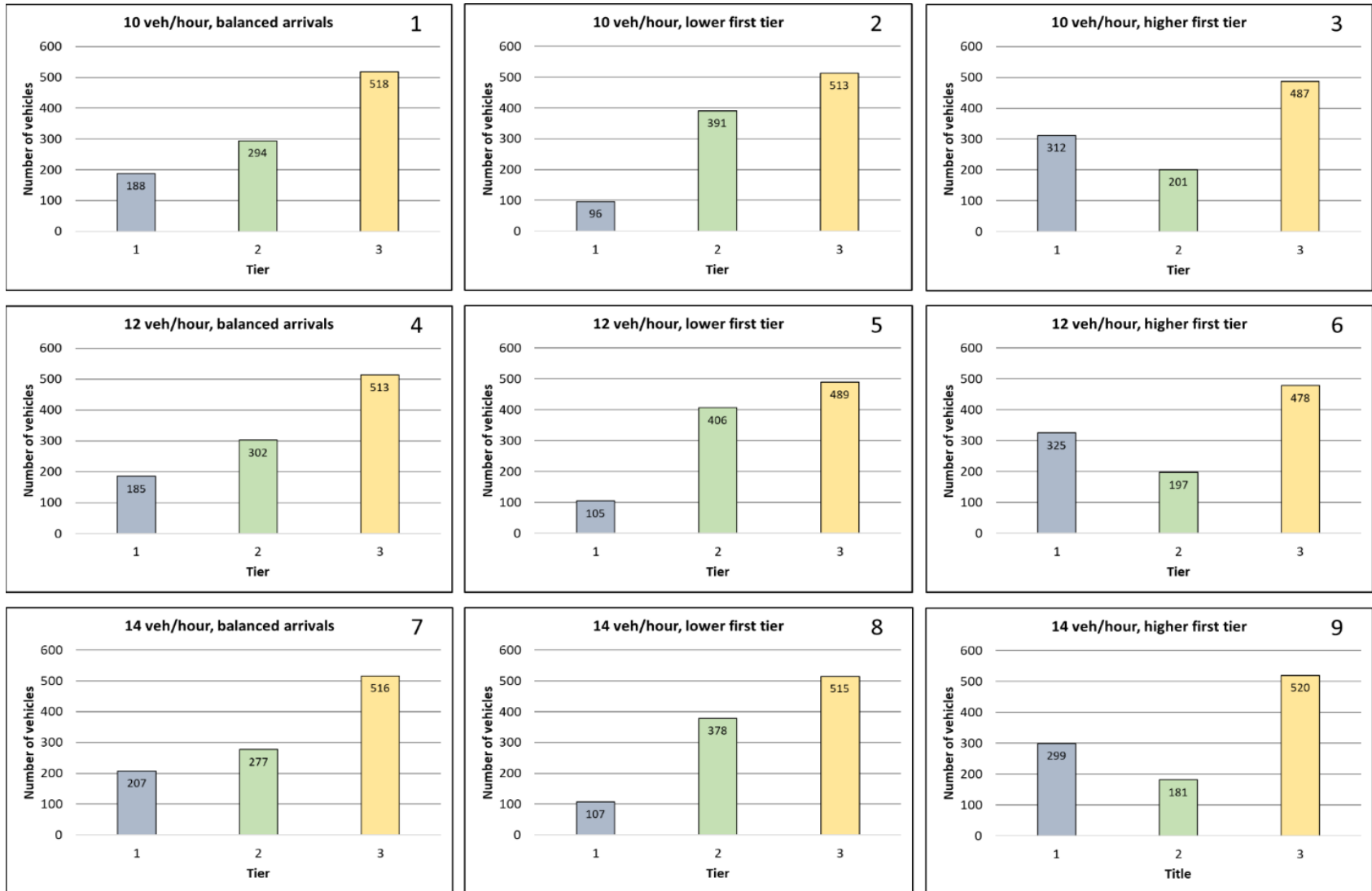
Energy requirement for each vehicle in this tier is $0.8 \cdot 81 = 64.8$ kWh

$$\lambda_1 = 0.2 \cdot 10 = 2 \text{ veh/hour} \quad \mu_1 = \lambda_1 / 0.8 = 0.8 \cdot 2 = 2.5 \text{ veh/hour} \quad \left(\frac{\lambda_i}{\mu_i} \sim 0.8 \right)$$

Required power delivery rate in tier 1: $2.5 \cdot 64.8 = 162$ kW \rightarrow **2 pumps**

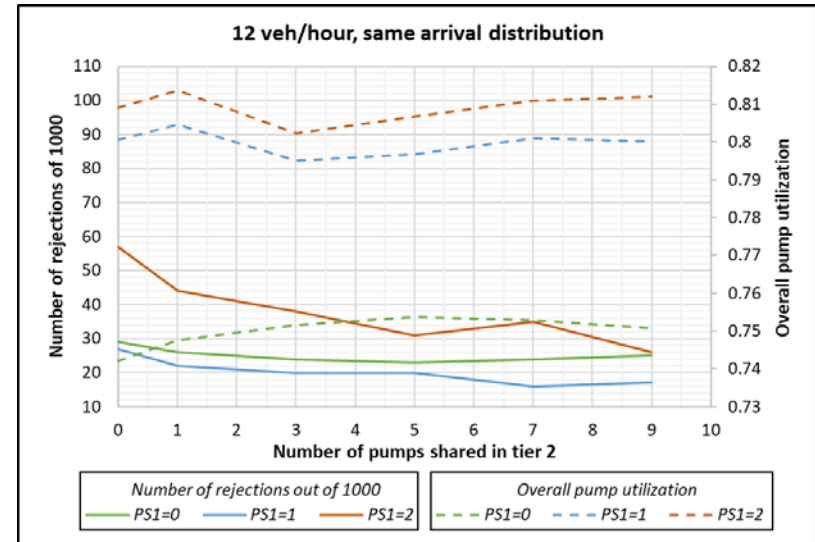


Arrival Rates and Distribution between Tiers

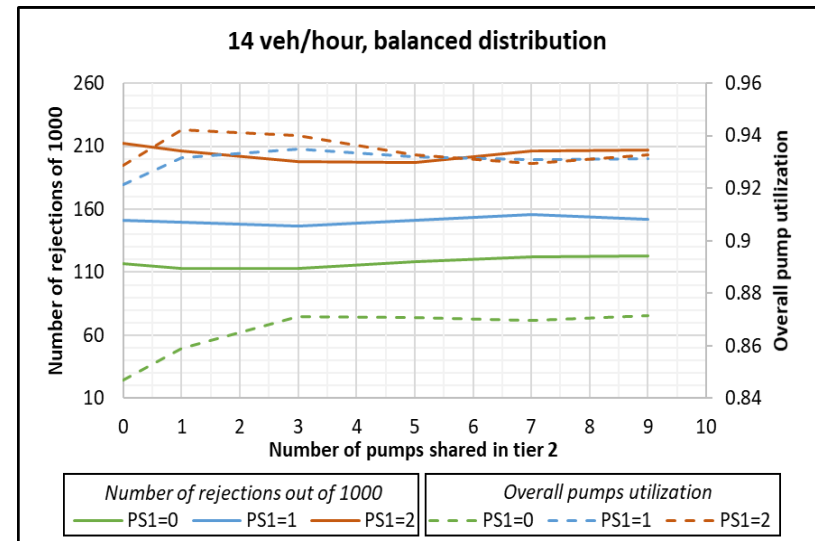


Number of Rejections and Pump Utilization

- No benefit from sharing for balanced scenario;
- Sharing of tier 1 pumps has higher impact than sharing of tier 2 pumps;
- Sharing of both pumps of tier 1 results in the highest number of rejections increasing it by 8 to 25 vehicles;
- Sharing only one pump in tier 1 results in the minimum number of rejections for any level of sharing in tier 2.



- Min number of rejections is achieved when none of pumps is shared;
- Sharing of one and two pumps in tier 1 increases the number of overall rejections by about 35 and 50 vehicles from the minimum.



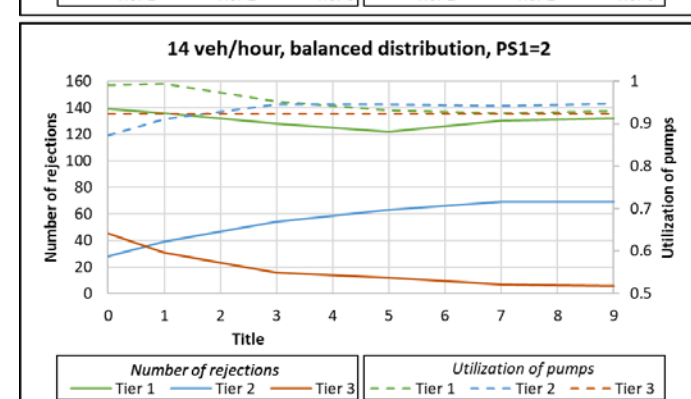
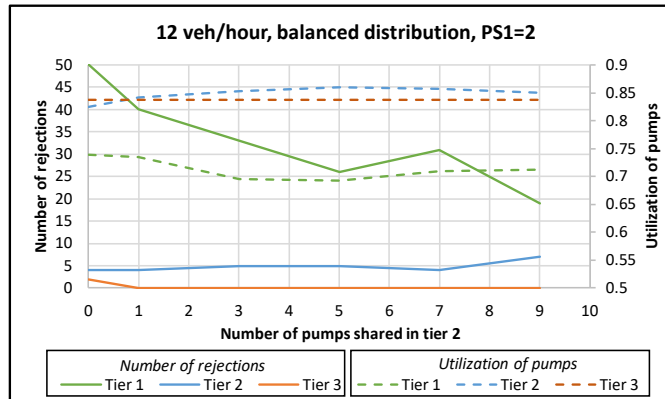
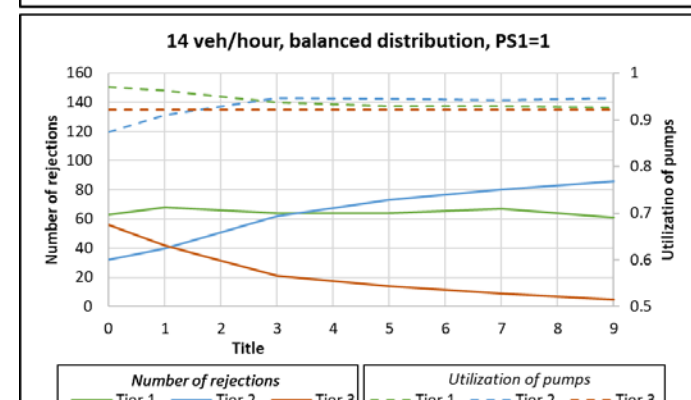
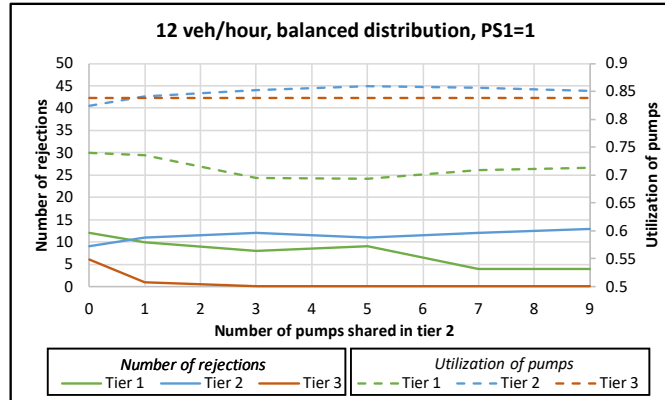
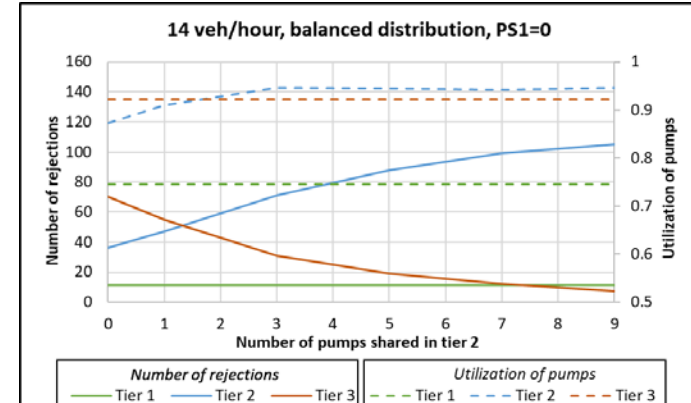
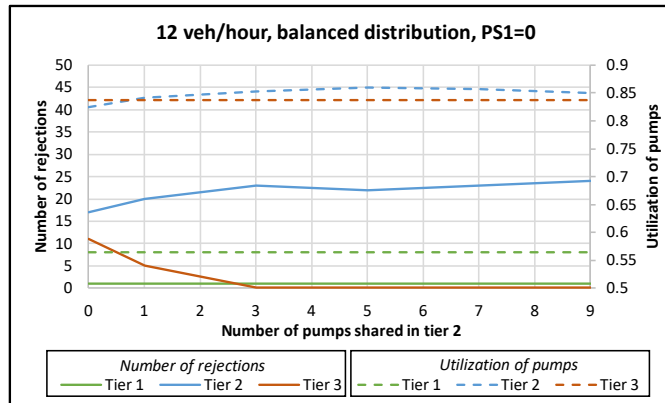
Tier-specific Parameters

12 veh/hour:

- Sharing one pump in tier 1 increases its utilization by up to 18% for 12 veh/hour;
- Sharing two pumps in tier 1 results in the highest number of rejections in this tier.

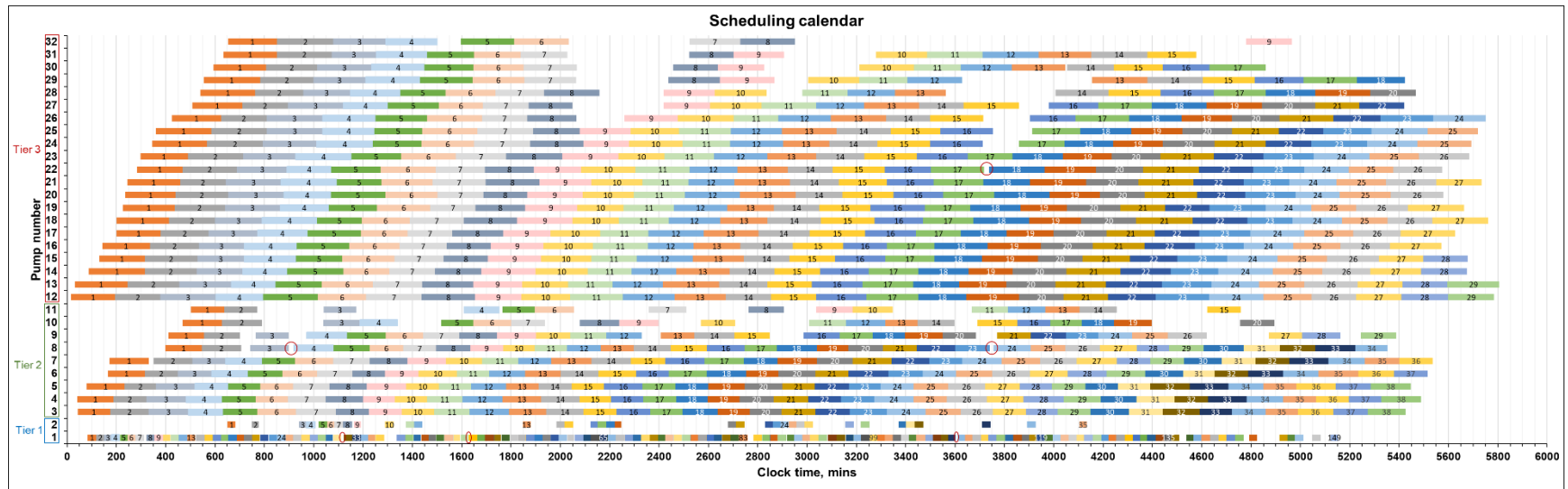
14 veh/hour:

- similar trends with a significant increase in number of rejections and the corresponding growth of pumps' utilization.

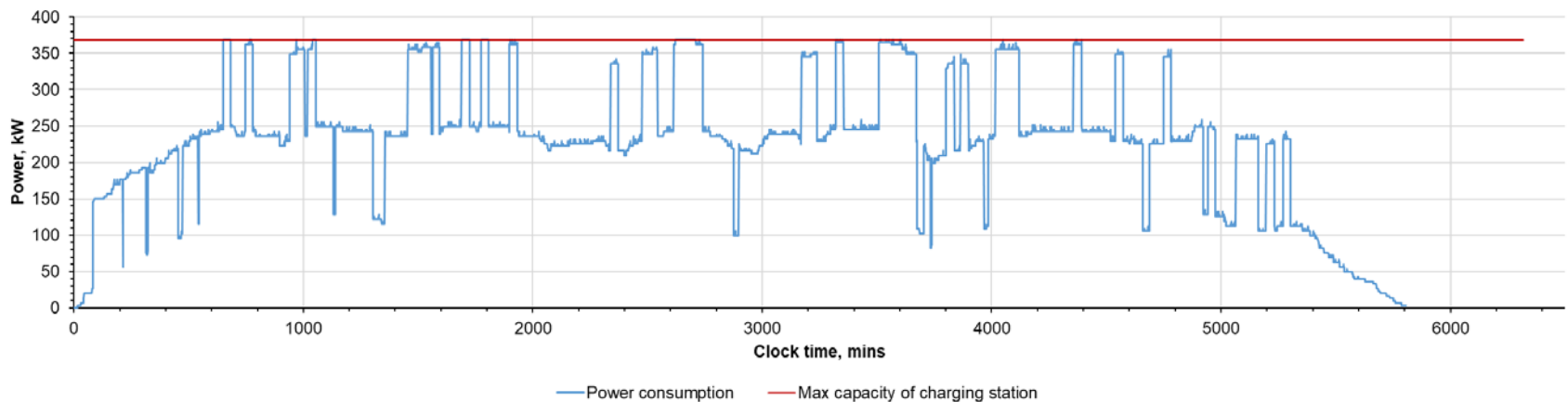


Testing Results and Analyses

Scheduling Calendar & Power Consumption Profile



Power consumption profile



Pumps' Assignment for Selection Rule

Sharing combination	Sharable pumps		Pump selection rule		
	Tier 1	Tier 2	Tier 1	Tier 2	Tier 3
1	0	0	1, 2	3, 4, ..., 11	12, 13, ..., 32
2	0	3	1, 2	4, 5, ..., 11, 3	12, 13, ..., 32, 3
...
6	0	3, 4, ..., 11	1, 2	3, 4, ..., 11	12, 13, ... 32, 3, 4, ..., 11
7	1	0	2, 1	3, 4, ..., 11, 1	12, 13, ... 32, 1
8	1	3	2, 1	4, 5, ..., 11, 3, 1	12, 13, ... 32, 3, 1
...
12	1	3, 4, ..., 11	2, 1	3, 4, ..., 11, 1	12, 13, ... 32, 3, 4, ..., 11, 1
13	1, 2	0	1, 2	3, 4, ..., 11, 1, 2	12, 13, ... 32, 1, 2
14	1, 2	3	1, 2	4, 5, ..., 11, 3, 1, 2	12, 13, ... 32, 3, 1, 2
...
18	1, 2	3, 4, ..., 11	1, 2	3, 4, ..., 11, 1, 2	12, 13, ... 32, 3, 4, ..., 11, 1, 2

