BuildCurves-July5

July 5, 2021

1 Initialization

In this section the notebook is initialized and usefull constants/functions are defined.

[1]:
```
import sys, os, logging
sys.path.append(os.getenv('VIRTUAL_ENV')+'/lib/python3.8/site-packages/')

import curves as c

import matplotlib.pyplot as plt
pc=1e-2
year = 1
month = year/12
day = month/30
week = 7*day

import pandas as pd
pd.set_option('display.max_colwidth', 100)
```

[2]:
```
def curve_build_info(curve):
    '''Show info on the curve'''
    data = []
    for instr in curve.GetInstruments():
        try:
            v1 = instr.Value()
            v2 = instr.Eval(curve)
            data.append({
                'maturity':instr.GetMaturity(),
                'Value':v1,
                'Eval(Curve)':v2,
                'diff':v1-v2,
                'instrument':str(instr)
            })
        except Exception as e:
            logging.error(f'Failed to eval {instr}: {e}')
    df = pd.DataFrame(data)
    df.style.set_caption(f'{curve}')
    return df
```

2 Curve Construction Example

Example of how a curve is constructed.

[3]:
```
# Declare a curve variable
curve = c.YieldCurve()

# Add some instruments. We use here only two forward rate agreements.

# FRA, starting at 0 with length 3 months and rate=1%
curve.Add(c.ForwardRateAgreement(0,3*month,0.01))

# FRA, starting at 0 with length 1 year and rate=2%
curve.Add(c.ForwardRateAgreement(0,1*year,0.02))

# Build the curve, cubic spline interpolation is used by default
curve.Build()

# The curve is ready!
```

[3]:
```
CubicSpline 3 points (x,y) = (0,-0.00538701) (0.25,0.0231954) (1,0.00374684)
```

2.1 Check

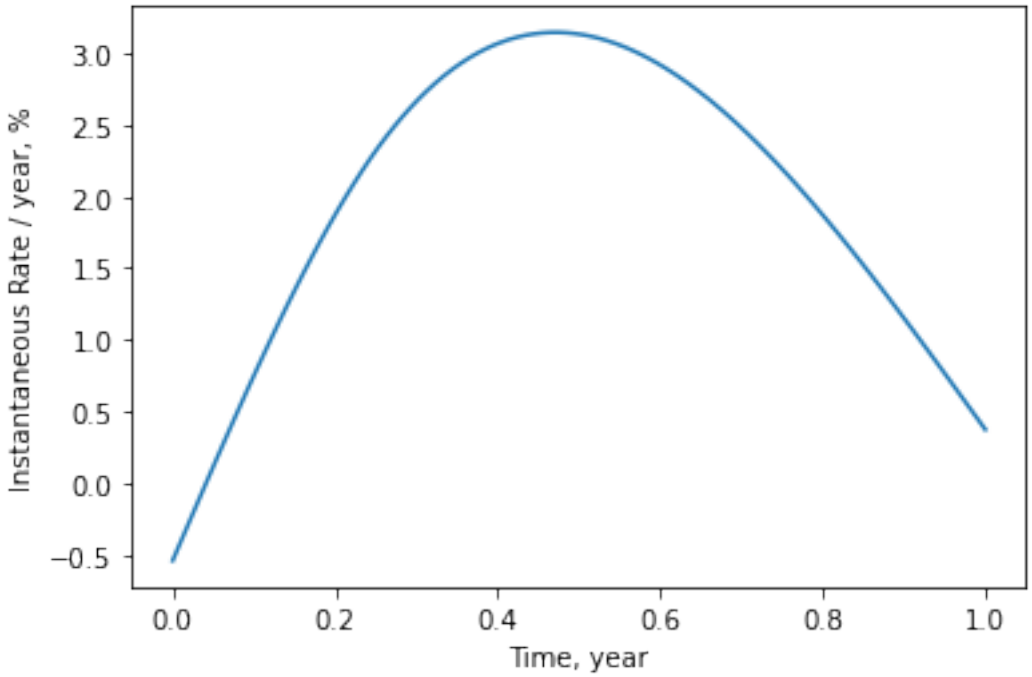
Here we check that the curve instruments are priced correctly.

[4]:
```
curve_build_info(curve)
```

[4]:
```
   maturity  Value  Eval(Curve)          diff  \
0      0.25   0.01         0.01 -8.288771e-08
1      1.00   0.02         0.02  1.303852e-07

                                          instrument
0  ForwardRateAgreement start=0 length=0.25 rate=0.01
1     ForwardRateAgreement start=0 length=1 rate=0.02
```

2.2 Plot the Curve

[5]:
```
plot_points = 100
fig, ax = plt.subplots()
ax.set_xlabel('Time, year')
ax.set_ylabel('Instantaneous Rate / year, %')
tmax = curve.GetX()[-1]
vx = [(tmax*i)/(plot_points-1) for i in range(plot_points)]
ax.plot(vx,[curve(t)/pc for t in vx])
plt.show()
```



3 Market Quotes

We want to check how the curve is constructed from the real market quotes. We first need to define a few functions which *adds* instruments to a curve.

[6]:
```
def add_eur_libor_quotes_1(curve):
    # https://www.global-rates.com/en/interest-rates/libor/libor.aspx

    # date: 06-25-2021
    data = [
        {'t0':0, 'length': 1*day,   'rate': -0.58471*pc},
        {'t0':0, 'length': 1*week,  'rate': -0.57571*pc},
        {'t0':0, 'length': 1*month, 'rate': -0.57786*pc},
        {'t0':0, 'length': 2*month, 'rate': -0.55857*pc},
        {'t0':0, 'length': 3*month, 'rate': -0.54614*pc},
        {'t0':0, 'length': 6*month, 'rate': -0.52614*pc},
        {'t0':0, 'length': 1*year,  'rate': -0.48357*pc},
    ]
    for v in data:
        curve.Add(c.ForwardRateAgreement(v['t0'],v['length'],v['rate']))
```

[7]:
```
def add_eur_libor_quotes_2(curve):
    # https://www.chathamfinancial.com/technology/european-market-rates
```

```
    # date: 06-25-2021
    data = [
        {'t0':0, 'length': 1*month, 'rate': -0.56200*pc},
        {'t0':0, 'length': 3*month, 'rate': -0.53800*pc},
        {'t0':0, 'length': 6*month, 'rate': -0.51300*pc},
        {'t0':0, 'length': 1*year,  'rate': -0.47800*pc},
    ]
    for v in data:
        curve.Add(c.ForwardRateAgreement(v['t0'],v['length'],v['rate']))
```

[8]:
```
def add_swaps(curve,curve_float_leg=None):
    if curve_float_leg is None:
        curve_float_leg = curve
    t0 = 0
    dt = year*0.25
    swaps = {
         1*year:  0.19*pc,
         2*year:  0.32*pc,
         3*year:  0.55*pc,
         5*year:  0.93*pc,
         7*year:  1.20*pc,
        10*year:  1.45*pc,
        15*year:  1.53*pc,
        30*year:  1.78*pc
    }
    for period,rate in swaps.items():

        swap = c.Swap()

        # fixed rate: payed quartely
        swap.lfix = c.LegFixed(t0,dt,round(period/dt),rate)

        # floating rate: payed quartely
        dt = year*0.25

        swap.lflt = c.LegFloat(t0,dt,round(period/dt),curve_float_leg)
        curve.Add(swap)
```

4 Curve Interpolation

There are several choices available to interpolate the curve. The most reasonable interpolations are: - piecewise constant - linear - cubic spline

We show that using different interpolations we are able to price the instruments correctly, though the curve shape changes dramatically from one interpolation type to another.

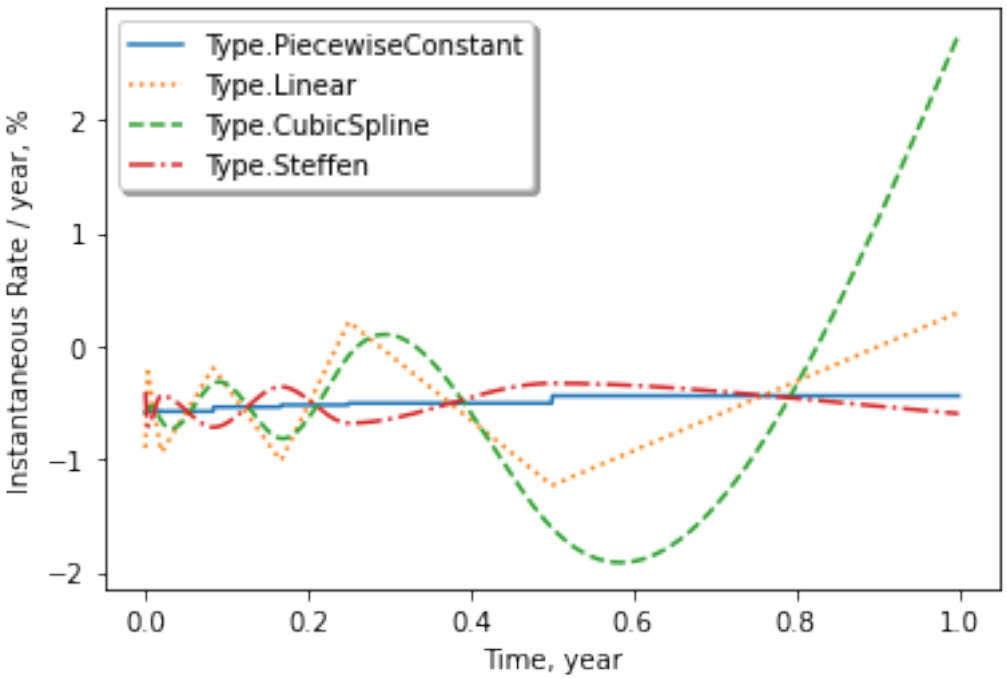
[9]:
```
def f(add_data,interpolations,plot_points=1000):
    curves = {}
    build_info = {}
    tmax = None

    for itype in interpolations:
        curve = c.YieldCurve()
        curves[itype] = curve
        try:
            add_data(curve)
            curve.Build(itype,0)
            tmax = curve.GetX()[-1]
        except Exception as e:
            logging.error(f'Problem with {itype} interpolation: {e}')
        build_info[itype] = curve_build_info(curve)

    if tmax:
        vx = [(tmax*i)/(plot_points-1) for i in range(plot_points)]
        def line_style(i):
            l = ['-',':','--','-.']
            return l [i%len(l)]

        fig1, ax1 = plt.subplots()
        ax1.set_xlabel('Time, year')
        ax1.set_ylabel('Instantaneous Rate / year, %')

        fig2, ax2 = plt.subplots()
        ax2.set_xlabel('Time, year')
        ax2.set_ylabel('Forward Rate / year, %')

        i=0
        for itype,curve in curves.items():
            ax1.plot(
                vx,
                [curve(t)/pc for t in vx],
                label=str(itype),
                linestyle=line_style(i)
            )
            ax2.plot(
                vx,
                [curve.GetForwardRate(0,t)/pc for t in vx],
                label=str(itype),
                linestyle=line_style(i)
            )
            i += 1
#         loc = 'upper bottom'
#         ax1.legend(loc=loc, shadow=True)
```

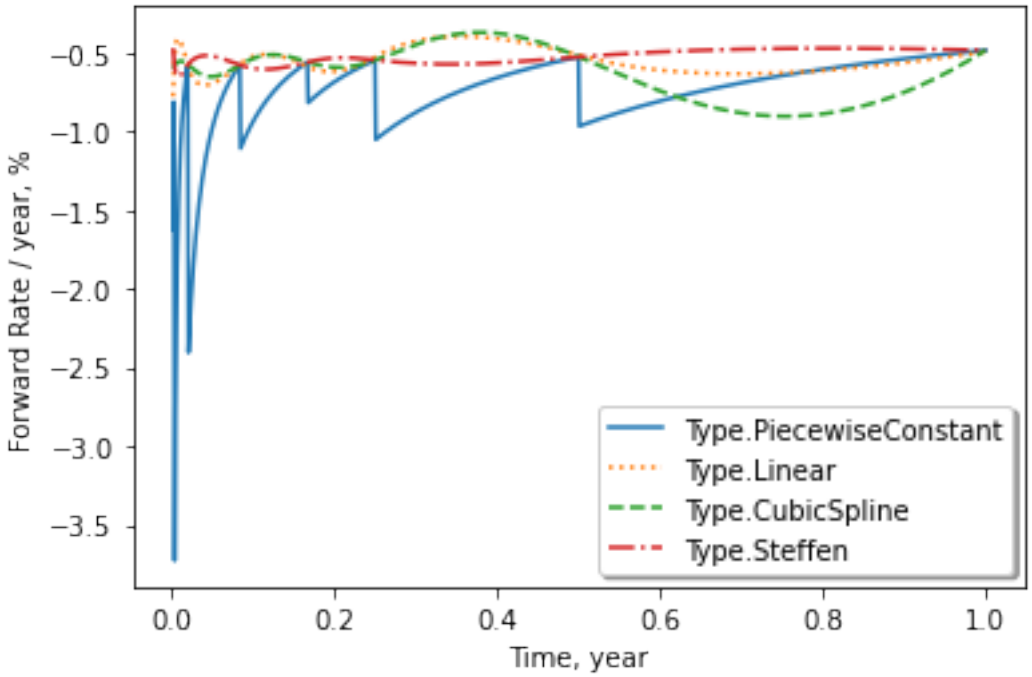
```
#         ax2.legend(loc=loc, shadow=True)
        ax1.legend(shadow=True)
        ax2.legend(shadow=True)

    return {
        'curves':curves,
        'build_info':build_info,
        'plots' :{
            'InstantaneousRates': fig1,
            'ForwardRates': fig2,
        }
    }
```

[10]:
```
interpolations = [
    c.Interpolator1D.Type.PiecewiseConstant,
    c.Interpolator1D.Type.Linear,
    c.Interpolator1D.Type.CubicSpline,
#    c.Interpolator1D.Type.Akima,
   c.Interpolator1D.Type.Steffen
]
```

4.1 Building with FRAs (data1)

[11]:
```
r = f(add_eur_libor_quotes_1,interpolations)
```





[12]:
```
r['build_info'][c.Interpolator1D.Type.PiecewiseConstant]
```

[12]:

	maturity	Value	Eval(Curve)	diff	instrument
0	0.002778	-0.005847	-0.005836	-1.066085e-05	ForwardRateAgreement start=0 length=0.00277778 rate=-0.0058471
1	0.019444	-0.005757	-0.005756	-6.356277e-07	ForwardRateAgreement start=0 length=0.0194444 rate=-0.0057571
2	0.083333	-0.005779	-0.005779	7.050112e-07	ForwardRateAgreement start=0 length=0.0833333 rate=-0.0057786
3	0.166667	-0.005586	-0.005586	-7.916242e-09	ForwardRateAgreement start=0 length=0.166667 rate=-0.0055857
4	0.250000	-0.005461	-0.005461	-9.918585e-08	ForwardRateAgreement start=0 length=0.25 rate=-0.0054614
5	0.500000	-0.005261	-0.005261	8.847564e-09	ForwardRateAgreement start=0 length=0.5 rate=-0.0052614
6	1.000000	-0.004836	-0.004836	-3.725290e-09	ForwardRateAgreement start=0 length=1 rate=-0.0048357

[13]:
```
r['build_info'][c.Interpolator1D.Type.Linear]
```

[13]:

	maturity	Value	Eval(Curve)	diff
0	0.002778	-0.005847	-0.005622	-0.000225
1	0.019444	-0.005757	-0.005793	0.000036

```
2  0.083333 -0.005779    -0.005719 -0.000059
3  0.166667 -0.005586    -0.005877  0.000291
4  0.250000 -0.005461    -0.005248 -0.000213
5  0.500000 -0.005261    -0.005165 -0.000096
6  1.000000 -0.004836    -0.004921  0.000085

                                                    instrument
0  ForwardRateAgreement start=0 length=0.00277778 rate=-0.0058471
1   ForwardRateAgreement start=0 length=0.0194444 rate=-0.0057571
2   ForwardRateAgreement start=0 length=0.0833333 rate=-0.0057786
3    ForwardRateAgreement start=0 length=0.166667 rate=-0.0055857
4        ForwardRateAgreement start=0 length=0.25 rate=-0.0054614
5         ForwardRateAgreement start=0 length=0.5 rate=-0.0052614
6           ForwardRateAgreement start=0 length=1 rate=-0.0048357
```

[14]:
```
r['build_info'][c.Interpolator1D.Type.CubicSpline]
```

[14]:
```
   maturity     Value  Eval(Curve)          diff  \
0  0.002778 -0.005847    -0.005836 -1.066085e-05
1  0.019444 -0.005757    -0.005763  5.494338e-06
2  0.083333 -0.005779    -0.005779  7.050112e-07
3  0.166667 -0.005586    -0.005586 -7.916242e-09
4  0.250000 -0.005461    -0.005460 -1.527835e-06
5  0.500000 -0.005261    -0.005260 -1.179986e-06
6  1.000000 -0.004836    -0.004836  1.150183e-07

                                                    instrument
0  ForwardRateAgreement start=0 length=0.00277778 rate=-0.0058471
1   ForwardRateAgreement start=0 length=0.0194444 rate=-0.0057571
2   ForwardRateAgreement start=0 length=0.0833333 rate=-0.0057786
3    ForwardRateAgreement start=0 length=0.166667 rate=-0.0055857
4        ForwardRateAgreement start=0 length=0.25 rate=-0.0054614
5         ForwardRateAgreement start=0 length=0.5 rate=-0.0052614
6           ForwardRateAgreement start=0 length=1 rate=-0.0048357
```

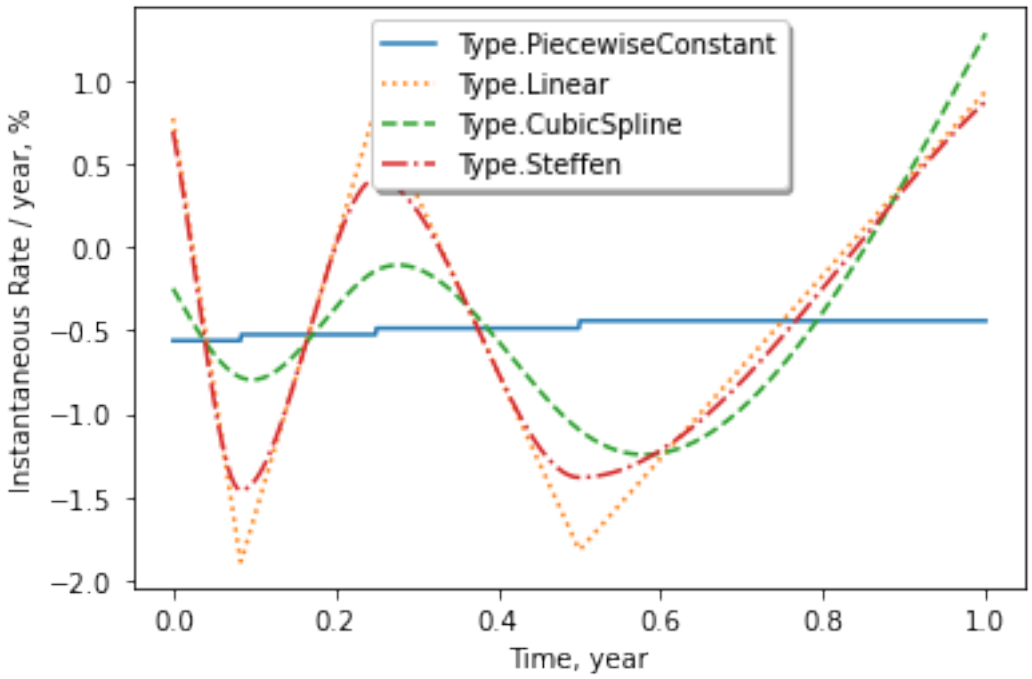
[15]:
```
r['build_info'][c.Interpolator1D.Type.Steffen]
```

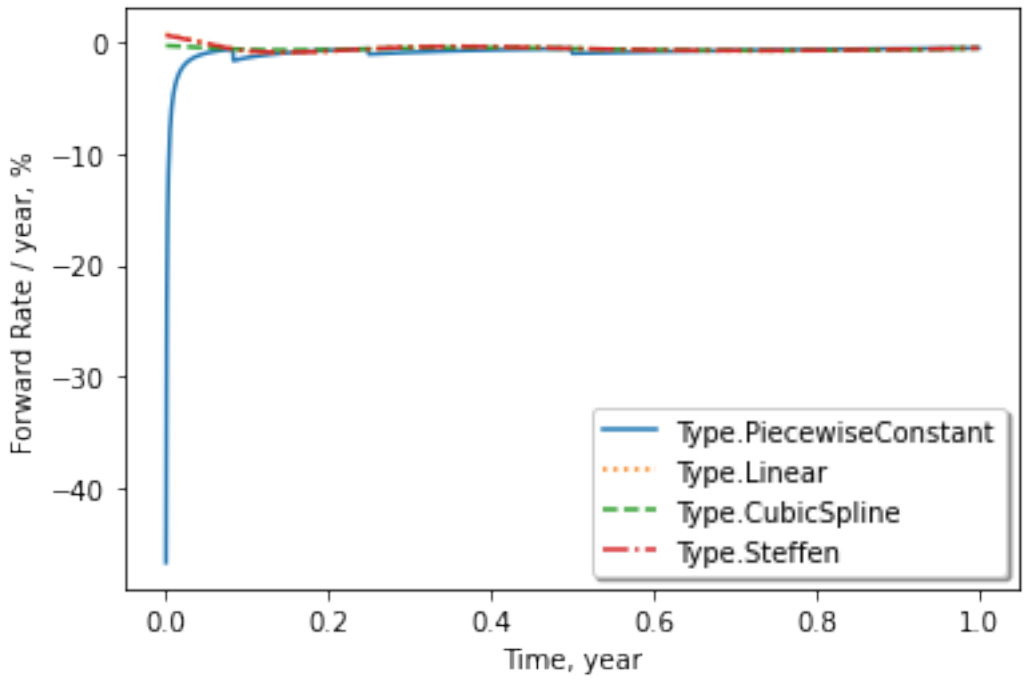
[15]:
```
   maturity     Value  Eval(Curve)          diff  \
0  0.002778 -0.005847    -0.005836 -1.066085e-05
1  0.019444 -0.005757    -0.005756 -6.356277e-07
2  0.083333 -0.005779    -0.005778 -7.250346e-07
3  0.166667 -0.005586    -0.005586 -7.916242e-09
4  0.250000 -0.005461    -0.005462  3.771856e-07
5  0.500000 -0.005261    -0.005261 -2.291054e-07
6  1.000000 -0.004836    -0.004835 -2.407469e-07

                                                    instrument
```

```
0  ForwardRateAgreement start=0 length=0.00277778 rate=-0.0058471
1   ForwardRateAgreement start=0 length=0.0194444 rate=-0.0057571
2   ForwardRateAgreement start=0 length=0.0833333 rate=-0.0057786
3    ForwardRateAgreement start=0 length=0.166667 rate=-0.0055857
4        ForwardRateAgreement start=0 length=0.25 rate=-0.0054614
5         ForwardRateAgreement start=0 length=0.5 rate=-0.0052614
6           ForwardRateAgreement start=0 length=1 rate=-0.0048357
```

4.2 Building with FRAs (data2)

[16]:
```
r = f(add_eur_libor_quotes_2,interpolations)
```





[17]:
```
r['build_info'][c.Interpolator1D.Type.PiecewiseConstant]
```

[17]:
```
   maturity    Value  Eval(Curve)          diff  \
0  0.083333 -0.00562    -0.005621  5.932525e-07
1  0.250000 -0.00538    -0.005380 -1.280569e-07
2  0.500000 -0.00513    -0.005130 -9.452924e-08
3  1.000000 -0.00478    -0.004780  5.541369e-08

                                                 instrument
0  ForwardRateAgreement start=0 length=0.0833333 rate=-0.00562
1       ForwardRateAgreement start=0 length=0.25 rate=-0.00538
2        ForwardRateAgreement start=0 length=0.5 rate=-0.00513
3          ForwardRateAgreement start=0 length=1 rate=-0.00478
```

[18]:
```
r['build_info'][c.Interpolator1D.Type.Linear]
```

[18]:
```
   maturity    Value  Eval(Curve)          diff  \
0  0.083333 -0.00562    -0.005619 -8.363277e-07
1  0.250000 -0.00538    -0.005379 -6.039627e-07
2  0.500000 -0.00513    -0.005131  6.188639e-07
3  1.000000 -0.00478    -0.004780 -3.003515e-07

                                                 instrument
0  ForwardRateAgreement start=0 length=0.0833333 rate=-0.00562
1       ForwardRateAgreement start=0 length=0.25 rate=-0.00538
```

```
2        ForwardRateAgreement start=0 length=0.5 rate=-0.00513
3          ForwardRateAgreement start=0 length=1 rate=-0.00478
```

[19]:
```
r['build_info'][c.Interpolator1D.Type.CubicSpline]
```

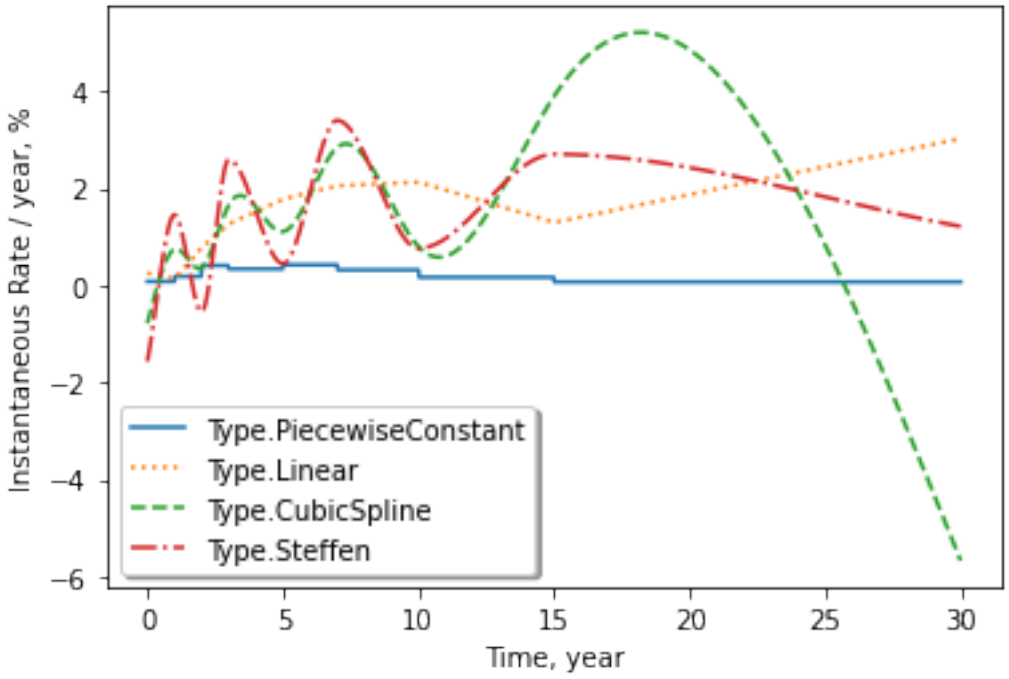
[19]:
```
   maturity    Value  Eval(Curve)          diff  \
0  0.083333 -0.00562    -0.005619 -8.363277e-07
1  0.250000 -0.00538    -0.005381  8.242205e-07
2  0.500000 -0.00513    -0.005129 -1.283828e-06
3  1.000000 -0.00478    -0.004781  6.486662e-07

                                                 instrument
0  ForwardRateAgreement start=0 length=0.0833333 rate=-0.00562
1       ForwardRateAgreement start=0 length=0.25 rate=-0.00538
2        ForwardRateAgreement start=0 length=0.5 rate=-0.00513
3          ForwardRateAgreement start=0 length=1 rate=-0.00478
```

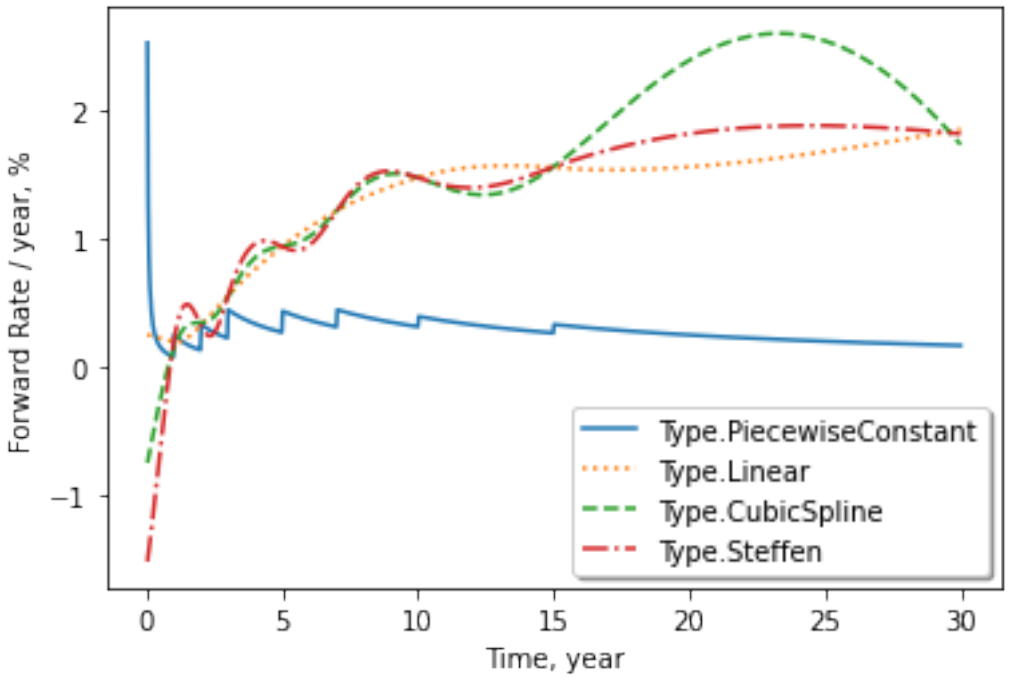
[20]:
```
r['build_info'][c.Interpolator1D.Type.Steffen]
```

[20]:
```
   maturity    Value  Eval(Curve)          diff  \
0  0.083333 -0.00562    -0.005619 -8.363277e-07
1  0.250000 -0.00538    -0.005379 -6.039627e-07
2  0.500000 -0.00513    -0.005131  6.188639e-07
3  1.000000 -0.00478    -0.004780 -1.820736e-07

                                                 instrument
0  ForwardRateAgreement start=0 length=0.0833333 rate=-0.00562
1       ForwardRateAgreement start=0 length=0.25 rate=-0.00538
2        ForwardRateAgreement start=0 length=0.5 rate=-0.00513
3          ForwardRateAgreement start=0 length=1 rate=-0.00478
```

4.3 Building with SWAPs

[21]:
```
r = f(add_swaps,interpolations)
```





[22]:
```
r['build_info'][c.Interpolator1D.Type.PiecewiseConstant]
```

[22]:

	maturity	Value	Eval(Curve)	diff	instrument
0	1.0	0.0	1.164153e-10	-1.164153e-10	Swap: (LegFixed t0=0 dt=0.25 n=4 rate=0.0019) (LegFloat t0=0 dt=0.25 n=4)
1	2.0	0.0	0.000000e+00	0.000000e+00	Swap: (LegFixed t0=0 dt=0.25 n=8 rate=0.0032) (LegFloat t0=0 dt=0.25 n=8)
2	3.0	0.0	1.862645e-09	-1.862645e-09	Swap: (LegFixed t0=0 dt=0.25 n=12 rate=0.0055) (LegFloat t0=0 dt=0.25 n=12)
3	5.0	0.0	-3.725290e-09	3.725290e-09	Swap: (LegFixed t0=0 dt=0.25 n=20 rate=0.0093) (LegFloat t0=0 dt=0.25 n=20)
4	7.0	0.0	0.000000e+00	0.000000e+00	Swap: (LegFixed t0=0 dt=0.25 n=28 rate=0.012) (LegFloat t0=0 dt=0.25 n=28)
5	10.0	0.0	1.490116e-08	-1.490116e-08	Swap: (LegFixed t0=0 dt=0.25 n=40 rate=0.0145) (LegFloat t0=0 dt=0.25 n=40)
6	15.0	0.0	0.000000e+00	0.000000e+00	Swap: (LegFixed t0=0 dt=0.25 n=60 rate=0.0153) (LegFloat t0=0 dt=0.25 n=60)
7	30.0	0.0	-1.192093e-07	1.192093e-07	Swap: (LegFixed t0=0 dt=0.25 n=120 rate=0.0178) (LegFloat t0=0 dt=0.25 n=120)

[23]:
```
r['build_info'][c.Interpolator1D.Type.Linear]
```

[23]:

	maturity	Value	Eval(Curve)	diff	instrument
0	1.0	0.0	-3.341120e-07	3.341120e-07	Swap: (LegFixed t0=0 dt=0.25 n=4 rate=0.0019) (LegFloat t0=0 dt=0.25 n=4)
1	2.0	0.0	1.018401e-06	-1.018401e-06	Swap: (LegFixed t0=0 dt=0.25 n=8 rate=0.0032) (LegFloat t0=0 dt=0.25 n=8)
2	3.0	0.0	-1.456589e-06	1.456589e-06	Swap: (LegFixed t0=0 dt=0.25 n=12 rate=0.0055) (LegFloat t0=0 dt=0.25 n=12)
3	5.0	0.0	-5.029142e-07	5.029142e-07	Swap: (LegFixed t0=0 dt=0.25 n=20 rate=0.0093) (LegFloat t0=0 dt=0.25 n=20)
4	7.0	0.0	1.490116e-08	-1.490116e-08	Swap: (LegFixed t0=0 dt=0.25 n=28 rate=0.012) (LegFloat t0=0 dt=0.25 n=28)
5	10.0	0.0	8.791685e-07	-8.791685e-07	Swap: (LegFixed t0=0 dt=0.25 n=40 rate=0.0145) (LegFloat t0=0 dt=0.25 n=40)
6	15.0	0.0	-2.533197e-07	2.533197e-07	Swap: (LegFixed t0=0 dt=0.25 n=60 rate=0.0153) (LegFloat t0=0 dt=0.25 n=60)
7	30.0	0.0	-1.490116e-07	1.490116e-07	Swap: (LegFixed t0=0 dt=0.25 n=120 rate=0.0178) (LegFloat t0=0 dt=0.25 n=120)

[24]:
```
r['build_info'][c.Interpolator1D.Type.CubicSpline]
```

[24]:

	maturity	Value	Eval(Curve)	diff
0	1.0	0.0	0.000489	-0.000489
1	2.0	0.0	-0.000404	0.000404

```
2       3.0    0.0     0.000224 -0.000224
3       5.0    0.0    -0.000108  0.000108
4       7.0    0.0     0.000076 -0.000076
5      10.0    0.0    -0.000021  0.000021
6      15.0    0.0    -0.000003  0.000003
7      30.0    0.0     0.000002 -0.000002
```

```
                                                                                instrument
0      Swap: (LegFixed t0=0 dt=0.25 n=4 rate=0.0019) (LegFloat t0=0 dt=0.25 n=4)
1      Swap: (LegFixed t0=0 dt=0.25 n=8 rate=0.0032) (LegFloat t0=0 dt=0.25 n=8)
2    Swap: (LegFixed t0=0 dt=0.25 n=12 rate=0.0055) (LegFloat t0=0 dt=0.25 n=12)
3    Swap: (LegFixed t0=0 dt=0.25 n=20 rate=0.0093) (LegFloat t0=0 dt=0.25 n=20)
4     Swap: (LegFixed t0=0 dt=0.25 n=28 rate=0.012) (LegFloat t0=0 dt=0.25 n=28)
5    Swap: (LegFixed t0=0 dt=0.25 n=40 rate=0.0145) (LegFloat t0=0 dt=0.25 n=40)
6    Swap: (LegFixed t0=0 dt=0.25 n=60 rate=0.0153) (LegFloat t0=0 dt=0.25 n=60)
7  Swap: (LegFixed t0=0 dt=0.25 n=120 rate=0.0178) (LegFloat t0=0 dt=0.25 n=120)
```

[25]:
```
r['build_info'][c.Interpolator1D.Type.Steffen]
```

[25]:
```
   maturity  Value   Eval(Curve)          diff  \
0       1.0    0.0 -3.253808e-07  3.253808e-07
1       2.0    0.0 -4.256144e-07  4.256144e-07
2       3.0    0.0 -2.793968e-07  2.793968e-07
3       5.0    0.0  6.332994e-07 -6.332994e-07
4       7.0    0.0  4.917383e-07 -4.917383e-07
5      10.0    0.0  2.831221e-07 -2.831221e-07
6      15.0    0.0  1.072884e-06 -1.072884e-06
7      30.0    0.0  2.384186e-07 -2.384186e-07
```

```
                                                                                instrument
0      Swap: (LegFixed t0=0 dt=0.25 n=4 rate=0.0019) (LegFloat t0=0 dt=0.25 n=4)
1      Swap: (LegFixed t0=0 dt=0.25 n=8 rate=0.0032) (LegFloat t0=0 dt=0.25 n=8)
2    Swap: (LegFixed t0=0 dt=0.25 n=12 rate=0.0055) (LegFloat t0=0 dt=0.25 n=12)
3    Swap: (LegFixed t0=0 dt=0.25 n=20 rate=0.0093) (LegFloat t0=0 dt=0.25 n=20)
4     Swap: (LegFixed t0=0 dt=0.25 n=28 rate=0.012) (LegFloat t0=0 dt=0.25 n=28)
5    Swap: (LegFixed t0=0 dt=0.25 n=40 rate=0.0145) (LegFloat t0=0 dt=0.25 n=40)
6    Swap: (LegFixed t0=0 dt=0.25 n=60 rate=0.0153) (LegFloat t0=0 dt=0.25 n=60)
7  Swap: (LegFixed t0=0 dt=0.25 n=120 rate=0.0178) (LegFloat t0=0 dt=0.25 n=120)
```