CERE, ACISE, EMA, M2-EES-EM, M2-EES-EnPA
Application of multi-variable synthesis using Matlab

Maneuvering a submarine

Objectives

The goal of the project is to synthesize insensitive and decoupled control laws for a submarine. Those controls will then be tested using computer simulation.

The complexity of output computation and digital simulation require the use of a software, in this case Matlab.

1 Modeling

The modeling of this submarine comes from the book "Commande des systèmes linéaires à plusieurs entrées" de F. Rotella, from l'Ecole Nationale d'Ingénieurs de Tarbes. Motion along the vertical plane (going up or down) is achieved by rudders on the bow (*proue*) and stern (*poupe*). As every dive or ascent should be executed with constant pitch (assiette), it is necessary to use both controls. This inevitably leads to the study of a multi-variable system having characteristics such as a input-output couplings and inputs with antagonistic effects : any action on one of both rudders changes the vertical position of the center of gravity of the submarine and its pitch at the same time. Additionally, moving both of the rudders in the same way results in qualitatively opposite effects. One can notice that similar characteristics would be observed on aircraft flight models. Figure 1 shows a submarine on the vertical plane. Pitch is the angle between the horizontal axis and the submarine's longitudinal axis.

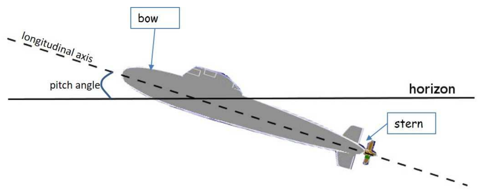

Figure 1 – Submarine on the vertical plane

Figure 2 [1] shows the motion of a submarine depending on the direction of the bow rudder. Regardless of the direction of the stern rudder, the submarine sinks in A and B, and ascends in C and D. The only difference is that it moves horizontally for A and D, and obliquely for B and C. The bow rudders enable the submarine to move up or down while the stern rudders help with the control of the pitch, that is the angle the submarine makes with the horizon.

1.1 Input selection

Inputs are naturally bow and stern angles.

This gives $u = \begin{bmatrix} pr \\ po \end{bmatrix}$ (pr stands for the bow angle and po for the stern angle)

1. from website subalien.free.fr

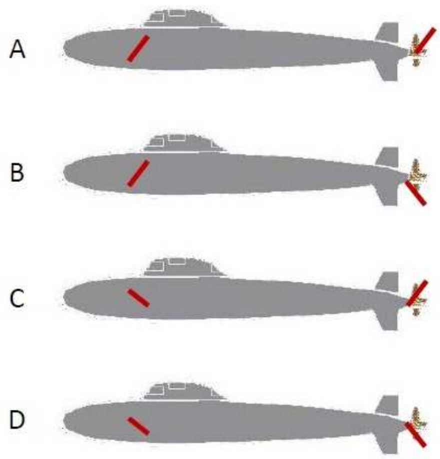

FIGURE 2 – Movement of a submarine

1.2 Output selection

In any case, outputs are linear combinations of the states, they are not as set as the inputs are. The output selection mainly depends on two factors :

- Target goal, that is variables that will naturally be used as outputs because they represent what needs to be controlled.
- Available measurements that are a part of the control law. They can be either states or easily measurable variables (such as load factor) related to states by linear combinations.

In our case, the chosen outputs are the depth h in meters and the pitch angle θ in radians ; $y = \begin{bmatrix} h \\ \theta \end{bmatrix}$

1.3 Numerical model

The linearized state-space model (centered around an operating point) can be written as : $U = \begin{bmatrix} \delta u_1 \\ \delta u_2 \end{bmatrix}$ $Y = \begin{bmatrix} \delta y_1 \\ \delta y_2 \end{bmatrix}$

$$
\begin{aligned}
\dot{X} &= \begin{bmatrix} a_{11} & a_{12} & 0 & a_{14} \\ a_{21} & a_{22} & 0 & a_{24} \\ 1 & 0 & 0 & a_{34} \\ 0 & 1 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} U \\
Y &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} X
\end{aligned}
$$

with a speed of 6 knots :

$$
\begin{array}{lll}
a_{11} = -0.038006; & a_{12} = 0.89604; & a_{14} = 0.0014673; \\
a_{21} = 0.0017105; & a_{22} = -0.091676; & a_{24} = -0.0056095; \\
a_{34} = -3.0867; & b_{11} = -0.007542; & b_{12} = -0.022859; \\
b_{21} = 0.0017323; & b_{22} = -0.0022217. &
\end{array}
$$

with a speed of 30 knots :

$$
\begin{array}{lll}
a_{11} = -0.19003; & a_{12} = 4.4802; & a_{14} = 0.0014673; \\
a_{21} = 0.0085526; & a_{22} = -0.45988; & a_{24} = -0.0056095; \\
a_{34} = -15.433; & b_{11} = -0.1855; & b_{12} = -0.57149; \\
b_{21} = 0.043308; & b_{22} = -0.055543. &
\end{array}
$$

2 Analysis of the natural system

Work to do on Matlab

1. Determine the system's state-space model for an operating point for either one of the two submarine models.
2. Study observability and controllability of this system.
3. Compute and display the system's modes in the Laplace plane. Characterize those modes (damping, natural frequency, etc). Is the system stable in open-loop ?
4. Display the step response and interpret it using its modes. Do the same for the impulse response.
5. Study couplings existing on the submarine using previously drawn curves.

3 Objectives of the control law

Motion along the vertical plane (going up or down) is achieved by rudders on the bow and stern. As every dive or ascent should be executed with constant pitch, it is necessary to use both controls. The set objective is to pilot the submarine along the vertical plane in order to control the depth δh and pitch angle $\delta\theta$ independently. Actually, as a residual coupling is unavoidable but practically not a problem, specifications will be more like :

— For a given bow angle step ($1°$ for example), pitch angle variation should be limited by a given $\Delta\theta$ (for example 0.2 rad).

— For a given stern angle step ($1°$ for example), depth variation should be limited by a given Δh (for example 0.5 m).

Independently of the necessary decoupling, submarine's closed-loop modes should be placed inside a trapeze defined by the usual constraints : minimal damping, shortest rise time and bandwidth limitation as shown in the figure 3.

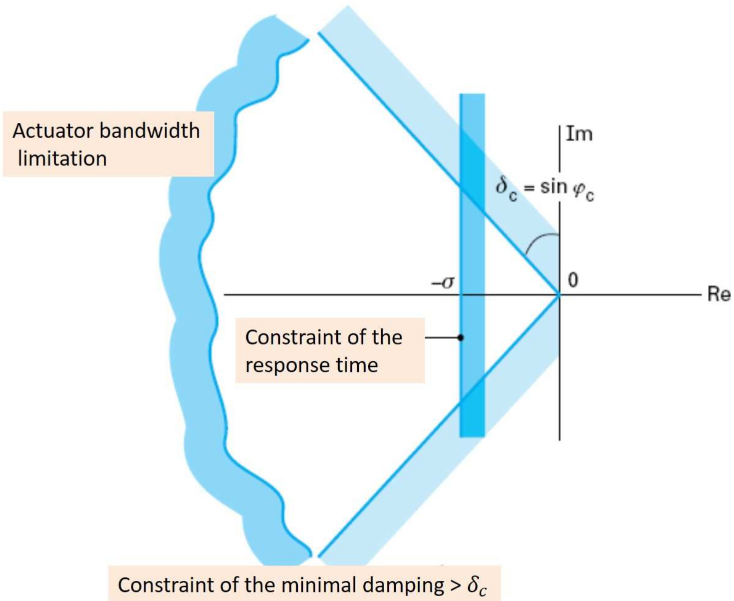

FIGURE 3 – Constraints on desired modes

For example, a rise time to 95% of $80s$ and $100s$ for δh and $\delta\theta$ will be considered. In the case of a study in which conjugate eigenvalues are considered, a damping greater than 0.65 is imposed.

4 Objectives' realization

4.1 Back to the specifications

The goal is to meet the set objectives by using a state-feedback control law :

$$U = -KX + HY_c$$

with

— U being the control vector of the system

— Y_c being the reference input vector $\begin{bmatrix} \delta h_c & \delta\theta_c \end{bmatrix}^T$.

In order to satisfy a decoupling constraint, two subsets of disjointed groups Σ_1 and Σ_2 are defined. The goals are to make the outputs δh and $\delta\theta$ dependent on only one of the subsets Σ_1 and Σ_2 and to make those modes stimulated by a single input.

As a first step, it is possible to assign two real modes to Σ_1 : λ_1, and λ_2. Same for Σ_2, assign it two real modes : λ_3 and λ_4. A study on conjugate complex poles can also be considered.

In order to get a rise time of about 80s for h, a λ_3 of about $-3/80$ is a good option. λ_4 is chosen so that its dynamic is negligible. Regarding θ, a λ_1 of about $-3/100$ and λ_2 with a negligible dynamic are chosen.

1. Justify the values of eigenvalues.

5 Eigenstructure assignment and decoupling

The goal is now to get the dynamics desired by the specifications and the decoupling specified above

1. Give the shape the eigenvector matrix should have in order to satisfy the right decoupling constraints.
2. Compute the state-feedback gain K that allows you to place the desired eigenvalues and right eigenvectors in the best possible way.
3. Write the required conditions for the matrix H to satisfy the left decoupling constraints. Give matrix H that leads to a static gain equal to 1.
4. Test the obtained results by simulating the closed-loop response to a step δh_c and then $\delta\theta_c$. What can be concluded from this simulation, especially regarding the decoupling ?

6 Decoupling in static mode

It is sometimes not possible to guarantee the left side decoupling exactly. In that case (it depends on the choice of the eigenvalues), it is possible to do a decoupling in the static mode (steady state, so $\dot{X} = 0$), coupled with the state-feedback done in the previous subsection.

1. The static mode decoupling should satisfy $Y_c = CX$. Demonstrate that such a decoupling can be obtained by choosing
$$H = \Omega_{22} + K\Omega_{12}$$
with
$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1}$$
2. Test the obtained results with the pre-control matrix H, by simulating the response to a step δh_c and then $\delta\theta_c$. What can be concluded from this simulation, especially regarding the decoupling ?

7 Insensitive mode placement

1. Explain how the function place.m works. Is the result of the function unique ? Why ?
2. Compute a state-feedback that allows you to place the modes on λ_1, λ_2, λ_3 and λ_4 with a minimal sensitivity as a requirement.
3. Are the decoupling constraints satisfied ?

8 Sensitivity study

1. Explain how the function cond.m works.
2. Study the sensitivity of the obtained mode placements (for each previous sections).
3. For the insentitive mode placement, do a study in a degraded mode : try with U_1 inactive (mono-variable control with U_2) and in a symmetrical way (mono-variable control with U_1), recomputing your state-feedback law. Is the degraded mode more or less sensitive than the one with both controls ? Is it normal ? why ?
4. Conclude.

9 Conclusions

Conclude on the feasibility of an insensitive control with decoupling on such a system.