1.1.8 Интерполяционная формула Ньютона в начале таблицы

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Дано: таблица точек и значений функции

In[176]:=
```
Clear@NewtonInterpolation
NewtonInterpolation[points_, temp_, grad_] :=
 Module[{f, x, dy, p, h = points[[1, 2]] - points[[1, 1]], t},
  Do[x[i - 1] = points[[1, i]];
   f[x[i - 1]] = points[[2, i]],
   {i, Length@points[[1]]}];
  dy[x_, 1] := f[x + h] - f[x];
  dy[x_, k_] := dy[x + h, k - 1] - dy[x, k - 1];
  t[y_] := (y - x[0])/h;
  p[y_, n_] :=
   f[x[0]] + Sum[(Times @@ Table[(t[y] - j), {j, 0, i - 1}]) * dy[x[0], i]/Factorial[i], {i, 1, n}];
  p[temp, grad]
 ]
```

Результаты

Возьмем несколько значений функции

In[•]:=
```
f[x_] := x^3 - 0.2 x^2 - 0.2 x - 1.2
```

In[•]:=
```
Clear@interPoint
(interPoint = {Table[x, {x, 1, 4}], Table[f[x], {x, 1, 4}]}) // TableForm
```

Out[•]//TableForm=

1	2	3	4
-0.6	5.6	23.4	58.8

In[•]:=
```
NewtonInterpolation[interPoint, x, 3]
```

Out[•]=
$$-0.6 + 6.2\,(-1 + x) + 5.8\,(-2 + x)\,(-1 + x) + 1.\,(-3 + x)\,(-2 + x)\,(-1 + x)$$

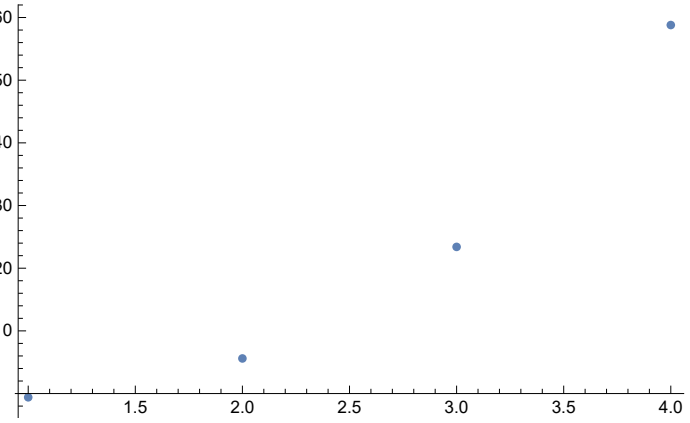
Проверка

In[•]:=
```
Table[NewtonInterpolation[interPoint, i, 3] == f[i], {i, 6}]
```

Out[•]=
```
{True, True, True, True, True, True}
```

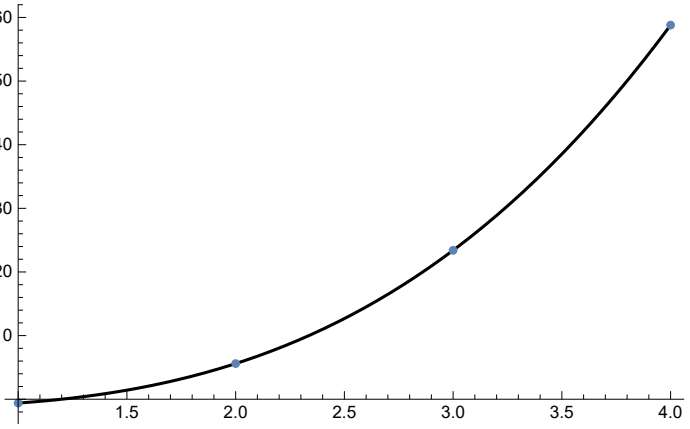
In[•]:=
```
ListPlot[interPoint[[2]], DataRange → {interPoint[[1, 1]], interPoint[[1, -1]]}]
```

Out[•]=



In[•]:=
```
Show[Plot[NewtonInterpolation[interPoint, x, 3],
  {x, interPoint[[1, 1]], interPoint[[1, -1]]}, PlotStyle → Black],
 ListPlot[interPoint[[2]], DataRange → {interPoint[[1, 1]], interPoint[[1, -1]]}]]
```

Out[•]=



Пример 2

In[•]:=
```
f2[x_] := x^4 + 3 x^3 - 11 x^2 - 3 x + 10
```

In[•]:=
```
(interPoint2 = {Table[x, {x, -15, 14, 6}], Table[f2[x], {x, -15, 14, 6}]}) // TableForm
```

Out[•]//TableForm=

-15	-9	-3	3	9
38 080	3520	-80	64	7840

In[•]:=
```
NewtonInterpolation[interPoint2, x, 4]
```

Out[•]=
$$38\,080 - 5760\,(15+x) + 2580\,(15+x)\left(-1+\frac{15+x}{6}\right) - 756\,(15+x)\left(-2+\frac{15+x}{6}\right)\left(-1+\frac{15+x}{6}\right) + 216\,(15+x)\left(-3+\frac{15+x}{6}\right)\left(-2+\frac{15+x}{6}\right)\left(-1+\frac{15+x}{6}\right)$$

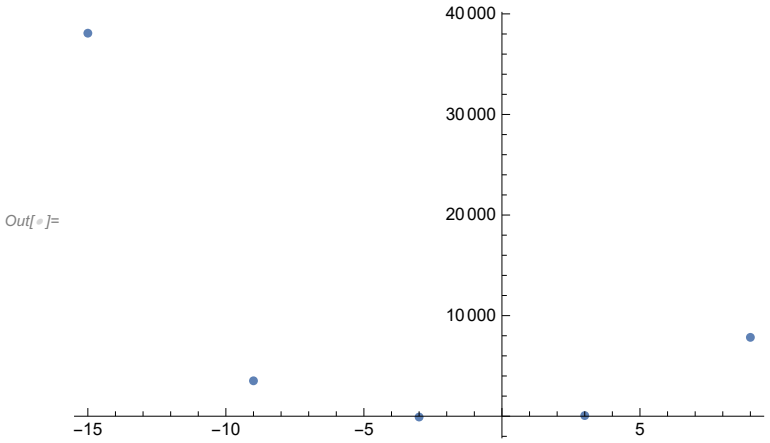
Проверка

In[•]:=
```
Table[NewtonInterpolation[interPoint2, i, 4] == f2[i], {i, 1, 9, 2}]
```

Out[•]=
```
{True, True, True, True, True}
```

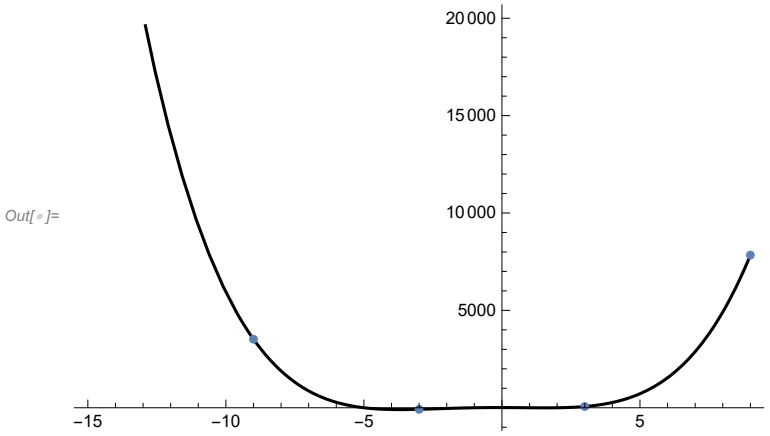
In[•]:=
```
ListPlot[interPoint2[[2]], DataRange → {interPoint2[[1, 1]], interPoint2[[1, -1]]}]
```

Out[•]=



In[•]:=
```
Show[Plot[NewtonInterpolation[interPoint2, x, 4],
  {x, interPoint2[[1, 1]], interPoint2[[1, -1]]}, PlotStyle → Black],
 ListPlot[interPoint2[[2]], DataRange → {interPoint2[[1, 1]], interPoint2[[1, -1]]}]]
```

Out[•]=



Пример 3

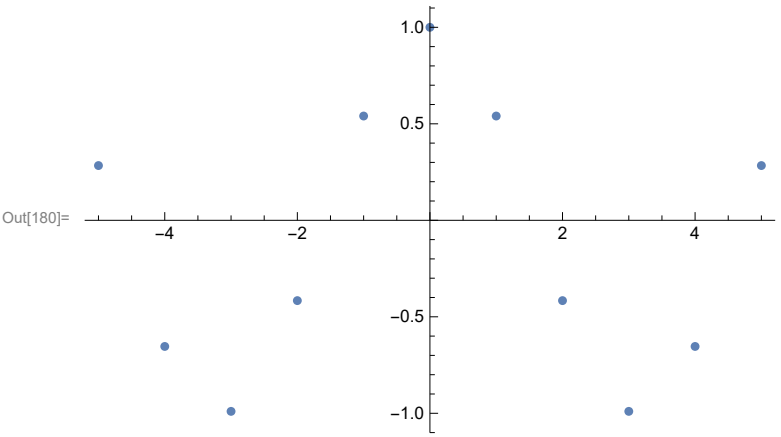
In[178]:=
```
f[x_] := Cos[x];
(interPoint3 = {Table[x, {x, -5, 5}], Table[f[x], {x, -5, 5}]}) // TableForm
```

Out[179]//TableForm=

−5	−4	−3	−2	−1	0	1	2	3	4
Cos[5]	Cos[4]	Cos[3]	Cos[2]	Cos[1]	1	Cos[1]	Cos[2]	Cos[3]	Co

In[180]:=
```
ListPlot[interPoint3[[2]], DataRange → {interPoint3[[1, 1]], interPoint3[[1, -1]]}]
```

Out[180]=



In[183]:=
```
ex3 = NewtonInterpolation[interPoint3, x, 10];
```

In[186]:=
```
Show[Plot[Cos[x], {x, -8, 8}, PlotStyle → Red, PlotLegends → {"cos"}],
 Plot[poly, {x, -8, 8}, PlotStyle → Black, PlotLegends → {"ex3"}],
 ListPlot[interPoint3[[2]], DataRange → {interPoint3[[1, 1]], interPoint3[[1, -1]]}]]
```

Out[186]=