

4.1

Domain: ~~All values of type order~~ -2 for domain

for values t of type tree and m of type int

Property $P(t)$: If $\max t = \text{Some } m$, then $\text{Member}(m, t)$ \wedge

Order: $R(t_1, t_2) : t_1$ is a proper substructure of t_2 .

Partition: $\{\text{Leaf}\} \{\text{Branch}(l, k, r)\}$ | l and r are values of type int tree and k is a value of type int \exists .

Case 1: $t = \text{Leaf}$

Property holds vacuously because $\max \{\text{Leaf}\} = \text{None}$

Case 2: $t = \text{Branch}(l, k, r)$

By assumption, we know $\max t = \text{Some } m$ no!

2nd Induction

Domain: ~~All values of type order~~ for int values m and k

Property $Q(o)$: if $\text{compare_int } m \ k = 0$, then $P(t)$

Order: $R(o_1, o_2)$ false (empty relation)

Partition: $\{\text{less}\}, \{\text{Equal}\}, \{\text{Greater}\}$

Case a: $o = \text{Less}$

This situation is not dealt with in \max , so holds

Case b: $o = \text{Equal}$

If $o = \text{Equal}$, by definition of \max
 $\max k = \text{Some } m$, by $P(t)$

$\text{Member}(m, k)$, and because k is a proper substructure of t , $\text{Member}(m, t)$

Case c: $o = \text{Greater}$

If $o = \text{greater}$, by definition of \max

$\max r = \text{Some } m$, by $P(t)$

$\text{Member}(m, r)$, because r is a proper substructure of t
 $\text{Member}(m, t)$

-5 inductive case needs work

this is the right idea, but your partitioning in this problem is wrong

Less Greater and Equal are not in the max function. You should have broken up the function into None and Some m, -6

Because either $o = \text{Equal}$ or $o = \text{Greater}$ will be true $P(t)$.

4.2

Domain: ~~All values of type order~~ -2 domain

Property $P(t)$: If $\text{max } t = \text{Some } m$, then $\text{AllLess}(m, \text{delete } m \ t)$
and If $\text{max } t = \text{None}$, then $t = \text{Leaf}$, for values of t of type tree and m ,
order: $R(t_1, t_2)$: t_1 is a proper substructure of t_2 . +type int

Partition: $\{\text{Some } m \mid m \text{ is a value of type int}\} \ \{\text{None}\}$

Case 1: $\text{max } t = \text{None}$

your partitions should be leaf and branch

-4 no leaf base case

by the definition of max, $\text{max } t$ returns None
when t is matched with leaf, therefore
 $t = \text{Leaf}$.

-18 many issues with the inductive case

You can't use intuition like m is the greatest
int value without proving it.

Case 2: $\text{max } t = \text{Some } m$

• by the definition of max on $\text{BST}(t)$, m is
the greatest int value in the tree t .

• $\text{AllLess}(m, \text{delete } m \ t)$ is true if m is greater
than every value in the tree $\text{delete } m \ t$.

This is intuition, and
not rigorous.

• We know m is the greatest value in t , so
if delete correctly removes m from t , $P(t)$
holds.

• By lemma 1, If $\text{max } t = \text{Some } m$, $\text{AllLess}(m, \text{delete } m \ t)$

Lemma 1: for values i of type int and tr of type tree,
if $\text{Member}(i, tr)$ delete i tr removes i from tr .

Domain: ~~All values of type order~~

Property $P(i)$: If $\text{Member}(i, tr)$, delete i tr removes i from tr

Order: $R(t_1, t_2)$: t_1 is a proper substructure of t_2

Partition: $\{\text{Member}(i, tr) = \text{true}\} \ \{\text{Member}(i, tr) = \text{false}\}$

Case 1: $\text{Member}(i, tr) = \text{false}$

by thm 4.1 this is an impossible case.

Case 2: $\text{Member}(i, tr) = \text{true}$

• by def of delete with the understanding that $\text{Member}(i, tr)$
delete will reach the 'Equal' case in the 2nd matching

• From here there are 2 cases the right side of the
tree matches with Leaf or —.

~ If r matches with leaf, this means there is nothing in the right side, so the only remaining part is the left side, so delete returns the left side of br .

~ If r matches with $—$, delete will then check if l is a leaf or not, it matches $max\ l$ with some m and None

* in the case of None, this means l is empty and delete will return r .

* in the case of some m , this means l is not empty, so delete returns a tree with both r and l .